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Implication of the $B \to (\rho, \omega)\,\gamma$ Branching Ratios for the CKM Phenomenology

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We study the implication of the recent measurement by the BELLE collaboration of the averaged branching fraction $\bar{\mathcal{B}}_{\rm exp}[B \to (\rho,\omega)\,\gamma] = (1.8^{+0.6}_{-0.5} \pm 0.1) \times 10^{-6}$ for the CKM phenomenology. Combined with the averaged branching fraction $\bar{\mathcal{B}}_{\rm exp}(B \to K^*\,\gamma) = (4.06 \pm 0.26) \times 10^{-5}$ measured earlier, this yields $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma] = (4.2 \pm 1.3)\%$ for the ratio of the two branching fractions. Updating earlier theoretical analysis of these decays based on the QCD factorization framework, and constraining the CKM-Wolfenstein parameters from the unitarity fits, our results yield $\bar{\mathcal{B}}_{\rm th}[B \to (\rho,\omega)\,\gamma] = (1.38 \pm 0.42) \times 10^{-6}$ and $\bar{R}_{\rm th}[(\rho,\omega)\,\gamma/K^*\gamma] = (3.3 \pm 1.0)\%$, in agreement with the BELLE data. Leaving instead the CKM-Wolfenstein parameters free, our analysis gives (at 68% C.L.) $0.16 \le |V_{td}/V_{ts}| \le 0.29$, which is in agreement with but less precise than the indirect CKM-unitarity fit of the same, $0.18 \le |V_{td}/V_{ts}| \le 0.22$. The isospin-violating ratio in the $B \to \rho\gamma$ decays and the $SU(3)$-violating ratio in the $B_d^0 \to (\rho^0,\omega)\,\gamma$ decays are presented together with estimates of the direct and mixing-induced CP-asymmetries in the $B \to (\rho,\omega)\,\gamma$ decays within the SM. Their measurements will overconstrain the angle α of the CKM-unitarity triangle.

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1. Introduction. Recently, the BELLE collaboration have presented evidence for the observation of the decays $B^+ \to \rho^+\gamma$, $B_d^0 \to \rho^0\gamma$ and $B_d^0 \to \omega\gamma$ (and their charged conjugates) [1]. Their observation based on an integrated luminosity of 140 fb^{-1} lacks the statistical significance in the individual channels, but combining the data in the three decay modes and their charged conjugates yields a signal at 3.5σ C.L. [1]:

$$\bar{\mathcal{B}}_{\rm exp}[B \to (\rho,\omega)\,\gamma] = (1.8^{+0.6}_{-0.5} \pm 0.1) \times 10^{-6}. \tag{1}$$

This result updates the previous upper bounds [2] by the BELLE collaboration, while the upper bound from the BABAR collaboration (at 90% C.L.) [3]:

$$\bar{\mathcal{B}}_{\rm exp}[B \to (\rho,\omega)\,\gamma] < 1.9 \times 10^{-6}, \tag{2}$$

remains to be updated. The experimental averages given above are defined as:

$$\bar{\mathcal{B}}[B \to (\rho,\omega)\,\gamma] \equiv \frac{1}{2}\left\{\mathcal{B}(B^+ \to \rho^+\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\left[\mathcal{B}(B_d^0 \to \rho^0\gamma) + \mathcal{B}(B_d^0 \to \omega\gamma)\right]\right\}, \tag{3}$$

and the world average [4] for the B-meson lifetime ratio:

$$\tau_{B^+}/\tau_{B^0} = 1.086 \pm 0.017, \tag{4}$$

has been used in arriving at the BELLE result (1). This is the first observation of the CKM-suppressed electromagnetic penguin $b \to d\gamma$ transition. The CKM-allowed $b \to s\gamma$ transition in the exclusive decays $B \to K^*\gamma$ was observed more than a decade ago by the CLEO collaboration [5], followed by the observation of the inclusive decay $B \to X_s\gamma$ in 1994 [6]. Since then, data on these decay modes have been provided by a number of experimental collaborations, and the current situation is summarized in Table 1. In getting the isospin-averaged branching ratio $\bar{\mathcal{B}}_{\rm exp}(B \to K^*\gamma)$, we used the following definition:

$$\bar{\mathcal{B}}_{\rm exp}(B \to K^*\gamma) \equiv \frac{1}{2}\left[\mathcal{B}_{\rm exp}(B^+ \to K^{*+}\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\,\mathcal{B}_{\rm exp}(B_d^0 \to K^{*0}\gamma)\right], \tag{5}$$

and the world average (4) for the B-meson lifetime ratio. Table 1 also contains the measurements of the inclusive decay $B \to X_s\gamma$ branching fraction, the resulting ratio of the exclusive-to-inclusive decay rates $R_{\rm exp}(K^*\gamma/X_s\gamma)$, for each experiment separately, and their world averages, with the errors added in quadrature.

The measurements from BELLE and the upper limit from BABAR on the $B \to (\rho,\omega)\gamma$ decays given in (1) and (2), respectively, can be combined with their respective measurements of the $B \to K^*\gamma$ decay rates to yield the following ratios:

$$R_{\rm exp}[(\rho,\omega)\gamma/K^*\gamma] < 0.047, \qquad \text{(BABAR)} \tag{6}$$

$$R_{\rm exp}[(\rho,\omega)\gamma/K^*\gamma] = 0.042 \pm 0.013, \qquad \text{(BELLE)} \tag{7}$$

where $R_{\rm exp}[(\rho,\omega)\gamma/K^*\gamma] = \bar{\mathcal{B}}_{\rm exp}[B \to (\rho,\omega)\,\gamma]/\bar{\mathcal{B}}_{\rm exp}(B \to K^*\gamma)$. In this paper, we do an analysis of the two quantities in Eqs. (1) and (7) in the context of the SM.

Table 1: The branching ratios averaged over the charge-conjugated modes (in units of 10^{-5}) of the exclusive decays $B^+ \to K^{*+}\gamma$ and $B^0_d \to K^{0*}\gamma$ and the inclusive decay $B \to X_s\gamma$ taken from Refs. [4, 7–10]. The averaged branching ratios defined in (5) and the ratio of the exclusive-to-inclusive branching ratios $R_{\rm exp}(K^*\gamma/X_s\gamma)$ are also tabulated.

Fraction	BABAR	BELLE	CLEO	Average
$\mathcal{B}_{\rm exp}(B^+ \to K^{*+}\gamma)$	$3.87 \pm 0.28 \pm 0.26$	$4.25 \pm 0.31 \pm 0.24$	$3.76^{+0.89}_{-0.83} \pm 0.28$	4.03 ± 0.26
$\mathcal{B}_{\rm exp}(B^0_d \to K^{*0}\gamma)$	$3.92 \pm 0.20 \pm 0.24$	$4.01 \pm 0.21 \pm 0.17$	$4.55^{+0.72}_{-0.68} \pm 0.34$	4.01 ± 0.20
$\bar{\mathcal{B}}_{\rm exp}(B \to K^*\gamma)$	4.06 ± 0.26	4.30 ± 0.25	4.35 ± 0.62	4.20 ± 0.17
$\mathcal{B}_{\rm exp}(B \to X_s\gamma)$	$38.8 \pm 3.6^{+5.7}_{-4.6}$	$35.5 \pm 3.2^{+3.0+1.1}_{-3.1-0.7}$	$32.1 \pm 4.3^{+3.2}_{-2.9}$	35.1 ± 3.0
$R_{\rm exp}(K^*\gamma/X_s\gamma)$	$0.105^{+0.021}_{-0.016}$	$0.121^{+0.019}_{-0.015}$	$0.136^{+0.033}_{-0.027}$	0.117 ± 0.012

2. Effective Hamiltonian. The starting point for the theoretical discussion of the radiative $b \to d\gamma$ decays (equivalently $B \to \rho\gamma$ and $B \to \omega\gamma$ decays) is an effective Hamiltonian obtained from the Standard Model (SM) by integrating out the heavy degrees of freedom (the top quark and $W^\pm$-bosons). The resulting expression at the scale $\mu = O(m_b)$, where m_b is the b-quark mass, is given by

$$\begin{aligned}\mathcal{H}^{b\to d}_{\rm eff} &= \frac{G_F}{\sqrt{2}} \Big\{ V_{ub}V^*_{ud} \left[C^{(u)}_1(\mu)\, \mathcal{O}^{(u)}_1(\mu) + C^{(u)}_2(\mu)\, \mathcal{O}^{(u)}_2(\mu) \right] \\ &\quad + V_{cb}V^*_{cd} \left[C^{(c)}_1(\mu)\, \mathcal{O}^{(c)}_1(\mu) + C^{(c)}_2(\mu)\, \mathcal{O}^{(c)}_2(\mu) \right] \\ &\quad - V_{tb}V^*_{td} \left[C^{\rm eff}_7(\mu)\, \mathcal{O}_7(\mu) + C^{\rm eff}_8(\mu)\, \mathcal{O}_8(\mu) \right] + \ldots \Big\}\,, \end{aligned} \tag{8}$$

where G_F is the Fermi coupling constant, and only the dominant terms are shown. The operators $\mathcal{O}^{(q)}_1$ and $\mathcal{O}^{(q)}_2$, $(q = u, c)$, are the standard four-fermion operators:

$$\mathcal{O}^{(q)}_1 = (\bar{d}_\alpha \gamma_\mu (1-\gamma_5) q_\beta)\, (\bar{q}_\beta \gamma^\mu (1-\gamma_5) b_\alpha), \qquad \mathcal{O}^{(q)}_2 = (\bar{d}_\alpha \gamma_\mu (1-\gamma_5) q_\alpha)\, (\bar{q}_\beta \gamma^\mu (1-\gamma_5) b_\beta), \tag{9}$$

and $\mathcal{O}_7$ and $\mathcal{O}_8$ are the electromagnetic and chromomagnetic penguin operators, respectively:

$$\mathcal{O}_7 = \frac{e m_b}{8\pi^2}\, (\bar{d}_\alpha \sigma^{\mu\nu} (1+\gamma_5) b_\alpha)\, F_{\mu\nu}, \qquad \mathcal{O}_8 = \frac{g_s m_b}{8\pi^2}\, (\bar{d}_\alpha \sigma^{\mu\nu} (1+\gamma_5) T^a_{\alpha\beta} b_\beta)\, G^a_{\mu\nu}. \tag{10}$$

Here, e and g_s are the electric and colour charges, $F_{\mu\nu}$ and $G^a_{\mu\nu}$ are the electromagnetic and gluonic field strength tensors, respectively, $T^a_{\alpha\beta}$ are the colour $SU(N_c)$ group generators, and the quark colour indices α and β and gluonic colour index a are written explicitly. Note that in the operators $\mathcal{O}_7$ and $\mathcal{O}_8$ the d-quark mass contributions are negligible and therefore omitted. The coefficients $C^{(q)}_1(\mu)$ and $C^{(q)}_2(\mu)$ in Eq. (8) are the usual Wilson coefficients corresponding to the operators $\mathcal{O}^{(q)}_1$ and $\mathcal{O}^{(q)}_2$, while the coefficients $C^{\rm eff}_7(\mu)$

and $C_8^{\rm eff}(\mu)$ include also the effects of the QCD penguin four-fermion operators which are assumed to be present in the effective Hamiltonian (8) and denoted by ellipses there. For details and numerical values of these coefficients, see [11] and reference therein. We use the standard Bjorken-Drell convention [12] for the metric and the Dirac matrices; in particular $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and the totally antisymmetric Levi-Civita tensor $\varepsilon_{\mu\nu\rho\sigma}$ is defined as $\varepsilon_{0123} = +1$.

For the $b \to s\gamma$ decay (equivalently the $B \to K^*\gamma$ decays), the effective Hamiltonian $\mathcal{H}_{\rm eff}^{b\to s}$ describing the $b \to s$ transition can be obtained by the replacement of the quark field d_α by s_α in all the operators in Eqs. (9) and (10) and by replacing the CKM factors $V_{qb}V_{qd}^* \to V_{qb}V_{qs}^*$ $(q = u, c, t)$ in $\mathcal{H}_{\rm eff}^{b\to d}$ (8). Noting that among the three factors $V_{qb}V_{qs}^*$, the combination $V_{ub}V_{us}^*$ is CKM suppressed, the corresponding contributions to the decay amplitude can be safely neglected. Thus, within this approximation, unitarity of the CKM matrix yields $V_{cb}V_{cs}^* = -V_{tb}V_{ts}^*$, the dependence on the CKM factors in the effective Hamiltonian $\mathcal{H}_{\rm eff}^{b\to s}$ factorizes, and the CKM factor is taken as $V_{tb}V_{ts}^*$. Note also that the three CKM factors shown in $\mathcal{H}_{\rm eff}^{b\to d}$ are of the same order of magnitude and, hence, the matrix elements in the decays $b \to d\gamma$ and $B \to (\rho, \omega)\gamma$ have non-trivial dependence on the CKM parameters.

3. Theoretical framework for the $B \to V\gamma$ decays. To get the matrix elements for the $B \to V\gamma$ $(V = K^*, \rho, \omega)$ decays, we need to calculate the matrix elements $\langle V\gamma|\mathcal{O}_i|B\rangle$, where $\mathcal{O}_i$ are the operators appearing in $\mathcal{H}_{\rm eff}^{b\to s}$ and $\mathcal{H}_{\rm eff}^{b\to d}$. At the leading order in α_s, this involves only the operators $\mathcal{O}_7$, $\mathcal{O}_1^{(u)}$ and $\mathcal{O}_2^{(u)}$, where the latter two are important only for the $B \to (\rho, \omega)\gamma$ decays. One also uses the terminology of the short-distance and long-distance contributions, where the former characterizes the top-quark induced penguin-amplitude and the latter includes the penguin amplitude from the u- and c-quark intermediate states and also the so-called weak annihilation and W-exchange contributions. There are also other topologies, such as the annihilation penguin diagrams, which, however, are small. For a recent discussion of the long-distance effects in $B \to V\gamma$ decays and references to earlier papers, see Ref. [13].

Including the $O(\alpha_s)$ corrections, all the operators listed in (9) and (10) have to be included. A convenient framework to carry out these calculations is the QCD factorization framework [14] which allows to express the hadronic matrix elements in the schematic form:

$$\langle V\gamma|\mathcal{O}_i|B\rangle = F^{B\to V}\mathcal{T}_i^I + \int \frac{dk_+}{2\pi} \int_0^1 du\, \phi_{B,+}(k_+) T_i^{II}(k_+, u)\phi_\perp^V(u), \qquad (11)$$

where $F^{B\to V}$ are the transition form factors defined through the matrix elements of the operator $\mathcal{O}_7$, $\phi_{B,+}(k_+)$ is the leading-twist B-meson wave-function with k_+ being a light-cone component of the spectator quark momentum, $\phi_\perp^V(u)$ is the leading-twist light-cone distribution amplitude (LCDA) of the transversely-polarized vector meson V, and u is the fractional momentum of the vector meson carried by one of the two partons. The quantities $\mathcal{T}_i^I$ and T_i^{II} are the hard-perturbative kernels calculated to order α_s, with

the latter containing the so-called hard-spectator contributions. The factorization formula (11) holds in the heavy quark limit, i.e., to order $\Lambda_{\rm QCD}/M_B$. This factorization framework has been used to calculate the branching fractions and related quantities for the decays $B \to K^*\gamma$ [15–17] and $B \to \rho\gamma$ [15,17]. The isospin violation in the $B \to K^*\gamma$ decays in this framework have also been studied [18]. (For applications to $B \to K^*\gamma^*$, see Refs. [16,19,20]). Very recently, the hard-spectator contribution arising from the chromomagnetic operator $\mathcal{O}_8$ have also been calculated in next-to-next-to-leading order (NNLO) in α_s showing that the spectator interactions factorize in the heavy quark limit [21]. However, the numerical effect of the resummed NNLO contributions is marginal and we shall not include this in our update.

In what follows we shall use the notations and results from Ref. [15], to which we refer for detailed derivations, and point out the changes (and corrections) that we have incorporated in this analysis. The branching ratio of the $B \to K^*\gamma$ decay corrected to $O(\alpha_s)$ can be written as follows [15]:

$$\mathcal{B}_{\rm th}(B \to K^*\gamma) = \tau_B \, \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2}{32\,\pi^4} \, m_{b,\rm pole}^2 \, M_B^3 \, \left[\xi_\perp^{(K^*)}\right]^2 \left[1 - \frac{m_{K^*}^2}{M_B^2}\right]^3 \left| C_7^{(0)\rm eff}(\mu) + A^{(1)}(\mu)\right|^2 , \tag{12}$$

where α is the fine-structure constant, $m_{b,\rm pole}$ is the b-quark pole mass, and M_B and m_{K^*} are the B- and K^*-meson masses, respectively. The quantity $\xi_\perp^{(K^*)}$ is the soft part of the QCD form factor $T_1^{K^*}(q^2)$ in the $B \to K^*$ transition, which is evaluated at $q^2 = 0$ in the HQET limit. For this study, we consider $\xi_\perp^{(K^*)}$ as a free parameter; its value will be extracted from the current experimental data on $B \to K^*\gamma$ decays. Note that the quantity $\xi_\perp^{(K^*)}$ used here is normalized at the scale $\mu = m_{b,\rm pole}$ of the pole b-quark mass. The corresponding quantity in Ref. [19] is defined at the scale $\mu = m_{b,\rm PS}$ involving the potential-subtracted (PS) b-quark mass [22, 23], which is numerically very close to the pole mass used here.

The function $C_7^{(0)\rm eff}(\mu)$ in Eq. (12) is the Wilson coefficient of the electromagnetic operator $\mathcal{O}_7$ in the leading order and the function $A^{(1)}(\mu)$ includes all the NLO corrections:

$$A^{(1)}(\mu) = A_{C_7}^{(1)}(\mu) + A_{\rm ver}^{(1)}(\mu) + A_{\rm sp}^{(1)K^*}(\mu_{\rm sp}), \tag{13}$$

where $A_{C_7}^{(1)}$, $A_{\rm ver}^{(1)}$ and $A_{\rm sp}^{(1)K^*}$ denote the $O(\alpha_s)$ corrections in the Wilson coefficient $C_7^{\rm eff}$, the $b \to s\gamma$ vertex, and the hard-spectator contributions, respectively. Their explicit expressions are given in Eqs. (5.9), (5.10) and (5.11) of Ref. [15]. The values used in the numerical analysis are collected in Table 2. Some comments on the input values are in order. The top-quark mass (interpreted here as the pole mass) has been recently updated and revised upwards by the Tevatron electroweak group [24], and the new world average $m_{t,\rm pole} = (178 \pm 4.3)$ GeV is being used in our analysis. The product $|V_{tb}V_{ts}^*|$ of the CKM matrix elements can be obtained from the estimate $|V_{cb}| = 0.0412 \pm 0.0021$ [25] using the relation $|V_{tb}V_{ts}^*| \simeq (1 - \lambda^2/2)|V_{cb}|$, which yields $|V_{tb}V_{ts}^*| = 0.0402 \pm 0.0020$ for $\lambda = 0.2224$. The $SU(3)$-breaking effects in the K- and K^*-meson LCDAs have been recently re-estimated by Ball and Boglione [26]. In this update, the transverse decay

Table 2: Input quantities and their values used in the theoretical analysis. The values of the masses, coupling constants and Λ_h given in the first four rows are fixed, and those of the others are varied in their indicated ranges to estimate theoretical uncertainties on the various observables discussed in the text.

Parameter	Value	Parameter	Value
M_W	80.423 GeV	M_Z	91.1876 GeV
M_B	5.279 GeV	m_{K^*}	894 MeV
G_F	1.16639×10^{-5} GeV^{-2}	α	1/137.036
$\alpha_s(M_Z)$	0.1172	Λ_h	0.5 GeV
$m_{t,\rm pole}$	(178.0 ± 4.3) GeV	$m_{b,\rm pole}$	(4.65 ± 0.10) GeV
$\lvert V_{tb}V_{ts}^*\rvert$	$(40.2 \pm 2.0) \times 10^{-3}$	$\sqrt{z} = m_c/m_b$	0.27 ± 0.06
f_B	(200 ± 20) MeV	$f_\perp^{(K^*)}$ (1 GeV)	(182 ± 10) MeV
$a_{\perp 1}^{(K^*)}$ (1 GeV)	-0.34 ± 0.18	$a_{\perp 2}^{(K^*)}$ (1 GeV)	0.13 ± 0.08
$\lambda_{B,+}^{-1}$ (1 GeV)	(2.15 ± 0.50) GeV^{-1}	$\sigma_{B,+}$ (1 GeV)	1.4 ± 0.4

constant of the K^*-meson, $f_\perp^{(K^*)}$, has remained practically unchanged, but the Gegenbauer coefficients in the K^*-meson leading-twist LCDA are effected significantly. The two Gegenbauer moments $a_{\perp 1}^{(K^*)}$ and $a_{\perp 2}^{(K^*)}$ used in the calculation of the hard-spectator contributions are now larger in magnitude, have larger errors and, moreover, the first Gegenbauer moment changes its sign. For comparison, previously, these coefficients were estimated as $a_{\perp 1}^{(K^*)}(1\ \text{GeV}) = 0.20 \pm 0.05$ and $a_{\perp 2}^{(K^*)}(1\ \text{GeV}) = 0.04 \pm 0.04$. The effect of these modifications on the QCD form factor $T_1^{K^*}(0)$, as well as of some other technical improvements [26], has not yet been worked out. Lastly, the first inverse moment $\lambda_{B,+}^{-1}(\mu)$ of the B-meson LCDA has also changed. In our previous analysis [15], we used the value $\lambda_{B,+}^{-1}(\mu_{\rm sp}) = (3.0 \pm 1.0)$ GeV^{-1} where the error effectively includes the scale dependence of the leading-twist light-cone B-meson wave-function $\phi_{B,+}(k,\mu)$. In a recent paper by Braun et al. [27], the scale dependence of this moment is worked out in the NLO with the result:

$$\lambda_{B,+}^{-1}(\mu) = \lambda_{B,+}^{-1}(\mu_0) \left\{ 1 - \frac{\alpha_s(\mu) C_F}{\pi} \left[\sigma_{B,+}(\mu_0) - \frac{1}{2} \right] \ln \frac{\mu}{\mu_0} \right\}, \tag{14}$$

where $(\alpha_s C_F/\pi) \ln(\mu/\mu_0) < 1$ and the quantities $\lambda_{B,+}^{-1}(\mu)$ and $\sigma_{B,+}(\mu)$ are defined as follows:

$$\lambda_{B,+}^{-1}(\mu) \equiv \int_0^\infty \frac{dk}{k}\, \phi_{B,+}(k,\mu), \qquad \sigma_{B,+}(\mu) \equiv \lambda_{B,+}(\mu) \int_0^\infty \frac{dk}{k}\, \ln \frac{\mu}{k}\, \phi_{B,+}(k,\mu). \tag{15}$$

At the initial scale $\mu_0 = 1$ GeV of the evolution, the above quantities were estimated by using the method of the Light-Cone-Sum-Rules (LCSR) and their values are presented in Table 2. At the typical scale $\mu_{\rm sp} = \sqrt{\Lambda_h m_{b,\rm pole}} \simeq 1.52$ GeV (here, $\Lambda_h = 0.5$ GeV is a typical hadronic scale) of the hard-spectator corrections, the first inverse moment is

now estimated as: $\lambda_{B,+}^{-1}(\mu_{\rm sp}) = (2.04 \pm 0.48)$ GeV^{-1}. Note that, while overlapping within errors with the previously used value, the updated estimate is substantially smaller as well as the current error on this quantity is now reduced by a factor of two.

Updating the analysis presented in Ref. [15], and using the experimental results on the branching ratios for the $B \to K^*\gamma$ and $B \to X_s\gamma$ decays given in Table 1, the phenomenological values of the soft part of the QCD form factor are: $\xi_\perp^{(K^{*0})}(0) = 0.28 \pm 0.02$, $\xi_\perp^{(K^{*\pm})}(0) = 0.27 \pm 0.02$ and $\xi_\perp^{(K^*/X_s)}(0) = 0.25 \pm 0.02$ resulting from the $B_d^0 \to K^{*0}\gamma$ and $B^\pm \to K^{*\pm}\gamma$ branching ratios and from the ratio $\bar{R}_{\rm exp}(K^*\gamma/X_s\gamma)$, respectively. The QCD form factor $\bar{T}_1^{K^*}(0)$ differs from its soft part $\bar{\xi}_\perp^{(K^*)}(0)$ by $O(\alpha_s)$ terms worked out in Ref. [19], which in our notation is given in Eq. (5.13) of Ref. [15]. However, the updated input parameters reduce this correction, yielding typically a correction of $2-4\%$ only, in contrast to about 8% previously. Thus, the QCD transition form factor $\bar{T}_1^{K^*}(0)$ now differs only marginally from its soft part, and is estimated as follows:

$$\bar{T}_1^{K^*}(0) = 0.27 \pm 0.02. \tag{16}$$

The central value of the QCD form factor (16) extracted from the current data has remained unchanged compared to the previous estimate $\bar{T}_1^{K^*}(0) = 0.27 \pm 0.04$ (see Eq. (5.25) of Ref. [15]), but the error is now reduced by a factor 2, mostly due to the reduction of the uncertainty on the input parameters. It remains an interesting and open theoretical question if improved theoretical techniques for the calculation of the transition form factor $\bar{T}_1^{K^*}(0)$ could accommodate this phenomenological result.

4. Results for $B \to (\rho,\omega)\,\gamma$ decays and comparison with the BELLE data. This part is devoted to an update of the theoretical predictions for the $B \to \rho\gamma$ and $B_d^0 \to \omega\gamma$ branching ratios, and their comparison with the BELLE data. Results for the direct and mixing-induced CP-violating asymmetries in these decays, the isospin-violating ratio in the $B \to \rho\gamma$ decays, and the $SU(3)$-violating ratio in the neutral $B_d^0 \to \rho^0\gamma$ and $B_d^0 \to \omega\gamma$ decays are also presented.

4.1. Branching ratios. We now proceed to calculate numerically the branching ratios for the $B^\pm \to \rho^\pm\gamma$, $B_d^0 \to \rho^0\gamma$ and $B_d^0 \to \omega\gamma$ decays. The theoretical ratios involving the decay widths on the r.h.s. of these equations can be written in the form:

$$R_{\rm th}(\rho\gamma/K^*\gamma) = \frac{\mathcal{B}_{\rm th}(B \to \rho\gamma)}{\mathcal{B}_{\rm th}(B \to K^*\gamma)} = S_\rho \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(M_B^2 - m_\rho^2)^3}{(M_B^2 - m_{K^*}^2)^3}\,\zeta^2\,[1 + \Delta R(\rho/K^*)]\,, \tag{17}$$

$$R_{\rm th}(\omega\gamma/K^*\gamma) = \frac{\overline{\mathcal{B}}_{\rm th}(B_d^0 \to \omega\gamma)}{\overline{\mathcal{B}}_{\rm th}(B_d^0 \to K^{*0}\gamma)} = \frac{1}{2}\left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(M_B^2 - m_\omega^2)^3}{(M_B^2 - m_{K^*}^2)^3}\,\zeta^2\,[1 + \Delta R(\omega/K^*)]\,, \tag{18}$$

where m_ρ and m_ω are the masses of the ρ- and ω-mesons, ζ is the ratio of the transition form factors, $\zeta = \bar{T}_1^\rho(0)/\bar{T}_1^{K^*}(0)$, which we have assumed to be the same for the ρ^0- and ω-mesons, and $S_\rho = 1$ and $1/2$ for the $\rho^\pm$- and ρ^0-meson, respectively. To get the theoretical

Table 3: Input parameters and their values used to calculate the branching fractions in the $B \to \rho\gamma$ and $B_d^0 \to \omega\gamma$ decays. The parameters entering in the $B \to K^*\gamma$ part in Eqs. (17) and (18) are given in Table 2.

Parameter	Value	Parameter	Value
m_ρ	771.1 MeV	m_ω	782.57 MeV
$f_\perp^{(\rho)}(1\text{ GeV})$	(160 ± 10) MeV	$a_{\perp 2}^{(\rho)}(1\text{ GeV})$	0.20 ± 0.10
ζ	0.85 ± 0.10	$\lvert V_{tb}V_{td}^*\rvert$	$(8.1 \pm 0.8) \times 10^{-3}$
$\epsilon_A^{(\pm)}$	$+0.30 \pm 0.07$	$\epsilon_A^{(0)} = -\epsilon_A^{(\omega)}$	$+0.03 \pm 0.01$
$\bar\rho$	0.17 ± 0.07	$\bar\eta$	0.36 ± 0.04

branching ratios for the decays $B \to \rho\gamma$ and $B_d^0 \to \omega\gamma$, the ratios (17) and (18) should be multiplied with the corresponding experimental branching ratio of the $B \to K^*\gamma$ decay.

The theoretical uncertainty in the evaluation of the $R_{\rm th}(\rho\gamma/K^*\gamma)$ and $R_{\rm th}(\omega\gamma/K^*\gamma)$ ratios is dominated by the imprecise knowledge of $\zeta = \bar T_1^\rho(0)/\bar T_1^{K^*}(0)$ characterizing the $SU(3)$ breaking effects in the QCD transition form factors. In the $SU(3)$-symmetry limit, $\bar T_1^\rho(0) = \bar T_1^{K^*}(0)$, yielding $\zeta = 1$. The $SU(3)$-breaking effects in these form factors have been evaluated within several approaches, including the LCSR and Lattice QCD. In the earlier calculations of the ratios [15,28], the following ranges were used: $\zeta = 0.76 \pm 0.06$ [15] and $\zeta = 0.76 \pm 0.10$ [28], based on the LCSR approach [29–33] which indicate substantial $SU(3)$ breaking in the $B \to K^*$ form factors. There also exists an improved Lattice estimate of this quantity, $\zeta = 0.9 \pm 0.1$ [34]. In the present analysis, we use $\zeta = 0.85 \pm 0.10$, given in Table 3 together with the values of the other input parameters entering in the calculation of the $B \to (\rho, \omega)\,\gamma$ decay amplitudes.

We now discuss the difference in the hadronic parameters involving the ρ^0- and ω-mesons. It is known that both mesons are the maximally mixed superpositions of the $\bar u u$ and $\bar d d$ quark states: $|\rho^0\rangle = (|\bar d d\rangle - |\bar u u\rangle)/\sqrt{2}$ and $|\omega\rangle = (|\bar d d\rangle + |\bar u u\rangle)/\sqrt{2}$. Neglecting the W-exchange contributions in the decays, the radiative decay widths are determined by the penguin amplitudes which involve only the $|\bar d d\rangle$ components of these mesons, leading to identical branching ratios (modulo a tiny phase space difference). The W-exchange diagrams from the $\mathcal{O}_1^{(u)}$ and $\mathcal{O}_2^{(u)}$ operators (in our approach, we are systematically neglecting the contributions from the penguin operators $\mathcal{O}_3, ..., \mathcal{O}_6$) yield contributions equal in magnitude but opposite in signs. In the numerical analysis, the LCSR results: $\epsilon_A^{(0)} = +0.03 \pm 0.01$ and $\epsilon_A^{(\omega)} = -0.03 \pm 0.01$ [30], are used, where the smallness of these numbers reflects both the colour-suppressed nature of the W-exchange amplitudes in $B_d^0 \to (\rho^0, \omega)\,\gamma$ decays, and the observation that the leading contributions in the weak annihilation and W-exchange amplitudes arise from the radiation off the d-quark in the B_d^0-meson, which is suppressed due to the electric charge. The parameter $\epsilon_A^{(\pm)}$ entering in $B^\pm \to \rho^\pm\gamma$ and $\epsilon_A^{(0)}$ in the $B_d^0 \to \rho^0\gamma$ decay have been estimated in the factorization approximation for the weak annihilation (and W-exchange) contribution, but this is expected to be a good approximation in the heavy quark limit, where the $O(\alpha_s)$ non-factorizable

Table 4: Updated theoretical estimates of the functions $\Delta R(\rho/K^*)$ and $\Delta R(\omega/K^*)$, and the ratios of the branching ratios $R_{\rm th}(\rho\gamma/K^*\gamma)$ and $R_{\rm th}(\omega\gamma/K^*\gamma)$ defined in Eqs. (17) and (18), respectively. The third and fourth rows give the branching ratios $\mathcal{B}_{\rm th}(B\to\rho\gamma)$ and $\mathcal{B}_{\rm th}(B_d^0\to\omega\gamma)$ (in units of 10^{-6}) and direct CP asymmetries in the $B\to\rho\gamma$ and $B_d^0\to\omega\gamma$ decays, respectively.

	$B^\pm\to\rho^\pm\gamma$	$B_d^0\to\rho^0\gamma$	$B_d^0\to\omega\gamma$
ΔR	0.116 ± 0.099	0.093 ± 0.073	0.092 ± 0.073
$R_{\rm th}$	0.0334 ± 0.0103	0.0164 ± 0.0049	0.0163 ± 0.0049
$\mathcal{B}_{\rm th}$	1.35 ± 0.42	0.66 ± 0.20	0.65 ± 0.20
$\mathcal{A}_{\rm CP}^{\rm dir}$	$(-11.6\pm3.3)\%$	$(-9.4^{+4.2}_{-3.8})\%$	$(-8.8^{+4.4}_{-3.9})\%$

corrections are found to be suppressed in the chiral limit [13]. Moreover, their magnitudes can be checked experimentally through the radiative decays $B^\pm\to\ell^\pm\nu_\ell\gamma$, as emphasized in Ref. [13]. These and the other parameters needed for calculating the branching ratios in the $B\to(\rho,\omega)\,\gamma$ decays are given in Table 3, where we have also given the default ranges for $|V_{tb}V_{td}^*|$ and the CKM-Wolfenstein parameters $\bar\rho$ and $\bar\eta$ obtained from the recent fit of the CKM unitarity triangle [25].

The individual branching ratios $\mathcal{B}_{\rm th}(B\to\rho\gamma)$ and $\mathcal{B}_{\rm th}(B_d^0\to\omega\gamma)$ and their ratios $R_{\rm th}(\rho\gamma/K^*\gamma)$ and $R_{\rm th}(\omega\gamma/K^*\gamma)$ with respect to the corresponding $B\to K^*\gamma$ branching ratios are presented in Table 4. Note that in our estimates there is practically no difference between the $B_d^0\to\rho^0\gamma$ and $B_d^0\to\omega\gamma$ branching fractions, as the two differ only in the signs of the weak-annihilation contributions in the decay amplitudes, but these contributions given in terms of the parameters $\varepsilon_A^{(0)}$ and $\varepsilon_A^{(\omega)}$ are small. Using the definition of the weighted average (3), we get:

$$\bar{\mathcal{B}}_{\rm th}[B\to(\rho,\omega)\gamma] \;=\; (1.38\pm0.42)\times10^{-6}, \tag{19}$$

$$\bar{R}_{\rm th}[(\rho,\omega)\gamma/K^*\gamma] \;=\; 0.033\pm0.010, \tag{20}$$

where the current experimental values of the $B\to K^*\gamma$ branching ratios given in Table 1 have been used in arriving at the result (19). These theoretical estimates, carried out in the context of the SM, are in the comfortable agreement with the current BELLE measurements (1) and (7).

4.2. CP-violating asymmetries. The direct CP-violating asymmetries in the decay rates for $B^+\to\rho^+\gamma$ and $B_d^0\to(\rho^0,\omega)\,\gamma$ decays and their charged conjugates are defined as follows:

$$\begin{aligned}
\mathcal{A}_{\rm CP}^{\rm dir}(\rho^\pm\gamma) &\equiv \frac{\mathcal{B}(B^-\to\rho^-\gamma)-\mathcal{B}(B^+\to\rho^+\gamma)}{\mathcal{B}(B^-\to\rho^-\gamma)+\mathcal{B}(B^+\to\rho^+\gamma)},\\
\mathcal{A}_{\rm CP}^{\rm dir}(\rho^0\gamma) &\equiv \frac{\mathcal{B}(\bar B_d^0\to\rho^0\gamma)-\mathcal{B}(B_d^0\to\rho^0\gamma)}{\mathcal{B}(\bar B_d^0\to\rho^0\gamma)+\mathcal{B}(B_d^0\to\rho^0\gamma)},
\end{aligned} \tag{21}$$

$$\mathcal{A}_{\rm CP}^{\rm dir}(\omega\gamma) \;\equiv\; \frac{\mathcal{B}(\bar{B}_d^0 \to \omega\gamma) - \mathcal{B}(B_d^0 \to \omega\gamma)}{\mathcal{B}(\bar{B}_d^0 \to \omega\gamma) + \mathcal{B}(B_d^0 \to \omega\gamma)}.$$

The explicit expressions for the first two of these asymmetries in terms of the individual contributions in the decay amplitude can be found in Ref. [15] and the one for the last, $\mathcal{A}_{\rm CP}^{\rm dir}(\omega\gamma)$ may be obtained from $\mathcal{A}_{\rm CP}^{\rm dir}(\rho^0\gamma)$ by obvious replacements. Their updated values in the SM, taking into account the parametric uncertainties and adding the various errors in quadrature, are presented in Table 4. The main contribution to the errors is coming through the scale dependence and the uncertainty in the c- to b-quark mass ratio, which is a NNLO effect. A complete NNLO calculation will certainly be required to reduce the theoretical errors. It should be noted that the predicted direct CP-asymmetries in all three cases are rather sizable (of order 10%) and negative. This differs from our earlier estimates [15,28], worked out for $\mathcal{A}_{\rm CP}^{\rm dir}(\rho^\pm\gamma)$ and $\mathcal{A}_{\rm CP}^{\rm dir}(\rho^0\gamma)$, where the explicit expressions were erroneously typed and used in the numerical program with the incorrect overall sign.

The dependence of the direct CP-asymmetry on the CKM unitarity-triangle angle α is presented in the left frame in Fig. 1. We note that the CP-asymmetries are calculated with the strong phases generated perturbatively in $O(\alpha_s)$ in the QCD factorization approach. In particular, they do not include any non-perturbative rescattering contribution. We recall that for the CP-asymmetries in non-leptonic decays, such as in $B \to \pi\pi$, current data point to the inadequacy of the perturbatively generated strong phases [25]. In radiative decays $B \to (\rho,\omega)\gamma$, such long-distance effects enter via the penguin amplitudes $P_u^{(i)}$, which are the $u\bar{u}$-loop contributions involving the operators $\mathcal{O}_i^{(u)}$ $(i = 1,2)$, and $P_c^{(i)}$, the corresponding $c\bar{c}$-loop contributions involving the operators $\mathcal{O}_i^{(c)}$ [13]. They are included in the estimates of the complete matrix elements to a given order [here, up to $O(\alpha_s)$]. In the hadronic language, they can be modelled via the hadronic intermediate states, such as $B^\pm \to \rho^\pm\rho^0 \to \rho^\pm\gamma$, $B^\pm \to D^{*\pm}\bar{D}^{*0} \to \rho^\pm\gamma$, etc. Their relative contribution at the amplitude level was estimated for the decay $B^- \to \rho^-\gamma$ as $|P_c/P_t| \simeq 0.06$ [13], with $|P_u| \ll |P_c|$. A recent model-dependent estimate [35] of the long-distance contribution in $B^0 \to \rho^0\gamma$ via the intermediate D^+D^- state, $B^0 \to D^+D^- \to \rho^0\gamma$, puts the relative contribution of the long-distance (LD) and short-distance (SD) contributions to the decay widths as $\Gamma_{\rm LD}/\Gamma_{\rm SD} \simeq 0.3$, using the lowest order result for $\Gamma_{\rm SD}$. Taking into account that the next-to-leading order contributions in $\Gamma_{\rm SD}$, updated in this paper, result in an enhancement by a factor of about 1.7, and noting further that the perturbative charm-penguin contribution should be subtracted from $\Gamma_{\rm LD}$ to avoid double counting, the remaining rescattering contributions are very likely below 10%. However, one can not exclude an enhanced charm-penguin contribution at this rate and the CP-asymmetry $\mathcal{A}_{\rm CP}^{\rm dir}(\rho^0\gamma)$ could beinfluenced from such long-distance contribution. Charm-penguin enhanced effects can be also tested in the Dalitz pair reaction $B^0 \to \rho^0\gamma^* \to \rho^0\, e^+e^-$ through measurements of the Stoke's vector components [35].

We now discuss the time-dependent (or mixing-induced) CP-asymmetry in the $B_d^0(t) \to (\rho^0,\omega)\,\gamma$ and $\bar{B}_d^0(t) \to (\rho^0,\omega)\,\gamma$ decays. Below, the equations for the B_d^0-meson decays into the final state with the ρ^0-meson are presented. Similar quantities for the decays with the ω-meson production can be obtained by the obvious replacement: $\varepsilon_A^{(0)} \to \varepsilon_A^{(\omega)}$.

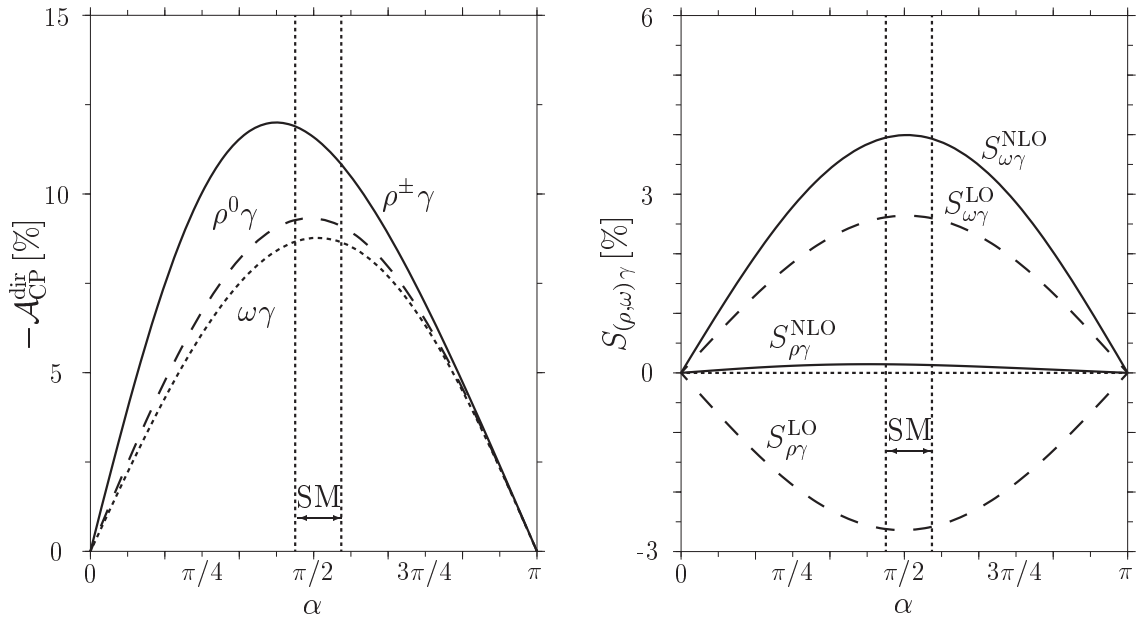

Figure 1: Left frame: The direct CP-violating asymmetries for the $B \to \rho\gamma$ and $B_d^0 \to \omega\gamma$ decays, defined in Eq. (21). Right frame: The mixing-induced CP-violating asymmetries for the $B_d^0 \to \rho^0\gamma$ and $B_d^0 \to \omega\gamma$ decays, defined in Eq. (22).

The time-dependent CP-asymmetry in the decays of neutral B_d^0-mesons and its CP-conjugate involves the interference of the $B_d^0 - \bar{B}_d^0$ mixing and decay amplitudes and is given by [36]:

$$a_{\rm CP}^{\rho\gamma}(t) = -C_{\rho\gamma}\cos(\Delta M_d\, t) + S_{\rho\gamma}\sin(\Delta M_d\, t), \tag{22}$$

where $\Delta M_d \simeq 0.503\,{\rm ps}^{-1}$ is the mass difference between the two mass eigenstates in the $B_d^0 - \bar{B}_d^0$ system. For getting explicit formulae for $C_{\rho\gamma}$ and $S_{\rho\gamma}$, it is convenient to introduce the quantity:

$$\lambda_{\rho\gamma} \equiv \frac{q}{p}\,\frac{A(\bar{B}_d^0 \to \rho^0\gamma)}{A(B_d^0 \to \rho^0\gamma)} = \frac{C_7^{(0)\rm eff} + A^{(1)t} - [C_7^{(0)\rm eff}\varepsilon_A^{(0)} + A^u]\,F\,{\rm e}^{+i\alpha}}{C_7^{(0)\rm eff} + A^{(1)t} - [C_7^{(0)\rm eff}\varepsilon_A^{(0)} + A^u]\,F\,{\rm e}^{-i\alpha}}, \tag{23}$$

where $p/q \simeq \exp(2i\beta)$ is the $B_d^0 - \bar{B}_d^0$ mixing parameter and $F = R_b/R_t$ with $R_b = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$ and $R_t = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2}$. In terms of $\lambda_{\rho\gamma}$, the direct and mixing-induced CP-violating asymmetries can be written as follows:

$$C_{\rho\gamma} = -\mathcal{A}_{\rm CP}^{\rm dir}(\rho^0\gamma) = \frac{1 - |\lambda_{\rho\gamma}|^2}{1 + |\lambda_{\rho\gamma}|^2}, \qquad S_{\rho\gamma} = \frac{2\,{\rm Im}(\lambda_{\rho\gamma})}{1 + |\lambda_{\rho\gamma}|^2}. \tag{24}$$

Thus, the direct CP-violating asymmetry $C_{\rho\gamma}$ is expressed by Eq. (6.6) in Ref. [15] while the mixing-induced CP-violating asymmetry $S_{\rho\gamma}$ in NLO can be presented in the form:

$$S_{\rho\gamma}^{\rm NLO} = S_{\rho\gamma}^{\rm LO} - \frac{2F\sin\alpha\,[1 - 2F\varepsilon_A^{(0)}\cos\alpha + (F\varepsilon_A^{(0)})^2\cos(2\alpha)]}{[1 - 2F\varepsilon_A^{(0)}\cos\alpha + (F\varepsilon_A^{(0)})^2]^2}\,\frac{A_R^u - \varepsilon_A^{(0)}A_R^{(1)t}}{C_7^{(0)\rm eff}}, \tag{25}$$

$$S_{\rho\gamma}^{\rm LO} = -\frac{2F\varepsilon_A^{(0)}\sin\alpha\,(1-F\varepsilon_A^{(0)}\cos\alpha)}{1-2F\varepsilon_A^{(0)}\cos\alpha+(F\varepsilon_A^{(0)})^2}, \tag{26}$$

where $A_R^{(1)t}$ and A_R^u are the real parts of the NLO contributions to the decay amplitudes entering Eq. (23). It is easy to see that, neglecting the weak-annihilation contribution ($\varepsilon_A^{(0)}=0$), the mixing-induced CP-asymmetry vanishes in the leading order. However, including the $O(\alpha_s)$ contribution, this CP-asymmetry is non-zero.

The dependence on the CKM unitarity-triangle angle α of the mixing-induced CP-asymmetry for the B_d^0-meson modes considered is presented in Fig. 1 (right frame). The dashed lines show the dependence in the LO while the solid lines correspond to the NLO result. Thus, fixing the parameters to their central values, one notices a marked effect from the NLO corrections on both $S_{\rho\gamma}^{\rm LO}$ and $S_{\omega\gamma}^{\rm LO}$. However, including the errors in the input parameters, the resulting allowed values for $S_{\rho\gamma}^{\rm NLO}$ and $S_{\omega\gamma}^{\rm NLO}$ are rather uncertain. This is worked out by taking into account the SM range $\alpha=(92\pm11)^\circ$ [25], and the numerical values for these asymmetries in the leading order and including the $O(\alpha_s)$ corrections are as follows:

$$S_{\rho\gamma}^{\rm LO} = (-2.7\pm1.0)\%, \qquad S_{\rho\gamma}^{\rm NLO} = (0.1^{+4.7}_{-4.3})\%, \tag{27}$$

$$S_{\omega\gamma}^{\rm LO} = (+2.7\pm1.0)\%, \qquad S_{\omega\gamma}^{\rm NLO} = (4.0^{+5.0}_{-4.6})\%. \tag{28}$$

Thus, the $\pm1\sigma$ ranges for the mixing-induced asymmetries in the SM are: $-0.04 \le S_{\rho\gamma}^{\rm NLO} \le 0.05$ and $-0.01 \le S_{\omega\gamma}^{\rm NLO} \le 0.09$. They are too small to be measured in the near future. Hence, the observation of a significant (and hence measurable) mixing-induced CP-asymmetries $S_{\rho\gamma}$ and $S_{\omega\gamma}$ would signal the existence of CP-violating phases beyond the SM.

4.3. Isospin-violating ratio. The charge-conjugated isospin-violating ratio is defined as follows:

$$\Delta \equiv \frac{1}{2}\left[\Delta^{+0}+\Delta^{-0}\right], \qquad \Delta^{\pm0} = \frac{\Gamma(B^\pm\to\rho^\pm\gamma)}{2\Gamma(B^0(\bar B^0)\to\rho^0\gamma)} - 1. \tag{29}$$

The explicit NLO expression in terms of the vertex, hard-spectator and weak-annihilation contributions to the decay amplitude can be found in Ref. [15]. The dependence of this ratio on the angle α is shown in the left frame in Fig. 2. With the improved input, the updated result is:

$$\Delta = (1.1\pm3.9)\%. \tag{30}$$

Thus, the isospin violation in $B\to\rho\gamma$ decays is expected to be small in the SM. The reason for this lies in the dependence $\Delta\propto\varepsilon_A^{(\pm)}\cos\alpha+O[(\varepsilon_A^{(\pm)})^2,\alpha_s]$, and we have used the current knowledge of the angle α from the CP-asymmetry in $B\to\pi\pi$ decays and the indirect unitarity fits, yielding $\alpha=(92\pm11)^\circ$ [25].

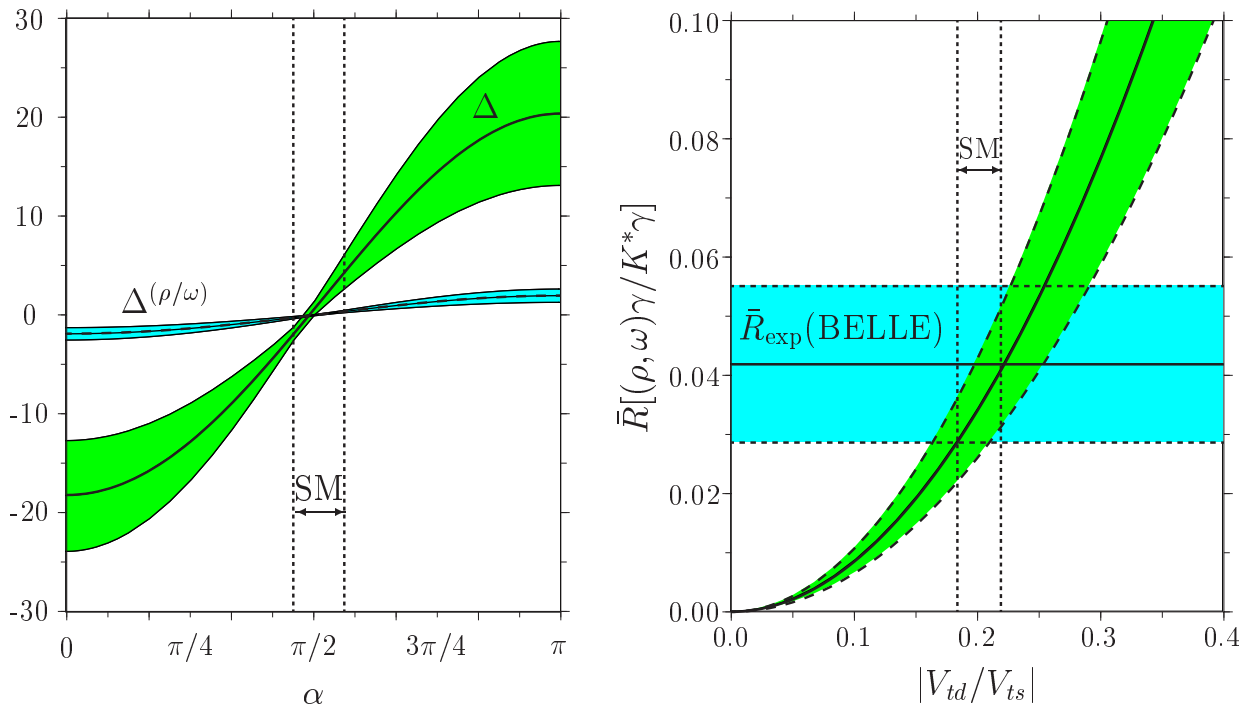

Figure 2: Left frame: The isospin-violating ratio Δ, defined in Eq. (29) for the decays $B \to \rho\gamma$, and the $SU(3)$-breaking ratio $\Delta^{(\rho/\omega)}$, defined in Eq. (31) and involving the $B_d^0 \to (\rho^0, \omega)\,\gamma$ decays, plotted as functions of the CKM unitarity-triangle angle α. Right frame: The ratio $\bar{R}_{\rm th}[(\rho,\omega)\gamma/K^*\gamma]$ plotted as a function of $|V_{td}/V_{ts}|$. The current experimental measurement with its $\pm 1\sigma$ range is shown as the horizontal band. The solid and dashed curves are the theoretical predictions in the SM and their $\pm 1\sigma$ errors, respectively. The vertical dotted lines show the SM-based best-fit interval for $|V_{td}/V_{ts}|$ from the CKM unitarity fits.

4.4. $SU(3)$-violating ratio. Another quantity of experimental interest is the ratio $\Delta^{(\rho/\omega)}$, involving the $B_d^0 \to (\rho,\omega)\gamma$ decays. It can be defined as:

$$\Delta^{(\rho/\omega)} \equiv \frac{1}{2}\left[\Delta_B^{(\rho/\omega)} + \Delta_{\bar{B}}^{(\rho/\omega)}\right], \tag{31}$$

with

$$\begin{aligned}
\Delta_B^{(\rho/\omega)} &\equiv \frac{(M_B^2 - m_\omega^2)^3\,\mathcal{B}(B_d^0 \to \rho^0\gamma) - (M_B^2 - m_\rho^2)^3\,\mathcal{B}(B_d^0 \to \omega\gamma)}{(M_B^2 - m_\omega^2)^3\,\mathcal{B}(B_d^0 \to \rho^0\gamma) + (M_B^2 - m_\rho^2)^3\,\mathcal{B}(B_d^0 \to \omega\gamma)}, \\
\Delta_{\bar{B}}^{(\rho/\omega)} &\equiv \frac{(M_B^2 - m_\omega^2)^3\,\mathcal{B}(\bar{B}_d^0 \to \rho^0\gamma) - (M_B^2 - m_\rho^2)^3\,\mathcal{B}(\bar{B}_d^0 \to \omega\gamma)}{(M_B^2 - m_\omega^2)^3\,\mathcal{B}(\bar{B}_d^0 \to \rho^0\gamma) + (M_B^2 - m_\rho^2)^3\,\mathcal{B}(\bar{B}_d^0 \to \omega\gamma)}.
\end{aligned}$$

The weighted factors in $\Delta_{B,\bar{B}}^{(\rho/\omega)}$ are introduced to suppress the effect of the phase space due to the difference in the ρ- and ω-meson masses. The expression for $\Delta_{B,\bar{B}}^{(\rho/\omega)}$, derived in

$O(\alpha_s)$ order, is rather lengthy:

$$\Delta_{B,\bar{B}}^{(\rho/\omega)} = \Delta_{\rm LO}^{(\rho/\omega)}\left[1+\frac{A_R^{(1)t}\cos\alpha - FA_R^u \mp A_I^{(1)t}\sin\alpha}{C_7^{(0)\rm eff}\left[\cos\alpha - F\bar{\varepsilon}_A\right]}\left(1-2\bar{\varepsilon}_A\Delta_{\rm LO}^{(\rho/\omega)}\right)\right. \tag{32}$$

$$\left. - \frac{2\left[A_R^{(1)t} - FA_R^u\cos\alpha \mp FA_I^u\sin\alpha\right]}{C_7^{(0)\rm eff}\left[1-2F\bar{\varepsilon}_A\cos\alpha+(F\bar{\varepsilon}_A)^2+(F\Delta\varepsilon_A)^2\right]}\right],$$

$$\Delta_{\rm LO}^{(\rho/\omega)} = \frac{-2F\Delta\varepsilon_A\left[\cos\alpha - F\bar{\varepsilon}_A\right]}{1-2F\bar{\varepsilon}_A\cos\alpha+(F\bar{\varepsilon}_A)^2+(F\Delta\varepsilon_A)^2}, \tag{33}$$

where $\bar{\varepsilon}_A = (\varepsilon_A^{(0)}+\varepsilon_A^{(\omega)})/2$ and $\Delta\varepsilon_A = (\varepsilon_A^{(0)}-\varepsilon_A^{(\omega)})/2$. In our approximation, $\bar{\varepsilon}_A = 0$ and, neglecting tiny corrections $\sim (F\Delta\varepsilon_A)^2$, the final expression is greatly simplified:

$$\Delta_{\rm NLO}^{(\rho/\omega)} = -\frac{2F\Delta\varepsilon_A}{C_7^{(0)\rm eff}}\left[(C_7^{(0)\rm eff}-A_R^{(1)t})\cos\alpha + FA_R^u\cos(2\alpha)\right]. \tag{34}$$

The dependence of this ratio on the angle α is shown in the left frame in Fig. 2. In the SM, with the input parameters specified above, this ratio can be estimated as:

$$\Delta_{\rm NLO}^{(\rho/\omega)} = (0.3\pm 3.9)\times 10^{-3}. \tag{35}$$

This value is an order of magnitude smaller than the isospin-violating ratio (30) in $B\to\rho\gamma$ decays due to the suppression of the weak-annihilation contributions in the decays of the neutral B-meson. In this case, the neglected subdominant long-distance contributions may become important. They can be estimated in a model-dependent way. In any case, the result in (35) should be improved by including the contributions of the penguin operators and the NNLO corrections. The ratio $\Delta^{(\rho/\omega)}$ in the SM is also too small to be measured. Both the ratios Δ and $\Delta^{(\rho/\omega)}$ are sensitive tests of the SM, and as argued in Refs. [28, 37] for the isospin-violating ratio Δ, their measurements significantly different from zero would reveal physics beyond the SM.

5. Determination of $|V_{td}/V_{ts}|$ from $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]$. To extract the value of $|V_{td}/V_{ts}|$ from the $B\to(K^*,\rho,\omega)\,\gamma$ decays, we use the ratio $\bar{R}_{\rm th}[(\rho,\omega)\,\gamma/K^*\gamma]$, which can be rewritten within the SM as follows:

$$\bar{R}_{\rm th}[(\rho,\omega)\,\gamma/K^*\gamma] = r_{\rm th}^{(\rho/\omega)}\left|\frac{V_{td}}{V_{ts}}\right|^2\zeta^2, \qquad r_{\rm th}^{(\rho/\omega)} = 1.18\pm 0.10, \tag{36}$$

where the error in $r_{\rm th}^{(\rho/\omega)}$ takes into account all the parametric uncertainties except in ζ and $|V_{td}/V_{ts}|$ which are treated as free variables. Applying this equation to the BABAR upper limit (6) and the BELLE experimental range (7), the product $\zeta\,|V_{td}/V_{ts}|$ can be restricted as follows:

$$\zeta\,|V_{td}/V_{ts}| > 0.19, \qquad \text{(BABAR)} \tag{37}$$

$$\zeta\,|V_{td}/V_{ts}| = 0.19\pm 0.03, \qquad \text{(BELLE)} \tag{38}$$

where the bound is at 90% C.L., following from the BABAR data. At present, the error in (38) is dominated by the experimental uncertainty. Using the range $\zeta = 0.85 \pm 0.10$ for the ratio of the transition form factors, one gets the following constraints on the CKM matrix element ratio $|V_{td}/V_{ts}|$:

$$|V_{td}/V_{ts}| > 0.19, \qquad \text{(BABAR)} \tag{39}$$

$$|V_{td}/V_{ts}| = 0.22 \pm 0.05, \qquad \text{(BELLE)} \tag{40}$$

where the lower limit from the BABAR data (6) corresponds to 90% C.L. In arriving at these numbers, the theoretical and experimental errors were considered as uncorrelated. Taking this correlation into account, the BELLE data yields the range $0.16 < |V_{td}/V_{ts}| < 0.29$, which is much larger than but in agreement with the SM range $|V_{td}/V_{ts}| = 0.20 \pm 0.02$.

The dependence of the ratio $\bar{R}_{\rm th}[(\rho,\omega)\gamma/K^*\gamma]$ on $|V_{td}/V_{ts}|$ is shown in the right frame in Fig. 2. The solid curve corresponds to the central values of the input parameters, and the dashed curves are obtained by taking into account the $\pm 1\sigma$ errors on the individual input parameters in $\bar{R}_{\rm th}[(\rho,\omega)\gamma/K^*\gamma]$ and adding the errors in quadrature. The current measurement for this quantity is also shown in this figure. Experimental error is currently large which renders the determination of $|V_{td}/V_{ts}|$ uncertain. However, in the long run, with greatly increased statistics, the impact of the measurement of $\bar{R}_{\rm exp}[(\rho,\omega)\gamma/K^*\gamma]$ on the CKM phenomenology, in particular the profile of the unitarity triangle, will depend largely on the theoretical accuracy of the ratio ζ. Note that using $|V_{td}/V_{ts}| = 0.20 \pm 0.02$, the estimates (37) and (38) result into the lower limit $\zeta > 0.81$ (at 90% C.L.) from the BABAR data and the range $0.71 < \zeta < 1.19$ from the BELLE measurement. These inferences are not precise enough to distinguish among models of SU(3)-breaking. We hope that with the first measurement of $\bar{R}_{\rm exp}[(\rho,\omega)\gamma/K^*\gamma]$ having been already posted [1], the ratio ζ will receive a renewed theoretical effort, in particular from the lattice community.

6. Current and potential impact of $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]$ on the CKM unitarity triangle. In this part we present the impact of the $B \to (\rho,\omega)\,\gamma$ branching ratio on the CKM parameters $\bar{\rho}$ and $\bar{\eta}$. For this purpose, it is convenient to rewrite the ratio $\bar{R}_{\rm th}[(\rho,\omega)\gamma/K^*\gamma]$ in the form in which the dependence on the CKM-Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$ is made explicit:

$$\begin{aligned} \bar{R}_{\rm th}[(\rho,\omega)\gamma/K^*\gamma] &= \frac{\lambda^2\zeta^2}{4}\,\frac{(M_B^2-m_\rho^2)^3}{(M_B^2-m_{K^*}^2)^3}\,\left[2\,G(\bar{\rho},\bar{\eta},\varepsilon_A^{(\pm)}) + G(\bar{\rho},\bar{\eta},\varepsilon_A^{(0)})\right] \\ &+ \frac{\lambda^2\zeta^2}{4}\,\frac{(M_B^2-m_\omega^2)^3}{(M_B^2-m_{K^*}^2)^3}\,G(\bar{\rho},\bar{\eta},\varepsilon_A^{(\omega)}). \end{aligned} \tag{41}$$

Here, the function $G(\bar{\rho},\bar{\eta},\varepsilon)$ encodes both the LO and NLO contributions:

$$G(\bar{\rho},\bar{\eta},\varepsilon) = [1-(1-\varepsilon)\,\bar{\rho}]^2 + (1-\varepsilon)^2\bar{\eta}^2 + 2\,{\rm Re}\left[G_0 - \bar{\rho}\,G_1(\varepsilon) + (\bar{\rho}^2+\bar{\eta}^2)\,G_2(\varepsilon)\right], \tag{42}$$

and the functions G_i $(i=0,1,2)$ are defined as follows:

$$G_0 = \left[A_{\rm sp}^{(1)\rho} - A_{\rm sp}^{(1)K^*}\right]/C_7^{(0)\rm eff}, \tag{43}$$

Table 5: Values (in units of 10^{-2}) of the real and imaginary parts of the functions G_i ($i = 0, 1, 2$), including the parametric uncertainties, are presented for three values of the weak-annihilation parameter ε_A.

ε_A	Re G_0	Im G_0	Re G_1	Im G_1	Re G_2	Im G_2
$+0.30$	4.63 ± 3.89	-0.48 ± 1.50	3.50 ± 7.90	6.20 ± 4.09	-1.84 ± 5.89	3.60 ± 3.38
$+0.03$	4.63 ± 3.89	-0.48 ± 1.50	10.80 ± 5.96	9.01 ± 3.28	6.10 ± 4.23	9.18 ± 3.55
-0.03	4.63 ± 3.89	-0.48 ± 1.50	12.43 ± 5.69	9.64 ± 3.13	7.86 ± 4.06	10.42 ± 3.60

Table 6: The input parameters used in the CKM-unitarity fits. Their explanation and discussion can be found, for example, in Ref. [41]. The parameter η_1 is evaluated at the scale of $\overline{\rm MS}$ mass $m_c(m_c) = 1.30$ GeV.

λ	0.2224 ± 0.002 (fixed)	$\lvert V_{cb}\rvert$	$(41.2 \pm 2.1) \times 10^{-3}$
$\lvert V_{ub}\rvert$	$(3.90 \pm 0.55) \times 10^{-3}$	$a_{\psi K_S}$	0.736 ± 0.049
$\lvert\epsilon_K\rvert$	$(2.280 \pm 0.13) \times 10^{-3}$	ΔM_{B_d}	(0.503 ± 0.006) ps^{-1}
η_1	1.32 ± 0.32	η_2	0.57 ± 0.01
η_3	0.47 ± 0.05	$m_c(m_c)$	(1.25 ± 0.10) GeV
$m_t(m_t)$	(168 ± 4) GeV	$\hat{B}_K$	0.86 ± 0.15
$f_{B_d}\sqrt{B_{B_d}}$	$(215 \pm 11 \pm 15^{+0}_{-23})$ MeV	η_B	0.55 ± 0.01
ξ	$1.14 \pm 0.03 \pm 0.02^{+0.13+0.03}_{-0.0\ -0.0}$	$\bar{R}_{\rm exp}[(\rho,\omega)\gamma/K^*\gamma]$	0.042 ± 0.013
ΔM_{B_s}	> 14.4 ps^{-1} at 95% C.L.		

$$G_1(\varepsilon) = 2G_0 - \left[A^u + \varepsilon\, A^{(1)t}\right]/C_7^{(0)\rm eff}, \tag{44}$$

$$G_2(\varepsilon) = G_0 - \left[(1-\varepsilon)\, A^u + \varepsilon\, A^{(1)t}\right]/C_7^{(0)\rm eff}. \tag{45}$$

Numerical values of the real and imaginary parts of the functions G_i ($i = 0, 1, 2$), and the parametric uncertainties, are given in Table 5. The three rows in this table correspond to the decays $B^\pm \to \rho^\pm\gamma$, $B_d^0(\bar{B}_d^0) \to \rho^0\gamma$, and $B_d^0(\bar{B}_d^0) \to \omega\gamma$, respectively. It should be noted that the function $G(\bar{\rho}, \bar{\eta}, \varepsilon)$ (42) is related with the dynamical function ΔR, introduced in Ref. [15] to account for the weak-annihilation and NLO corrections, with: $G(\bar{\rho}, \bar{\eta}, \varepsilon) = R_t^2\,(1 + \Delta R)$.

To undertake the fits of the CKM parameters, we adopt a Bayesian analysis method. Systematic and statistical errors are combined in quadrature. We add a contribution to the χ^2-function for each of the input parameters presented in Table 6. Other input quantities are taken from their central values given in the PDG review [38]. The lower bound on the mass difference ΔM_{B_s} in the $B_s^0 - \bar{B}_s^0$ system is implemented using the modified χ^2-method (as described in the CERN CKM Workshop proceedings [39]), which makes use of the amplitude technique [40]. The $B_s \leftrightarrow \bar{B}_s$ oscillation probabilities are modified to have the dependence $P(B_s \to \bar{B}_s) \propto [1 + \mathcal{A}\cos(\Delta M_{B_s} t)]$ and $P(B_s \to B_s) \propto [1 - \mathcal{A}\cos(\Delta M_{B_s} t)]$.

Table 7: The 68% C.L. ranges for the CKM-Wolfenstein parameters, $R_b = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$, $R_t = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2}$, CP-violating phases, ΔM_{B_s} and $\bar{R}[(\rho,\omega)\,\gamma/K^*\gamma]$ from the CKM-unitarity fits.

λ	0.2224	A	$0.79 \div 0.86$
$\bar{\rho}$	$0.10 \div 0.24$	$\bar{\eta}$	$0.32 \div 0.40$
R_b	$0.37 \div 0.43$	R_t	$0.83 \div 0.98$
$\sin(2\alpha)$	$-0.44 \div +0.30$	α	$(81 \div 103)^\circ$
$\sin(2\beta)$	$0.69 \div 0.78$	β	$(21.9 \div 25.5)^\circ$
$\sin(2\gamma)$	$0.50 \div 0.96$	γ	$(54 \div 75)^\circ$
ΔM_{B_s}	$(16.6 \div 20.3)$ ps^{-1}	$\bar{R}[(\rho,\omega)\,\gamma/K^*\gamma]$	$(2.3 \div 4.3)\%$

The contribution to the χ^2-function is then:

$$\chi^2(\Delta M_{B_s}) = 2\left[\mathrm{Erfc}^{-1}\left(\frac{1}{2}\,\mathrm{Erfc}\,\frac{1-\mathcal{A}}{\sqrt{2}\,\sigma_{\mathcal{A}}}\right)\right]^2, \tag{46}$$

where $\mathcal{A}$ and $\sigma_{\mathcal{A}}$ are the world average amplitude and error, respectively. The resulting χ^2-function is then minimized over the following parameters: $\bar{\rho}$, $\bar{\eta}$, A, $\hat{B}_K$, η_1, η_2, η_3, $m_c(m_c)$, $m_t(m_t)$, η_B, $f_{B_d}\sqrt{B_{B_d}}$, ξ. Further details can be found in Ref. [41].

We present the output of the fits in Table 7, where we show the 68% C.L. ranges for the CKM parameters A, $\bar{\rho}$ and $\bar{\eta}$, the angles of the unitarity triangle α, β and γ, as well as $\sin(2\phi_i)$ with $\phi_i = \alpha, \beta, \gamma$, and ΔM_{B_s}. The allowed profile (at 95% C.L.) of the unitarity triangle from the resulting fit is shown in Fig. 3 as shaded region. Here we also show the 95% C.L. range of the ratio $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma] = \bar{\mathcal{B}}_{\rm exp}[B \to (\rho,\omega)\,\gamma]/\bar{\mathcal{B}}_{\rm exp}(B \to K^*\gamma)$, which is used as an input in the fits now. We find that the current measurement of $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]$ is in comfortable agreement with the fits of the CKM unitarity triangle resulting from the measurements of the five quantities (R_b, ϵ_K, ΔM_{B_d}, ΔM_{B_s}, and $a_{\psi K_S}$). The resulting contour in the $\bar{\rho} - \bar{\eta}$ plane practically coincides with the shaded region, and hence not shown. We conclude that due to the large experimental error on $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]$, but also due to the significant theoretical errors, the impact of the measurement of $B \to (\rho,\omega)\gamma$ decays on the profile of the CKM unitarity triangle is currently small. How this could change in future is illustrated by reducing the current experimental error on $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]$ by a factor 3, which is a realistic hope for the precision on this quantity from the B-factory experiments in a couple of years from now. The resulting (95% C.L.) contours are shown as dashed-dotted curves, which result in reducing the currently allowed $\bar{\rho} - \bar{\eta}$ parameter space. This impact will be enhanced if the theoretical errors, dominated by $\Delta\zeta/\zeta$, are also brought under control.

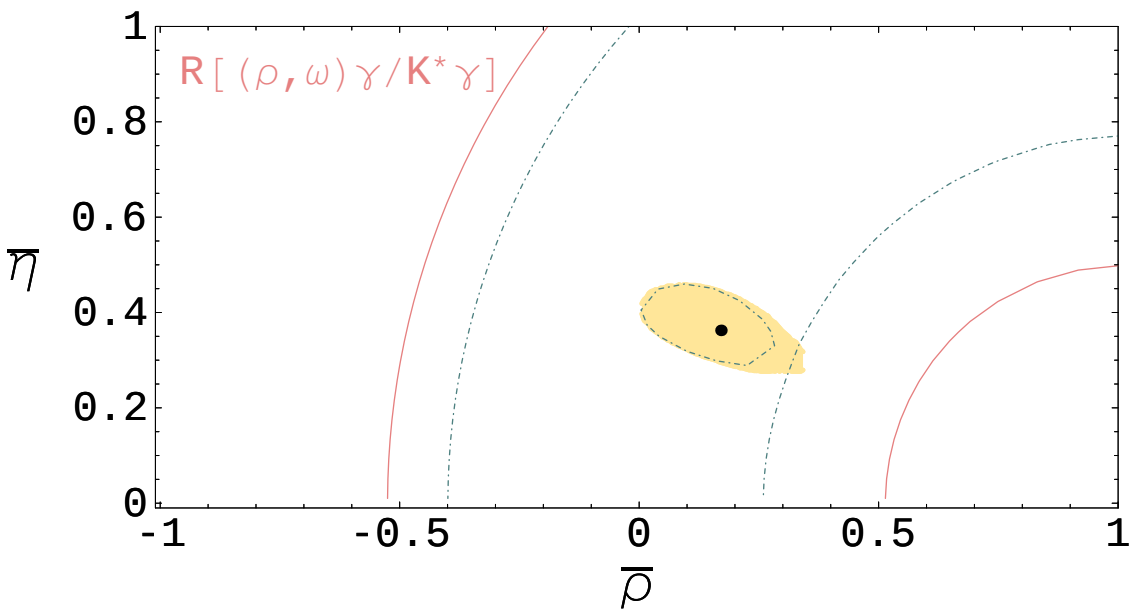

Figure 3: Allowed $\bar{\rho}-\bar{\eta}$ regions following from the six measurements (R_b, ϵ_K, ΔM_{B_d}, ΔM_{B_s}, $a_{\psi K_S}$, and $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]$), corresponding to 95% C.L., with the dot showing the best-fit values. The shaded region shows the current profile. The two outer (solid) curves give the 95% C.L. constraints in the $\bar{\rho}-\bar{\eta}$ plane from the current measurement of $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]$. The inner (dashed-dotted) curves are the 95% C.L. constraints from an assumed measurement of $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]$ having the current central value but the error reduced by a factor 3. The contour shows the potential impact of this assumed measurement in the $\bar{\rho}-\bar{\eta}$ plane.

7. Summary We have studied the implication of the first measurement of the averaged branching fraction $\bar{\mathcal{B}}_{\rm exp}[B\to(\rho,\omega)\,\gamma]$ by the BELLE collaboration for the CKM phenomenology in the SM. Updating the earlier theoretical calculations [15], carried out in the QCD factorization framework, in which several input parameters have changed, we have calculated the averaged branching ratios for the exclusive $B\to(K^*,\rho,\omega)\,\gamma$ decays and the ratio $\bar{R}_{\rm th}[(\rho,\omega)\,\gamma/K^*\gamma]$. Using the CKM-Wolfenstein parameters $\bar{\rho}=0.17\pm0.07$ and $\bar{\eta}=0.36\pm0.04$ from the unitarity fits [25], we find $\bar{\mathcal{B}}_{\rm th}[B\to(\rho,\omega)\,\gamma]=(1.38\pm0.42)\times10^{-6}$ and $\bar{R}_{\rm th}[(\rho,\omega)\,\gamma/K^*\gamma]=(3.3\pm1.0)\%$, to be compared with the experimental numbers $\bar{\mathcal{B}}_{\rm exp}[B\to(\rho,\omega)\,\gamma]=(1.8^{+0.6}_{-0.5}\pm0.1)\times10^{-6}$, and $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]=(4.2\pm1.3)\%$, respectively. We see a quantitative agreement between the SM and the BELLE measurement in the radiative penguin $b\to d$ transitions. Leaving the CKM parameters $\bar{\rho}$ and $\bar{\eta}$ as free, we determine (at 68% C.L.) $0.16\le|V_{td}/V_{ts}|\le0.29$ (at 68% C.L.), which is in agreement with but less precise than the corresponding range from the CKM fits $|V_{td}/V_{ts}|=0.20\pm0.02$ [25]. This is, however, expected to change as the experimental precision on the branching ratios and $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]$ improves. We emphasize that the measurement of $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]$ provides the first direct determination of the ratio $|V_{td}/V_{ts}|$ in rare B-meson decays. We have also presented updated estimates of a

number of the isospin-violating and SU(3)-violating ratios and CP-violating asymmetries in the $B \to (\rho, \omega)\,\gamma$ decays. Their measurements will either overconstrain the angle α of the unitarity triangle, or they may lead to the discovery of physics beyond the SM in the radiative $b \to d\gamma$ decays.

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