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# Structure of the Nucleon*

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Generalized parton distributions provide information on the longitudinal and transverse distribution of partons in the fast moving nucleon. Furthermore, they contain information on the spin structure of the nucleon. First results of a lattice study of generalized parton distributions are presented.

## 1. INTRODUCTION

Understanding the internal structure of hadrons in terms of quarks and gluons (partons), and in particular how quarks and gluons provide the binding and spin of the nucleon, is one of the outstanding problems in particle physics.

Electromagnetic form factors provide information on the spatial distribution of charge and magnetization in the nucleon, irrespective of the partons' momenta and independent of the resolution scale. Parton distributions measure the probability $|\psi(x)|^2$ of finding a parton with fractional longitudinal momentum $x$ in the fast moving nucleon at a given transverse resolution $1/Q$, while no information on the transverse distribution of partons is provided. Generalized parton distributions (GPDs) [1] describe the coherence of two different hadron wave functions $\psi^\dagger(x+\xi/2)\,\psi(x-\xi/2)$, one where the parton carries fractional momentum $x+\xi/2$ and one where this fraction is $x-\xi/2$, from which further information on the transverse distribution of partons can be drawn [2]. In the limit where the momentum transfer $\Delta$ on the nucleon is purely transverse, i.e. $\Delta = (0, \boldsymbol{\Delta}_\perp)$ and $\xi = \eta = 0$, GPDs regain a probabilistic interpretation [3]. When Fourier transformed to impact parameter

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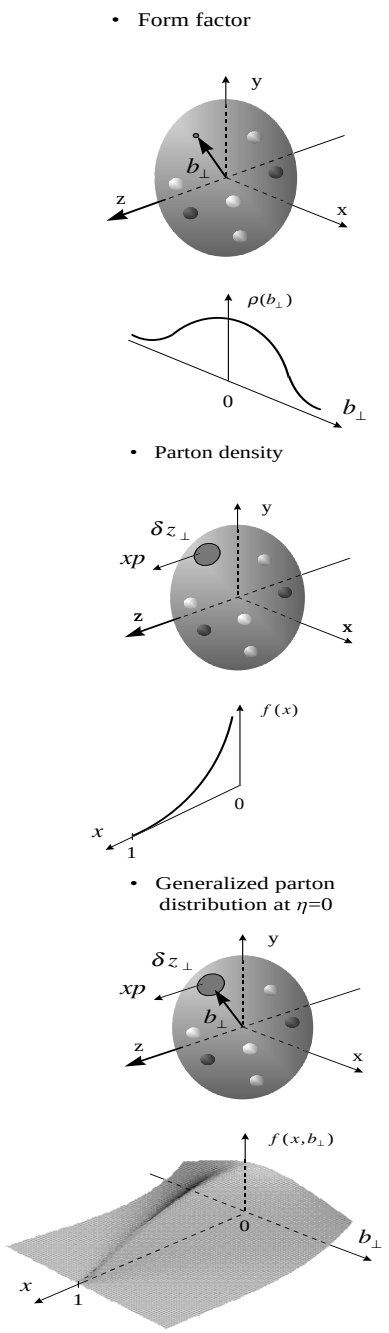


Figure 1. Probabilistic interpretation of form factors, parton distributions and generalized parton distributions at $\eta = 0$ [4] in the infinite momentum frame (resolution: $\delta z_\perp = 1/Q$).

space, they measure, for example, the probability of finding a parton of momentum fraction $x$ at the impact parameter $\mathbf{b}_\perp$. In Fig. 1 we compare the information provided by form factors, parton distributions and GPDs. In other words, GPDs extend the well-known Feynman parton distribution and electromagnetic form factors of the nucleon to new kinematic dimensions. In the forward limit ($\Delta = 0$), these distributions reduce to the Feynman parton distributions.

Moments of GPDs are amenable to lattice calculations [5,6]. Thus, they offer promising ways to link phenomenological observations to first principle theoretical considerations. In this talk we will report on recent quenched results obtained by the QCDSF collaboration [5]. The calculation employs non-perturbatively improved Wilson fermions and non-perturbative renormalization factors [7] of the operators whose matrix elements are involved. Generally, improved Wilson fermions show only a very mild cut-off dependence [8]. We may therefore restrict ourselves to calculations done on the $16^3\,32$ lattice at $\beta = 6.0$.

## 2. (GENERALIZED) FORM FACTORS

We shall consider the GPDs $H_q$ and $E_q$ in the nucleon, where $q = u, d, \cdots$ denotes the flavor of the struck quark. We will not consider the gluon sector here.

The zeroth moments of $H_q$ and $E_q$ give the electromagnetic Dirac and Pauli form factors:

$$\begin{aligned} \int_{-1}^{1} \mathrm{d}x\, H_q(x,\xi,\Delta^2) &= F_1^q(\Delta^2)\,, \\ \int_{-1}^{1} \mathrm{d}x\, E_q(x,\xi,\Delta^2) &= F_2^q(\Delta^2)\,. \end{aligned} \tag{1}$$

Both form factors have been computed on the lattice [8,9]. In Fig. 2 we show a typical result for the Dirac form factor of the proton, $F_1 = F_1^u + F_1^d$. The solid curve is a dipole fit of the form

$$F_1(\Delta^2) = F_1(0)/(1 - \Delta^2/M_1^2)^2\,. \tag{2}$$

In Fig. 3 the dipole mass $M_1$ is extrapolated to the physical pion mass. We find good agreement

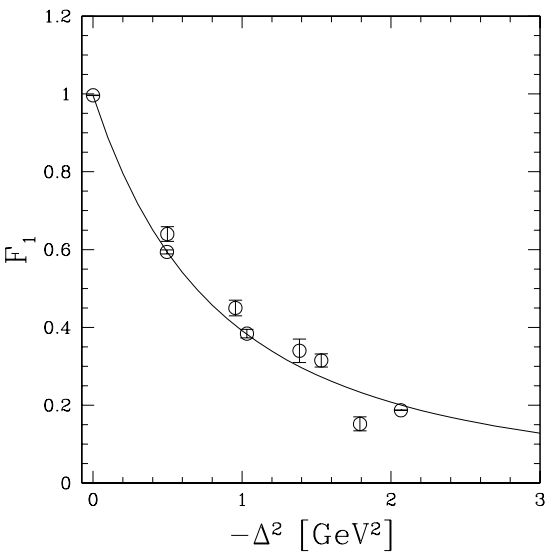


Figure 2. The Dirac form factor $F_1(\Delta^2)$ of the proton at $m_\pi = 450$ MeV, together with a dipole fit.

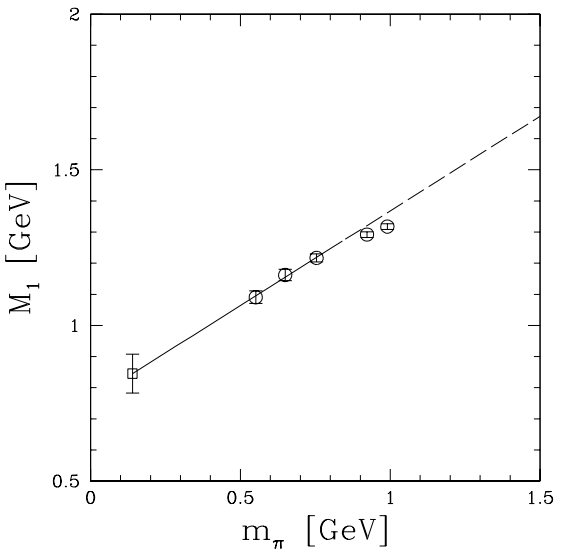


Figure 3. The dipole mass $M_1$ as a function of $m_\pi$, together with a linear extrapolation to the physical pion mass ($\Box$).

with the phenomenological value, which is approximately equal to the physical $\rho$ and $\omega$ mass.

The first moment gives us generalized form factors of a class of tensor operators:

$$\begin{aligned}\int_{-1}^{1} \mathrm{d}x\, x\, H_q(x,\xi,\Delta^2) &= A_2^q(\Delta^2) + \xi^2 C_2^q(\Delta^2)\,,\\ \int_{-1}^{1} \mathrm{d}x\, x\, E_q(x,\xi,\Delta^2) &= B_2^q(\Delta^2) - \xi^2 C_2^q(\Delta^2)\,,\end{aligned} \tag{3}$$

where $A_2^q(\Delta^2)$, $B_2^q(\Delta^2)$ and $C_2^q(\Delta^2)$ are given by the nucleon matrix elements of the energy-momentum tensor (EMT):

$$\begin{aligned}\langle p'|\mathcal{O}^q_{\{\mu\nu\}}|p\rangle &\equiv \frac{\mathrm{i}}{2}\langle p'|\bar{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}q|p\rangle\\ &= A_2^q(\Delta^2)\,\bar{u}(p')\gamma_{\{\mu}\bar{p}_{\nu\}}u(p)\\ &- B_2^q(\Delta^2)\,\frac{\mathrm{i}}{2m_N}\bar{u}(p')\Delta^\alpha\sigma_{\alpha\{\mu}\bar{p}_{\nu\}}u(p)\\ &+ C_2^q(\Delta^2)\,\frac{1}{m_N}\bar{u}(p')u(p)\Delta_{\{\mu}\Delta_{\nu\}}\,.\end{aligned} \tag{4}$$

Here $m_N$ denotes the nucleon mass, $\bar{p} = \frac{1}{2}(p'+p)$, $\Delta = p'-p$, and curly brackets refer to symmetrization of indices and subtraction of traces. The EMT has twist two and spin two. It is assumed to be renormalized at the scale $Q$, which makes $A_2^q(\Delta^2)$, $B_2^q(\Delta^2)$ and $C_2^q(\Delta^2)$ scale and scheme dependent. For the classification of states of definite $J^{PC}$ contributing to (4) in the $t$-channel see [10]. The so-called skewedness parameter $\xi$ is defined by $\xi = -n\cdot\Delta$, where $n$ is a light-like vector with $n\cdot\bar{p} = 1$, and bounded by $|\xi| \le 2\sqrt{\Delta^2/(\Delta^2 - 4m_N^2)}$. In the forward limit, $\Delta^2 = \xi = 0$, the first moment of $H_q$ reduces to the first moment of the unpolarized parton distribution:

$$A_2^q(0) = \langle x_q\rangle \equiv \int_0^1 \mathrm{d}x\, x\,\big(q_\uparrow(x) + q_\downarrow(x)\big)\,, \tag{5}$$

where $q_{\uparrow(\downarrow)}(x)$ is the distribution with quark spin parallel (antiparallel) to the spin of the nucleon. Furthermore, we have [11]

$$\frac{1}{2}\big(A_2^q(0) + B_2^q(0)\big) = J_q\,, \tag{6}$$

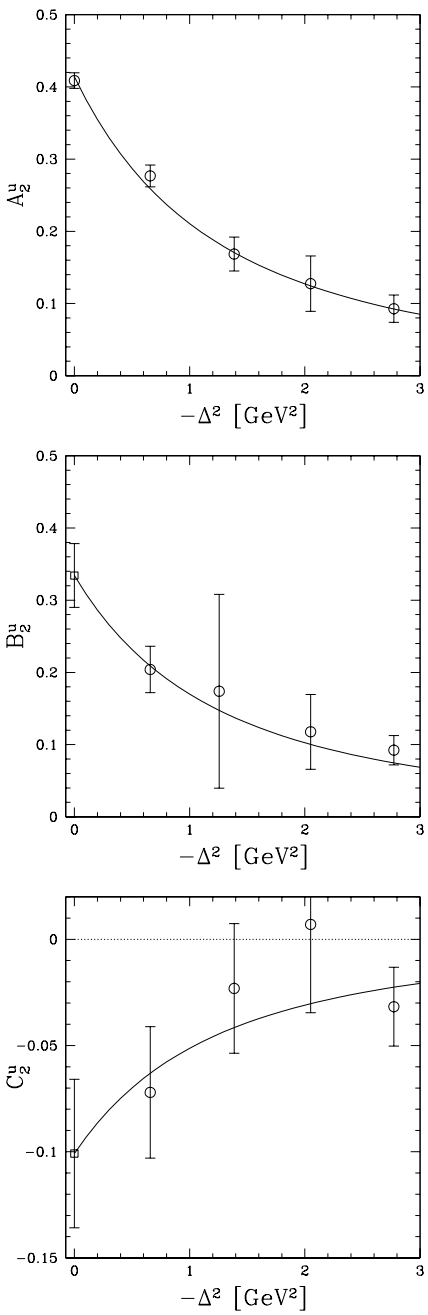


Figure 4. The generalized form factors $A_2^u$, $B_2^u$ and $C_2^u$ at $m_\pi = 450$ MeV, together with the dipole fit and the extrapolated values at $\Delta^2 = 0$ ($\Box$).

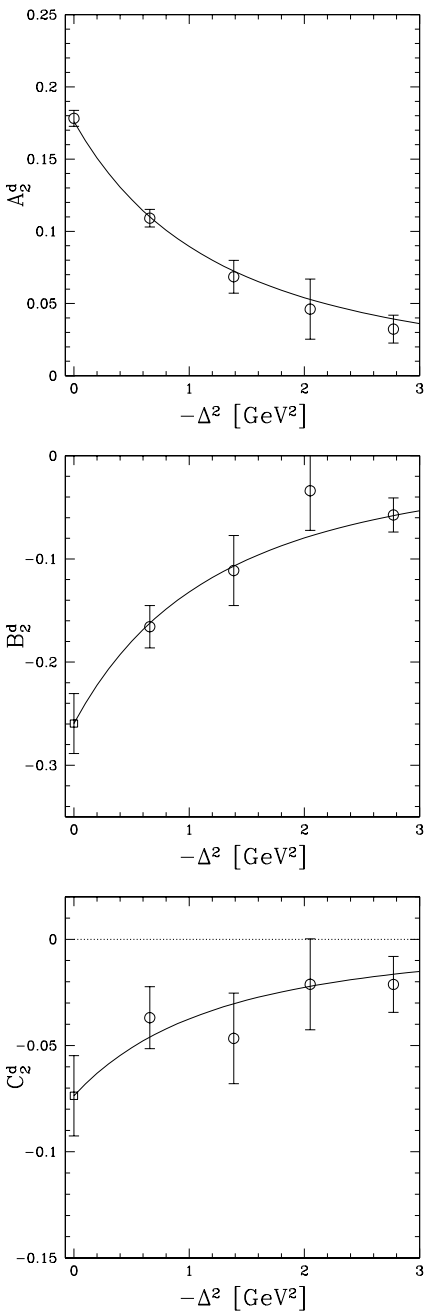


Figure 5. The generalized form factors $A_2^d$, $B_2^d$ and $C_2^d$ at $m_\pi = 450$ MeV, together with the dipole fit and the extrapolated values at $\Delta^2 = 0$ ($\Box$).

where $J_q$ is the angular momentum of the $q$ quark, and

$$J = \sum_q J_q \tag{7}$$

is the total angular momentum of the nucleon carried by the quarks.

In Figs. 4 and 5 we show the generalized form factors $A_2^u(\Delta^2)$, $B_2^u(\Delta^2)$, $C_2^u(\Delta^2)$ and $A_2^d(\Delta^2)$, $B_2^d(\Delta^2)$, $C_2^d(\Delta^2)$ of the proton, again for $m_\pi = 450$ MeV. As for the nucleon form factors, the generalized form factors can be well described by the dipole ansatz

$$A_2^q(\Delta^2) = A_2^q(0)/(1-\Delta^2/M^2)^2\,, \tag{8}$$

and similarly for $B_2^q$ and $C_2^q$. The numbers refer to the $\overline{MS}$ scheme at the renormalization scale $Q = 2\,\mathrm{GeV}$. Fits of $A_2^u(\Delta^2)$ and $A_2^d(\Delta^2)$ give the same dipole mass $M$ within errors. We therefore fit our data by a common dipole mass $M$, only depending on the quark mass. For a reliable extrapolation to $\Delta^2 = 0$ it is important to cover a wide enough range of $\Delta^2$. Our data do not favor a monopole behavior.

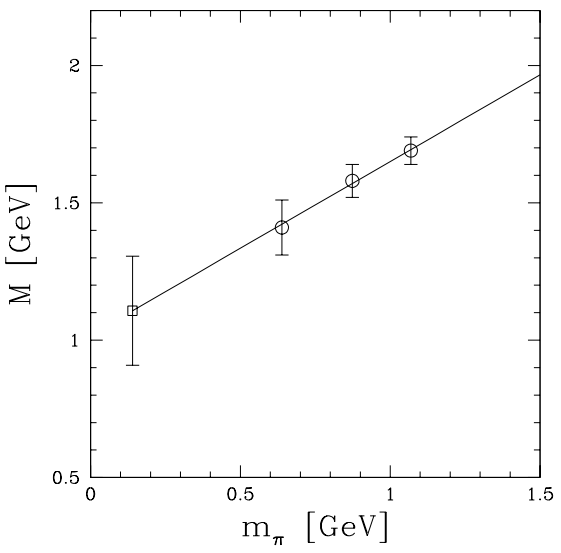


Figure 6. The dipole mass $M$ as a function of $m_\pi$, together with a linear extrapolation to the physical pion mass ($\Box$).

In Fig. 6 we show the dipole mass $M$ as a function of the pion mass. Again, the numbers appear to lie on a straight line. A linear extrapolation in $m_\pi$ to the physical pion mass gives $M = 1.1(2)\,\mathrm{GeV}$. This value is close to the physical masses of the $f_2$ and $a_2$ mesons, which supports the hypothesis of tensor meson dominance. The numbers $A_2^q(0)$, $B_2^q(0)$ and $C_2^q(0)$ show little variation with the quark mass and are extrapolated quadratically in $m_\pi$ to the physical pion mass. We obtain

| $q$ | $A_2^q(0)$ | $B_2^q(0)$ | $C_2^q(0)$ |
|---|---|---|---|
| $u$ | 0.400(22) | 0.334(113) | -0.134(81) |
| $d$ | 0.147(11) | -0.232(77) | -0.071(42) |

Higher moments of $H_q$ and $E_q$ may be obtained in a similar way. Knowing sufficiently many moments, we can reconstruct $H_q(x,\xi,\Delta^2)$ and $E_q(x,\xi,\Delta^2)$ by inverse Mellin transform [12].

## 3. NUCLEON ANGULAR MOMENTUM

The angular momentum decomposes, in a gauge invariant way, into two contributions:

$$J_q = L_q + S_q\,, \tag{9}$$

where $L_q$ is the orbital angular momentum and

$$S_q = \frac{1}{2}\Delta q \equiv \frac{1}{2}\int_0^1 \mathrm{d}x\,\left(q_\uparrow(x) - q_\downarrow(x)\right) \tag{10}$$

is the spin of the quark. We know $\Delta q$ from separate calculations [13,14], so that $L_q$ can be computed from (6).

In Fig. 7 we show the total angular momentum $J = J_u + J_d$. The dependence on the pion mass is rather flat, as expected [15]. The errors are due to the relatively large statistical errors of $B_2^u$ and $B_2^d$ and the fact that $B_2^u$ and $B_2^d$ cancel to a large extent. A quadratic extrapolation in $m_\pi$ to the physical pion mass gives

| $J$ | $J_u$ | $J_d$ | $S_u$ | $S_d$ |
|---|---|---|---|---|
| 0.33(7) | 0.37(6) | -0.04(4) | 0.42(1) | -0.12(1) |

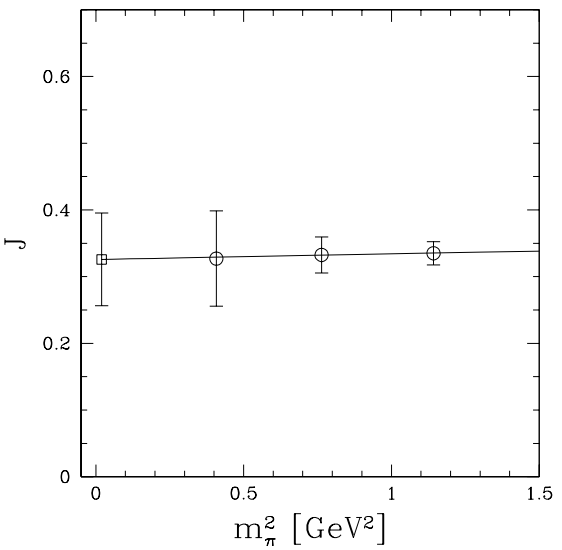


Figure 7. The total angular momentum $J$, together with a quadratic extrapolation to the physical pion mass ($\Box$).

The numbers for $S_q$ refer to our latest results [16], computed from the non-perturbatively improved axial vector current with non-perturbative renormalization factors. It turns out that the total angular momentum $J$ carried by the quarks amounts to $\approx 70\%$ of the spin of the (quenched) proton, leaving a contribution of $\approx 30\%$ for the gluons. The major contribution is given by the $u$ quark, while the contribution of the $d$ quark is found to be negligible, which hints at strong pairing effects.

From $J$ and $S$ we can compute $L_q$ now. The total orbital angular momentum of the (valence) quarks turns out to be consistent with zero:

$$L \equiv L_u + L_d = 0.03(7). \tag{11}$$

This indicates that (at virtuality $Q = 2\,\mathrm{GeV}$) the parton's transverse momentum in the (quenched) proton is small.

## 4. GENERALIZED PARTON DISTRIBUTION

If the dipole behavior (2), (8) continues to hold for the higher moments as well, and if we assume that the dipole masses continue to grow in a Regge-like fashion, we may write

$$\int_{-1}^{1} \mathrm{d}x\, x^n\, H_q(x,0,\Delta^2) = \langle x_q^n \rangle / (1-\Delta^2/M_{n+1}^2)^2\,, \tag{12}$$

with

$$M_l^2 = \text{const.} + l/\alpha'\,, \tag{13}$$

where const. $\approx -0.5$ GeV$^2$ and $1/\alpha' \approx 1.1$ GeV$^2$. We know $\langle x_q^n \rangle$ from previous lattice calculations for $0 \le n \le 3$ [17]. That is sufficient to compute $H_q(x,0,\Delta^2)$ by means of an inverse Mellin transform [12].

Having done so, the desired probability distribution of finding a parton of momentum fraction $x$ at the impact parameter $\mathbf{b}_\perp$ (see Fig. 1) is then obtained by the Fourier transform of $H_q(x,0,\Delta^2)$:

$$H_q(x,0,\mathbf{b}_\perp^2) = \frac{1}{(2\pi)^2} \int d^2\mathbf{\Delta}_\perp\, \mathrm{e}^{\mathrm{i}\mathbf{b}_\perp \mathbf{\Delta}_\perp} H_q(x,0,\mathbf{\Delta}_\perp^2)\,. \tag{14}$$

In Fig. 8 we show a first attempt at a lattice calculation of $H_q(x,0,\mathbf{b}_\perp^2)$ for the $u$ quark distribution in the proton. The volume under the two-dimensional surface (taken over the full kinematical range $0 \le x \le 1$ and $-\infty \le b \le \infty$) gives us back $\langle x_u \rangle$. We see that at smaller values of $x$ the distribution in $\mathbf{b}_\perp$ is rather broad (and different from the distribution sketched in Fig. 1), while at larger values of $x$ it becomes somewhat more narrow. We also notice that the peak of the distribution is shifted slightly towards larger values of $x$ as $|\mathbf{b}_\perp|$ is increased.

## 5. CONCLUSION

We have presented a first quenched lattice estimate of the generalized parton distribution

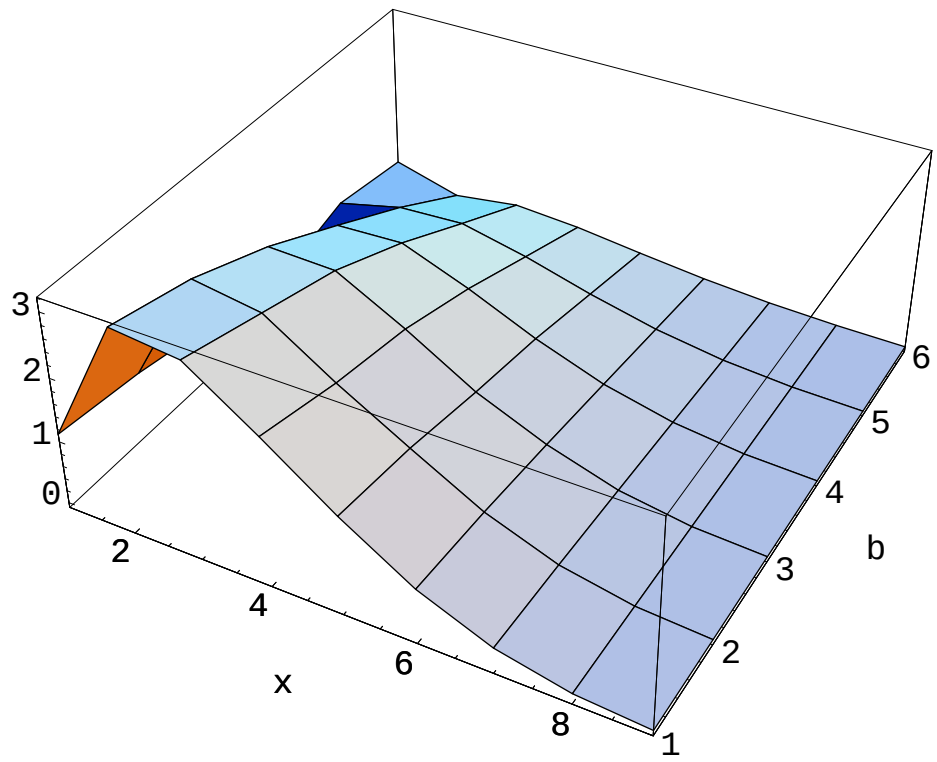


Figure 8. The generalized parton distribution $xH_u(x, 0, \mathbf{b}_\perp^2)$ as a function of $x$ and the impact parameter $b = |\mathbf{b}_\perp|$. The plot extends over the range $0.1 \leq x \leq 0.9$ and 0.1 fm $\leq b \leq$ 0.6 fm.

$H_q(x, 0, \mathbf{b}_\perp^2)$, assuming that the generalized form factors of the higher operators (i.e. higher than the tensor operator) can be described by a dipole ansatz as well, with dipole masses lying on a Regge trajectory. As a by-product we obtained the total and orbital angular momentum of the nucleon carried by quarks. A calculation of the missing generalized form factors is in progress.

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