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Gauge Unification in Extra Dimensions: Power Corrections vs. Higher-Dimension Operators

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Abstract

Power-like loop corrections to gauge couplings are a generic feature of higher-dimensional field theories. In supersymmetric grand unified theories in $d = 5$ dimensions, such corrections arise only in the presence of a vacuum expectation value of the adjoint scalar of the gauge multiplet. We show that, using the analysis of the exact quantum effective action by Intriligator, Morrison and Seiberg, these power corrections can be understood as the effect of higher-dimension operators. Such operators, both classical and quantum, are highly constrained by gauge symmetry and supersymmetry. As a result, even non-perturbatively large contributions to gauge coupling unification can be unambiguously determined within 5d low-energy effective field theory. Since no massive hypermultiplet matter exists in 6 dimensions, the predictivity is further enhanced by embedding the 5d model in a 6d gauge theory relevant at smaller distances. Thus, large and quantitatively controlled power-law contributions to gauge couplings arise naturally and can, in the most extreme case, lead to calculable TeV-scale power law unification. We identify a simple 5d SU(5) model with one massless **10** in the bulk where the power-law effect is exactly MSSM-like.

1 Introduction

The elegance with which the Standard Model (SM) gauge and matter fields fit into the groups SU(5) [1] or SO(10) [2] and their smallest representations forms the main piece of evidence for the idea of Grand Unified Theories or GUTs (also [3]). In the conventional view, this is further supported by quantitative gauge coupling unification at $\sim 10^{16}$ GeV due to logarithmic running in 4d supersymmetric (SUSY) field theory [4, 5]. However, given the excellent theoretical and phenomenological motivation of extra dimensions, especially in the context of string-theoretic [6] or field-theoretic [7–10] unified models, it is important to understand novel features introduced by loop-effects in higher dimensions.

It is by now well-known that power-like loop corrections in higher dimensions can affect gauge unification in a dramatic way [11] (see also [12]). However, their quantitative analysis requires the detailed knowledge of the high-scale unified model [13, 14]. Related issues have also been discussed in the context of dimensional deconstruction (see, e.g., [15]).

As described in detail in [16] (cf. also [17]), the dominant power-law effects are calculable within effective field theory (i.e., are independent of the UV completion) if the 5- or 6-dimensional unified gauge theory is broken by the vacuum expectation value (VEV) of a bulk scalar field. The relevant finite loop correction, linear in the mass or momentum scale in $d = 5$, was also obtained in the context of AdS models [18–20]. In particular, the linear correction to gauge unification is implicitly contained in [19] (cf. also [21]).

Specifically, the loop corrections to the differences of the inverse SM gauge couplings α_4^{-1} in 4 dimensions, evaluated directly below the compactification scale $M_c \sim 1/R$, take the form [16]

$$\Delta\alpha_4^{-1}(M_c) \sim \begin{cases} |\Phi| R & , d = 5 \\ (\,|\Phi| R\,)^2 \cdot \ln\left(\Lambda/|\Phi|\right) & , d = 6\,. \end{cases} \qquad (1)$$

Here Φ is the relevant GUT-breaking VEV and Λ is the UV cutoff scale (implying that, in $d = 6$, only the leading-log term is predicted). Calculability holds in the same sense as for threshold corrections of 4d GUTs [22–24], i.e., they are uniquely specified by group theory once the field content and relevant mass thresholds of the model are given.

A potential problem comes from higher-dimension operators such as $(1/\Lambda)^n \,\mathrm{tr}[\Phi^n\, F^2]$. If Φ develops a VEV, they contribute to gauge coupling differences at the tree level and compete with the calculable loop effects [19, 20]. The most dangerous such operators may be forbidden or restricted by symmetries [16]. Nevertheless, their presence limits the quantitative control over power-law corrections to the region $|\Phi|/\Lambda \ll 1$ in the non-SUSY case. In particular, consider a 5-dimensional model where $\Lambda \sim g_5^{-2}$ sets the fundamental scale of the bulk theory. Then the requirement $|\Phi| \ll \Lambda$ implies that the corrections of Eq. (1) can not change the tree-level relation $(4\pi\alpha_4)^{-1} = g_4^{-2} = 2\pi R g_5^{-2}$ at the 100% level.

The present paper demonstrates that, in the supersymmetric case, the analysis of the exact quantum effective action by Intriligator, Morrison and Seiberg [25] drastically

improves the situation outlined above. The statements of [16] concerning the predictive power and validity range of power-law calculations are strengthened very significantly. Here the crucial point is that the quantum effective action at the two-derivative level is completely known. Technically, this follows from the SUSY-based restrictions on higher-dimension operators, including the absence of two-derivative operators of mass dimension 6 and higher in the super Yang-Mills (SYM) lagrangian. (Higher-derivative operators can be present but do not affect low-energy gauge couplings.) In particular, low-energy 4d gauge couplings may receive 100% corrections from higher-dimensional power-law effects which are nevertheless controlled within effective field theory. In the most extreme case, this allows scenarios with quantitative TeV-scale power-law unification.

The paper is organized as follows. In Sect. 2 we recall the quantum exact prepotential of 5d SYM theory discussed in detail in [25] (for earlier results see [26] and, in particular, [27]). For our purposes, it is essential that the only classical operators are the (SUSY versions of) gauge kinetic and Chern-Simons (CS) terms and that quantum corrections, which arise only at the 1-loop-level, are known explicitly.

In Sect. 3 it is shown how the by now familiar power-corrections to gauge unification (sometimes referred to as 'power-law running') arise in the above framework. They correspond to higher-dimension operators which are, in general, non-analytic in the symmetry-breaking VEV. Corrections induced by a bulk hypermultiplet become analytic (in fact, identical to a classical CS term) if the hypermultiplet mass is sufficiently large. However, the tuning of finite hypermultiplet masses comparable to the bulk VEV allows the realization of almost any desired power law effect. This ambiguity is avoided if the 5d model arises as the low-energy limit of a 6d construction because of the absence of massive hypermultiplets in 6 dimensions.

Section 4 introduces a realistic SU(5) model on S^1/Z_2, where the GUT group is broken to the SM group at both boundaries. The bulk field Φ breaking the symmetry in 5d is the adjoint scalar of the 5d vector multiplet. Its VEV, which is stabilized, e.g., by boundary Fayet-Iliopoulos (FI) terms, induces large power-law corrections to gauge unification. At the same time, this VEV gives masses to the A_5 zero modes of X, Y gauge bosons which would otherwise plague an SU(5) orbifold GUT on S^1/Z_2. Power-law corrections to gauge unification are given in terms of the Φ-VEV, the bulk hypermultiplet masses, and the bulk CS term, the latter being fixed by brane anomaly cancellation. Intriguingly, a bulk field content of just the gauge multiplet and a massless **10** of SU(5) induces a power-law effect that is identical to the logarithmic running within the Minimal Supersymmetric Standard Model (MSSM). The extreme lightness of one of the three SM generations emerges naturally in this context.

In Sect. 5, power-like loop corrections in 6-dimensional unified models are considered [28, 29] (for related earlier string theory results, especially including Wilson lines, see, e.g., [30] and [31,32]). If the geometry is equivalent to T^2/Z_2 and $R_5 \gg R_6$, the phenomenology of such models becomes very similar to the 5d case. In particular, the finite 5d version of Eq. (1) rather than the logarithmically UV-sensitive 6d version applies. The role of the Φ-VEV is taken over by an A_6 Wilson line wrapping the cylinder-like central part of the compact space. The value of this Wilson line is fixed, e.g., by the

orbifold breaking of the gauge symmetry at the 4d fixed points. As far as power-law corrections are concerned, such effectively 5d scenarios arising from the compactification of 6d theories are more predictive than pure 5d models because of the absence of massive gauged hypermultiplets and 6d anomaly constraints on massless bulk matter.

The summary and our conclusions can be found in Sect. 6 and some technical details are given in the Appendix.

2 Prepotential of the 5d SYM theory

In this Section, we collect the relevant results of [25–27] and set up the notation used in the rest of the paper. Consider a 5d SYM theory with massive gauged hypermultiplet matter. In addition to the vector field and gaugino, the 5d vector multiplet contains an adjoint scalar field Φ. The theory is conveniently described as a 4d $\mathcal{N}=2$ SYM theory depending on the extra parameter x^5. Its low-energy effective action is thus completely characterized by its holomorphic prepotential $\mathcal{F}(\Sigma)$ [33] (see also [34]). The scalar component of the chiral superfield Σ is $\Phi + iA_5$, where we use the conventions of [35] (see also [36]) but interchange the names Σ and Φ to facilitate comparison with [25]. Given the prepotential, the lagrangian of a 4d $\mathcal{N}=2$ SYM theory can be written in conventional $\mathcal{N}=1$ superfield notation as

$$\mathcal{L} = \frac{1}{2}\left\{ \int d^4\theta \; \frac{\partial \mathcal{F}(\Sigma)}{\partial \Sigma^a} \; \left(\bar{\Sigma} e^{2V}\right)^a \; + \; \int d^2\theta \; \frac{\partial^2 \mathcal{F}(\Sigma)}{\partial \Sigma^a \, \partial \Sigma^b} \; W^a W^b \right\} \; + \; \text{h.c.} \tag{2}$$

Here $\Sigma = \Sigma^a T_a$ and the generators of the gauge group G are normalized by $2\,\mathrm{tr} T_a T_b = \delta_{ab}$ (traces are taken in the fundamental representation unless otherwise specified).

Under the constraints of SUSY and 5d Lorentz invariance, the 4d lagrangian of Eq. (2) extends in a unique way to a 5d lagrangian. However, 5d gauge invariance now constrains the prepotential to be at most cubic in Σ. In our context, this is crucial since it ensures the absence of higher-dimension operators beyond the CS term (see Appendix A for more details). Following [25], we can also write the prepotential as a function of Φ. Requiring the prepotential to be analytic, the most general form is now

$$\mathcal{F}(\Phi) = \frac{1}{2 g_{5,cl.}^2} \,\mathrm{tr}\, \Phi^2 + \frac{c_{cl.}}{48\pi^2} \,\mathrm{tr}\, \Phi^3 \,. \tag{3}$$

The coefficients of these two terms determine the coefficients of the classical F^2 term and of the classical CS term, all other terms in the component lagrangian then being fixed by supersymmetry. (The normalization is chosen such that, in the absence of charged matter, $c_{cl.}$ is integer due to the boundary anomaly constraint. This will become evident below.) In the present context of gauge coupling unification, it is crucial that the SUSY CS term includes an operator $\sim \Phi F^2$, which clearly has the potential of affecting low-energy gauge couplings if Φ develops a VEV. Thus, the most important two terms of the component lagrangian derived from Eq. (3) are

$$\mathcal{L} \supset -\frac{1}{2 g_{5,cl.}^2} \,\mathrm{tr}\, F^2 - \frac{c_{cl.}}{16\pi^2} \,\mathrm{tr}\, \Phi F^2 \,. \tag{4}$$

The field Φ has a flat potential and one can consider the low-energy effective field theory in the presence of a Φ-VEV. It will become clear from the discussion in Sect. 5 that such a Φ-VEV (rather than just hypermultiplet VEVs) is necessary in order for loop corrections to gauge unification to arise. Without loss of generality, we write $\Phi = \phi^i H_i$, where H_i are the Cartan generators of the gauge group G and $i \in \{1, \cdots, r = \mathrm{rank}(G)\}$. We choose the H_i to be the first r elements of the set of generators T_a. Since a generic VEV breaks G to $\mathrm{U}(1)^r$, the relevant quantity is the prepotential of this abelian gauge theory. Including quantum corrections induced by the vector and hypermultiplets and choosing counterterms such that $g_{5,cl.}$ and $c_{cl.}$ remain unchanged, it reads [25, 37]

$$\mathcal{F}(\Phi) = \frac{1}{4g_{5,cl.}^2}\delta_{ij}\phi^i\phi^j + \frac{c_{cl.}}{48\pi^2}d_{ijk}\phi^i\phi^j\phi^k + \frac{1}{96\pi^2}\left(\sum_\alpha |\alpha_i\phi^i|^3 - \sum_f\sum_\lambda |\lambda_i\phi^i + m_f|^3\right) . \quad (5)$$

Given the definition

$$d_{abc} = \frac{1}{2}\,\mathrm{tr}\, T_a\{T_b, T_c\}\,, \quad (6)$$

it is clear that the first two terms of Eq. (5) are simply a restriction of Eq. (3) to the $\mathrm{U}(1)^r$ subgroup. The remaining terms are the 1-loop-effects resulting from integrating out the heavy vector multiplets (corresponding to the broken directions of G) and the hypermultiplets with masses m_f labelled by their 'flavour' f. The other sums run over the roots α of Lie(G) and the weights λ of the relevant matter representations[1]. Our notation implies that

$$[H_i, E_\alpha] = \alpha_i E_\alpha \qquad \text{and} \qquad H_i|\lambda\rangle = \lambda_i|\lambda\rangle\,, \quad (7)$$

where E_α is the Lie algebra element (root) corresponding to the root vector α and $|\lambda\rangle$ is a representation vector with weight vector λ (see, e.g., [38]). It is important that Eq. (5) is interpreted as defining a locally *holomorphic* prepotential, i.e., the modulus-signs merely determine whether a given cubic term is to be multiplied by $+1$ or -1 in a given region of the multi-dimensional space parameterized by ϕ^i. Note also that the coefficient of the last term in Eq. (5) differs from Ref. [25] due to our different normalization of $c_{cl.}$.

For our purposes, it is essential that Eq. (5) specifies the complete low-energy effective action – no higher-loop contributions arise and no other classical terms are allowed at the two-derivative level.

As done before in Eq. (4) for the classical non-abelian theory, we now give the gauge-kinetic term of the component lagrangian for each of the surviving U(1) factors. For the U(1) group generated by H_i the relevant piece of the component lagrangian reads

$$\mathcal{L}_i \supset -\frac{1}{4}F_i^2\left\{\frac{1}{g_{5,cl.}^2} + \frac{c_{cl.}}{4\pi^2}d_{iij}\phi^j + \frac{1}{8\pi^2}\left(\sum_\alpha \alpha_i^2|\alpha_j\phi^j| - \sum_f\sum_\lambda \lambda_i^2|\lambda_j\phi^j + m_f|\right)\right\} . \quad (8)$$

[1] If several representation vectors have the same weight vector λ, this weight vector contributes with the appropriate multiplicity.

3 Power-law corrections from higher-dimension operators

It is now very easy to see that the above result corresponds precisely to the power-like loop corrections to gauge unification considered recently by many authors following the proposal of [11]. The one-loop correction to a U(1) gauge coupling induced by massive particles is a standard result in quantum field theory. It is the basic building block of GUT threshold calculations [22, 23] (see also [24]). In particular, the dimensionally regularized result of [22] lends itself to an immediate implementation in the 5d situation. Combining the effects two complex scalars and a Dirac fermion, as appropriate for a massive hypermultiplet with mass m and charge q, the correction reads

$$\delta\left(\frac{1}{g_5^2}\right) = -\frac{q^2}{8\pi^2}\, m\,. \tag{9}$$

This dimensionally regularized result hides a mass-independent, linearly divergent piece. However, this piece is irrelevant in the present context since it is universal with respect to the different U(1) subgroups emerging from a spontaneously broken simple group G. It gives rise to a renormalization of the original non-abelian gauge coupling.

In the context of the previous section, the hypermultiplet component corresponding to the weight λ has mass $|\lambda_j\phi^j + m_f|$ and, with respect to the U(1) subgroup generated by H_i, charge λ_i. Thus, Eq. (9) precisely reproduces the matter contribution in Eq. (8). Furthermore, the correction from massive vector multiplets in Eq. (8) is also unambiguously determined since, apart from the different charges and masses, its contribution must be equal and opposite in sign compared to the hypermultiplet. This is clear since a vector- and hypermultiplet with the same mass and charge combine to a 4d $\mathcal{N}=4$ multiplet and therefore induce no gauge coupling correction.

Of course, we are not really interested in the breaking of a gauge group G of rank r to $\mathrm{U}(1)^r$ but rather in its breaking to a set of simple subgroups and U(1) factors, for which we use the common notation G_i. For example, G_i with $i=1,2,3$ may be the three gauge groups of the SM. The relevant gauge coupling corrections can be immediately read off from Eq. (8) by choosing an appropriate Φ-VEV, i.e., appropriately degenerate ϕ^i. It is useful to present the corresponding result in a different form, using traces of representation generators. In this form, the correction to the low-energy gauge coupling of the subgroup G_i reads

$$\delta\left(\frac{1}{g_{5,i}^2}\right) = \frac{c_{cl.}}{4\pi^2}\,\mathrm{tr}\, H_i^2\Phi \;+\; \frac{1}{8\pi^2}\left(\sum_{r_i(a)} T_{r_i(a)} M_{r_i(a)} - \sum_f \sum_{r_i(f)} T_{r_i(f)} M_{r_i(f)}\right)\,. \tag{10}$$

Here H_i is one of the Cartan generators of G that fall into G_i. The G_i-representations emerging from the adjoint of G and from the representation of the hypermultiplet f are labelled by $r_i(a)$ and $r_i(f)$ respectively. As usual, T_{r_i} is defined by $\mathrm{tr}_{r_i}[T_a T_b] = \delta_{ab} T_{r_i}$, with the trace taken in the representation r_i. Furthermore, $M_{r_i(a)}$ and $M_{r_i(f)}$ denote the masses of the vector multiplet in the representation $r_i(a)$ and the hypermultiplet in the

representation $r_i(f)$ respectively. Given these definitions and the relations

$$\sum_{\alpha \in r_i(a)} \alpha_i^2 = \mathrm{tr}_{r_i(a)} H_i^2 = T_{r_i(a)} \qquad , \qquad \sum_{\lambda \in r_i(f)} \lambda_i^2 = \mathrm{tr}_{r_i(f)} H_i^2 = T_{r_i(f)} \, , \tag{11}$$

the derivation of Eq. (10) from Eq. (8) is straightforward.

So far, we have just recovered the 5d threshold formulae of [16], based on the 4d results of [22, 23], in the prepotential language of [25], which is based on the anomaly calculation of [37]. However, this deeper conceptual understanding of power-like threshold corrections is crucial for their phenomenological applicability. The main point here is that the above prepotential formulae are quantum exact, which implies that the by now familiar 1-loop power-law contributions to gauge unification are not subject to further corrections. More specifically, while higher-loop contributions are absent because of $\mathcal{N} = 2$ SUSY, the only competing tree-level higher-dimension operator is the SUSY CS term, corresponding to the first term on the r.h. side of Eq. (10). (Note also that the holomorphic gauge couplings discussed here coincide with the canonical gauge couplings in $\mathcal{N} = 2$ SUSY [39].) Moreover, in the phenomenologically relevant case of a compactification on an interval, the CS term induces anomalies at the boundaries [40] (see [41] for a recent review). These induced anomalies must precisely cancel possible boundary anomalies coming from gauged bulk or brane fields. Thus, the value of the coefficient $c_{cl.}$ is completely determined by the field content of the model. This will be worked out in more detail in Sect. 4.

The high predictivity of this scenario relies on the uniqueness of the tree-level dimension-5 operator, i.e., the SUSY CS term. This uniqueness is clearly based on the analyticity of the prepotential as a function of Φ and the uniqueness of the third-order symmetric invariant tensor d_{abc} [42] (in fact, such an invariant exists only for SU(N) groups). Furthermore, it is also clear that the quantum corrected prepotential is not globally analytic (it is only analytic away from points where certain charged particle masses vanish). This allows for the distinct group-theoretical structures appearing in the quantum part of Eq. (5). However, for a given Φ-VEV, any of the hypermultiplet contributions becomes analytic in the limit $|m_f| \to \infty$. In fact, because of the relations

$$2 \sum_{\lambda} \lambda_i \lambda_j \sim \delta_{ij} \qquad \text{and} \qquad \sum_{\lambda} \lambda_i \lambda_j \lambda_k \sim d_{ijk} \, , \tag{12}$$

it simply corrects the already existing tree-level operators $\sim \delta_{ij}$ and $\sim d_{ijk}$. (For the fundamental representation the proportionalities in Eq. (12) become equalities.) In this sense, heavy matter effectively decouples from gauge unification corrections, i.e., its only trace is a contribution to the CS term which, however, is anyway fixed by low-energy anomaly constraints.

Finally, one may consider the following somewhat exotic possibility. If a certain hypermultiplet is in a large representation, then $\lambda_i \phi^i$ can balance even a very large m_f and a non-analytic contribution to Eq. (5) may result. However, this does not contradict the above claim of effective decoupling since, given the spread of the values of λ_i in a large representation, many relatively light states (with a mass comparable to $|\Phi|$) will

automatically also be present. Thus, the presence of a large-representation hypermultiplet will be known to the low-energy effective field theorist even if its mass is very large.

We can now conclude that in a 5d SYM theory with hypermultiplet matter which is broken by the VEV of the scalar adjoint Φ, power-law corrections to gauge unification are calculable in low-energy effective field theory.

4 5d GUT phenomenology

4.1 Basic structure

The simplest scenario in which the above power-law corrections to 5d low-energy gauge couplings become relevant for a realistic GUT model is that of a field-theoretic S^1/Z_2 orbifold [43] (see also the slightly different later models of [7, 8]). Specifically, consider a 5d SYM theory with gauge group G and hypermultiplet matter compactified on an S^1 parameterized by $x^5 \in [0, 2\pi R)$ and restrict the field space by requiring invariance under the reflection $x^5 \to -x^5$. If the space-time action of this Z_2 is accompanied by an inner automorphism of G (characterized by an element $P \in G$ with $P^2 = 1$) acting in field-space, the gauge group is broken at both boundaries. In general, the surviving subgroup contains a U(1) factor which contains P, i.e., $G \supset G' \times \mathrm{U}(1)$. We now assume that boundary interactions stabilize a VEV of the adjoint scalar Φ which points in the direction of the U(1) generator (cf. [44]). This breaks the gauge group in the bulk in the same way as the orbifolding does at the two boundaries.

The Φ-VEV can, for example, be stabilized by introducing a FI term within the U(1) subgroup surviving at each boundary. This term is, in general, generated by loop effects [45] but may also be present at the classical level. Thus, we can treat its coefficient as a free parameter. However, to be consistent with 4d supergravity (which we of course require although, at the technical level, the present paper uses only rigid SUSY), the coefficients at the two boundaries are assumed to sum up to zero. As discussed in detail in [46], the FI terms induce the desired constant bulk VEV of the scalar adjoint Φ.

Alternatively, the 5d model may be considered as the small-R_6 limit of a 6d theory, in which case the Φ-VEV corresponds to a Wilson line wrapping the 6th dimension. It is stabilized by the boundary conditions at the conical singularities of the 6d model. A more detailed discussion will be provided in the next section.[2]

In the above setting, the 4d gauge couplings observed just below the compactification scale $M_c = 1/R$ read

$$\frac{1}{g_{4,i}^2(M_c)} = \frac{\pi R}{g_{5,i}^2} + \frac{1}{g_{bd.,i}^2} \,. \tag{13}$$

[2] This alternative possibility is interesting in view of the following possible criticism of FI-term-stabilization: As argued in [47], the FI-terms can be understood in supergravity as arising from a mixed gauge-graviphoton CS term in the bulk. However, in the constructions considered here the brane U(1) arises from a non-Abelian gauge symmetry in the bulk and we are not aware that the required mixed CS term has been discussed in this case.

Here the 5d gauge couplings are defined at zero momentum (i.e., as in the low-energy effective action of Sect. 2) and the last term accounts for the (presumably sub-dominant) effect of boundary gauge-kinetic terms. From the results of the last two sections, it is now clear that power-law corrections to inverse 4d gauge couplings are of the order $\sim |\Phi| R$ and can thus be as large as the tree-level term $\sim R/g^2_{5,cl.}$.

To be more specific, we focus on the situation where $G = \mathrm{SU}(5)$ and $P = \mathrm{diag}(1,1,1,-1,-1)$ so that the breaking is to the SM gauge group. In this case, the above S^1/Z_2 orbifold of a pure 5d SYM theory gives, at the zero mode level, the SM gauge multiplet and a chiral superfield with the quantum numbers of the X, Y gauge bosons. The latter one becomes massive when Φ develops a VEV and is therefore phenomenologically harmless. SM matter and Higgs fields can be added at the branes and/or in the bulk making the model as realistic (and arguably even somewhat simpler and more generic) as the more widely discussed $S^1/(Z_2 \times Z_2')$ models of [7,8].

4.2 Power-law corrections and consistency with boundary anomaly cancellation

For the purpose of this subsection, we treat boundary gauge-kinetic terms and the corresponding logarithmic running as sub-dominant. Thus, our analysis is based entirely on Eq. (10), where the gauge group is SU(5) and we consider the possibility of hypermultiplet matter in the **5**, **10** and **24**. (Recall that, for example, a hypermultiplet in the **5** contains, in 4d $\mathcal{N}=1$ language, one 4d chiral superfield in the **5** and one in the $\bar{\mathbf{5}}$.) All the group theory we need is the familiar decomposition of the simplest SU(5) representations under SU(3)×SU(2)×U(1):

$$\begin{aligned} \mathbf{5} &= (\mathbf{3},\mathbf{1})_{-2} + (\mathbf{1},\mathbf{2})_3 && (14)\\ \mathbf{10} &= (\mathbf{3},\mathbf{2})_1 + (\bar{\mathbf{3}},\mathbf{1})_{-4} + (\mathbf{1},\mathbf{1})_6 && (15)\\ \mathbf{24} &= (\mathbf{8},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 + (\mathbf{1},\mathbf{1})_0 + (\mathbf{3},\mathbf{2})_{-5} + (\bar{\mathbf{3}},\mathbf{2})_5\,. && (16) \end{aligned}$$

The U(1) charges q' given here, in the conventions of [38], correspond to charges $q = q'/\sqrt{60}$ if the U(1) generator is normalized consistently with the other SU(5) generators. For easy reference we also collect in Table 1 the relevant group-theoretical factors $T_{r_i(f)}$ in a hopefully self-explanatory notation.

We are now in a position to write down explicitly the corrections $\Delta\alpha_1^{-1}$, $\Delta\alpha_2^{-1}$, and $\Delta\alpha_3^{-1}$ to the inverse couplings of the three SM gauge groups U(1), SU(2), and SU(3) (as usual, $\alpha_1 = g_1^2/(4\pi)$ etc.). For example, a **5** hypermultiplet with bulk mass $m_{\mathbf{5}}$ (parameterized by $\xi_{\mathbf{5}} = m_{\mathbf{5}}/M_V$) induces corrections

$$\begin{aligned} \Delta\alpha_1^{-1} &= -\left(\tfrac{1}{5}\left|\xi_{\mathbf{5}} - \tfrac{2}{5}\right| + \tfrac{3}{10}\left|\xi_{\mathbf{5}} + \tfrac{3}{5}\right|\right)\tfrac{M_V}{2M_c}\ , \\ \Delta\alpha_2^{-1} &= -\tfrac{1}{2}\left|\xi_{\mathbf{5}} + \tfrac{3}{5}\right|\tfrac{M_V}{2M_c} \qquad , \qquad \Delta\alpha_3^{-1} = -\tfrac{1}{2}\left|\xi_{\mathbf{5}} - \tfrac{2}{5}\right|\tfrac{M_V}{2M_c}\ . \end{aligned} \tag{17}$$

This and corresponding formulae for the **10** hypermultiplet and the **24** hypermultiplet

			$T^{\mathrm{U}(1)}$	$T^{\mathrm{SU}(2)}$	$T^{\mathrm{SU}(3)}$	VEV-induced mass
$(\mathbf{3},\mathbf{1})$	of	$\mathbf{5}$	$1/5$	0	$1/2$	$-(2/5)M_V$
$(\mathbf{1},\mathbf{2})$	of	$\mathbf{5}$	$3/10$	$1/2$	0	$(3/5)M_V$
$(\mathbf{3},\mathbf{2})$	of	$\mathbf{10}$	$1/10$	$3/2$	1	$(1/5)M_V$
$(\bar{\mathbf{3}},\mathbf{1})$	of	$\mathbf{10}$	$4/5$	0	$1/2$	$-(4/5)M_V$
$(\mathbf{1},\mathbf{1})$	of	$\mathbf{10}$	$3/5$	0	0	$(6/5)M_V$
$(\mathbf{8},\mathbf{1})$	of	$\mathbf{24}$	0	0	3	0
$(\mathbf{1},\mathbf{3})$	of	$\mathbf{24}$	0	2	0	0
$(\mathbf{1},\mathbf{1})$	of	$\mathbf{24}$	0	0	0	0
$(\mathbf{3},\mathbf{2})+(\bar{\mathbf{3}},\mathbf{2})$	of	$\mathbf{24}$	5	3	2	M_V

Table 1: Group-theoretical factors $T_{r_i(f)}$ of the simplest SU(5) representations relevant for the evaluation of Eq. (10). The last column contains the masses which the different representations acquire in the presence of a Φ-VEV (parameterized by the mass M_V of the 5d X, Y gauge bosons).

or vector multiplet are easily read off from Table 1 and Eq. (10) after compactification on an interval with length $\pi R = \pi/M_c$.

Finally, we need to deal with the effect of a classical SUSY CS term parameterized by $c_{cl.}$. This term is constrained by boundary anomaly cancellation [36,40]. As can be seen explicitly from Eqs. (5) and (12), a bulk $\mathbf{5}$ in the limit $m_{\mathbf{5}} \to \pm\infty$ induces an effective CS term with $c_{cl.} = \mp 1/2$ [25]. The boundary anomalies induced by this term can be found as follows (see, e.g., [47,48]):

Consider first two massless bulk hypermultiplets $\mathbf{5}$ and $\mathbf{5}'$, each with the same boundary conditions at $x^5 = 0$ and $x^5 = \pi R$, but with the sign flipped between the two hypermultiplets. The model is anomaly-free, not just at the zero-mode level but also at each of the two boundaries taken separately. This is clear since the zero-mode matter is vector-like, so that there is no 4d anomaly, and the boundary anomaly is simply $1/2$ of the 4d anomaly. (Recall that there are no anomalies in 5d.) Furthermore, the consistency is not destroyed by continuously varying one of the mass parameters, e.g., taking $m_{\mathbf{5}} \to \infty$ while keeping $m_{\mathbf{5}'} = 0$.

Thus, the CS term induced by the infinitely heavy $\mathbf{5}$ precisely cancels the boundary anomalies coming from brane localized zero-modes emerging in the limiting procedure $m_{\mathbf{5}} \to \infty$ and from the massless $\mathbf{5}'$. The latter are half-integer-valued in units corresponding to a 4d chiral fermion in the $\mathbf{5}$. This is obvious since, again, the zero-mode anomaly is split equally between the two identical boundaries. Postponing a more explicit discussion to the next subsection, we can now already conclude that $c_{cl.} = \mp 1/2$ induces half-integer boundary anomalies. Thus, in the absence of charged bulk matter, $c_{cl.}$ must be integer and, to achieve gauge invariance, appropriate brane fields cancelling the induced integer-valued anomalies must be present. This argument for the value of $c_{cl.}$ could have also been made on the basis of the $m_{\mathbf{10}} \to \infty$ limit of a $\mathbf{10}$ hypermultiplet, which induces a CS term identical to that induced by a $\mathbf{5}$.

From Eq. (17) and corresponding formulae for the matter in the **10** and **24**, it is clear that almost any ratio of low-energy gauge couplings can be realized by tuning appropriately the bulk masses of the matter fields. We therefore now focus on the arguably more natural case where bulk fields are either massless or extremely heavy, i.e., contribute only via an analytic CS term. The relevant contributions to the differences of inverse 4d gauge couplings $\alpha_{ij} = \alpha_i^{-1} - \alpha_j^{-1}$ are given in Table 2.

massless fields or operator	$\alpha_{12} \times 2M_c/M_V$	$\alpha_{23} \times 2M_c/M_V$
24 vector	2	1
24 hypermultiplet	-2	-1
5 hypermultiplet	$1/25$	$-1/10$
10 hypermultiplet	$-27/25$	$3/10$
CS term with $c_{cl.} = \mp 1/2$	$\pm 1/5$	$\mp 1/2$

Table 2: Corrections to inverse gauge coupling differences (in units of $M_V/(2M_c)$) induced by massless fields in the simplest representations and by the smallest possible CS terms.

At this point, some basic phenomenological implications can already be derived. Note first that anomaly cancellation by boundary fields is only possible if the boundary anomalies induced by bulk fields and operators are integer-valued. Thus, the sum of the numbers of bulk **5**s, **10**s and "CS-term-quanta" (i.e., CS term contributions with $c_{cl.} = \pm 1/2$) has to be even.

Recall that 4d MSSM running gives $\alpha_{12}/\alpha_{23} = 7/5 = 1.4$, which is known to agree very well with the observed low-energy gauge couplings. The effect of just the gauge sector gives, both in the 4d logarithmic and in the above power-law case, $\alpha_{12}/\alpha_{23} = 2$. In the 4d case, this is then corrected by the contribution from the two Higgs doublets. As noted in [16], a single bulk hypermultiplet in the **5**, with $m_{\mathbf{5}}$ tuned such that, in the presence of the Φ-VEV, the doublet is massless in 5d, reproduces the approximately correct ratio $\alpha_{12}/\alpha_{23} = 1.2$ of [11]. However, as the anomaly argument above shows, such a single bulk **5** has to be supplemented with a CS term. Unfortunately, this destroys the approximately correct power-law effect of [11] (this important point was missed in [16]).

Now, coming back to the more restrictive framework of Table 2, we can look for simple configurations which give the MSSM prediction of $\alpha_{12}/\alpha_{23} = 7/5$ as a power-law effect. It is interesting to observe that, indeed, the vector multiplet together with a massless bulk **10** and the minimal required CS term (choosing the negative sign, $c_{cl.} = -1/2$) gives precisely $\alpha_{12}/\alpha_{23} = 7/5$. Thus, this combination of bulk fields and operators generates a power-law effect mimicking MSSM 1-loop running. Furthermore, replacing the **10** with a **5** and changing the sign of the CS term, one finds $\alpha_{12}/\alpha_{23} = 46/35 \simeq 1.31$, which is also quite close to the desired value 1.4. For the moment, we are satisfied with the two above examples and leave it to the reader to explore other, more complicated, matter field and CS term configurations. We believe that such a complete analysis should be performed in a more constrained context, e.g., in the search for a realistic flavour model or in the framework of a first-principles string construction.

4.3 Low-energy field content

It remains to be shown how, on the basis of a given bulk matter content and CS term, a full anomaly free model is built. We illustrate this construction using the particularly attractive scenario with SU(5) vector multiplet, **10** hypermultiplet and CS term with $c_{cl} = -1/2$ in the bulk, where the power law effect is equivalent to logarithmic MSSM running.[3]

As before, we compactify on S^1/Z_2 breaking SU(5) to the SM at both boundaries. We start with no bulk CS term but with a **10** and **10′** bulk hypermultiplet with opposite boundary conditions. In this situation, the spectrum of fermionic fields which are non-zero at any of the two boundaries is vector-like, i.e., no boundary anomalies arise. By continuity, the consistency of this model is not destroyed if, while keeping $m_{\mathbf{10}} = 0$, the limit $m_{\mathbf{10}'} \to \infty$ is taken. We now have an anomaly-free model with the desired content of light bulk fields and a CS term with $c_{cl} = -1/2$. A specific brane field content arises from the **10′** in the limit $m_{\mathbf{10}'} \to \infty$ due to the presence of localized zero-modes [49] (see also [50]).

To discuss these brane fields explicitly recall that, in $\mathcal{N}=1$ language, the **10′** hypermultiplet contains two chiral superfields in complex-conjugate representations, which we denote by **10′** and $\mathbf{10}'^c$. Assume that the sign-conventions of the 5d lagrangian are such that positive $m_{\mathbf{10}'}$ implies a localization[4] of the **10′** at $y = 0$ and of the $\mathbf{10}'^c$ at $y = \pi$. Furthermore, we define the SU(5)-breaking boundary conditions such that the $(\mathbf{3},\mathbf{2})'$ is non-zero while the $(\bar{\mathbf{3}},\mathbf{1})'$ and $(\mathbf{1},\mathbf{1})'$ vanish at both branes. It is now clear that, in the limit $m_{\mathbf{10}'} \to \infty$, the only light fields are the zero mode of $(\mathbf{3},\mathbf{2})'$, completely localized at $y = 0$, and the zero modes $(\bar{\mathbf{3}},\mathbf{1})'^c$ and $(\mathbf{1},\mathbf{1})'^c$, completely localized at $y = \pi$.

Phenomenologically, it also essential to know what zero modes arise from the **10** hypermultiplet and at which brane they are peaked. (Note that, in contrast to the complete localization of the zero modes arising from the **10′** hypermultiplet, we have strong but finite peaking characterized by $\exp[\pm ym]$.) Given our conventions for the relative sign between bulk mass and Φ-VEV, as specified by Eq. (17), and the signs in the last column of Table 1, the direction of the peaking of the various fields of the **10** hypermultiplet is easily determined. Recalling that the boundary conditions of the **10** hypermultiplet are opposite to those of the **10′** hypermultiplet, we find a $(\mathbf{1},\mathbf{1})$ zero mode peaked at $y = 0$ as well as $(\mathbf{3},\mathbf{2})^c$ and $(\bar{\mathbf{3}},\mathbf{1})$ zero modes localized at $y = \pi$.

To make the model realistic without destroying the MSSM-like power law contribution from the bulk, matter has to be introduced in the form of brane fields. We begin by localizing a **10** chiral superfield at $y = \pi$. Allowing all gauge-invariant mass terms and recalling that, from the previous construction, we also have a $(\mathbf{3},\mathbf{2})^c$, $(\bar{\mathbf{3}},\mathbf{1})$, $(\bar{\mathbf{3}},\mathbf{1})'^c$

[3] It is interesting to speculate that this field content arises from a (possibly even-higher-dimensional) SO(10) model where the adjoint decomposes as $\mathbf{45} = \mathbf{24} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{1}$ and the $\overline{\mathbf{10}}$ becomes heavy in the process of gauge-symmetry and SUSY breaking.

[4] By this we mean that the relevant bulk equations of motion for the $\mathcal{N} = 1$ superfields are $(\partial_y + m_{\mathbf{10}'})\mathbf{10}' = 0$ and $(\partial_y - m_{\mathbf{10}'})\mathbf{10}'^c = 0$, implying bulk solutions $\mathbf{10}' \sim \exp(-m_{\mathbf{10}'}y)$ and $\mathbf{10}'^c \sim \exp(+m_{\mathbf{10}'}y)$. In particular, if a zero mode is allowed by the boundary conditions, it will then be localized as described in the main text above.

and $(\mathbf{1},\mathbf{1})'^c$ peaked or localized at $y=\pi$, we find that all fields except for a partnerless $(\bar{\mathbf{3}},\mathbf{1})_{-4}$ become massive. Together with the $(\mathbf{3},\mathbf{2})_1$ and $(\mathbf{1},\mathbf{1})_6$ left over at $y=0$ from the previous construction, we now have a full **10** in zero modes. Of course, the introduction of a **10** chiral superfield at $y=\pi$ demands, by anomaly cancellation, the further introduction of a $\bar{\mathbf{5}}$ at the same brane. Now we have a full SM generation, with all fields except for the left-handed quarks and right-handed electron peaked at $y=\pi$. Amusingly, this matter distribution excludes all (not exponentially suppressed) mass terms within this generation. Thus, our construction has produced an anomaly-free setting with MSSM-like power law correction and one naturally light generation. The two Higgs doublets and the heavy generations are now easily added at $y=\pi$ without affecting any of the attractive features achieved so far.

To be completely explicit, we now calculate the gauge couplings at the Z-pole in the above model, including logarithmic terms. Below the compactification scale $M_c=1/R$ we have conventional MSSM running; above that scale we have the power-like effects discussed in this paper and further corrections associated with the logarithmic running of brane-localized gauge-kinetic terms (see, e.g., [8, 13, 16, 51]). This logarithmic running above M_c is cut off at some UV-scale Λ where the singular boundary is resolved. For simplicity, we assume $\Lambda/M_V=\mathcal{O}(1)$ and thus disregard logarithms of this ratio. In particular, this implies that only the Kaluza-Klein (KK) modes of the 5d vector multiplet within the SM gauge group contribute to the logarithmic running above M_c.

Note that, from the point of view of the bulk theory and the power-like terms, the existence of a UV scale Λ is immaterial since our calculation of inverse gauge coupling differences is entirely UV-insensitive. In fact, this was to be expected in view of the possible existence of a non-trivial UV fixed-point of the 5d theory discussed in [25], i.e., the possibility of taking $\Lambda\to\infty$ (see [52] for more general analyses, including in particular the 6d case, and [53] for a recent application in unified models). However, we emphasize that our calculations, although quite consistent with the fixed point proposal, do not rely on it or on the limit $\Lambda\to\infty$ since all dangerous higher-dimension operators are forbidden by symmetries.

The low-energy inverse gauge couplings are given by

$$\alpha_{4,i}^{-1}(m_Z)=\pi R\alpha_{5,cl.}^{-1}+b_i\left(\frac{1}{10}\frac{M_V}{M_c}+\frac{1}{2\pi}\ln\frac{M_c}{m_Z}\right)+\frac{\tilde{b}_i}{2\pi}\ln\frac{M_V}{M_c}+\{i\text{-indep. terms}\}\qquad(18)$$

where $\alpha_{5,cl.}=g_{5,cl.}^2/(4\pi)$. The coefficients $b_i=(0,-6,-9)+2\,(3/10,1/2,0)$ govern the familiar gauge and Higgs contributions to the MSSM running and, in our specific example, also the power-law term. Their S^1/Z_2 counterparts governing the modified running above M_c are $\tilde{b}_i=(0,-4,-6)+2\,(3/10,1/2,0)$. Note that, to simplify Eq. (18), we have chosen the "i-independent terms" to ensure that the familiar coefficients b_i multiply both the power-law term and $\ln(M_c/m_Z)$. This is possible because the power-law corrections respect the MSSM relation $\alpha_{12}/\alpha_{23}=7/5$.

The main technical statement to be made is the harmlessness of this modified logarithmic contribution, which is sufficiently similar to MSSM running and parametrically much smaller than the power law term. To see this explicitly, consider the

most extreme case of $M_c \sim m_Z$ (i.e., disregard the term $\sim \ln(M_c/m_Z)$) and choose $M_V = 48.5\, M_c$. Equation (18) then gives $\alpha_{12}(m_Z) = 29.4$ and $\alpha_{23}(m_Z) = 21.3$ in almost perfect agreement with what is needed to accommodate the low-energy values $\alpha_i^{-1}(m_Z) = (59.0, 29.6, 8.4)$.

5 Power corrections from 6 dimensions

In this section, we discuss power-like corrections to gauge unification in 6d SYM theories. To begin, consider uncompactified, flat, 6-dimensional space with minimal SUSY (corresponding to $\mathcal{N} = 2$ in 4d), in which case the vector multiplet contains just the gauge field and a 6d-chiral spinor [54]. We may add 6d gauged hypermultiplets, the spinors of which must be of opposite 6d chirality relative to the gaugino. The reason for this is the presence of a Yukawa-like interaction term in the 6d lagrangian. This term combines the gaugino with the charged matter fermion, forcing them to have opposite chirality. (In this context, it is useful to recall that, unlike in 4d, in 6d complex conjugation does not change the chirality of a spinor. Thus, 6d chirality is an 'absolute concept' in the sense that it does not depend on whether one views the spinor or its complex conjugate as the basic degree of freedom.)

The above implies that no mass terms connecting gauged hypermultiplets are allowed in 6d. Indeed, all the fermions involved have the same chirality making fermionic mass terms impossible. Independent of the gauging, the absence of masses in 6d simply follows from the fact that, in a hypermultiplet, the SUSY variation of a fermion is proportional to the SUSY generator, implying that all the fermions have the same chirality. This is very interesting from the model building perspective since it implies that the 5d hypermultiplet masses, which could in principle be used for an arbitrary tuning of 5d power-like unification corrections (cf. Eqs. (8) and (17)) have no 6d analogue.

However, it would be premature to conclude that there is no massive gauged matter in 6d. Indeed, mass terms linking a 6d hypermultiplet with a 6d vector multiplet, both charged under some gauge group, are possible. Such mass terms arise, for example, in the KK mode description of d-dimensional theories, where $d > 6$, compactified to 6d. They also appear in situations where a 6d gauge symmetry is broken by the VEV of the scalar component of one of the gauged hypermultiplets. Mass terms of this type, involving vector and hypermultiplet in the same representation, automatically produce a full $\mathcal{N} = 4$ multiplet at a given mass level. Thus, they are irrelevant in the present context of loop corrections to gauge coupling unification and we can from now on focus on massless 6d models.[5]

The 6d vector multiplet contains no scalar (the adjoint Φ of the corresponding 5d theory being promoted to the gauge field component A_6). Thus, soft gauge symmetry breaking in a 6d Lorentz-invariant setting has to rely on the VEV of one of the scalars of a gauged hypermultiplet. As explained above, massive fields can be collected in full

[5] More generally, according to Tables 4 and 5 of [55] all massive representations of minimal 6d SUSY with spin ≤ 1 automatically have a 4d $\mathcal{N} = 4$ spectrum.

$\mathcal{N}=4$ SUSY multiplets for any given mass and representation and no power corrections to gauge unification arise. This ends our discussion of the uncompactified 6d theory. What is more, it also implies that the only interesting situation in 5d is the one where the gauge symmetry breaking is driven by the adjoint scalar from the vector multiplet. Indeed, a 5d theory broken by a hypermultiplet VEV can be thought of as arising via dimensional reduction from a 6d theory, in which case the above argument demonstrates the absence of power-like loop corrections. This is the reason why our 5d analysis is focussed entirely on situations with gauge symmetry breaking by the adjoint scalar Φ. It may, however, be interesting to consider situations where bulk hypermultiplet VEVs are present in addition to the VEV of the adjoint scalar.

Given the absence of power-law corrections in the Lorentz-invariant 6d situation, we now focus on 6d theories compactified on an S^1 of radius R_6 to 5 dimensions. Any possible further compactification (with compactification radius R_5) leading to a realistic 4d model is assumed to occur at a lower energy scale, $R_5 \gg R_6$. In the 5d effective theory, the gauge symmetry can be broken by the VEV of the adjoint scalar Φ. The latter has to be identified with the VEV of A_6, i.e., the Wilson line wrapping the S^1 [56]. Thus, one can straightforwardly apply the analysis of the previous sections and obtain the power-law corrections for any given 6d model. Important new features are the absence of a classical CS term and of hypermultiplet masses in 6d, which makes the setting more predictive, and the appearance of a tower of KK modes, the loop contributions of which have to be summed. The remainder of this section is devoted to a detailed discussion of power-law effects in this effectively 5-dimensional situation.

Before coming to the actual calculation, another conceptual issue – the stabilization of the Wilson line – has to be addressed. For the simplest geometrical setting, a rectangular torus T^2 with radii R_5 and R_6, the Wilson line in x^6-direction, which is the analogue of the Φ-VEV of the 5d models above, is a modulus protected by SUSY. However, in an appropriate orbifold of the type T^2/Z_2, $T^2/(Z_2 \times Z_2')$ etc., the Wilson lines have certain fixed, discrete values determined by the gauge twists associated with the various orbifold actions [9]. In fact, this is quite analogous to the discrete or quantized Wilson lines of string-theoretic orbifold models [57]. To be specific, recall that a T^2/Z_2 orbifold can be visualized as the surface of a 'pillow' [58]. It has the topology of a sphere and 4 conical singularities with deficit angle π. In various field- or string-theoretic orbifold constructions, gauge symmetry breaking on this space arises from the non-trivial gauge holonomy associated with loops surrounding the 'corners' of this pillow. By Gauss' theorem, two of these Wilson lines surrounding two adjacent corners combine into a Wilson line going around the center of the pillow, which will therefore in many cases have a non-zero, quantized value. It is now straightforward to imagine an extremely elongated pillow ($R_5 \gg R_6$) equipped with a fixed Wilson line in x^6 direction. The conical singularities are simply boundary effects (from the effective 5d point of view) stabilizing the Wilson line VEV. In fact, as discussed in [59], in field theory the Wilson lines surrounding each of the conical singularities do not have to be quantized but can vary continuously and each possible value can be stabilized by local physics at the fixed point (brane). An example for such a local stabilization mechanism is provided by brane-localized FI terms inducing locally a non-zero field strength [60].

5.1 Power corrections in a 6d theory compactified on a circle

We have now identified an interesting and realistic setting for 6d power corrections: the effectively 5-dimensional case with a Wilson line wrapping the compactified dimension of length $2\pi R_6$. In fact, the corrections to gauge coupling unification arising in this setup could be extracted from the more general analysis of arbitrary tori with two Wilson lines performed in [28, 29] (including a discussion of the connection to string theory [32]). Terms linear in the Wilson line VEV appear, for example, in taking the appropriate limits of Eq. (27) in [29]. However, we find it useful to give an independent and extremely simple derivation, based on the 5d results obtained above, which adequately describes the dominant part of large, power-like corrections to gauge unification. It is important to note that the analysis of [28] supports the expectation (based, e.g., on the symmetry arguments or the UV fixed-point conjecture of [25]) that the field theory results for gauge coupling differences are recovered in string theory in the limit of infinite string tension.

Consider first, as at the beginning of Sect. 3, a supersymmetric 6d U(1) gauge theory with a gauged hypermultiplet of charge q. This will be a useful building block for the following realistic calculation although, without appealing to the Green-Schwarz mechanism, the simple U(1) model is inconsistent since it is anomalous.[6] After compactification, we have a KK tower of 5d hypermultiplets with masses $m_n = |n/R_6|$ with n integer. Turning on a Wilson line in x^6 direction, the former zero mode acquires a non-zero mass $m = qA_6$ (where we have chosen a gauge with constant A_6-VEV). A corresponding Wilson-line-induced effective mass correction is also added to the masses of the higher KK modes (which is particularly evident in the fermionic part of the lagrangian). The resulting KK spectrum is $m_n = |n/R_6 + m|$ with n running over all integers. Thus, the loop correction of Eq. (9) is replaced by

$$\delta\left(\frac{1}{g_5^2}\right) = -\frac{q^2}{8\pi^2}\sum_{n=-\infty}^{+\infty}|nR_6^{-1} + m|\,. \tag{19}$$

As before, we are only interested in the mass dependence of this correction. This mass dependence is finite and can be easily extracted from the above divergent sum using dimensional regularization. It is convenient to introduce the dimensionless parameter $c = mR_6 = qA_6R_6$ assuming $0 < c < 1$ for the moment. The result, derived in Appendix B, then reads

$$\delta\left(\frac{1}{g_5^2}\right) = -\frac{q^2}{8\pi^2 R_6}\,c(1-c) = -\frac{q^2}{8\pi^2}\,m\,(1-c)\,, \tag{20}$$

where, we emphasize again, an m-independent divergent contribution has been dropped.

This very simple formula has manifestly the correct limiting behaviour as $R_6 \to 0$ for fixed m. Furthermore, viewed as a function of R_6 and c, it is invariant under the substitution $c \to (1-c)$. This is a manifestation of the fact that the KK spectrum is

[6] For a 6d U(1) model, the anomaly induced by the box diagram is always non-zero since it is proportional to the sum of the fourth powers of the charges of the fermions, which all have the same chirality because of 6d SUSY. This will be different in non-abelian models (see below) since the gaugino is charged and has opposite chirality.

completely determined once the lightest mode is known. In other words, the point $n = 0$ has no absolute meaning and a shift of the label n or a reflection $n \to -n$ do not affect the physics. This last comment makes it obvious that Eq. (20) is extended to values of c outside the interval $(0, 1)$ by simply demanding reflection symmetry with respect to any point where c is integer. This is illustrated in Fig. 1.

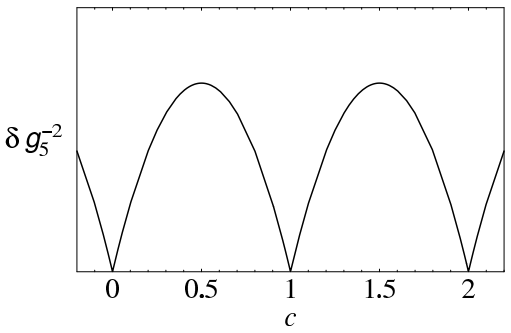

Figure 1: Illustration of the dependence of the 1-loop correction to the inverse gauge coupling in the 5d effective theory, g_5^{-2}, on the value of the Wilson line, parameterized by $c = qA_6R_6$.

It is evident from Fig. 1 that, locally, the inverse gauge coupling squared depends quadratically on the Wilson line VEV and thus, from the 5d point of view, on Φ. However, we know from SUSY and gauge invariance (cf. Sect. 2 and Appendix A) that the 5d prepotential is at most cubic and thus the Φ dependence is at most linear. This inconsistency is directly linked to the non-zero anomaly, as can be easily seen from Eq. (20). Indeed, for a model with several hypermultiplets the term quadratic in A_6 is proportional to the sum of the fourth powers of the charges, i.e., the anomaly, which is necessarily non-zero. We will see shortly that this problem disappears in an anomaly free, non-abelian model.

The non-abelian version of the above result can be written down without any further calculation. Recall that, at the beginning of Sect. 3, we have given a rederivation of Eq. (8) on the basis of Eq. (9) and simple group theory. Following this line of reasoning, the 6d version of Eq. (8) can now immediately be given:

$$\mathcal{L}_i \supset -\frac{1}{4}F_i^2 \left\{ \frac{2\pi R_6}{g_{6,cl.}^2} + \frac{1}{8\pi^2} \left(\sum_\alpha \alpha_i^2 \, |\alpha_j A_6^j| \left(1 - |\alpha_j A_6^j| R_6\right) - \sum_f \sum_\lambda \lambda_i^2 \, |\lambda_j A_6^j| \left(1 - |\lambda_j A_6^j| R_6\right) \right) \right\} . \tag{21}$$

It is obtained from the original expression by identifying each 5d mass m and replacing it by $m(1 - mR_6)$. This is the same procedure that leads from Eq. (9) to its 6d version Eq. (20). Of course in addition, the components ϕ^i of the field Φ are replaced by the corresponding components A_6^i of A_6 and the classical CS term as well as the hypermultiplet masses are dropped.

Similarly, the 6d analogue of Eq. (10), which is most directly useful for GUT phe-

nomenology, reads

$$\delta\left(\frac{1}{g_{5,i}^2}\right) = \frac{1}{8\pi^2}\left(\sum_{r_i(a)} T_{r_i(a)} M_{r_i(a)}\left(1 - M_{r_i(a)} R_6\right) - \sum_f \sum_{r_i(f)} T_{r_i(f)} M_{r_i(f)}\left(1 - M_{r_i(f)} R_6\right)\right) . \tag{22}$$

It is clear from the structure of Eq. (21) that each of the 5d low-energy $U(1)$ gauge couplings depends on A_6 like a sum of functions of the type displayed in Fig. 1. In fact, both Eq. (21) and Eq. (22) can be taken at face value only in a certain neighbourhood of the point $A_6 = 0$. They are extended to all values of A_6 along a certain direction in the Cartan subalgebra by extending each of the terms of the form $m(1 - mR_6)$ as illustrated in Fig. 1. Locally, the sum of these terms must be a linear function since the 5d prepotential is at most cubic. The required cancellation of the coefficient of $(A_6)^2$ is indeed possible because of the relative sign between the vector multiplet and the hypermultiplet contributions in Eq. (21). This cancellation is intimately linked to the absence of 6d anomalies. To see this more explicitly, let A_6^i (with i fixed) be the only non-zero component of A_6 and consider the gauge coupling correction to the U(1) subgroup generated by H_i as specified by Eq. (21). The coefficient of $(A_6^i)^2$ is now manifestly proportional to the box anomaly coefficient. It vanishes whenever the sum of the fourth powers of charges (specified by α_i) of fermions of the gaugino-chirality minus the sum of the fourth powers of charges (specified by λ_i) of fermions of matter-chirality is equal to zero. This is a nice consistency check of the present analysis.

5.2 A 6d SO(10) example

As an illustration of the general discussion above we now explicitly calculate the power corrections to gauge unification in a 6d SO(10) model compactified to 5d on an S^1. The group is broken to SU(3)×SU(2)×U(1)×U(1)′ by an A_6 Wilson line along the hypercharge direction, which corresponds to the first of the two U(1)s above. (For a detailed discussion of the corresponding group theory and the various breaking possibilities see, e.g., Sect. 3.2 of [59].)

One possible special case of 5d effective theories of this type arises in the orbifold models of [9]. These models have a pillow-like fundamental space with gauge symmetries SO(10), SU(5)×U(1), SU(5)′×U(1)′ and SU(4)×SU(2)×SU(2)) at the four corners. One can now imagine stretching this space in one direction such that the SO(10) and the Pati-Salam fixed points are at one of the two boundaries of the resulting effectively 5-dimensional model (while the SU(5) and the flipped SU(5) fixed points are at the other boundary). Away from the boundaries, we have a cylinder wrapped by a Wilson line in hypercharge-direction, which is precisely the 6 to 5d compactification discussed above. In this specific orbifold realization, the Wilson line is quantized such that it corresponds to a Z_2 gauge twist and correspondingly the gauge symmetry in the 5d bulk is enhanced from the generic case, SM×U(1)′, to the Pati-Salam group (cf. the 5d the models of [61]). However, one can clearly imagine other similar constructions with different values of the A_6 Wilson line (see, e.g., the models of [59] and [60] where Wilson lines encircling conical

singularities take on continuous values not related to the geometrical deficit angle).

Restricting ourselves to hypermultiplet matter in the **10** and **16** of SO(10), there is only one model without irreducible or reducible gauge anomalies. It contains, in addition to the vector multiplet in the **45**, 6 hypermultiplets in the **10** and 4 hypermultiplets in the **16** of SO(10) [62]. The existence and uniqueness of this solution is easily checked using the formulae of [63] (based on [64] and [65]). More possibilities exist if one only requires that the irreducible anomaly cancels, appealing to the Green-Schwarz mechanism [66] for the cancellation of the reducible anomalies. We leave the investigation of power-law corrections in this context to future work. We also do not discuss 4d boundary anomalies arising at the conical singularities of the full model [67] since they are not an intrinsic part of the effective 5d theory in which the power-corrections arise. However, we emphasize that an example of a realistic SUSY GUT with the above anomaly-free 6d bulk matter content has been given in [62].

In principle, the calculation of the power-law corrections in the anomaly-free 6d SO(10) model is a straightforward application of Eq. (22). The analysis becomes particularly simple if one uses the 5d results of Table 1 together with the familiar decomposition of SO(10) representations in SU(5) language. Specifically, the matter content of a vector **45** and hypermultiplets $6 \times \mathbf{10} + 4 \times \mathbf{16}$ of SO(10) corresponds to vector multiplets $\mathbf{24} + 2 \times \mathbf{10}$ and hypermultiplets $16 \times \mathbf{5} + 4 \times \mathbf{10}$ of SU(5). (Note that, as far as gauge coupling corrections are concerned, we do not need to distinguish between **5** and $\bar{\mathbf{5}}$ etc.) The effective 5d masses of the various fields are completely specified by these SU(5) representations since the symmetry-breaking Wilson line lies within the SU(5) subgroup. In particular, the two vector **10**s cancel the effect of two of the hypermultiplet **10**s of SU(5) because of effective $\mathcal{N}=4$ SUSY in the spectrum.

Of course, the modification of corrections of the type displayed in Eq. (17) arising from the summation of the full KK tower has to be taken into account as described in Sect. 5.1. For example, the fields of one 6d bulk hypermultiplet in the **5** give a correction

$$\Delta\alpha_2^{-1} = -\frac{1}{2}\left(\frac{3}{5}M_V\right)\left(1-\frac{3}{5}M_V R_6\right)\frac{R_5}{2} + \cdots = -\frac{1}{2}\left(\frac{3}{5}c\right)\left(1-\frac{3}{5}c\right)\frac{R_5}{2R_6} + \cdots , \quad (23)$$

where $c = M_V R_6$. Here we have assumed that the effective 5d theory is further compactified to 4d on an interval of length πR_5 following as closely as possible the purely 5-dimensional situation of Sect. 4. As before, the typical A_6 dependence arising from the structure $m(1-mR_6)$ has to be continued to all values of A_6 as shown in Fig. 1. In the anomaly-free SO(10) model under discussion, we have contributions corresponding to a vector multiplet **24**, 2 hypermultiplet **10**s and 16 hypermultiplet **5**s in SU(5) language. Thus, the full correction reads

$$\Delta\alpha_2^{-1} = \frac{R_5}{2R_6}\left\{3c(1-c) - 3\left(\frac{c}{5}\right)\left(1-\frac{c}{5}\right) - 8\left(\frac{3}{5}c\right)\left(1-\frac{3}{5}c\right)\right\} , \quad (24)$$

with $c = M_V R_6$. Similar formulae for $\Delta\alpha_1^{-1}$ and $\Delta\alpha_3^{-1}$ are easily derived using the data of Table 1. For illustration, we plot the inverse gauge coupling differences relevant to unification in Fig. 2. This figure nicely illustrates the piecewise linear functional dependence on A_6 that results from a sum of functions of the type displayed in Fig. 1 in an

anomaly-free model. The figure also shows that, in the specific model under consideration, realistic gauge unification cannot be driven by just the power-law effect since the ratio $\alpha_{12}/\alpha_{23} \simeq 1.4$ is not realized for any value of A_6. This may be different for models with other matter content and corresponding Green-Schwarz anomaly cancellation. It may also be changed if other Wilson lines or bulk hypermultiplet VEVs affect the mass spectrum of the model. However, since the main aim of the present paper is not the construction of realistic GUT models but rather the conceptual and technical understanding of power-law corrections to unification, we end our brief excursion into SO(10) phenomenology here.

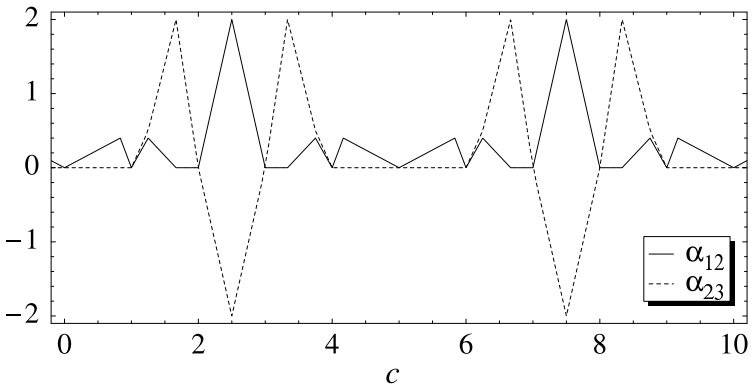

Figure 2: Power corrections to the inverse gauge coupling differences α_{12} and α_{23} in units of $(R_5/2R_6)$ as functions of A_6 (parameterized by $c = M_V R_6$, where M_V is the 6d X, Y gauge boson mass). Note the piecewise linear form related to 6d anomaly cancellation.

Finally, we emphasize that the structure of Eq. (24) justifies, a posteriori, our assumption of an intermediate, effectively 5-dimensional theory, i.e., the assumption $R_5 \gg R_6$. Indeed, given that $c(1-c)$ and the other terms of this type are at most $\mathcal{O}(1)$, power corrections to inverse gauge coupling differences can only become parametrically larger than the familiar 4d threshold effects if $R_5/R_6 \gg 1$ (cf. [32]).

6 Conclusion

We have demonstrated that power-like loop corrections to gauge coupling unification arising in generic supersymmetric 5d unified models are exactly calculable in the framework of the 5d low-energy effective field theory. Such power-law corrections are induced, for example, by the loop effects of charged bulk matter fields. They are also introduced by higher-dimension operators which contain the symmetry-breaking bulk Higgs field together with the field strength tensor. In fact, one can equivalently view the loop effect of bulk matter as coming from higher-dimension operators introduced when these fields are integrated out. These operators then change low-energy gauge couplings at the tree level.

The crucial points underlying calculability are the following: On the one hand, min-

imal 5d SUSY, which corresponds to $\mathcal{N}=2$ SUSY in 4d language, ensures that no corrections arise beyond the one-loop level. On the other hand, possible higher-dimension operators are extremely restricted by the combination of 5d SUSY and 5d gauge invariance. In fact, there is only one globally analytic higher-dimension operator at the two-derivative level, which is the SUSY version of the Chern-Simons (CS) term. Knowledge of the light 5d field content and the coefficient of the CS term determines the low-energy gauge couplings completely.

Realistic 5d models can arise by compactification on an interval, e.g., as an S^1/Z_2 orbifold. Since the 5d CS term induces boundary anomalies, a given bulk and brane field content together with the requirement that boundary anomalies cancel fixes the coefficient of this higher-dimension operator. Thus, power-like corrections to gauge coupling differences are completely fixed. Because of the absence of higher-loop effects or other higher-dimension operators, this calculability is not lost at strong coupling, i.e., if the gauge symmetry is broken at a scale where the 5d gauge theory is strongly coupled. In this case, power-law corrections are parametrically large and can be of the same size as the conventional logarithmic running from GUT scale to weak scale. In particular, we find that, in an SU(5) model with a single **10** hypermultiplet in the bulk and the CS term required by anomaly cancellation, the power-law effect is group-theoretically equivalent to MSSM running. Thus, calculable TeV-scale unification is possible.

We have also considered the possibility that a 5d model arises as the low-energy effective theory of a 6d model compactified on an S^1. In this case the 5d bulk breaking, realized in all interesting cases by the bulk VEV of the scalar adjoint from the vector multiplet, can be traced to a 6d Wilson line wrapping the S^1. Such models correspond to familiar 6d T^2/Z_2 constructions where the ratio of the two torus radii, R_5 and R_6, is taken to be large. Power-like gauge coupling corrections are calculable in close analogy to the 5d case and produce contributions to differences of inverse gauge couplings of the order $\sim R_5/R_6$. These effective 5d theories coming from 6d are highly constrained by 6d anomaly cancellation and because of the absence of bulk hypermultiplet masses in 6d. For $d \geq 7$ the minimal SUSY corresponds to $\mathcal{N}=4$ in 4d language and no loop corrections to gauge coupling unification arise.

Thus, we have found that large and fully calculable power-like loop corrections to gauge unification arise in the context of 5d and 6d grand unified theories. Their phenomenological relevance may be as striking as a lowering of the unification scale by many orders of magnitude or as modest as an interesting field theoretic contribution to the detailed GUT dynamics in a string-derived high-scale model. In any case, we believe that the field theoretic calculability of such power-like loop corrections, based on higher-dimensional SUSY, gauge symmetry, and anomaly cancellation, is an interesting phenomenon.

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Appendix A: Cubic order of the 5d prepotential

The fact that the holomorphic prepotential $\mathcal{F}$ characterizing a 5d SYM theory in 4d $\mathcal{N}=2$ language is a polynomial of at most cubic order [25] lies at the very heart of the results presented in this paper. We therefore include a brief discussion of this known fact.

The lagrangian of a the 4d $\mathcal{N}=2$ SYM theory obtained at the zero mode level from a 5d theory can be written as an $\mathcal{N}=2$ superspace integral [68] (for a review see, e.g., [69]). It is proportional to [33]

$$\int d^2\theta\, d^2\tilde{\theta}\, \mathcal{F}(\mathcal{A}) \,+\, \text{h.c.}\,, \tag{A.1}$$

where $\mathcal{A}$ is a Lie-algebra valued, constrained $\mathcal{N}=2$ superfield and θ, $\tilde{\theta}$ are the two independent Grassmann coordinates. The function $\mathcal{F}$ is a power series in $\mathcal{A}$, where the nth order term

$$d_{a_1\cdots a_n}\mathcal{A}^{a_1}\cdots\mathcal{A}^{a_n} \tag{A.2}$$

is specified by a totally symmetric invariant tensor $d_{a_1\cdots a_n}$ of the Lie algebra. The $\mathcal{N}=1$ superspace expression for the lagrangian has been given in Eq. (2).

It can now be easily shown that, if such a 4d lagrangian comes from a 5d theory, no terms with $n>3$ can be present. Assuming the presence of a non-zero term with $n>3$, we first restrict our attention to a U(1) subgroup on whose generator the corresponding term does not vanish. The rest of the argument can now be given for this abelian $\mathcal{N}=2$ gauge theory. It is clear from Eq. (2) that the relevant prepotential term induces, among others, a bosonic lagrangian term

$$\sim \Phi^{n-3} A_5\; dA\wedge dA\,, \tag{A.3}$$

where A is the 4d vector potential and A_5 is, for the moment, just a 4d scalar. More explicitly, this term involves the contraction $\epsilon^{\mu\nu\rho\sigma}\partial_\mu A_\nu \partial_\rho A_\sigma$. The expression in Eq. (A.3) has to come from one of the terms of the original 5d component lagrangian, and this implies that the 4d ϵ-tensor comes from a 5d ϵ-tensor. Thus, the 5d component lagrangian contains a term

$$\sim \Phi^{n-3}\, A\wedge dA\wedge dA\,, \tag{A.4}$$

where now A is the 5d vector potential. Under (abelian) gauge transformations, the Φ-independent part transforms into a total derivative, so that gauge invariance of this term can not be realized unless $n=3$. It is also easy to see that none of the other terms allowed in a completely generic 5d lagrangian with Φ and a vector field A can compensate the gauge variation of Eq. (A.4). This completes our argument.

Appendix B: 6d Wilson line corrections

In this appendix, we present a very simple and self-contained derivation of the mass dependence, i.e., the Wilson line dependence, of the KK sum in Eq. (19) (for a more detailed discussion of this and related calculations see, e.g., [28]). Recalling that each

term in this sum comes from a 5d 1-loop integral whose only dimensionful parameter is the KK mass of the relevant 5d multiplet, it is immediately clear that performing the same calculation in $5-\epsilon$ dimensions yields

$$\delta\left(\frac{1}{g_{5-\epsilon}^2}\right) = -\frac{q^2}{8\pi^2}\, f(\epsilon) \sum_{n=-\infty}^{+\infty} |n R_6^{-1} + m|^{1-\epsilon}\,. \tag{B.1}$$

The function $f(\epsilon)$, the exact form of which can not be obtained from the above simple dimensional argument, has the limit $f(\epsilon) \to 1$ for $\epsilon \to 0$. However, as will become obvious shortly, the m-dependent part of the infinite sum in Eq. (B.1) is continuous in this limit so that the factor $f(\epsilon)$ can simply be dropped. Introducing the important dimensionless parameter $c = m R_6$ (we assume $0 < c < 1$ for the moment) and splitting off the $n = 0$ contribution, the above correction can be written as

$$\begin{aligned}
\delta\left(\frac{1}{g_{5-\epsilon}^2}\right) &= -\frac{q^2}{8\pi^2 R_6}\left\{c + \sum_{n=1}^{\infty}\left[(n+c)^{1-\epsilon} + (n-c)^{1-\epsilon}\right]\right\} \qquad \text{(B.2)}\\
&= -\frac{q^2}{8\pi^2 R_6}\int_0^c dc'\left\{1 + \sum_{n=1}^{\infty} n^{-\epsilon}\left[(1+c'/n)^{-\epsilon} - (1-c'/n)^{-\epsilon}\right]\right\} + \{c\text{-indep.}\}.
\end{aligned}$$

Here we have suppressed terms $\mathcal{O}(\epsilon)$ whenever possible without affecting the final result. In the second line, an irrelevant c-independent part (the value of the sum at $c = 0$) has been separated. Expanding the terms within the square brackets in ϵ and using the relation

$$\lim_{\epsilon\to 0} \epsilon \sum_{n=1}^{\infty} n^{-1-\epsilon} = 1\,, \tag{B.3}$$

familiar in the context of Riemann's zeta function (see, e.g., [70]), one finds the result given in Eq. (20).

Before closing, we would like to comment on the regularization independence of the above result. In fact, returning to the level of actual loop integrations, the calculation can be performed without ever introducing a regularization. It is clear that the desired A_6 dependence of the gauge coupling can be extracted from the c dependence of a sum of 5d one-loop integrals, conveniently written as the integral of a sum, of the form

$$\begin{aligned}
I(c) &= \int \frac{d^5k}{(2\pi)^5} \sum_{n=-\infty}^{+\infty} \frac{1}{\left[k^2 + (n+c)^2\right]^2} = \int \frac{d^5k}{(2\pi)^5} \sum_{n=-\infty}^{+\infty} \left\{-\frac{\partial}{\partial k^2}\left(\frac{1}{k^2+(n+c)^2}\right)\right\} \qquad \text{(B.4)}\\
&= \int \frac{d^5k}{(2\pi)^5}\left\{-\frac{\partial}{\partial |k|^2}\left(\frac{1}{|k|}\cdot\frac{\pi\,\sinh(2\pi|k|)}{\cosh(2\pi|k|) - \cos(2\pi c)}\right)\right\} = -\frac{c(1-c)}{16\pi^2} + \{c\text{-indep.}\}\,.
\end{aligned}$$

Here we have focussed on the simplest scalar integral appearing in the detailed calculation, rescaled the 5-momentum according to $k \to k/R_6$, and suppressed an overall A_6-independent factor. Thus, all we need is $I'(c)$, which is finite simply because the first derivative with respect to c of the integrand in Eq. (B.4) falls exponentially for $|k| \to \infty$. We have used *Mathematica* [71] for evaluating the sum and the integral. Note also that dropping all higher KK modes (i.e., restricting the sum to $n = 0$) corresponds to the replacement $c(1-c) \to c$ in the final answer.

Performing the sum before the (in general divergent) 5d loop integration is crucial because in this way we are sure to respect the non-locality of the Wilson line effect in the 6d theory. This non-locality is the reason for finiteness. Regularization is just useful for finding the explicit result in a somewhat simpler way, not necessary at the conceptual level. Of course, it has to respect the non-local structure of the Wilson line wrapping the S^1. Clearly, dimensional regularization in the 5 non-compact dimensions satisfies this requirement. We could have reduced all relevant loop corrections to the form of Eq. (B.4) to arrive at our result. However, in this paper we have emphasized the dimensional regularization approach as the simplest way to obtain the answer directly from the known 5d formula.

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