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Gravity mediated supersymmetry breaking in six dimensions

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Abstract

We study gravity mediated supersymmetry breaking in four-dimensional effective theories derived from six-dimensional brane-world supergravity. Using the Noether method we construct a locally supersymmetric action for a bulk-brane system consisting of the minimal six-dimensional supergravity coupled to vector and chiral multiplets located at four-dimensional branes. Couplings of the bulk moduli to the brane are uniquely fixed, in particular, they are flavour universal. We compactify this system on $T_2/\mathbb{Z}_2$ and derive the four-dimensional effective supergravity. The tree-level effective Kähler potential is not of the sequestered form, therefore gravity mediation may occur at tree-level. We identify one scenario of moduli stabilization in which the soft scalar masses squared are postive.

1 Introduction

Supersymmetry breaking and its mediation to the observable sector is an important issue for any supersymmetric extension of the Standard Model. In the Minimal Supersymmetric Standard Model supersymmetry breaking is parametrized by a set of soft terms: gaugino, squark, slepton and Higgs masses and trilinear A-terms. But, the allowed pattern of soft terms is strongly constrained by the observed features of the low-energy physics. Therefore an underlying theory of soft terms generation is needed.

Gravity mediation [1] is a simple and economical possibility because it is a generic consequence of local supersymmetry. Let us denote the three generations of visible matter superfields by Q^i and assume supersymmetry breaking is parametrized by an F-component of a chiral superfield Σ. The four-dimensional (4d) supergravity action is specified by three functions: the Kähler potential $K = -3\log\Omega$, the superpotential W and the gauge kinetic function f. In this paper we concentrate on soft scalar masses and these are encoded in Ω. Quite generally, the Kähler potential can be expanded in powers of $|Q|^2$,

$$\Omega = \Omega_0(\Sigma^\dagger, \Sigma) - \frac{1}{3M_p^2} Q^{i\dagger} Q^j \left(\delta_{ij} + C_{ij}(\Sigma, \Sigma^\dagger)\right) + \mathcal{O}(Q^4)\,. \tag{1}$$

Without loss of generality we can assume $C_{ij}(\langle\Sigma\rangle, \langle\Sigma^\dagger\rangle) = 0$. The functions C_{ij} are called the contact terms as they control supersymmetry breaking mediation to the visible scalar sector. Indeed, in a vacuum with the vanishing cosmological constant the supergravity F-term potential contributes to soft scalar masses as

$$(m_Q^2)_{ij} = -|F_\Sigma|^2 \left(\partial_\Sigma \partial_{\overline{\Sigma}} C_{ij} - \partial_\Sigma C_{ik} \partial_{\overline{\Sigma}} C_{kj}\right)\,. \tag{2}$$

But in order to avoid excessive flavour changing neutral currents we need $(m_Q^2)_{ij}$ to be approximately diagonal (in the basis in which quark and lepton masses are diagonal). To this end we need the contact terms of a very special form, either universal, $C_{ij} \approx \delta_{ij}$, or aligned with the Yukawa matrices. To justify this ad-hoc assumption it is desirable to have a 'theory of contact terms'. That is, 4d supergravity should be embedded in a more fundamental set-up in which the contact terms could be actually computed and the universalness (or alignment) could be explained.

Higher dimensional brane-world supergravities offer promising candidates for such a theory of contact terms [2]. First of all, the supersymmetry breaking sector can be spatially separated from the visible sector. This is achieved by assuming that visible matter fields are confined to a 4d brane (the visible brane) while supersymmetry breaking dynamics takes place on a different brane (the hidden brane). The couplings between the two sectors that could arise from integrating out heavy fields from the UV completion of the theory can be neglected if separation between the branes is large enough (in units of the fundamental scale). In such case, the couplings arise only from integrating out Kaluza-Klein (KK) modes of bulk degrees of freedom and are calculable within the field theoretical framework. Still there is no guarantee that the resulting soft terms are flavour blind, as the bulk may contain matter fields coupling non-universally to SM generations. However, the degrees of freedom belonging to the higher dimensional gravity multiplet necessarily couple universally. If the

corresponding moduli dominate supersymmetry breaking mediation (that is, either bulk matter fields are absent or their F-terms are negligible) we end up with calculable and flavour blind soft terms. Of course, the flavour problem can be really solved only if the issue of generating Yukawa hierarchy in the visible sector is also addressed. If the correct Yukawa pattern is generated at the fundamental scale then the non-universal corrections to soft terms can be neglected, provided the compactification scale is well below the fundamental scale. But when flavour structures are generated at low energies by the Froggatt-Nielsen mechanism one indeed expects non-negligible contributions to A-terms that could induce dangerous FCNC [3].

Concrete realizations of this idea were extensively studied in the settings of 5d supergravity compactified on $S_1/\mathbb{Z}_2$ [4–7]. It turns out that the issue of supersymmetry breaking mediation can be most conveniently studied within the 4d effective theory after all Kaluza-Klein modes are integrated out. In the minimal version with only gravity in the bulk such effective theory contains only one modulus T corresponding to the size of the fifth dimension. Apart from that there are visible and hidden brane matter fields, Q_V^i and Q_H. In the simplest scenario, when the bulk cosmological constant vanishes, the Kähler potential of the effective 4d supergravity at tree-level is of the no-scale form:

$$\Omega_{5d} = \frac{1}{2}(T+\overline{T}) - \frac{1}{3}\Omega_V(Q_V^i, Q_V^{i\,\dagger}) - \frac{1}{3}\Omega_H(Q_H, Q_H^\dagger)\,. \tag{3}$$

In this Kähler potential sequestering is explicit - the contact terms between visible and hidden matter are absent. The T modulus does not couple to the visible matter as well, therefore in the 5d scenario there is no tree-level gravity mediation. The contact terms are however generated at loop level after one loop corrections from Kaluza-Klein modes of the gravity multiplet are included. Then supersymmetry breaking can be mediated to the visible brane but, unfortunately, the sign of these gravity mediated soft masses squared is negative. Therefore the minimal 5d set-up does not provide for a phenomenologically viable theory of contact terms. Generalizing the set-up to warped supergravity does not improve the situation [8,9]. Positive soft scalar masses can be however obtained in models with sizable brane gravity kinetic terms [7,8].

In this paper we extend the previous studies to 6d supergravities compactified on $T_2/\mathbb{Z}_2$. Using the Noether method [10] we construct a locally supersymmetric action for a bulk-brane system containing the minimal $N = 2$ 6d supergravity and $N = 1$ chiral and vector multiplets confined to a 4d brane at an orbifold fixed point. It should be stressed that our approach is purely field-theoretical. Embedding this set-up in string theory would presumably result in additional constraints on the spectrum and couplings, but this issue is beyond the scope of this paper. Our results hold irrespectively of which string theory provides a UV completion to our model, as long as local supersymmetry and 6d locality remain valid below the string scale.

After deriving bulk-brane couplings we identify the 4d effective supergravity describing the dynamics of this system at energies below the compactification scale. Except for the visible and hidden brane matter fields, Q_V^i and Q_H, the effective theory contains three

moduli T, S and τ. The Kähler potential is not of the sequestered form:

$$\Omega_{6d} = \frac{1}{2}\left(T+\overline{T} - 2\Omega_V(Q_V^i, Q_V^{i\,\dagger}) - 2\Omega_H(Q_H, Q_H^\dagger)\right)^{1/3}\left(S+\overline{S}\right)^{1/3}(\tau+\overline{\tau})^{1/3} \qquad (4)$$

Expanding this Kähler potential in powers of $|Q_V|^2$ we find the contact terms involve not only the moduli but also the hidden brane fields. Therefore this Kähler potential allows for both moduli and brane-to-brane mediation of supersymmetry breaking at tree-level.

Absence of sequestering in higher dimensional brane world models was already discussed by Anisimov et al. in ref. [11]. These authors argue that higher dimensional locality does not have to show up below the compactification scale. The reason is that brane world models typically involve bulk fields with non-trivial coupling to the branes. Decoupling KK modes of these bulk fields may lead to seemingly 'non-local' contact interactions involving fields from different branes. These operators are suppressed by the volume of the extra dimensions, but the gravitino mass, which sets the scale of supersymmetry breaking, is suppressed by the same factor. The conclusion of ref. [11] is that in generic brane world models contact terms arise at tree-level leading to (in general non-diagonal) soft masses of order the gravitino mass.

Non-sequestering is indeed a feature of our Kähler potential (4). We will argue however that the situation is slightly more subtle. Compactification of the 6d action we consider does not contain 'non-local' operators (except for vector-vector interactions that are also present in the sequestered case and play no role in supersymmetry breaking). More precisely, scalar-scalar 'non-local' interactions are present in the off-shell formulation, but vanish when the auxiliary fields are integrated out. This conclusion relies not only on the form of the Kähler potential (4), but also on the fact that the superpotential describing the compactified action does not depend on the T modulus. As a consequence, tree-level supersymmetry breaking mediation does not operate, in spite of the non-sequestered form of eq. (4). 'Non-local' interactions and tree-level mediation does however occur when the minimal 6d action is supplemented by certain higher order operators. In the effective 4d description these higher order operators induce dependence of the superpotential on the T modulus. Thus our conclusion is that information about sequestering in brane world models is encoded not only in the contact terms of the Kähler potential, but also in the structure of the superpotential.

This paper is organized as follows. In Section 2 we construct minimal 6d supergravity coupled to matter on codimension two branes. In Section 3 we derive the Kähler potential, superpotential and gauge kinetic function of the resulting low energy 4d supergravity. In Section 4 and Section 5 we discuss supersymmetry breaking mediation and moduli stabilization and in Section 6 we present our conclusions.

2 Bulk-brane action

In this section we construct a locally supersymmetric action for the 6d minimal $N = 2$ (eight supercharges) supergravity coupled to $N = 1$ chiral and vector multiplets that are confined to a 4d brane at an orbifold fixed point. The construction is done for just one

chiral and one abelian vector multiplet with minimal kinetic terms, but it can be easily extended to more general cases. We use the Noether method, which is a simple but very efficient way to derive the bulk-brane couplings. Starting with a locally supersymmetric bulk action and a globally supersymmetric brane action we systematically add new terms to the action and supersymmetry transformations until the whole set-up becomes locally supersymmetric. We work out all necessary zero- and two-fermion terms so that all two-fermion supersymmetric variations of the bulk-brane action cancel. It turns out that the analysis at the two-fermion level is sufficient to read off unambiguously the form of the 4d low energy effective supergravity, which is our main objective in this paper. Therefore we stop our procedure at this point and do not derive four-fermion terms in the brane action. This means that we tacitly assume that our construction can be completed such that all supersymmetric variations cancel and supersymmetry algebra closes on-shell.

The starting point is to write down the bulk action. The minimal set-up contains 6d $N = 2$ gravity+tensor multiplet that includes a sechsbein e_A^a, a gravitino ψ_A, a Kalb-Ramond form B_{AB} with a three-form field strength $H_{ABC} = 3\partial_{[A}B_{BC]}$, a fermionic dilatino χ and a real scalar Φ called the dilaton. The indices A, B, C, a run over $0 \ldots 3, 5, 6$ and the fermions obey 6d chirality conditions. The conventions used in this paper are collected in A. The bulk action up to four-fermion terms reads [12, 13]

$$\begin{aligned}
\mathcal{L}_{\rm bulk} = M_6^4 e_6 \Big[& \frac{1}{2}R_6 - i\overline{\psi_A}\Gamma^{ABC}D_B\psi_C + \frac{1}{12}e^{-2\Phi}H_{ABC}H^{ABC} + i\overline{\chi}\Gamma^A D_A\chi \\
& + \frac{1}{2}\partial_A\Phi\partial^A\Phi - \frac{i}{12\sqrt{2}}\overline{\psi_A}\Gamma^{ABCDE}\psi_B e^{-\Phi}H_{CDE} \\
& - \frac{i}{2\sqrt{2}}\overline{\psi_A}\Gamma^C\psi_B e^{-\Phi}H_{ABC} + \frac{1}{12\sqrt{2}}\overline{\psi_A}\Gamma^{ABCD}\chi e^{-\Phi}H_{BCD} + \text{h.c.} \\
& - \frac{1}{4\sqrt{2}}\overline{\psi_A}\Gamma^{BC}\chi e^{-\Phi}H_{ABC} + \text{h.c.} \\
& + \frac{i}{12\sqrt{2}}\overline{\chi}\Gamma^{ABC}\chi e^{-\Phi}H_{ABC} - \frac{1}{2}\overline{\chi}\Gamma^A\Gamma^B\psi_A\partial_B\Phi + \text{h.c.}\Big] \,,
\end{aligned} \tag{5}$$

while the supersymmetry transformations, up to three-fermion terms are given by

$$\begin{aligned}
\delta e_A^a &= \frac{i}{2}\overline{\psi_A}\Gamma^a\epsilon + \text{h.c.}\,, \\
\delta\psi_A &= D_A\epsilon + \frac{1}{24\sqrt{2}}(\Gamma_{ABCD} - 3g_{AB}\Gamma_{CD})\epsilon\, e^{-\Phi}H_{BCD}\,, \\
\delta B_{AB} &= \frac{i}{\sqrt{2}}e^{\Phi}\overline{\psi}_{[A}\Gamma_{B]}\epsilon - \frac{1}{2\sqrt{2}}e^{\Phi}\overline{\chi}\Gamma_{AB}\epsilon + \text{h.c.}\,, \\
\delta\chi &= -\frac{i}{2}\Gamma^A\epsilon\partial_A\Phi + \frac{i}{12\sqrt{2}}\Gamma_{ABC}\epsilon e^{-\Phi}H_{ABC}\,, \\
\delta\Phi &= \frac{1}{2}\overline{\epsilon}\,\chi + \text{h.c.}\,.
\end{aligned} \tag{6}$$

The model as it stands is inconsistent because of gravitational anomalies. Extension to anomaly-free spectrum is possible [14], but it requires a large number of additional bulk

multiplets (gravitational anomalies vanish when $244 + n_V - n_H - n_T = 0$, where $n_{V,H,T}$ denote the number of vector, hyper- and tensor multiplets, respectively [15]). It is of course not possible to give 6d masses to all of these multiplets and their zero modes will survive below the compactification scale. It is however conceivable that these multiplets turn out to be irrelevant for the issue of supersymmetry breaking mediation. This is the case when they are stabilized by low energy 4d dynamics such that their masses are much larger than the supersymmetry breaking scale and all F-terms in this sector are negligible. We will make this ad hoc assumption in what follows and we continue working with the minimal set-up.

Next, we need to define the orbifold action of $\mathbb{Z}_2$ in a way consistent with the 6d action. The torus T_2 is parametrized by $x_5, x_6 \in (-\pi R, \pi R]$ and the $\mathbb{Z}_2$ acts as $(x_5, x_6) \to (-x_5, -x_6)$. There are four orbifold fixed points: $(0,0)$, $(0,\pi R)$, $(\pi R, 0)$ and $(\pi R, \pi R)$. We choose the field components e_μ^m, e_α^β, $B_{\mu\nu}$, $B_{\alpha\beta}$, ϕ ψ_μ^+, ψ_α^+ and χ^+ to be even under $\mathbb{Z}_2$, $f(-x_5,-x_6) = f(x_5,x_6)$, while e_μ^α, e_α^m, $B_{\mu\alpha}$, ψ_μ^-, ψ_α^- and χ^- to be odd, $f(-x_5,-x_6) = -f(x_5,x_6)$. Here $\mu,\nu,m = 0\ldots3$. See A for definitions of the fermionic components.

We move to the Noether construction of the brane action. In fact, the procedure we employ here is very similar to that of coupling 4d brane to 5d supergravity [9] or 10d brane to 11d supergravity [16]. It can be summarized in the following steps:

1. *Global brane action.* Here we write a globally supersymmetric action for a chiral multiplet (Q, ψ_Q) charged under a vector multiplet (A_μ, λ) (in our notation ψ_Q and λ are 4d Dirac spinors, respectively left- and right-handed). Next, we promote the infinitesimal supersymmetry transformation parameter to the one depending on the 4d spacetime coordinates x_μ. We also couple the brane fields to the 4d metric induced at the brane, as dictated by general coordinate invariance.

2. *Noether current.* After completing the first step, supersymmetric variations of the brane action no longer vanish. Firstly, there is a variation of the form $\delta\mathcal{L} = -\delta(x_5)\delta(x_6)\partial_\mu \epsilon\, j^\mu_{\rm SC}$, where $j^\mu_{\rm SC}$ is the supercurrent of brane matter and gauge fields. In order to cancel this variation we need to couple the brane supercurrent to the positive parity gravitino ψ_μ^+.

3. *Bianchi identity.* The other uncanceled supersymmetric variations after the first step are those of the metric fields in the brane action. It turns out that the modifications needed to cancel these variations can be concisely summarized as a redefinition of the three-form field strength H. That is, we replace the three-form H in the 6d bulk action (5) and supersymmetry transformations (6) by $\hat{H}$ defined such that $d\hat{H} \neq 0$. With appropriate choice of the right hand side of the Bianchi identity all variations of the metric at the brane are canceled.

4. *Moduli coupling.* In the final step we study two-fermion variations containing derivatives of the dilaton and higher dimensional metric fields. From cancellation of these terms we are able to determine how these moduli should couple to the brane action.

Performing this program we arrive at the bulk-brane action ($\delta(x_{56}) \equiv \delta(x_5)\delta(x_6)$):

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\rm bulk}(H \to \hat{H}) + \delta(x_{56})\mathcal{L}_{\rm brane}\,, \\
\mathcal{L}_{\rm brane} &= e_4 \Big[e^{-\Phi} D^\mu Q^\dagger D_\mu Q + \frac{i}{2} e^{-\Phi} \overline{\psi}_Q \gamma^\mu D_\mu \psi_Q - \frac{i}{2} e^{-\Phi} \overline{D}_\mu \psi_Q \gamma^\mu \psi_Q \\
&\quad - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\overline{\lambda}\gamma^\mu D_\mu \lambda - ig\sqrt{2} e^{-\Phi} Q \overline{\psi}_Q \lambda + \text{h.c.} - \frac{1}{2} g^2 e^{-2\Phi} |Q|^4 \\
&\quad - \frac{1}{\sqrt{2}} e^{-\Phi} \overline{\psi}_Q \gamma^\mu \gamma^\nu \psi_\mu D_\nu Q - \frac{i}{\sqrt{2}} e^{-\Phi} \overline{\psi}_Q \gamma^\mu \chi D_\mu Q + \text{h.c.} \\
&\quad - \frac{i}{4} \overline{\lambda} \gamma^\mu \gamma^{\nu\rho} \psi_\mu F_{\nu\rho} + \frac{1}{2} g |Q|^2 e^{-\Phi} \overline{\lambda} \gamma^\mu \psi_\mu - ig|Q|^2 e^{-\Phi} \overline{\lambda} \chi + \text{h.c.} \Big]\,.
\end{aligned}
\tag{7}
$$

In the above formula we used the following definitions

$$
\begin{aligned}
\hat{H}_{\mu\dot{5}\dot{6}} &= H_{\mu\dot{5}\dot{6}} + \frac{1}{M_6^4} \delta(x_{56}) j_\mu\,, \\
\hat{H}_{\tau\rho\sigma} &= H_{\tau\rho\sigma} + \frac{1}{M_6^4} \delta(x_{56}) j_{\tau\rho\sigma}\,, \\
j_\mu &= -\frac{i}{\sqrt{2}} (Q^\dagger D_\mu Q - D_\mu Q^\dagger Q) - \frac{1}{2\sqrt{2}} \overline{\psi}_Q \gamma_\mu \psi_Q + \frac{1}{2\sqrt{2}} e^{\Phi} \overline{\lambda} \gamma_\mu \lambda\,, \\
j_{\tau\rho\sigma} &= -\frac{i}{2\sqrt{2}} \overline{\psi}_Q \gamma_{\tau\rho\sigma} \psi_Q + \frac{1}{\sqrt{2}} A e^{\Phi} \overline{\lambda} \gamma_{\tau\rho\sigma} \lambda\,, \\
D_\mu Q &= \partial_\mu Q + igA_\mu Q\,, \qquad D_\mu \psi_Q = \partial_\mu \psi_Q + igA_\mu \psi_Q + \frac{1}{4} \omega_{\mu mn} \gamma^{mn} \psi_Q\,, \\
D_\mu \lambda &= \partial_\mu \lambda + \frac{1}{4} \omega_{\mu mn} \gamma^{mn} \lambda\,.
\end{aligned}
\tag{8}
$$

The supersymmetry transformations of the bulk fields are those of eq. (6) with $H \to \hat{H}$. In addition we need to modify the transformation law of the Kalb-Ramond form by

$$
\delta B_{\dot{5}\dot{6}} = -\frac{i}{2} \frac{1}{M_6^4} \delta(x_{56}) \overline{\psi}_Q \epsilon Q + \text{h.c.}\,. \tag{9}
$$

The supersymmetry transformations of the brane fields up to three-fermion terms are simply

$$
\begin{aligned}
\delta Q &= \frac{1}{\sqrt{2}} \overline{\epsilon} \psi_Q\,, & \delta \psi_Q &= -\frac{1}{\sqrt{2}} i \gamma^\mu D_\mu Q\, \epsilon\,, \\
\delta A_\mu &= -\frac{i}{2} \overline{\lambda} \gamma_\mu \epsilon + \text{h.c.}\,, & \delta \lambda &= \frac{1}{4} \gamma^{\mu\nu} \epsilon F_{\mu\nu} + \frac{i}{2} g |Q|^2 e^{-\Phi} \epsilon\,.
\end{aligned}
\tag{10}
$$

One comment is in order here. Substituting $H \to \hat{H}$ in eq. (7) results in appearance of singular $\delta(x_{56})^2 = \delta(x_5)^2 \delta(x_6)^2$ terms in the bulk-brane action. A similar thing happens in 5d [9] and in that case the situation is well understood. Firstly, these singular terms are absent in the low energy effective theory after integrating out the Kaluza-Klein modes.

Secondly, in the 5d set-up the δ^2 terms provide for necessary counterterms to cancel divergences in certain one-loop diagrams [17]. In 6d the latter issue was studied in [18] with the same conclusion. We will show in Section 3 that the former is also true in 6d: the $\delta(x_{56})^2$ terms cancel after compactification to 4d and integrating out the Kaluza-Klein modes of the Kalb-Ramond form.

We can further generalize the setup by allowing for a superpotential W for the brane chiral multiplet. Here we give the supersymmetric bulk-brane action in the simplest case when $W = W_0 =$ constant. In other words, we supersymmetrize a gravitino brane mass term. The brane action in addition to (5) should contain

$$\mathcal{L}_w = \frac{1}{2} e_4 \delta(x_{56}) \sqrt{\frac{A}{\tau_2}} W_0 \left(-\overline{\psi}_\mu \gamma^{\mu\nu} C \overline{\psi}_\nu^T + i\overline{\psi}_{5+6} \gamma^\mu C \overline{\psi_\mu}^T + i\overline{\psi}_{5-6} \gamma^\mu C \overline{\psi_\mu}^T + \text{h.c.} \right) , \quad (11)$$

where $\psi_{5\pm6} = -(\psi_5 \pm i\psi_6)$. Moreover, we need to modify the transformation laws of the higher dimensional components of the gravitino as

$$\delta\psi_{5\pm6} \;=\; \pm \frac{1}{M_6^4} \delta(x_{56}) \frac{1}{\sqrt{A\tau_2}} W_0 C \overline{\epsilon}^T \quad (12)$$

In the above A and τ_2 are related to the volume and the shape of the compact torus. Their definition in terms of higher dimensional components of the 6d metric reads: $A/\tau_2 = g_{55}^{\perp\perp}$, $A^2 = \det g_{\alpha\beta}$.

One very important conclusion is that the couplings of the bulk moduli to the brane are completely fixed and non-trivial. Namely, we found that the dilaton must couple to the kinetic terms of the brane chiral multiplet via the exponential factor $e^{-\Phi}$. On the other hand, the dilaton does not couple to the kinetic terms of the brane gauge fields. In the presence of a brane superpotential also the volume and the shape of the compact torus must couple to the brane. One interesting consequence follows. If some brane dynamics develops a vev of an F-term and/or D-term potential then the bulk moduli couple to the resulting brane tension as

$$\mathcal{L} = -e_4 \delta(x_{56}) \left(\frac{1}{2} e^{-2\Phi} g^2 D^2 + \frac{A e^\Phi}{\tau_2} |F|^2 \right) . \quad (13)$$

These couplings will be important for our discussion of moduli stabilization further in this paper. More generally, they should affect the analysis of domain wall solutions in 6d supersymmetric brane-worlds.

3 Low energy effective supergravity

At energies below the compactification scale M_c we can integrate out all Kaluza-Klein modes and maintain only the zero modes in the spectrum. If the scale of spontaneous supersymmetry breaking is below M_c the effective theory has to reduce to 4d $N = 1$ supergravity. In this section we identify the Kähler potential, superpotential and gauge kinetic function of the effective supergravity.

We compactify the 6d theory on the flat background

$$ds^2 = \frac{1}{A} g_{\mu\nu} dx^\mu dx^\nu - g_{\alpha\beta} dx^\alpha dx^\beta \tag{14}$$

where $\langle g_{\mu\nu} \rangle = \eta_{\mu\nu}$ and the metric on the two-torus is parametrized as

$$g_{\alpha\beta} = \frac{A}{\tau_2} \begin{bmatrix} 1 & \tau_1 \\ \tau_1 & \tau_1^2 + \tau_2^2 \end{bmatrix} . \tag{15}$$

The modulus A describes the volume while $\tau = \tau_2 + i\tau_1$ the shape of the toroidal extra dimensions.

The light degrees of freedom in the compactified theory are the zero modes (those independent of x_5, x_6) of the positive parity fields. They include the metric $g_{\mu\nu}$, the gravitino ψ_μ^+, the volume modulus A, the shape modulus τ, the dilaton Φ, the Kalb-Ramond form $B_{\mu\nu}$ and $B_{\dot{5}\dot{6}}$, the dilatino χ_+ and the two extra dimensional components of the gravitino ψ_5, ψ_6. Counting the degrees of freedom we find that the zero modes can be collected into a gravity multiplet and three chiral multiplets (the moduli multiplets). Apart from that there is the brane chiral multiplet (Q, ψ_Q) and the brane gauge multiplet (A_μ, λ).

Let us first discuss the relevant scales in our setup. The input parameters in 6d are M_6, R and the moduli vevs A and τ. In fact, R is a superfluous parameter that can be set to any value by coordinate transformations, but we find it convenient to keep it explicit. For simplicity, we will assume $\tau \sim 1$, so that the two extra dimensions are of comparable size. In the 6d Einstein frame the fundamental scale at which gravity gets strong is the 6d Planck mass M_6, the KK excitations start at $A^{-1/2}R^{-1}$, and the zero-mode graviton exchange is controlled by $2\pi R A^{1/2} M_6^2$. The 6d field theory description is sensible when the physical volume of the torus $V_{T_2} = (2\pi R)^2 A$ is large enough, so that the compactification scale and the cutoff scale are separated by at least an order of magnitude: $M_6/(A^{-1/2}R^{-1}) \sim V_{T_2}^{1/2} M_6 \gg 1$. This condition also ensures perturbativity of gravitational loop corrections. The 4d effective action is formulated in the 4d Einstein frame (14), in which all mass parameters are rescaled by the factor $A^{-1/2}$ wrt the 6d Einstein frame. In particular the zero-mode graviton exchange is controlled by the 4d Planck scale $M_p = 2\pi R M_6^2$, while the compactification scale is $M_c \sim (AR)^{-1}$. Of course the ratios of physical scales in both frames are the same and depend only on the physical volume in fundamental units.

3.1 Tree-level effective action

Let us study kinetic terms of the bosonic fields. We obtain

$$\begin{aligned}\mathcal{L}_{\text{b kin}} = M_p^2\sqrt{-g}\Bigg[&\frac{1}{2}R(g) + \frac{1}{2A^2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu\Phi)^2 + \frac{1}{4\tau_2^2}(\partial_\mu\tau_2)^2 + \frac{1}{4\tau_2^2}(\partial_\mu\tau_1)^2 \\ &+ \frac{e^{-\Phi}}{M_p^2 A}\partial_\mu Q^\dagger\partial^\mu Q - \frac{1}{4M_p^2}F_{\mu\nu}F^{\mu\nu} + \frac{e^{-2\Phi}}{12A^2}(H_{\mu\nu\rho})^2 \\ &+\frac{e^{-2\Phi}}{2A^2}\left(\partial_\mu B_{\dot5\dot6} + \frac{i}{\sqrt{2}M_p^2}(Q^\dagger\partial_\mu Q - \partial_\mu Q^\dagger Q)\right)^2\Bigg]\,,\end{aligned} \tag{16}$$

All the terms except for the last one are trivially obtained by inserting the background (14) into the 6d action (7) and integrating over the extra dimensions. The last term requires a more careful derivation. The 6d bulk-brane action contains

$$\begin{aligned}\mathcal{L} &= \frac{1}{12}e_6 M_6^4 e^{-2\Phi}\hat{H}_{ABC}\hat{H}^{ABC} \\ &\longrightarrow \frac{e^{-2\Phi}}{2A^2}\sqrt{-g}M_6^4\left(\partial_\mu B_{\dot5\dot6} + \partial_6 B_{\mu\dot5} + \partial_5 B_{\dot6\mu} + \frac{1}{M_6^4}\delta(x_{56})j_\mu\right)^2\,,\end{aligned} \tag{17}$$

where $j_\mu = \frac{i}{\sqrt{2}}(Q^\dagger\partial_\mu Q - \partial_\mu Q^\dagger Q)$ + fermionic terms. Compactifying to 4d one cannot simply set the odd components $B_{\mu\alpha}$ of the Kalb-Ramond form to zero. The reason is they couple to the brane matter via $\delta(x_{56})\partial_\alpha B_{\mu\beta}j^\mu \to m_n B^{(n)}_{\mu\beta}j^\mu$. As the coupling is proportional to the mass, the diagram with an exchange of the massive vector $B^{(n)}_{\mu\beta}$ does not decouple at low energies, and thus integrating out these modes is non-trivial. Restricting to the tree-level diagrams and two-derivative terms in the effective action there is a simple prescription to take into account these non-decoupling diagrams. Namely, one should first solve the equations of motion for $B_{\mu\alpha}(x_5, x_6)$ with j_μ and $\partial_\mu B_{\dot5\dot6}$ treated as constant sources, then insert the solution back into the 6d action and integrate over the extra dimensions. The non-trivial equations of motion read $\partial^5\hat{H}_{\mu\dot5\dot6} = \partial^6\hat{H}_{\mu\dot5\dot6} = 0$, thus $\hat{H}_{\mu\dot5\dot6} = C_\mu$ where C_μ is a constant. Let us choose an ansatz: $B_{\mu\dot5} = -\partial_6 W_\mu$, $B_{\mu\dot6} = \partial_5 W_\mu$. Then the equations of motion reduce to

$$(\partial_5^2 + \partial_6^2)W_\mu = \partial_\mu B_{\dot5\dot6} - C_\mu + \frac{1}{M_6^4}\delta(x_{56})j_\mu\,. \tag{18}$$

We can solve this equation (with appropriate periodicity conditions) if we choose the integration constant as

$$C_\mu = \partial_\mu B_{\dot5\dot6} + \frac{1}{(2\pi R)^2 M_6^4}j_\mu\,. \tag{19}$$

This follows from integrating the equation eq. (18) over the torus and noticing that the left-hand side vanishes by the Stokes theorem. The equation of motion can now be solved [19], but this is not important for our purpose. Inserting $\hat{H}_{\mu\dot5\dot6} = C_\mu$ back into the 6d action and integrating over the extra dimensions we reproduce the last term in eq. (16).

The kinetic terms (16) are not diagonal yet as both A and Φ couple to the axions $B_{\dot{5}\dot{6}}$ and $H_{\mu\nu\rho}$. Therefore we define new variables [20]:

$$s = Ae^{-\Phi}\,, \qquad t = Ae^{\Phi}\,. \tag{20}$$

We also dualize the two-form to a scalar: $e^{-2\Phi}H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\tau}\partial^{\tau}\sigma$. Then the kinetic terms take the form

$$\begin{aligned}\mathcal{L}_{\rm b\,kin} = M_p^2\sqrt{-g}\Bigg[&\frac{1}{2}R(g) + \frac{1}{4t^2}(\partial_\mu t)^2 + \frac{1}{4s^2}(\partial_\mu s)^2 + \frac{1}{4\tau_2^2}(\partial_\mu \tau_2)^2 + \frac{1}{4\tau_2^2}(\partial_\mu \tau_1)^2 \\ &+ \frac{1}{M_p^2 t}\partial_\mu Q^\dagger \partial^\mu Q - \frac{1}{4M_p^2}F_{\mu\nu}F^{\mu\nu} \\ &+ \frac{1}{2s^2}(\partial_\mu\sigma)^2 + \frac{1}{2t^2}\left(\partial_\mu B_{\dot{5}\dot{6}} + \frac{i}{\sqrt{2}M_p^2}(Q^\dagger\partial_\mu Q - \partial_\mu Q^\dagger Q)\right)^2\Bigg]\,.\end{aligned} \tag{21}$$

These kinetic terms are reproduced by the Kähler potential and the gauge kinetic function:

$$\begin{gathered}K = -\log\left(\frac{1}{2}(T+\overline{T}) - \frac{1}{M_p^2}|Q|^2\right) - \log\left(\frac{1}{2}(S+\overline{S})\right) - \log\left(\frac{1}{2}(\tau+\overline{\tau})\right)\,, \\ f = 1\,,\end{gathered} \tag{22}$$

with the following definition of the scalar components of the moduli multiplets:

$$T = t + \frac{1}{M_p^2}|Q|^2 + i\sqrt{2}B_{\dot{5}\dot{6}}\,, \qquad S = s + i\sqrt{2}\sigma\,, \qquad \tau = \tau_2 + i\tau_1\,. \tag{23}$$

Note that the admixture of the brane matter in the T-modulus is necessary to correctly reproduce the $|Q|^2(\partial Q)^2$ terms in eq. (21).

When the brane superpotential is present, compactification on the background of eq. (14) yields the gravitino mass term[1]

$$\mathcal{L} = -\frac{1}{2A\sqrt{\tau_2}}\sqrt{-g}W_0\overline{\psi}_\mu\gamma^{\mu\nu}C\overline{\psi}_\nu^T + \text{h.c.}\,. \tag{24}$$

Comparing it with the gravitino mass term in 4d supergravity , $-\frac{1}{2}\sqrt{-g}e^{K/2}W\overline{\psi}_\mu\gamma^{\mu\nu}C\overline{\psi}_\nu^T$ and the Kähler potential (22) we find that (24) corresponds to a constant superpotential:

$$W = W_0\,. \tag{25}$$

[1]This formula is true at lowest order in W_0. At higher orders in W_0 the mass of the gravitino zero mode receives divergent corrections that require tree-level renormalization [21]. This UV sensitivity of the effective theory can be neglected when W_0/M_p^2 is much smaller than the compactification scale, which we always assume in this paper.

We can now easily generalize these results to the case when brane matter with arbitrary kinetic terms is present on all four $\mathbb{Z}_2$ fixed points. The low energy effective supergravity is then described by

$$\begin{aligned} K &= -\log\left(\frac{T+\overline{T}}{2} - \Omega_{\rm branes}\right) - \log\left(\frac{S+\overline{S}}{2}\right) - \log\left(\frac{\tau+\overline{\tau}}{2}\right), \\ \Omega_{\rm branes} &= \sum_a \Omega_a(Q_i^a, Q_i^{a\dagger}), \qquad f = \sum_a f_a(Q_i^a), \qquad W = \sum_a W_a(Q_i^a), \end{aligned} \tag{26}$$

where $a = 1 \ldots 4$ labels the $\mathbb{Z}_2$ fixed points. One also needs to generalize the definition of T as

$$\mathrm{Re}\, T = t + \Omega_{\rm branes}. \tag{27}$$

The effective supergravity defined by eq. (26) is the starting point for our study of supersymmetry breaking mediation, which we perform in Section 4. Here we point out several interesting features of (26):

1. The Kähler potential eq. (26) is not of the sequestered form. Still, the higher dimensional locality is encoded in the fact that $\Omega_{\rm branes}$, f and W consist of separate contributions from different branes. One consequence of this special structure is that the 4d effective action derived from (26) does not contain 'non-local' interactions that would couple scalar operators involving fields from different branes. On the other hand, there are 'non-local' vector-vector interactions in $\mathcal{L} \sim (\partial_i \Omega \partial_\mu Q_i)^2$ that arise from integrating out the KK modes of the bulk 2-form field.

2. The bulk moduli T, S and τ couple universally to all brane fields. In our setup there is no possibility to introduce non-universal couplings of these moduli, as the form of eq. (26) is uniquely fixed by 6d local supersymmetry. Therefore soft terms mediated by these moduli will necessarily be flavour blind.

3. The superpotential W does not depend on the moduli T, S, τ. This statement remains true as long as only perturbative effects are considered. Later in Section 5 we will argue that non-perturbative bulk physics (e.g. gaugino condensation) can generate moduli dependence of the superpotential.

4. The Kähler potential is of the special no-scale form such that the coefficients of the logs sum up to three and therefore the factor $-3|W|^2$ in the scalar potential cancels out. In consequence, for W independent of the moduli, the scalar potential takes the form

$$V = \frac{1}{s\tau_2} g^{i\overline{j}} \partial_i W \overline{\partial_j W}, \tag{28}$$

   where the indices i, j run over brane matter fields only and $g^{i\overline{j}}$ is the inverse sigma-model metric in the brane sector.

5. The parametrization (15) of the compact manifold is redundant, so that the shape moduli related by a $SL(2,\mathbb{Z})$ transformation $\tau \to \frac{a\tau - ib}{ci\tau + d}$ with integer a, b, c, d and

$ad - bc = 1$ parametrize the same torus. Under the two generators of $SL(2,\mathbb{Z})$ transformations $g_1 : \tau \to \tau + i$ and $g_2 : \tau \to 1/\tau$ the Kähler potential (26) transforms as

$$g_1 : \ K \to K \,, \qquad\qquad g_2 : \ K \to K + \log\tau + \log\overline{\tau} \,. \tag{29}$$

This ensures that the kinetic terms are invariant under the $SL(2,\mathbb{Z})$ transformations. However if the superpotential is non-vanishing it should transform as $g_1 : \ W \to W$, $g_2 : \ W \to \frac{1}{\tau} W$ in order for the entire supergravity action to be invariant. Since the superpotential in (26) is independent of τ its presence breaks the g_2 generator of $SL(2,\mathbb{Z})$. Similarly, the presence of vector fields on the branes breaks the $SL(2,\mathbb{Z})$ invariance.

6. It may be surprising that the structure of the Kähler potential (26) is so different than that of the Kähler potential (3) derived from 5d minimal supergravity. Note however that compactifying our 6d model to 5d we obtain a 5d supergravity coupled to a tensor and a vector multiplet. Therefore there is no a priori reason why the minimal 6d model should be similar to the minimal 5d model.

3.2 One-loop corrections

Integrating out heavy Kaluza-Klein modes at one-loop order corrects the Kähler potential of the low-energy effective supergravity given in eq. (26). In 5d the minimal supergravity model yields the no-scale tree-level Kähler potential, therefore one loop corrections are crucial for a discussion of supersymmetry breaking [5–7]. In 6d the no-scale structure is absent and generically we do not expect loop corrections to play a prominent role. However we will see in Section 5 that in certain circumstances tree-level mediation of supersymmetry breaking may be absent also in 6d. In such case it is important to know the precise form of one loop corrections. Our result for the one-loop Kähler potential may also be interesting for future applications, as e.g. it encodes the Casimir energy contribution to the scalar potential.

As discussed in refs. [7,9] there is a simple formula for the one loop Kähler potential:

$$\Delta\Omega = \frac{N}{3} \frac{\Gamma(1-d/2)}{M_p^2 (4\pi)^{d/2}} \cdot \sum m_n^{d-2} \tag{30}$$

where $K = -3\log(\Omega_{\rm tree} + \Delta\Omega)$, $d = 4 + \epsilon$ and m_n denotes the Kaluza-Klein spectrum in the 4d conformal frame. The numerical factor N can be determined by counting the number of fermions and gauge bosons (from the 4d perspective) at each Kaluza-Klein level: $N = N_{1/2} - 2N_1$. In the present case we obtain $N = -4$. The spectrum in the conformal frame is given by:

$$m^2_{n_5,n_6} = \frac{1}{A^{4/3}\tau_2^{2/3} R^2} |n_6 + i n_5 \tau|^2 \tag{31}$$

Using the standard methods [22] we find the sum:

$$\Sigma(s) \equiv \frac{1}{2}\sum_{n_5,n_6\in Z}{}' m_{n_5,n_6}^{2s} = \frac{1}{A^{4/3}\tau_2^{2/3}R^2}\left(\Sigma_1(s)+\Sigma_2(s)+\Sigma_3(s)\right),$$
$$\Sigma_1(s) = \zeta(-2s), \qquad \Sigma_2(s) = \sqrt{\pi}\tau_2^{2s+1}\frac{\Gamma(-s-1/2)\zeta(-2s-1)}{\Gamma(-s)}, \tag{32}$$
$$\Sigma_3(s) = \frac{2\tau_2^{s+1/2}}{\pi^s\Gamma(-s)}\sum_{n_5,p_6>0}\left(\left(\frac{n_5}{p_6}\right)^{s+1/2}K_{s+1/2}(2\pi\tau_2 n_5 p_6)e^{-2\pi i\tau_1 n_5 p_6}+\text{h.c.}\right).$$

Inserting $s = d/2-1$ and then $d = 4+\epsilon$ we find that all Σ_i's are proportional to ϵ, which cancels the $1/\epsilon$ pole of $\Gamma(1-d/2)$ in eq. (30). Thus our expression for the correction to the Kähler potential is finite and we find:

$$\Delta\Omega = -\frac{32}{3(4\pi)^2}\frac{1}{(2\pi RM_p)^2}\left(T+\overline{T}-\frac{2}{M_p^2}\Omega_{\text{branes}}\right)^{-2/3}\left(S+\overline{S}\right)^{-2/3}(\tau+\overline{\tau})^{-2/3}$$
$$\times\left[\zeta(3)+\frac{\pi^3(\tau+\overline{\tau})^3}{360}+\sum_{n_5,p_6>0}\frac{1+\pi n_5 p_6(\tau+\overline{\tau})}{p_6^3}(e^{-2\pi\tau n_5 p_6}+\text{h.c.})\right] \tag{33}$$

For $\tau\sim 1$ this is suppressed wrt to the tree level term by the factor $\frac{1}{(4\pi)^2(2\pi RM_pA)^2}$ or, equivalently, $\frac{1}{(4\pi)^2(V_{T_2}M_6)^4}$. The volume suppression signals that $\Delta\Omega$ corresponds to (truly) non-local operators in 6d. Apart from the finite part (33) there is a UV sensitive contribution to $\Delta\Omega$ but it simply renormalizes the tree-level parameters: the Planck scale and Ω_{branes} in the Kähler potential. One can also compute one-loop gravitational corrections to the gauge kinetic function f and find that they vanish (except for a UV sensitive renormalization of the tree-level f).

4 Gravity mediated supersymmetry breaking

We have gathered all necessary tools to analyze supersymmetry breaking in 6d brane-worlds. This issue can be most efficiently studied within an effective theory after all Kaluza-Klein modes are integrated out, that is within 4d supergravity defined by eq. (26). We will see that, unlike the tree-level Kähler potential in the 5d case, the Kähler potential (26) may lead to gravity mediation of supersymmetry breaking. In order to recognize what is special about the 6d case let us first review the corresponding analysis in 5d [4,7]. For the minimal flat 5d brane-world supergravity the tree-level Kähler potential of the low energy 4d supergravity reads:

$$\Omega_{5d} = \frac{1}{2}(T+\overline{T}) - \frac{1}{3}\Omega_V(Q_V,Q_V^\dagger) - \frac{1}{3}\Omega_H(Q_H,Q_H^\dagger). \tag{34}$$

Here Q_V are superfields living on the visible brane, while Q_H correspond to the hidden brane sector. We also expand the visible matter Kähler potential as $\Omega_V = |Q_V|^2/M_p^2+\ldots$.

The visible matter couples neither to the hidden brane matter nor to the moduli in eq. (34). This so-called sequestering guarantees the absence of tree-level gravity mediation of supersymmetry breaking. This is most easily seen in the conformal compensator formalism in which the relevant part of the action can be written as $\mathcal{L} = -3M_p^2 \int d^4\theta C^\dagger C \Omega$, with $C = 1 + \theta^2 F_C$. Equivalently, we can study the soft scalar masses directly in the on-shell formulation. The standard expression for the scalar potential in 4d supergravity in the Einstein frame reads [25]

$$V = \frac{1}{M_p^2} e^K (K^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2) + \frac{M_p^4}{2\mathrm{Re}\, f_a} D_a^2 \,. \tag{35}$$

The scalar masses are defined as

$$m_{Q_V}^2 = \left. \frac{\frac{\partial^2 V}{\partial Q_V \partial Q_V^\dagger}}{\frac{\partial^2 K}{\partial Q_V \partial Q_V^\dagger}} \right|_{Q_V = 0} \,. \tag{36}$$

The soft masses can be expressed in terms of the supersymmetry breaking order parameters, the F-terms and the gravitino mass defined by:

$$F_i = -e^{K/2} (K^{-1})_{ij} D_j W \,, \qquad\qquad m_{3/2} = \frac{1}{M_p^2} e^{K/2} W \,, \tag{37}$$

where $D_j W = \partial_j W + \partial_j K \, W$ and K and W should be evaluated for moduli vevs at the minimum. Tuning the cosmological constant to zero allows to relate $m_{3/2}$ and F_i, therefore the soft masses can be expressed solely by F-terms (and D-terms if present).

Performing this procedure in the 5d setup we find at tree-level

$$m_{Q_V}^2 = 0 \tag{38}$$

for arbitrary values of F_T, F_H and $m_{3/2}$. Independence of the soft scalar masses of supersymmetry breaking order parameters is the consequence of the no-scale structure of eq. (34). At one-loop level the Kähler potential (34) gets corrected by [7,9] :

$$\begin{aligned} \Delta\Omega_{5d} &= \epsilon (T + \overline{T} - \frac{2}{3}\Omega_V - \frac{2}{3}\Omega_H)^{-2} \\ &\approx \epsilon \left[(T+\overline{T})^{-2} + \tfrac{4}{3M_p^2} (T+\overline{T})^{-3} |Q_V|^2 + \tfrac{8}{3M_p^4} (T+\overline{T})^{-4} |Q_V|^2 |Q_H|^2 + \dots \right] , \end{aligned} \tag{39}$$

where $\epsilon = -\frac{16\zeta(3)}{3(4\pi)^2} \frac{1}{(2\pi R M_p)^2}$. We see that the sequesterd structure of eq. (34) is not respected by loop corrections. One finds (here and in the following we assume that field vevs in the hidden brane sector are negligible):

$$m_{Q_0}^2 = 48\epsilon \frac{|F_T|^2}{(T+\overline{T})^5} + 8\epsilon \frac{|F_H|^2}{M_p^2 (T+\overline{T})^4} \,, \tag{40}$$

so at one-loop level both radion and brane-to-brane mediation are present. But, since ϵ is negative in the minimal 5d set-up, this contribution to scalar soft masses squared is negative

and thus gravity cannot be the leading source of supersymmetry breaking mediation. Note also that the anomaly mediation, which is another mechanism operating here, by itself cannot save the day as it generates negative slepton masses.

We move to the 6d case. For simplicity we consider matter only on two of the four fixed points. At tree-level:

$$\Omega_{6d} = \frac{1}{2}\left(T+\overline{T}-2\Omega_V-2\Omega_H\right)^{1/3}\left(S+\overline{S}\right)^{1/3}(\tau+\overline{\tau})^{1/3}\,. \tag{41}$$

It is clear that sequestering is not present in the Kähler potential (41). This is confirmed by a direct computation of the scalar soft masses:

$$\begin{aligned} m^2_{Q_V} &= \frac{1}{3}\frac{|F_S|^2}{(S+\overline{S})^2}+\frac{1}{3}\frac{|F_\tau|^2}{(\tau+\overline{\tau})^2}-\frac{2}{3}\frac{|F_T|^2}{(T+\overline{T})^2}-\frac{4}{3}\frac{|F_H|^2}{M_p^2(T+\overline{T})}\,,\\ m^2_{3/2} &= \frac{1}{3}\left(\frac{|F_T|^2}{(T+\overline{T})^2}+\frac{|F_S|^2}{(S+\overline{S})^2}+\frac{|F_\tau|^2}{(\tau+\overline{\tau})^2}+2\frac{|F_H|^2}{M_p^2(T+\overline{T})}\right)\,. \end{aligned} \tag{42}$$

Thus in 6d tree-level gravity mediation can be present. This result might be a little surprising, as the effective action we derived in Section 3 does not contain any interactions that could mediate supersymmetry breaking. However, the analysis in Section 3 did not include possible non-perturbative effects. In absence of non-perturbative effects the effective superpotential cannot depend on the bulk moduli and the resulting action has a special structure that reflects the higher-dimensional locality. Actually, gravity mediation is absent as long as the superpotential is T independent. The Kähler potential depends on T and the brane superfields Q only via the combination $T+\overline{T}-2\Omega_{\rm branes}$. For $\partial_T W=\partial_Q W=0$, T and Q enter the scalar potential V only in this combination. Once the potential stabilizes T, that is $\partial_T V=0$ at the minimum, for $\Omega_{\rm branes}\sim Q^\dagger Q+\ldots$ we find

$$m^2_Q \sim \partial_Q\partial_{Q^\dagger}V \sim \partial_T V = 0\,. \tag{43}$$

For $\partial_Q W\neq 0$ we could, in general, obtain $m^2_Q\neq 0$, but in such case the mass terms would be supersymmetric. We conclude that gravity mediated supersymmetry breaking does not operate as long as $\partial_T W=0$ and moduli are stabilized. This situation is in fact similar as in the 5d set-up (without large gravity brane kinetic terms). In that case, the T and Q enter the Kähler potential in an analogous combination (see eqs. eq. (34) and eq. (39)) and soft masses appear (at one loop) only for $\partial_T W\neq 0$.

In the 5d case gaugino condensation in the bulk induces a T dependent superpotential. As we discuss in the next section, in the 6d case a T dependent superpotential in the effective 4d supergravity can arise only after including in the 6d action certain higher order operators, whose presence can be motivated by cancellation of gravitational anomalies in a fully consistent set-up. Only in these very special circumstances the 4d effective action may contain gravitational interactions that mediate supersymmetry breaking. In such setup, depending on the relation between the F-terms, the tree-level mediated soft masses can be either positive or negative.

In conclusion, the 4d effective supergravity derived from the 6d minimal model has different property than that derived from 5d. Most importantly, the Kähler potential is

not of the sequestered form. In spite of that the action does not lead to gravity mediation at tree-level, unless 6d action is augmented by certain higher order operators and non-perturbative effects introduce T dependence in the effective superpotential.

We close this section with a more general formula for scalar soft mass terms in the presence of non-zero D -term vev on the hidden brane. From eq. (13), the D-term vev contributes to the action as $\mathcal{L} = -\frac{1}{2}e_4\delta(x_{56})e^{-2\Phi}D^2$. After compactification to 4d on the background (14) we obtain:

$$V_D = \frac{1}{2}D^2\left[(T+\overline{T})/2 - \Omega_V - \Omega_H\right]^{-2} \tag{44}$$

When tuning the cosmological constant to zero it is convenient to eliminate D^2. Then we obtain:

$$\begin{aligned} m^2_{Q_V} &= 4m^2_{3/2} - 2\frac{|F_T|^2}{(T+\overline{T})^2} - \frac{|F_S|^2}{(S+\overline{S})^2} - \frac{|F_\tau|^2}{(\tau+\overline{\tau})^2} - 4\frac{|F_H|^2}{M_p^2(T+\overline{T})}\,, \\ \frac{D^2}{2M_p^2(T/2+\overline{T}/2)^2} &= \left(3m^2_{3/2} - \frac{|F_T|^2}{(T+\overline{T})^2} - \frac{|F_S|^2}{(S+\overline{S})^2} - \frac{|F_\tau|^2}{(\tau+\overline{\tau})^2} - 2\frac{|F_H|^2}{M_p^2(T+\overline{T})}\right). \end{aligned} \tag{45}$$

For $D=0$ we can express $m_{3/2}$ by the F-terms and we recover the previous formula (42).

5 Moduli stabilization

In this section we discuss possible ways of stabilizing moduli in the 6d set-up. This certainly requires going beyond the minimal model we constructed in Section 2. Indeed, in the model with the Kähler potential and the moduli independent superpotential of eq. (26), the scalar potential for the moduli, schematically, is given by

$$V = \frac{F^2}{(S/2+\overline{S}/2)(\tau/2+\overline{\tau}/2)} + \frac{D^2}{2(T/2+\overline{T}/2)^2}\,, \tag{46}$$

which is of the runaway type. To conjure up a stabilization mechanism we have to modify the bulk field content. The simplest way is to add non-abelian vector multiplets in the bulk. Kinetic terms for a bulk gauge field are of the form $\mathcal{L} = -\frac{1}{4}e_6 e^{-\Phi}F_{AB}F^{AB}$ [13, 30]. After compactification on the background (14) it becomes $\mathcal{L} = -\frac{1}{4}\sqrt{-g}Ae^{-\Phi}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4}SF_{\mu\nu}F^{\mu\nu}$. Therefore a bulk vector multiplet corresponds at low energies to a vector multiplets with the gauge kinetic function $f = S$. Furthermore, one loop corrections yield [23] $\Delta f = \frac{b}{4\pi^2}\log\eta(i\tau)$, where b is the beta function coefficient and η denotes the Dedekind function. Gaugino condensation in this sector can be described in 4d supergravity by the effective superpotential $W_{\text{eff}} = \Lambda\eta(i\tau)^{-2}e^{-\frac{8\pi^2}{b}S}$. Note that the presence of the Dedekind η function is necessary to render the superpotential the correct transformation under $SL(2,\mathbb{Z})$ invariance $\tau \to \tau + i$ and $\tau \to 1/\tau$ (bulk dynamics, as opposed to that of branes, respects the $SL(2,\mathbb{Z})$ invariance). We assume that there are two condensing

bulk gauge groups and that a vev of the brane superpotential is negligible. Then the superpotential is of the racetrack [24] form:

$$W = \eta(i\tau)^{-2}\left(\Lambda_1 e^{-a_1 S} - \Lambda_2 e^{-a_2 S}\right) . \tag{47}$$

A model with such superpotential and the Kähler potential eq. (41) stabilizes S, τ at an AdS_4 stationary point. The equations of motion $\partial_S V = 0$, $\partial_\tau V = 0$ are satisfied for S and τ solving $F_S = 0$ and $F_\tau = 0$. We find that $F_S = 0$ for S solving the constraint

$$\Lambda_1 e^{-a_1 S}(1 + a_1(S + \overline{S})) = \Lambda_2 e^{-a_2 S}(1 + a_2(S + \overline{S})) . \tag{48}$$

This fixes both $\mathrm{Re}\, S$ and $\mathrm{Im}\, S$. As is well known, for the racetrack model the beta functions of the two condensing groups have to be similar, $|a_1 - a_2| \ll a_1$, in order for the solution to eq. (48) occur for large $\mathrm{Re}\, S$ where the expression (47) for the superpotential is reliable. $F_\tau = 0$ is solved for τ satisfying:

$$\hat{G}_2(\tau, \overline{\tau}) \equiv -2\pi\left(\partial_\tau \log \eta(i\tau) + \frac{1}{\tau + \overline{\tau}}\right) = 0 . \tag{49}$$

Here $\hat{G}_2$ is the modified Eisenstein series. It has zeros in the fundamental τ domain at the self dual fixed points $\tau = 1$ and $\tau = \sqrt{3}/2 + i/2$. It turns out that the latter solution is a saddle point of the potential while the former is a minimum (in the S, τ plane; in the full potential T is for the time being a runaway direction).

We now discuss mechanisms of stabilizing T. One possibility, that relies solely on perturbative effect, is to assume that some dynamics has generated vevs of D-terms and F-terms of hidden brane fields, which contributes to the moduli potential as in (46). In the presence of such hidden brane vevs the vacuum values of S and τ are slightly shifted, so that F_S and F_τ no longer vanish at the true minimum. After stabilizing S and τ the potential for T is of the form:

$$V/M_p^2 = -\frac{2\hat{m}_{3/2}^2 - |\hat{F}_S|^2 - |\hat{F}_\tau|^2}{(T + \overline{T})} + \frac{2}{M_p^2}|\hat{F}_H|^2 + \frac{2D^2}{(T + \overline{T})^2} \tag{50}$$

where $(\hat{m}_{3/2}, \hat{F}_S, \hat{F}_\tau) = (m_{3/2}, F_S/(S + \overline{S}), F_\tau/(\tau + \overline{\tau}))(T + \overline{T})^{1/2}$ and $\hat{F}_H = F_H(T + \overline{T})^{-1/2}$ are T-independent. Then the solution to $\partial_T V = 0$ with $V = 0$ at the minimum requires

$$\frac{2D^2}{(T + \overline{T})^2} = \frac{2}{M_p^2}|\hat{F}_H|^2 = \frac{\hat{m}_{3/2}^2 - \frac{1}{2}|\hat{F}_S|^2 - \frac{1}{2}|\hat{F}_\tau|^2}{(T + \overline{T})} \tag{51}$$

One of these equations determines $(T + \overline{T})$ and the other is a fine-tuning needed to arrive at $V = 0$ at the minimum. This solution is always a local minimum for $(T + \overline{T})$.

There are, however, two problems with this model of T stabilization. Firstly, the axion $(T - \overline{T})$ is not stabilized. This could be solved if the axion transformed under some gauge symmetry analogously as in ref. [20]. In such case the axion would be eaten by the gauge field. Secondly, since the superpotential in this model is T independent the soft masses

vanish by the argument of the preceding section. Note that this remains true after taking into account the one-loop corrections (33).[2] Therefore in this model gravity mediation cannot be the leading source of supersymmetry breaking in the visible sector and one needs to rely on gaugino mediation [26] or Scherk-Schwarz breaking [18].

Another mechanism of T stabilization relies on including 6d anomaly corrections. As discussed in ref. [20] the Green-Schwarz mechanism of anomaly cancellation in 6d requires extending the 6d supergravity action (5) by certain higher dimensional operators, including the coupling of the form $\mathcal{L} \sim \alpha B \wedge F \wedge F$. At low energies the effect of such coupling can be described by an order α correction to the gauge kinetic function, $f = S + \alpha T$. When gauginos condense, the effective superpotential picks up T dependence. We find that in certain multiple condensates models T is indeed stabilized, with F_T, F_S and F_τ much smaller than $m_{3/2}$. The moduli stabilization yields a negative contribution of order $-3M_p^2 m_{3/2}^2$ to the scalar potential that must be canceled by a contribution from the brane sector. In absence of hidden brane D-term vevs we need F_H of order $m_{3/2}$. In such case, by eq. (42) the gravity mediated soft masses are negative. However if the hidden brane D-term vev is of order $m_{3/2}$ and F_H is negligible then by eq. (45) we arrive at positive scalar soft masses squared. Furthermore, gaugino masses of similar order can be generated by anomaly mediation. Although anomaly mediated gaugino masses are suppressed by a loop factor with respect to the gravitino mass, they can be comparable to the tree-level mediated scalar masses. The reason is that, typically, non-perturbative moduli stabilization results in suppression of the corresponding F-terms with respect to the gravitino mass, and the suppression factor can be comparable to the loop factor $\sim 4\pi^2$. In such case the soft terms acquire comparable contribution from anomaly and gravity mediation, which leads to interesting phenomenology. In fact, a similar pattern of soft terms was recently analyzed (in the framework of KKLT compactification [27]) in refs. [28, 29].

6 Conclusions

In this paper we studied gravity mediated supersymmetry breaking in the 4d effective theory derived from 6d brane-world supergravity compactified on $T_2/\mathbb{Z}_2$. The main motivation for invoking extra dimensions is that they offer an opportunity of constructing a viable and predictable theory of contact terms between the visible and hidden sectors. Realizations of this idea in the settings of 5d brane-world supergravity have proven unsatisfactory. The 5d models yield negative soft scalar masses squared, unless they are augmented with large gravity brane kinetic terms. Therefore it is interesting to go beyond five dimensions.

Using the Noether method we constructed a locally supersymmetric action for a bulk-brane system consisting of the minimal $N = 2$ 6d supergravity and $N = 1$ vector and chiral

[2] It was argued in ref. [9] that there is an ambiguity in the one-loop Kähler potential at order $\epsilon\Omega_{\rm branes}^3$, depending on regularization of the bulk fields behavior near the delta-like branes. If $\Omega_{\rm branes}$ contains a constant (corresponding to brane field vevs or a brane gravity kinetic term), $\Omega_{\rm branes} = L + |Q_V|^2/M_p^2 + \ldots$, then this ambiguity would result in appearance of regularization dependent soft masses of order L^2F^2. This ambiguity can become relevant only when brane vevs or coefficients of brane gravity kinetic terms are bigger than the compactification scale. See also refs. [7, 8], which argue that the one-loop computation is non-ambiguous.

multiplets on codimension-two branes located at the orbifold fixed points. Then we derived an effective action describing physics at energies below the compactification scale. As the coupling of the bulk to the branes turn out to be uniquely fixed by local supersymmetry, the moduli dependence of the low energy effective action (and hence the contact terms) is also fixed.

The Kähler potential of the 4d low energy effective theory is not of the sequestered form. As the consequence, contact terms between the visible and hidden sectors can be present already at tree-level. The soft terms mediated by the bulk moduli originating from the 6d gravity+tensor multiplet are necessarily flavour blind. We find however that, typically, the 4d effective action exhibits similar sequestering properties as that derived from minimal 5d models. More precisely, in 6d tree-level gravity mediation does not occur as long as the effective superpotential is T independent. In order to generate a T dependent superpotential one needs to invoke, simultaneously, higher order operators and non-perturbative physics. Turning to such models, we identified one scenario in which all moduli are stabilized at a vacuum with the vanishing cosmological constant and the gravity mediated soft scalar masses squared are positive. This is an example of a successful embedding of 4d supergravity into a more fundamental set-up such that phenomenologically viable contact terms can be predicted. In this scenario stabilization of the T modulus requires rather contrived physics (multiple gaugino condensates). We believe however that generalizing the 6d bulk-brane set-up (e.g. to flux [20,30,31] or warped [32,33] compactifications) will lead to more appealing scenarios.

It would be interesting to explore the relation of the present work to string theory compactifications [34]. For example, compactification of heterotic string theory on the T_6/Z_4 orbifold [35] should, in an appropriate limit, yield a similar set-up to ours (but of course with a number of additional multiplets). However, to our knowledge, a 4d effective description of 6d supergravity subsector of such orbifold string theories has not been given in the literature.

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A Notation and conventions

We use the mostly minus metric signature $(+,-,-,-,-,-)$. The index conventions are the following: 6d Einstein indices $A,B,C,\ldots = 0\ldots3,5,6$, 6d Lorentz indices $a,b,c,\ldots = 0\ldots3,5,6$, 4d Einstein indices $\mu,\nu,\rho,\ldots = 0\ldots3$, 4d Lorentz indices $m,n,\ldots = 0\ldots3$.

Furthermore $\alpha, \beta, \ldots = 5, 6$. When necessary, we also distinguish Einstein indices with an overdot, e.g. in $B_{\dot{5}\dot{6}}$.

6d gamma matrices denoted by Γ^a satisfy $\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}$. They are 8×8 matrices and the 6d spinors are 8-dimensional. The 4d 4×4 gamma matrices denoted by γ^m satisfy $\{\gamma^m, \gamma^n\} = 2\eta^{mn}$. The convention for γ^5 is $\gamma^5 = \mathrm{diag}(-1, -1, 1, 1)$ and the chirality projection operators are $P_L = (1 - \gamma^5)/2$, $P_R = (1 + \gamma^5)/2$. 4d spinors are always written in the four-component Dirac notation. The 4d charge conjugation matrix $C = i\gamma^0\gamma^2\gamma^5$ satisfies $C^{-1} = C^T = C^\dagger = -C$, $C\gamma^m C^{-1} = (\gamma^m)^T$.

In our choice of basis the connection between the 6d and 4d gamma matrices is given by

$$\Gamma^m = \begin{bmatrix} \gamma^m & 0 \\ 0 & \gamma^m \end{bmatrix}, \quad \Gamma^5 = \begin{bmatrix} 0 & i\gamma^5 \\ i\gamma^5 & 0 \end{bmatrix}, \quad \Gamma^6 = \begin{bmatrix} 0 & \gamma^5 \\ -\gamma^5 & 0 \end{bmatrix}. \tag{A.1}$$

Furthermore $\Gamma^A = \Gamma^a e_a{}^A$, $\gamma^\mu = \gamma^m e_m{}^\mu$. It is also convenient to define 'Lorentz index gravitinos', $\psi_a = e_a{}^A \psi_A$.

The fermions in the 6d supergravity action are chiral. The 6d chirality projectors are $P = (1 \pm \Gamma^7)/2$ where $\Gamma^7 = \mathrm{diag}(-1, -1, 1, 1, 1, 1, -1, -1)$. The gravitino ψ_A and the dilatino χ have opposite 6d chirality. Written in terms of 4d spinors the 6d fermions look as follows

$$\psi_\mu = \begin{bmatrix} \psi_\mu^- \\ \psi_\mu^+ \end{bmatrix}, \quad \psi_\alpha = \begin{bmatrix} \psi_\alpha^+ \\ \psi_\alpha^- \end{bmatrix}, \quad \chi = \begin{bmatrix} \chi^- \\ \chi^+ \end{bmatrix}. \tag{A.2}$$

Here ψ_μ^+, ψ_α^-, χ^- are right-handed chiral (in the 4d sense), $P_R\psi = \psi$, while ψ_μ^-, ψ_α^+, χ^+ are left-handed chiral $P_L\psi = \psi$. In the paper we use the following linear combinations of the extra dimensional components of the (Lorentz index) gravitinos:

$$\psi_{5+6} \equiv -(\psi_5^+ + i\psi_6^+), \qquad \psi_{5-6} \equiv -(\psi_5^+ - i\psi_6^+). \tag{A.3}$$

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