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Two-loop helicity amplitudes for fermion-fermion scattering

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We discuss the calculation of two-loop helicity amplitudes for quark-quark scattering in QCD and four-gluino scattering in $\mathcal{N} = 1$ supersymmetric Yang-Mills theory. We study the dependence of the results on different variants of dimensional regularization. In particular, we consider the 't Hooft-Veltman and four-dimensional helicity (FDH) schemes. We also discuss ambiguities in continuing the Dirac algebra to D dimensions. For the four-gluino case, once the infrared divergent part of the amplitude is subtracted, the finite remainder is free of these ambiguities.

The level of precision that will be achieved in experiments at the LHC poses a challenge for theorists to match. In principle, new physics (or hints of it) may be found in small discrepancies between theoretical predictions and experimental results and it is therefore important that the theoretical predictions be robust. As one component of this, it would be helpful to know the cross section for the production of hadronic jets through next-to-next-to-leading order (NNLO) in the QCD coupling. There are a number of improvements to be expected from such calculations, for example, reducing the renormalization and factorization scale uncertainties in production rates. It should also allow a better understanding of energy flows within jets, as a jet may consist of up to to three partons at NNLO. It also allows for better matching between parton-level and hadron-level jet algorithms. Very importantly, it is also the first order where honest assessments of theoretical uncertainties can be made. See, for example, ref. [1] for further discussions.

There has been great progress in the past few years in the calculation of two-loop matrix elements, especially for $2 \to 2$ scattering processes [2]-[16]. This progress has been possible thanks to new developments in loop integration [17]-[23] and in understanding the infrared divergences of the theory [24]-[26]. In particular, in a series of papers the Durham group calculated the interference of the two-loop amplitudes with the tree level ones, summed over all external color and helicity states, for all $2 \to 2$ parton processes [5]-[9].

Two-loop amplitudes have also been calculated keeping full information on the color and helicity states of the external particles [10]-[16]. This additional information is not needed for the main phenomenological application, namely, NNLO jet production in collisions of unpolarized hadrons. However, experiments at RHIC involve the scattering of polarized protons, for which the helicity amplitudes are directly relevant. Other advantages of having the amplitudes in a helicity basis include the study of a number of formal properties of scattering amplitudes, such as supersymmetry Ward identities [27]-[28], collinear and high energy behavior [29]-[32], and the link between color decomposed QCD amplitudes, twistor space and topological string theories, recently uncovered by Witten [33].

We present here a summary of our calculation for quark-quark scattering amplitudes in QCD, as well as four-gluino scattering in $\mathcal{N} = 1$ super-Yang-Mills theory [34]. Our results for the two-loop quark-quark helicity amplitudes agree with

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Glover's results [16] (after correction of minor errors).

The three QCD processes considered here are,

$$q(p_1,\lambda_1) + \bar{q}(p_2,\lambda_2) \to \bar{Q}(p_3,\lambda_3) + Q(p_4,\lambda_4), \quad (1)$$
$$q(p_1,\lambda_1) + \bar{Q}(p_2,\lambda_2) \to q(p_3,\lambda_3) + \bar{Q}(p_4,\lambda_4), \quad (2)$$
$$q(p_1,\lambda_1) + Q(p_2,\lambda_2) \to q(p_3,\lambda_3) + Q(p_4,\lambda_4). \quad (3)$$

For the four-gluino case we consider

$$\tilde{g}(p_1,\lambda_1) + \tilde{g}(p_2,\lambda_2) \to \tilde{g}(p_3,\lambda_3) + \tilde{g}(p_4,\lambda_4), \quad (4)$$

where we use a "standard" (not all outgoing) convention for the external momentum (p_i) and helicity labeling (λ_i).

The amplitudes are calculated using dimensional regularization. We use the following prescription when a trace of the Minkowski metric is encountered,

$$\eta^\mu{}_\mu \equiv D_s \equiv 4 - 2\epsilon\,\delta_R\,. \quad (5)$$

In this way, we have a continuous set of schemes labeled by δ_R. Setting $\delta_R = 1$ corresponds to the 't Hooft-Veltman scheme [35], while setting $\delta_R = 0$ corresponds to the FDH scheme [36,28]. The FDH scheme has improved supersymmetry properties by virtue of fixing the number of internal gluon states to two.

We evaluate the amplitudes using the spinor helicity formalism [37]. In this formalism, the amplitudes will be proportional to helicity-dependent phase-containing factors written in terms of spinor inner products. These spinor inner products are defined as $\langle ij\rangle = \langle i^-|j^+\rangle$ and $[ij] = \langle i^+|j^-\rangle$, where $|i^\pm\rangle$ are massless Weyl spinors of momentum p_i, labeled with the sign of the helicity.

The L-loop amplitudes are color decomposed as,

$$\mathcal{M}^{(L)} = S \times \sum_{c=1}^{3} \mathrm{Tr}^{[c]} \times M^{(L),[c]}\,, \quad (6)$$

where S is the spinor factor mentioned above. For example, for process (1), with $\lambda_1 = \lambda_4 = +$ and $\lambda_2 = \lambda_3 = -$, we have $S = i\langle 31\rangle/\langle 42\rangle$. The factors for other helicity configurations and for process (4) are similar. The quantities $M^{(L),[c]}$ depend only on the Mandelstam variables $s = (p_1+p_2)^2$, $t = (p_1-p_4)^2$, and $u = (p_1-p_3)^2$.

For the quark process (1) the color basis is

$$\mathrm{Tr}^{[1]} = \delta^{i_4}_{\bar{i}_1}\delta^{i_2}_{\bar{i}_3}, \qquad \mathrm{Tr}^{[2]} = \delta^{i_2}_{\bar{i}_1}\delta^{i_4}_{\bar{i}_3}. \quad (7)$$

The color bases for processes (2) and (3) are similar. For the four-gluino case the color decomposition is the same as for the four-gluon case, and is given in terms of traces of color matrices (or products of them). Using the notation $\mathrm{tr}(T^{a_i}T^{a_j}T^{a_k}T^{a_l}) = \mathrm{tr}_{ijkl}$ we can write

$$\begin{aligned}
&\mathrm{Tr}^{[1]} = \mathrm{tr}_{1234}, \quad \mathrm{Tr}^{[2]} = \mathrm{tr}_{1243}, \quad \mathrm{Tr}^{[3]} = \mathrm{tr}_{1423}\\
&\mathrm{Tr}^{[4]} = \mathrm{tr}_{1324}, \quad \mathrm{Tr}^{[5]} = \mathrm{tr}_{1342}, \quad \mathrm{Tr}^{[6]} = \mathrm{tr}_{1432},\\
&\mathrm{Tr}^{[7]} = \mathrm{tr}_{12}\mathrm{tr}_{34}, \quad \mathrm{Tr}^{[8]} = \mathrm{tr}_{13}\mathrm{tr}_{24},\\
&\mathrm{Tr}^{[9]} = \mathrm{tr}_{14}\mathrm{tr}_{23}.
\end{aligned} \quad (8)$$

The two-loop Feynman diagrams were generated using `QGRAF` [38]. A `MAPLE` program was then used to evaluate each diagram. Some of the diagrams were also evaluated using `FORM` [39] as a cross-check.

When the interference method is used, i.e., when one calculates the two-loop amplitudes interfered with the tree-level ones, summed over helicities and colors, one can use standard trace techniques to put the integrand into a form containing only dot products of momenta. Integral reduction algorithms then give the loop integrals in terms of a minimal set of master integrals. In our case we want to keep full information over helicity and color states. In order to put the integrals into a form suitable for applying the general reduction algorithms, we multiply and divide by appropriate spinor inner products constructed from the external momenta. These spinor inner products effectively play the role of the tree-level amplitudes in the interference method, except that in this method the helicity information is maintained when converting the spinor strings into traces over γ matrices.

An important way of checking the correctness of the calculation is by comparing the infrared divergence of the renormalized amplitudes with the ones predicted by Catani's formula for two-loop n-point amplitudes [24],

$$\begin{aligned}
|\mathcal{M}_n^{(2)}\rangle = \boldsymbol{I}^{(1)}\,|\mathcal{M}_n^{(1)}\rangle + \boldsymbol{I}^{(2)}\,|\mathcal{M}_n^{(0)}\rangle\\
+ |\mathcal{M}_n^{(2)\mathrm{fin}}\rangle\,,
\end{aligned} \quad (9)$$

where the "ket" notation $|\mathcal{M}_n^{(L)}\rangle$ indicates that the L-loop amplitude is treated as a vector in color space. The components of this vector are given by the $M_h^{(L),[c]}$ of eq. (6). The divergences of $\mathcal{M}_n^{(1)}$ are encoded in the color operator $\boldsymbol{I}^{(1)}$, while those of $\mathcal{M}_n^{(2)}$ also involve the scheme-dependent operator $\boldsymbol{I}^{(2)}$. Catani's formula (9) not only allows us to check the results, but also to organize them in terms of finite and divergent parts. Proofs of Catani's formulas have appeared in refs. [25,26].

For each process and each color basis we will have a different $\boldsymbol{I}^{(1)}$ matrix. For the basis (7) we have,

$$\boldsymbol{I}^{(1)} = \frac{e^{-\epsilon\psi(1)}}{\Gamma(1-\epsilon)}\left(\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}\right)\times \qquad (10)$$
$$\begin{pmatrix} 2C_F T - \frac{1}{N}(\mathtt{S}-\mathtt{U}) & \mathtt{T}-\mathtt{U} \\ \mathtt{S}-\mathtt{U} & 2C_F\mathtt{S} - \frac{1}{N}(\mathtt{T}-\mathtt{U}) \end{pmatrix},$$

where N is the number of colors, $C_F = (N^2-1)/(2N)$ and

$$\mathtt{S} = \left(\frac{\mu^2}{-s}\right)^{\epsilon}, \quad \mathtt{T} = \left(\frac{\mu^2}{-t}\right)^{\epsilon}, \quad \mathtt{U} = \left(\frac{\mu^2}{-u}\right)^{\epsilon}.$$

The corresponding operator for $q\bar{Q} \to q\bar{Q}$ is obtained by changing $\mathtt{S} \to \mathtt{U}$, $\mathtt{T} \to \mathtt{S}$ and $\mathtt{U} \to \mathtt{T}$ in eq. (10). Similarly, the operator for $qQ \to qQ$ is obtained by exchanging $\mathtt{S}$ and $\mathtt{U}$ in (10). For the gluino case we get a nine-by-nine matrix, given in eq. (2.18) of ref. [13].

The operator $\boldsymbol{I}^{(2)}$ is given by [24]

$$\begin{aligned}\boldsymbol{I}^{(2)} = &-\frac{1}{2}\boldsymbol{I}^{(1)}\left(\boldsymbol{I}^{(1)}+\frac{2b_0}{\epsilon}\right)\\ &+\frac{e^{+\epsilon\psi(1)}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{b_0}{\epsilon}+K\right)\boldsymbol{I}^{(1)}\\ &+\boldsymbol{H}^{(2)}, \end{aligned} \qquad (11)$$

where b_0 is the first coefficient of the QCD β-function, and the coefficient K depends on δ_R and is given by [24,13]

$$\begin{aligned}K = &\left[\frac{67}{18}-\frac{\pi^2}{6}-\left(\frac{1}{6}+\frac{4}{9}\epsilon\right)(1-\delta_R)\right]N\\ &-\frac{10}{18}N_f, \end{aligned} \qquad (12)$$

with N_f being the number of massless fundamental representation quarks. $\boldsymbol{H}^{(2)}$ is a universal operator of order $1/\epsilon$. A general expression for $\boldsymbol{H}^{(2)}$ with an arbitrary number of external legs was recently presented in [26].

The two-loop remainders, as defined by eq. (9) will have the following form

$$\begin{aligned}M^{(2),[c]\mathrm{fin}} = &\left[Q_1 - b_0^2\left(\ln\left(\frac{s}{\mu^2}\right)-i\pi\right)^2\right.\\ &\left.-b_1\left(\ln\left(\frac{s}{\mu^2}\right)-i\pi\right)\right]M^{(0),[c]}\\ &+\left[-2b_0\left(\ln\left(\frac{s}{\mu^2}\right)-i\pi\right)\right.\\ &\left.+Q_2\right]M^{(1),[c]\mathrm{fin}}\\ &+Q_3 M^{(1),[c],\epsilon,\delta_R}+P^{[c]}, \end{aligned} \qquad (13)$$

where Q_1, Q_2 and Q_3 depend on the number of colors N, number of flavors N_f, and scheme label δ_R (in particular, $Q_1 = Q_2 = Q_3 = 0$ when $\delta_R = 1$). The parameter μ is the renormalization scale. The $M^{(1),[c]\mathrm{fin}}$ are the one-loop finite remainders, and the $M^{(1),[c],\epsilon,\delta_R}$ are the δ_R-dependent parts of the $\mathcal{O}(\epsilon^1)$ coefficients of the one-loop amplitudes. Finally, $P^{[c]}$ consists of powers of N and N_f multiplying functions (logarithms, polylogarithms, as well as rational functions) of the Mandelstam variables. The explicit form of these functions is given in ref. [34].

For the four-gluino case in $\mathcal{N}=1$ super-Yang-Mills theory, the two-loop finite remainders in the FDH scheme ($\delta_R = 0$) are,

$$\begin{aligned}M^{(2),[c]\mathrm{fin}} = &\left[-\tilde{b}_0^2\left(\ln\left(\frac{s}{\mu^2}\right)-i\pi\right)^2\right.\\ &\left.-\tilde{b}_1^2\left(\ln\left(\frac{s}{\mu^2}\right)-i\pi\right)\right]M^{(0),[c]}\\ &-2\tilde{b}_0\left(\ln\left(\frac{s}{\mu^2}\right)-i\pi\right)M^{(1),[c]\mathrm{fin}}\\ &+N^2A^{[c]}+B^{[c]}, \end{aligned} \qquad (14)$$

for $c = 1\ldots6$, and

$$\begin{aligned}M^{(2),[c]\mathrm{fin}} = &-2\tilde{b}_0\left(\ln\left(\frac{s}{\mu^2}\right)-i\pi\right)M^{(1),[c]\mathrm{fin}}\\ &+NG^{[c]}, \end{aligned} \qquad (15)$$

for $c = 7, 8, 9$. Here $\tilde{b}_0$ and $\tilde{b}_1$ are the first two coefficients of the $\mathcal{N} = 1$ super-Yang-Mills β-function. $A^{[c]}$, $B^{[c]}$ and $G^{[c]}$ are functions of s, t and u. Again the explicit form of all these function is given in ref. [34].

In previous papers [28,13,14] we showed that the following supersymmetry Ward identities are satisfied at two loops through $\mathcal{O}(\epsilon^0)$ when one uses the FDH scheme:

$$\mathcal{M}^{\rm SUSY}(g_1^{\pm}, g_2^-, g_3^+, g_4^+) \;=\; 0, \tag{16}$$

$$\mathcal{M}^{\rm SUSY}(\tilde{g}_1^+, \tilde{g}_2^-, g_3^+, g_4^+) \;=\; 0, \tag{17}$$

$$\mathcal{M}^{\rm SUSY}(\tilde{g}_1^+, \tilde{g}_2^-, g_3^-, g_4^+) \;=\; \frac{\langle 2\,3\rangle}{\langle 1\,3\rangle}\,\mathcal{M}^{\rm SUSY}(g_1^+, g_2^-, g_3^-, g_4^+)\,. \tag{18}$$

With the four-gluino amplitudes, we may also check the supersymmetry identity relating the four-gluon amplitude to the four-gluino one,

$$\mathcal{M}^{\rm SUSY}(\tilde{g}_1^+, \tilde{g}_2^-, \tilde{g}_3^-, \tilde{g}_4^+) \;=\; \frac{\langle 2\,4\rangle}{\langle 1\,3\rangle}\,\mathcal{M}^{\rm SUSY}(g_1^+, g_2^-, g_3^-, g_4^+). \tag{19}$$

(Note that in ref. [13] an all outgoing definition of helicity is used for the four-gluon amplitudes, while we use the 'standard' one here where legs 1 and 2 are incoming.) It turns out that this last identity does not work immediately at two loops [34]. The problem is related to ambiguities arising from continuing the Dirac algebra to D dimensions. At one-loop the ambiguity is harmless because it affects only $\mathcal{O}(\epsilon)$ terms. From the Catani formula a one-loop ambiguity at $\mathcal{O}(\epsilon)$ leads to an ambiguity in the $\mathcal{O}(1/\epsilon)$ terms at two loops. It is of course possible to arrange for a prescription to fix the ambiguities to restore the manifest supersymmetry Ward identities, but, in any case, these two-loop ambiguities may all be absorbed into Catani's formula for the divergent parts, leaving well defined finite parts. As expected, in the FDH scheme the finite remainders A, B and G in eqs. (14) and (15) satisfy supersymmetry identities and agree with the corresponding functions given in ref. [13] for pure glue scattering.

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