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Lattice Gauge Actions for Fixed Topology

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Abstract. We test a set of lattice gauge actions for QCD that suppress small plaquette values and in this way also suppress transitions between topological sectors. This is well suited for simulations in the ε-regime and it is expected to help in numerical simulations with dynamical quarks.

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Our aim is to study the possibility of simulating lattice QCD with a gauge action that strongly reduces the occurrence of small plaquette values. A gauge background with such a feature is expected to improve the locality properties [1] of the Overlap-Dirac operator $D_{\rm ov}$ [2]. By the same argument one also expects to ease the numerical evaluation of $D_{\rm ov}$ itself, and help in general dynamical simulations. It can be proven [1, 3] that as long as all plaquette values in a gauge configuration satisfy: $S_P := 1 - \frac{1}{3}\mathrm{Re\ Tr}(U_P) < 1/20.5$ then no change of the topological sector is possible. Hence a suppression of low plaquette values entails a suppression of $Q_{\rm top}$ changes.

Simulations constrained in a fixed topological sector can be problematic for evaluating physical observables in QCD, where all sectors have to be taken into account with the correct weight. However such a constraint is perfectly suited for studying QCD in the ε-regime [4], where predictions exist for observables defined in fixed topological sectors [5], which turns the limitation mentioned above into an advantage. However there are some conditions. The physical volume should be at least $L \gtrsim 1.1$ fm [6, 7]. Moreover, in order to reach very small pion masses, the chiral properties of the Dirac operator are crucial. Finally a sound definition of $Q_{\rm top}$ is important to compare with predictions in fixed topological sectors. All of these requirements are provided by Ginsparg-Wilson fermions. They have an exact, lattice modified chiral symmetry [8], and the fermionic index defines $Q_{\rm top}$ [9]. Results in the $\varepsilon-$regime with Ginsparg-Wilson fermions were obtained for the Dirac spectrum [6, 10, 11] and for meson correlation functions [7, 12, 13, 14], which were compared with quenched Chiral Perturbation Theory [15, 16, 17].

Simple examples of gauge actions that suppress small plaquette values (still expected to be in the same universality class as S_P) are

$$\beta\, S^{\rm hyp}_{\varepsilon,n}(U_P) = \beta\, \frac{S_P}{(1-\varepsilon^{-1}S_P)^n} \qquad \text{if } S_P < \varepsilon, \text{ and } +\infty \text{ otherwise} \tag{1}$$

$$\beta\, S^{\rm pow}_{\varepsilon,n}(U_P) = \beta\, S_P + \varepsilon^{-1} S_P^n \tag{2}$$

$$\beta\, S^{\rm exp}_{\varepsilon,n}(U_P) = \beta\, S_P \exp\left[\varepsilon^{-1} S_P^n\right] \tag{3}$$

TABLE 1. *Results for the $S^{hyp}_{\varepsilon,1}$ on a 16^4 lattice for various ε^{-1} and β. β_W is the coupling resulting in the same r_0 with the Wilson action.*

ε^{-1}	β	r_0/a	β_W	$\tau^{\rm plaq}$	$\tau^{\rm plaq}(\beta_W)$	f_J	dt	Acceptance
0	6.18	7.14(3)	6.18	7(1)	7(1)	*0.015*	0.1	$>99\%$
1.00	1.5	6.6(2)	6.13(2)	2.0(1)	n.a.	0.0027	0.05	$>99\%$
1.18	1.0	7.2(2)	6.18(2)	1.3(1)	7(1)	0.0014	0.02 - 0.01	$>99\%$
1.25	0.8	7.0(1)	6.17(1)	1.1(1)	9(1)	0.0025	0.1	$>99\%$
1.52	0.3	7.3(4)	6.19(4)	0.8(1)	7(1)	0.0008	0.1	$\sim 95\%$
1.64	0.1	6.8(3)	6.15(3)	1.0(1)	n.a.	0.0007	0.1	$\sim 65\%$

The first choice above (for $n = 1$) was introduced by M. Lüscher for conceptual purposes [18], and applied by Fukaya and Onogi in Schwinger model simulations [19, 20]. The question is whether one can conciliate the advantages mentioned above, with reasonable lattice sizes (say $La \sim 1-2$ fm), without increasing lattice artifacts and with a correct and reasonably decorrelated sampling of interesting observables. A first report of our ongoing study was presented in Ref. [21].

Results. Since gauge actions of the type (1,2,3) are non-linear in the link variables, the heat-bath algorithm cannot be applied. Instead we use a local HMC algorithm [22], which is competitive with heat-bath in the standard case. HMC trajectories have discretization dt quoted in Table 1, and the trajectory length is 1. The volume is chosen to 16^4 in order to allow a reliable determination of $r_0/a \simeq 7$ and in order have mild finite volume effects. We computed r_0/a following a standard procedure [23]. Our preliminary results are summarized in Table 1. We estimated the topological charge with cooling and searching for the first plateau [24]. Since we cannot reliably measure the autocorrelation of the $Q_{\rm top}$, we quote – as an indicator of stability – the number of jumps of $Q_{\rm top}$ divided by the number of trajectories in the full history (f_J). Since we save a configuration only every 50 trajectories, the measured f_J is only a lower bound on the frequency of jumps, which can be reliable only for $f_J \ll 0.01$ (which is the interesting case for us). In particular for the Wilson action at $\beta \sim 6.17$ we expect $f_J \gg 0.015$, which we measure. The stability of $Q_{\rm top}$ has to be compared with the autocorrelation of a typical observable (we quote the plaquette value under $\tau^{\rm plaq}$). It is interesting to see that the latter is strongly decreased for non-zero ε^{-1} (at fixed r_0/a). We have also studied actions of type (3), which have a smooth bound on the plaquette value, and therefore are better suited for efficient simulations with global HMC and dynamical fermions. Results will be presented elsewhere. We also checked that the results are consistent independently on the starting configuration. This is important because the constraint on the plaquette value could in principle generate more obstructions than the topological ones, and this would not be noticed simply from the autocorrelation.

It has been pointed out [25] that the actions (1) do not allow for the existence of a positive definite transfer matrix. However we checked that all the actions (1,2,3) have *site*-reflection positivity, which at least ensures the existence of a positive squared transfer matrix [26]. Moreover we have checked that the unphysical behaviour of the short distance force – which was observed in some cases, and related to the lack of a

positive transfer matrix [27] – does not appear in our cases.

We stress that – inspired by Ref. [1] and by simplicity – our search is restricted to actions which are functions of the plaquette value, thus excluding other alternatives, which also affect the stability of $Q_{\rm top}$, as for example in Ref. [28, 29].

Conclusions. Topology conserving gauge actions could be highly profitable in QCD simulations. The suppression of small plaquette values may speed up the simulations with dynamical quarks. A stable $Q_{\rm top}$ is useful in particular in the ε-regime. We are investigating such actions, in view of the physical scale and the topological stability.

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