Revised Version

DESY 04-095
SFB/CPP-04-21

arXiv:hep-ph/0406203 v2 1 Jul 2004

A complete set of scalar master integrals for massive 2-loop Bhabha scattering: where we are *

M. Czakon[a b], J. Gluza[a b] and T. Riemann[a]

[a]Deutsches Elektronen-Synchrotron, DESY Zeuthen, Platanenallee 6, 15738 Zeuthen, Germany

[b]Institute of Physics, University of Silesia, ul. Uniwersytecka 4, 40007 Katowice, Poland

We define a complete set of scalar master integrals (MIs) for massive 2-loop QED Bhabha scattering. Among others, there are thirty three 2-loop box type MIs. Five of them have been published in (semi-)analytical form, one is determined here, the rest remains to be calculated. Further, the last four so far unknown 2-loop 3-point MIs are identified and also computed here.

1. INTRODUCTION

The determination of luminosity at high energy colliders is a backbone linking theoretical calculations with experimental data. Bhabha scattering, $e^+e^- \to e^+e^-(n\gamma)$, is the process that allows to accomplish this task. So far the complete NNLO corrections in massive QED have not been computed. There are several reasons to fill this gap. First, the future experimental accuracy is expected to be of the order of 10^{-4} [1]. Obviously, the theoretical error should be smaller. Second, the current Monte Carlo generators for real photon radiation assume massive external fermions. Analytical results of NNLO corrections in massive QED are clearly needed there for consistency. There are also technical reasons to perform the calculation. First, as argued in [2], 2-loop QED results are a useful testing ground for 2-loop QCD physics containing more than one kinematic invariant. Second, the full analytic results can be easily approximated to various kinematic regions; this is not possible for massless results. Finally, the complete calculation of virtual 2-loop corrections in massive QED is a prelude to the determination of analogous effects within the Standard Model.

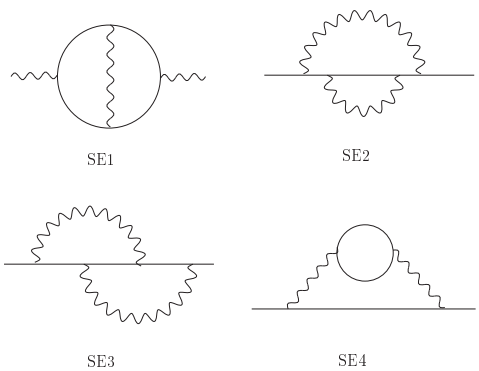

Figure 1. The 2-loop self energy diagrams.

In this contribution, we report on first results of our calculation of virtual 2-loop corrections in massive QED. We have determined a complete set of scalar master integrals to be calculated for the problem at hand, including 33 2-loop box master integrals[2]. Subsequently, we have calculated all 2- and 3-point MIs, four of which were still lacking in the literature. We also started the calculation of boxes.

*Work supported in part by European's 5-th Framework under contract HPRN–CT–2000–00149 Physics at Colliders, by TMR, EC-Contract No. HPRN-CT-2002-00311 (EURIDICE), by Deutsche Forschungsgemeinschaft under contract SFB/TR 9–03, and by the Polish State Committee for Scientific Research (KBN) under contract Nos. 2P03B01025 and 1P03B00927.

[2]More detailed information, including the MIs computed so far, can be found at the webpage **bhabha.html** in [5].

2. 2–LOOP DIAGRAMS AND IDENTIFICATION OF MIs

In Figs. 1 to 3 we show the set of 2-loop self energy, vertex and box diagrams for which the scalar MIs are needed.

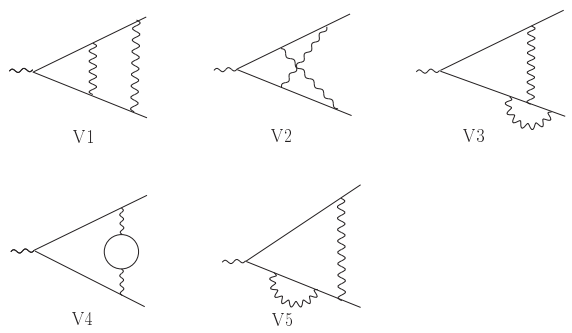

Figure 2. The 2-loop vertex diagrams.

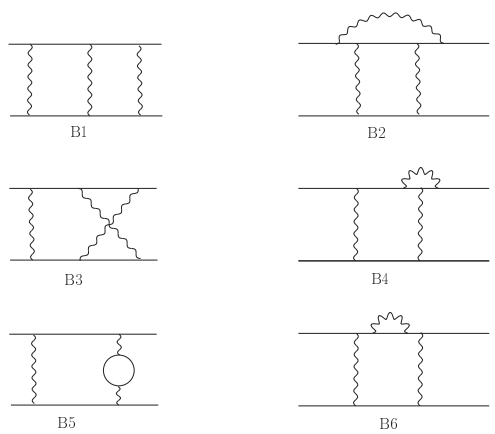

Figure 3. The 2-loop box diagrams.

Following the Laporta-Remiddi approach [6,7], we have determined a set of scalar MIs for the virtual 2-loop corrections to Bhabha scattering. We use the C++ library `DiaGen/IdSolver`[3], which determines and solves an appropriate set of algebraic equations, derived with integration by parts (IBP) [10] and Lorentz invariance (LI) identities [11]. The latter have been useful for algorithmic optimization but did not reduce the number of MIs. In Table 1 we give a list of the net numbers of 2-loop master integrals needed for the evaluation of the various 2-loop vertex and box diagrams. Obviously, the 1-loop masters add up. A complete list of 2-loop MIs is given in Figs. 4 to 6.

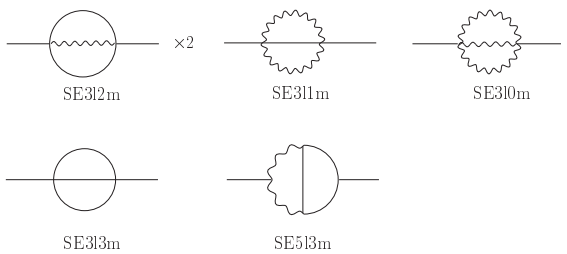

Figure 4. The six 2-loop 2-point MIs. The multiplication factor stands for the number of masters for the given diagram when different from 1.

Strictly speaking, we show prototypes, i.e. topologies with masses assigned to the lines. Some of the prototype diagrams are multiplied by an integer. This indicates that masters appear with dotted lines (higher powers of propagators)[4]. As an example, we give the master list for the prototype `V5l2m` (i.e. *V*-ertex with *5l*-ines, among them *2m*-assive). All the other masters may be found in [5]. There, we also give tables indicating which MIs contribute to SE1-SE4, V1-V5, B1-B6.

We have started to solve the MIs in a systematic way with the differential equation approach proposed in [12] and developed in [13][5]. With $(p_1 + p_2) = (p_3 + p_4)$ and $p_i^2 = m_e^2$, we use $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$. The analytical results depend on the variables x and y,

$$x = \frac{\sqrt{-s+4} - \sqrt{-s}}{\sqrt{-s+4} + \sqrt{-s}}, \qquad (1)$$

and y is obtained by replacing s by t. We set everywhere the electron mass to unity, $m_e = 1$.

[3] `DiaGen` has already been used in several other projects [8,9], `IdSolver` is a new software package by M.C. We are also using Fermat [4], FORM [3], Maple, Mathematica.

[4] One might also use irreducible numerators. However, this introduces an explicit dependence of the master on the specific momentum distribution chosen.

[5] We put the results obtained so far in the `Mathematica` file `MastersBhabha.m` in [5]. Recalculating some of the known results, we found some misprints in Eqs. (80), (85), (121), (128), (131), (144) in [14] and in Eqs. (36), (38), (39),(B.9) in [15] for the corresponding masters. We have been informed by the authors that an Erratum is in preparation.

Table 1
The number of 2-loop master integrals needed to calculate the 2-loop vertex diagrams (Fig. 2) and box diagrams (Fig. 3)

Diagram	V1	V2	V3	V4	V5	B1	B2	B3	B4	B5	B6
2-point MIs	3	4	4	1	3	4	5	5	4	3	3
3-point MIs	4	10	5	2	1	7	11	13	10	4	4
Box type MIs						9	15	22	11	2	3

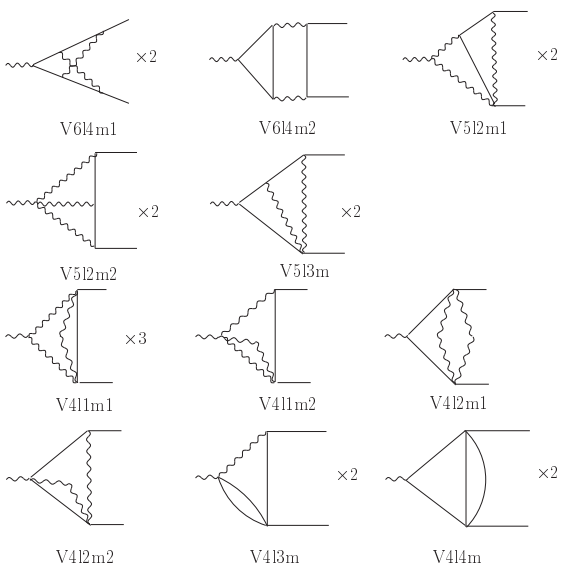

Figure 5. The nineteen 2-loop 3-point MIs (the multiplication factors as in Fig. 4.)

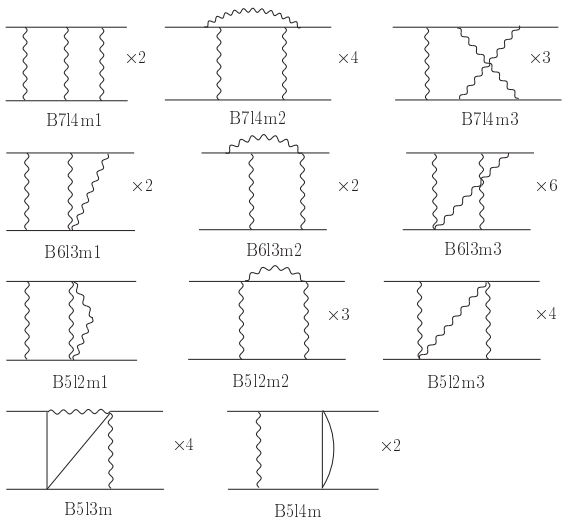

Figure 6. The thirty three 2-loop box MIs (the multiplication factors as in Fig. 4.)

Our normalization of the momentum integrals is chosen such that the one-loop tadpole is ($d = 4 - 2\epsilon$):

$$\texttt{T1l1m} = \frac{(\pi e^{\gamma_E})^\epsilon}{i\pi^2}\int\frac{d^dq}{q^2-1} = \frac{1}{\epsilon}+1+\left(1+\frac{\zeta_2}{2}\right)\epsilon+\ldots \quad (2)$$

3. NEW 2-LOOP 3-POINT MASTERS

Most of the nineteen 2-loop 3-point MIs needed for the massive Bhabha process have been calculated already: `V4l2m` (2 masters), `V4l3m` (2 masters), `V4l4m` (2 masters), `V5l3m` (2 masters), `V6l4m1` (2 masters) in [14]; `V4l1m` (4 masters) and `V6l4m2` in [16]. The last four masters completing the list, needed for box diagrams B1 and B3, are of the type `V5l2m` (see Fig. 7). We obtain the following analytical expressions:

$$\begin{aligned}\texttt{V5l2m1} = \frac{x}{(-1+x^2)}\Big[&-\frac{5}{2}\zeta_4+\zeta_2H(0,0,x)\\ &-2\zeta_2H(1,0,x)+2H(0,0,0,1,x)\\ &+H(0,0,1,0,x)+2H(0,0,1,1,x)\\ &+2H(0,1,0,0,x)-3H(1,0,0,0,x)\\ &+2H(1,0,0,1,x)+2H(1,0,1,0,x)\\ &+4H(1,0,1,1,x)-4\zeta_3H(0,x)-2\zeta_3H(1,x)\Big]\\ &+\mathcal{O}(\epsilon), \quad (3)\end{aligned}$$

$$\texttt{V5l2m1d} = -\frac{2x}{(-1+x)^2(1+x)}\Big[xH(0,0,x)$$

$$+(-1+x)\left[2\zeta_2 + H(0,1,x)\right]\Bigg] + \mathcal{O}(\epsilon), \quad (4)$$

$$\texttt{V5l2m2} = \frac{2x}{(-1+x^2)}\Bigg[8\zeta_4 + 2\zeta_2 H(0,0,x) - 4\zeta_2 H(0,1,x) + H(0,0,0,0,x) + 2H(0,0,0,1,x)\Bigg] + \mathcal{O}(\epsilon), \quad (5)$$

$$\begin{aligned}\texttt{V5l2m2d} = &-\frac{1}{\epsilon^2}\,\frac{x}{4(-1+x)^2} \\ &-\frac{1}{\epsilon}\,\frac{x}{2(-1+x)^2}\Big[-1+H(0,x)+2H(1,x)\Big] \\ &+\frac{x}{4(-1+x)^2(1+x)}\Big[-4-15\zeta_2-4x \\ &+\zeta_2 x + 4(1+x)\left[H(0,x)+2H(1,x)\right] \\ &-2(x+3)\left[H(0,0,x)+2H(0,1,x)\right] \\ &-8(1+x)\left[H(1,0,x)+2H(1,1,x)\right]\Big] + \mathcal{O}(\epsilon). \quad (6)\end{aligned}$$

The $H(0,\ldots,x)$ etc. are harmonic polylogarithms as introduced in [17]. We have performed a variety of cross-checks on the results, e.g. numerical evaluations at fixed kinematical points in the Euclidean region using sector decomposition following the approach described in [18]. The numerical integrations are done with `CUBA` [19]. See also [20] for further cross checks.

4. A NEW MASSIVE 2–LOOP BOX MI

As mentioned, there are thirty three 2-loop box MIs needed. By now, few of them are known analytically. For diagram B1, the masters `B7l4m1` and `B7l4m1N` are given in [21]; for diagram B2, the master `B7l4m2` is known as two-dimensional integral representation [22]; for diagram B3, the leading divergent part of master `B7l4m3` is published [22]. For diagram B5, the two double box masters, `B5l4m` and `B5l4md`, have been derived recently with the restriction to $N_f = 1$ (one flavor) in [15,5][5].

A further double box master integral, `B5l2m1` (see Fig. 6), is given here; it contributes to diagrams B1 and B3 and is the solution of the following differential equation:

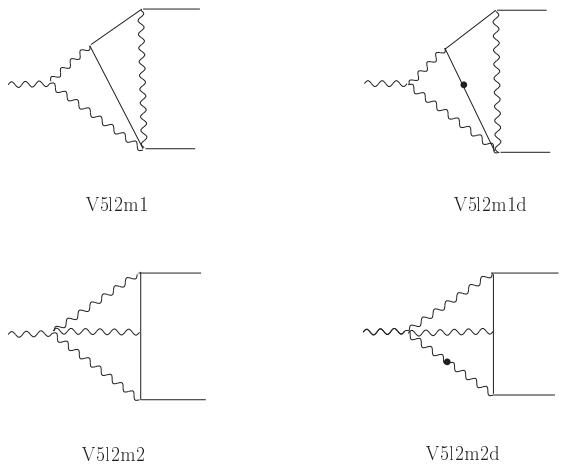

Figure 7. Four MIs of type V5l2m.

$$\begin{aligned}s\frac{\partial}{\partial s}\,\texttt{B5l2m1(s,t)} = &\left[\frac{8+s^2-2t+s(-6+t+\epsilon t)}{(-4+s)(-4+s+t)}\right]\texttt{B5l2m1(s,t)} \\ &+\left[\frac{2-9\epsilon+9\epsilon^2}{\epsilon(-4+s)t}\right]\texttt{SE3l0m(t)} \\ &+\left[\frac{(-1+3\epsilon)(-4+t)}{(-4+s)(-4+s+t)}\right]\texttt{V4l1m2(t)} \\ &-\left[\frac{s(1-3\epsilon)}{(-4+s)(-4+s+t)}\right]\texttt{V4l2m1(s)}. \quad (7)\end{aligned}$$

To get this equation we use a simple representation for differential operators. In the s-channel (for the t-channel operator: $p_2 \leftrightarrow -p_3$)

$$s\frac{\partial}{\partial s} = \frac{1}{2}\left\{(p_1^\mu + p_2^\mu) + \frac{s\,(p_2^\mu - p_3^\mu)}{s+t-4m^2}\right\}\frac{\partial}{\partial p_2^\mu}. \quad (8)$$

The boundary condition[6] is fixed at $x = 1$. The solution is:

$$\begin{aligned}\texttt{B5l2m1} = &\frac{1}{\epsilon^2}\,\frac{x}{(-1+x^2)}H(0,x) \\ &-\frac{1}{\epsilon}\,\frac{x}{(-1+x^2)}\Big[\zeta_2 - H(0,x)\left[2+H(0,y)\right.\end{aligned}$$

[6] We have also used the point $x = -1$ or on-shell conditions with cross checks with the `ONSHELL2` package [23].

$$+2H(1,y)] + 2H(-1,0,x) - H(0,0,x)\Big]$$
$$+ \frac{x}{(-1+x^2)}\Big[-G(-y,x)\,[3\zeta_2$$
$$+H(0,0,y) + 2H(0,1,y)] + G(-1/y,x)\,[5\zeta_2$$
$$+H(0,0,y) + 2H(0,1,y)] - 2\zeta_2$$
$$+2G(-1/y,-1,0,x) - G(-1/y,0,0,x)$$
$$+2G(-y,-1,0,x) - G(-y,0,0,x)$$
$$+2\zeta_2 H(-1,x) + 4H(0,x) - 4\zeta_2 H(0,y)$$
$$+G(-1/y,0,x)H(0,y) + G(-y,0,x)H(0,y)$$
$$+2H(0,x)H(0,y) + 2G(-1/y,0,x)H(1,y)$$
$$+2G(-y,0,x)H(1,y) + 4H(0,x)H(1,y)$$
$$-4H(-1,0,x) - 2H(0,y)H(-1,0,x)$$
$$-4H(1,y)H(-1,0,x) + 2H(0,0,x)$$
$$+2H(0,x)H(0,0,y) + 4H(0,x)H(0,1,y)$$
$$+4H(0,x)H(1,0,y) + 8H(0,x)H(1,1,y)$$
$$+4H(-1,-1,0,x) - 2H(-1,0,0,x)$$
$$-4H(0,-1,0,x) + 2H(0,0,0,x)$$
$$-H(0,0,0,y) - 2H(0,0,1,y) - 2\zeta_3\Big] + \mathcal{O}(\epsilon). \quad (9)$$

The 2-dimensional harmonic polylogarithms G are introduced in Appendix A of [24]. Here, we use the notations of [15]; the $G(-y,0,x)$ and $G(-y,-1,0,x)$ may be directly taken from (C.27) and (C.31). Further it is:

$$G(-y,0,0,x) \equiv \frac{1}{2}\int_0^x \frac{dv\,\ln^2 v}{v+y}$$
$$= \frac{\ln^2 x}{2}\ln(1+\frac{x}{y}) + \ln x \mathrm{Li}_2(-\frac{x}{y}) - \mathrm{Li}_3(-\frac{x}{y}). \quad (10)$$

To summarize, the next, certainly ambitious task is the calculation of the remaining box type master integrals. Finally, for applications at a linear collider, the complete 2-loop QED amplitudes, the 1-loop Standard Model corrections [25,26], and cut dependent real photon radiation effects (determined with Monte Carlo generators) must be evaluated.

REFERENCES

1. W. Lohmann, in: Proc. of 2003 IEEE Nucl. Sci. Symp., Portland, USA, to appear; M. Battaglia et al., eConf C010630 (2001) E3015.
2. Z. Bern, L. Dixon and A. Ghinculov, Phys. Rev. D63 (2001) 053007.
3. J. Vermaseren, "New features of FORM", math-ph/0010025.
4. R. H. Lewis, Fermat, www.bway.net/~lewis/.
5. M. Czakon, J. Gluza and T. Riemann, **http://www-zeuthen.desy.de/theory/research/bhabha/bhabha.html**.
6. S. Laporta and E. Remiddi, Phys. Lett. B379 (1996) 283.
7. S. Laporta, Int. J. Mod. Phys. A15 (2000) 5087.
8. M. Czakon et al., Nucl. Phys. B642 (2002) 157; Nucl. Phys. Proc. Suppl. 116 (2003) 230;
9. M. Awramik et al., Phys. Rev. Lett. 89 (2002) 241801; Phys. Rev. D68 (2003) 053004; Phys. Lett. B568 (2003) 48.
10. K.G. Chetyrkin and F.V. Tkachov, Nucl. Phys. B192 (1981) 159.
11. T. Gehrmann and E. Remiddi, Nucl. Phys. B580 (2000) 485.
12. A.V. Kotikov, Phys. Lett. B259 (1991) 314.
13. E. Remiddi, Nuovo Cim. A110 (1997) 1435.
14. R. Bonciani, P. Mastrolia and E. Remiddi, Nucl. Phys. B661 (2003) 289.
15. R. Bonciani et al., Nucl. Phys. B681 (2004) 261.
16. R. Bonciani et al., Nucl. Phys. B690 (2004) 138.
17. E. Remiddi and J.A.M. Vermaseren, Int. J. Mod. Phys. A15 (2000) 725.
18. T. Binoth and G. Heinrich, Nucl. Phys. B585 (2000) 741.
19. T. Hahn, hep-ph/0404043.
20. J. Gluza, transparencies of talk, http://www-zeuthen.desy.de/~lldesy/gluza.pdf.
21. V.A. Smirnov, Phys. Lett. B524 (2002) 129.
22. V.A. Smirnov, hep-ph/0406052; G. Heinrich and V.A. Smirnov, hep-ph/0406053.
23. J. Fleischer and M. Kalmykov, Comput. Phys. Commun. 128 (2000) 531.
24. T. Gehrmann and E. Remiddi, Nucl. Phys. B601 (2001) 248.
25. A. Lorca and T. Riemann, these proceedings.
26. J. Fleischer et al., Nucl. Phys. Proc. Suppl. 116 (2003) 43.