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An "all–poles" approximation to collinear resummations in the Regge limit of perturbative QCD

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Abstract

The procedure to improve the convergence in transverse momentum space of the NLL BFKL kernel using a ω–shift is revisited. An accurate approximation to this shift only depending on transverse momenta is presented. This approximation is based on a Bessel function of the first kind with argument depending on the strong coupling and a double logarithm of the ratio of transverse scales. A comparison between different renormalization schemes is also included.

1 Introduction

In recent years there has been an intense activity trying to understand which are the effective degrees of freedom driving the strong interaction at large energies. For processes where the center–of–mass energy is much larger than the other scales in the scattering an instructive picture emerges from the Balitsky–Fadin–Kuraev–Lipatov (BFKL) approach [1]. The effective picture in this framework relies upon t–channel "Reggeized" gluons interacting with each other via standard gluons in the s–channel. Although this simple structure must be modified at higher energies in order to introduce unitarization corrections, there is a window at present and future colliders where the BFKL predictions should hold.

The leading logarithmic (LL) approximation in BFKL resums terms of the form $(\alpha_s \ln s)^n$ to all orders. In such a limit diagrams contributing to the running of the strong coupling do not appear and α_s remains as a constant free parameter. Also the factor needed to scale the energy in the logarithms is not fixed, therefore the predictability of the LL approximation is limited. The situation improves already in the next–to–leading logarithmic (NLL) approximation

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where diagrams with an extra power in the coupling without introducing an extra logarithm in energy are taken into account. In this case the coupling is allowed to run and the energy scale is fixed.

The calculation of the NLL corrections to the BFKL equation was lengthy and completed in Ref. [2]. It was needed to evaluate corrections to the gluon trajectory [3] and real emission kernel [4], as well as to check that the bootstrap conditions are fulfilled in order to ensure that the complete set of NLL contributions are indeed correctly resummed [5].

Lately, to be able to perform phenomenological studies, the NLL calculation and numerical implementation of particular impact factors, describing the nature of the interaction between the Reggeized gluons and the scattering particles for different physical processes, is a remaining challenge [6].

In the construction of BFKL cross–sections the gluon Green's function is the element carrying the energy dependence and therefore it deserves a careful study. There has been a lot of theoretical work trying to understand its structure which, since the appearance of the NLL kernel, has revealed as quite non–trivial. It was shown in Ref [7] that, in the transverse momentum space, the NLL Green's function presents some instability. The origin of this instability and more convergent kernels including renormalization–group (RG) generated terms were presented in Ref. [8]. Based on this type of RG–improved kernels, including more sophisticated approaches, several studies of the Green's function have been performed in Ref. [9]. Other analysis of different aspects of the NLL kernel can be found in Ref. [10].

The aim of the present work is to revisit the approach of Ref. [8] and to extract the structure in transverse momentum space of the double logarithms there resummed. In order to do so, an approximation to the original ω–shift will be performed. It will be shown how this approximation is an accurate one and the corresponding expression in transverse momentum space very simple. The main feature of the new RG–improved kernel will be that it does not mix transverse with longitudinal momentum components. From the practical point of view it also allows for its straightforward implementation in the method of solution of the NLL BFKL equation proposed in Ref. [11] (for reviews see Ref. [12]).

In Section 2 the NLL BFKL kernel will be presented and its representation in γ–space will be introduced. The particular RG–based scheme under investigation will be described at the end of this section. In Section 3 the approximated solution to this scheme will be derived and an analysis of its accuracy in the minimal substraction ($\overline{\rm MS}$) renormalization scheme will be shown. In Section 4 the corresponding RG–improved kernel is derived in transverse momentum space and its asymptotic behaviour in terms of the ratio of transverse scales discussed. Section 5 is a brief one analysing the effect of changing the renormalization to the gluon–bremsstrahlung (GB) scheme. Finally, the Conclusions are presented.

2 NLL BFKL kernel and modifications based on the renormalization group

The starting point of this analysis is the following representation of the NLL BFKL kernel acting on a smooth function [2]:

$$\int d^2\vec{q}_2\,\mathcal{K}\left(\vec{q}_1,\vec{q}_2\right) f\left(q_2^2\right) = \tag{1}$$
$$\int \frac{d^2\vec{q}_2}{|q_1^2-q_2^2|}\left\{\left[\bar{\alpha}_s+\bar{\alpha}_s^2\left(\mathcal{S}-\frac{\beta_0}{4N_c}\ln\left(\frac{\left|q_1^2-q_2^2\right|^2}{\max\left(q_1^2,q_2^2\right)\mu^2}\right)\right)\right]\right.$$
$$\left.\times\left(f\left(q_2^2\right)-2\frac{\min\left(q_1^2,q_2^2\right)}{\left(q_1^2+q_2^2\right)}f\left(q_1^2\right)\right)-\frac{\bar{\alpha}_s^2}{4}\left(\mathcal{T}\left(q_1^2,q_2^2\right)+\ln^2\left(\frac{q_1^2}{q_2^2}\right)\right)f\left(q_2^2\right)\right\},$$

where $\bar{\alpha}_s \equiv g_\mu^2 N_c/(4\pi^2)$, $\beta_0 = \left(11N_c-2n_f\right)/3$, $\mathcal{S} = \left(4-\pi^2+5\beta_0/N_c\right)/12$, and the function $\mathcal{T}\left(q_1^2,q_2^2\right)$ can be obtained from Ref. [2]. As it stands this expression is written in the $\overline{\rm MS}$ renormalisation scheme. Alternatively, as in other calculations where a resummation of soft gluons is involved, the GB scheme [13] can also be used:

$$\bar{\alpha}_s^{\rm GB} \;=\; \bar{\alpha}_s\left(1+\mathcal{S}\,\bar{\alpha}_s\right). \tag{2}$$

Both schemes will be under analysis in this work.

As it is well–known the collinear structure of this kernel can be extracted by projecting it on the modified LL eigenfunctions $\left(\bar{\alpha}_s\left(q^2\right)/\bar{\alpha}_s(\mu^2)\right)^{-1/2} q^{2(\gamma-1)}$, *i.e.*

$$\int d^2\vec{q}_2\,\mathcal{K}\left(\vec{q}_1,\vec{q}_2\right)\left(\frac{\bar{\alpha}_s\left(q_2^2\right)}{\bar{\alpha}_s\left(q_1^2\right)}\right)^{-\frac{1}{2}}\left(\frac{q_2^2}{q_1^2}\right)^{\gamma-1} = \bar{\alpha}_s\left(q_1^2\right)\chi_0\left(\gamma\right)+\bar{\alpha}_s^2\chi_1\left(\gamma\right). \tag{3}$$

Note that the prefactor $\left(\bar{\alpha}_s\left(q_2^2\right)/\bar{\alpha}_s(q_1^2)\right)^{-1/2} \simeq 1+\bar{\alpha}_s\left(q_1^2\right)\left(\beta_0/8N_c\right)\ln\left(q_2^2/q_1^2\right)$ generates a NLL term which makes the γ–representation of the kernel invariant under the $\gamma\to 1-\gamma$ transformation [2].

Although $\bar{\alpha}_s\left(q_1^2\right) \simeq \bar{\alpha}_s\left(\mu^2\right)\left(1-\bar{\alpha}_s\left(\mu^2\right)\frac{\beta_0}{4N_c}\ln\frac{q_1^2}{\mu^2}\right)$ breaks the scale invariance, for the collinear analysis of the kernel it is enough to work with the scale invariant pieces which, at LL and NLL accuracy, read

$$\chi_0\left(\gamma\right) \;=\; 2\psi(1)-\psi\left(\gamma\right)-\psi\left(1-\gamma\right), \tag{4}$$

$$\chi_1\left(\gamma\right) \;=\; \mathcal{S}\chi_0\left(\gamma\right)+\frac{1}{4}\left(\psi''\left(\gamma\right)+\psi''\left(1-\gamma\right)\right)-\frac{1}{4}\left(\phi\left(\gamma\right)+\phi\left(1-\gamma\right)\right) \tag{5}$$
$$-\frac{\pi^2\cos\left(\pi\gamma\right)}{4\sin^2(\pi\gamma)(1-2\gamma)}\left(3+\left(1+\frac{n_f}{N_c^3}\right)\frac{\left(2+3\gamma(1-\gamma)\right)}{(3-2\gamma)(1+2\gamma)}\right)+\frac{3}{2}\zeta_3-\frac{\beta_0}{8N_c}\chi_0^2\left(\gamma\right).$$

All these expressions are re–written here just to set the notation. $\psi(\gamma) = \Gamma'(\gamma)/\Gamma(\gamma)$ and

$$\phi(\gamma) + \phi(1-\gamma) = \sum_{m=0}^{\infty} \left(\frac{1}{\gamma+m} + \frac{1}{1-\gamma+m} \right) \left(\psi' \left(\frac{2+m}{2} \right) - \psi' \left(\frac{1+m}{2} \right) \right). \quad (6)$$

For future reference it is important to remark that the terms with second derivatives in the ψ function directly stem from the double logarithm in Eq. (1). The pole structure of this kernel around $\gamma = 0, 1$ is as follows:

$$\chi_0(\gamma) \simeq \frac{1}{\gamma} + \{\gamma \to 1-\gamma\}, \quad (7)$$

$$\chi_1(\gamma) \simeq \frac{\mathrm{a}}{\gamma} + \frac{\mathrm{b}}{\gamma^2} - \frac{1}{2\gamma^3} + \{\gamma \to 1-\gamma\}. \quad (8)$$

The cubic poles come directly from ψ'' and

$$\mathrm{a} = \frac{5}{12}\frac{\beta_0}{N_c} - \frac{13}{36}\frac{n_f}{N_c^3} - \frac{55}{36}, \quad \mathrm{b} = -\frac{1}{8}\frac{\beta_0}{N_c} - \frac{n_f}{6N_c^3} - \frac{11}{12}. \quad (9)$$

Throughout this paper for numerical analysis the value $n_f = 3$ will be taken.

In this γ–space representation it is known that the cubic poles do compensate for the equivalent terms appearing when the symmetric Regge–like energy scale $s_0 = q_1 q_2$ is shifted to the $s_0 = q_{1,2}^2$ choice in DIS–like processes. This compensation takes place at NLL, the accuracy at which the BFKL kernel is known. It turns out that higher order terms beyond NLL, not compatible with RG evolution, are also generated by this change of scale. The NLL truncation of the perturbative expansion is then the reason why the gluon Green's function in the BFKL formalism develops oscillations in the q_1^2/q_2^2 ratio when this ratio is very far from unity. Along these oscillations the Green's function can have negative values.

At present collider energies it remains to be seen how important these oscillations are when a full NLL BFKL–resummed calculation of a physical cross section, including impact factors, is carried out. Nevertheless there have been attempts in the literature to improve the behaviour of the BFKL resummation in the q_1^2/q_2^2 variable. One of the original proposals, the subject of the present analysis, shows how it is possible to remove the most dominant poles in γ–space incompatible with RG evolution by simply shifting the ω–pole present in the BFKL scale invariant eigenfunction. This shift has to be performed with care not to double count terms and to match the NLL accuracy of the original BFKL calculation. In the present study the focus will be on the scheme proposed in Ref. [8]:

$$\omega = \bar{\alpha}_s \left(1 + \left(\mathrm{a} + \frac{\pi^2}{6} \right) \bar{\alpha}_s \right) \left(2\psi(1) - \psi\left(\gamma + \frac{\omega}{2} - \mathrm{b}\,\bar{\alpha}_s \right) - \psi\left(1 - \gamma + \frac{\omega}{2} - \mathrm{b}\,\bar{\alpha}_s \right) \right)$$
$$+ \bar{\alpha}_s^2 \left(\chi_1(\gamma) + \left(\frac{1}{2}\chi_0(\gamma) - \mathrm{b} \right) (\psi'(\gamma) + \psi'(1-\gamma)) - \left(\mathrm{a} + \frac{\pi^2}{6} \right) \chi_0(\gamma) \right). \quad (10)$$

By iterating this expression new terms beyond the original BFKL calculation appear improving the convergence in q_1^2/q_2^2 of the expansion. It is worth noting that the shift immediately resums these new terms to all orders in the coupling. Therefore, any attempt to modify this approach should retain this feature.

3 "All–poles" resummation

The main idea underlying the present analysis is that the numerical solution to the equation

$$\omega = \bar{\alpha}_s \left(2\psi(1) - \psi\left(\gamma + \frac{\omega}{2}\right) - \psi\left(1 - \gamma + \frac{\omega}{2}\right)\right) \tag{11}$$

can be approximated remarkably well by the all–orders resummed expression

$$\omega = \int_0^1 \frac{dx}{1-x}\left\{\left(x^{\gamma-1} + x^{-\gamma}\right)\sqrt{\frac{2\bar{\alpha}_s}{\ln^2 x}} J_1\left(\sqrt{2\bar{\alpha}_s \ln^2 x}\right) - 2\bar{\alpha}_s\right\}, \tag{12}$$

where $J_1(z)$ is the Bessel function of the first kind.

Before showing the calculations leading to this formula the expression in Eq. (12) is compared to the numerical solution of Eq. (11) in Fig. 1. The values corresponding to the LL BFKL kernel are also included in the plot. The ω–shift removes the poles at $\gamma = 0, 1$ and affects the saddle point region by reducing the LL intercept from about 0.55 at a fixed coupling of $\bar{\alpha}_s = 0.2$ to a value close to 0.31. The approximated value obtained from the expression containing $J_1(z)$ is 0.35. It will be shown below how this approximation to the shift is even more accurate when the full NLL corrections are included. On the last plot of Fig. 1 one can see that the approximation is a stable one under variations of the strong coupling values.

In the following the full NLL scale invariant terms are taken into account. In general one can consider the ω–shift of the form

$$\omega = \bar{\alpha}_s \left(1 + \mathrm{A}\,\bar{\alpha}_s\right)\left(2\psi(1) - \psi\left(\gamma + \frac{\omega}{2} + \mathrm{B}\,\bar{\alpha}_s\right) - \psi\left(1 - \gamma + \frac{\omega}{2} + \mathrm{B}\,\bar{\alpha}_s\right)\right). \tag{13}$$

It is convenient to use the following representation of the LL eigenvalue of the BFKL kernel in Eq. (4):

$$\chi_0(\gamma) = \sum_{m=0}^{\infty}\left(\frac{1}{\gamma + m} + \frac{1}{1 - \gamma + m} - \frac{2}{m+1}\right). \tag{14}$$

This representation reveals the singularity structure of $\chi_0(\gamma)$ which has an infinite number of poles at the $\gamma = -m, 1 + m$ points along the real axis. In this way the general shift can be written as

$$\omega = \tag{15}$$
$$\bar{\alpha}_s \left(1 + \mathrm{A}\bar{\alpha}_s\right) \sum_{m=0}^{\infty}\left(\frac{1}{\gamma + m + \frac{\omega}{2} + \mathrm{B}\,\bar{\alpha}_s} + \frac{1}{1 - \gamma + m + \frac{\omega}{2} + \mathrm{B}\,\bar{\alpha}_s} - \frac{2}{m+1}\right).$$

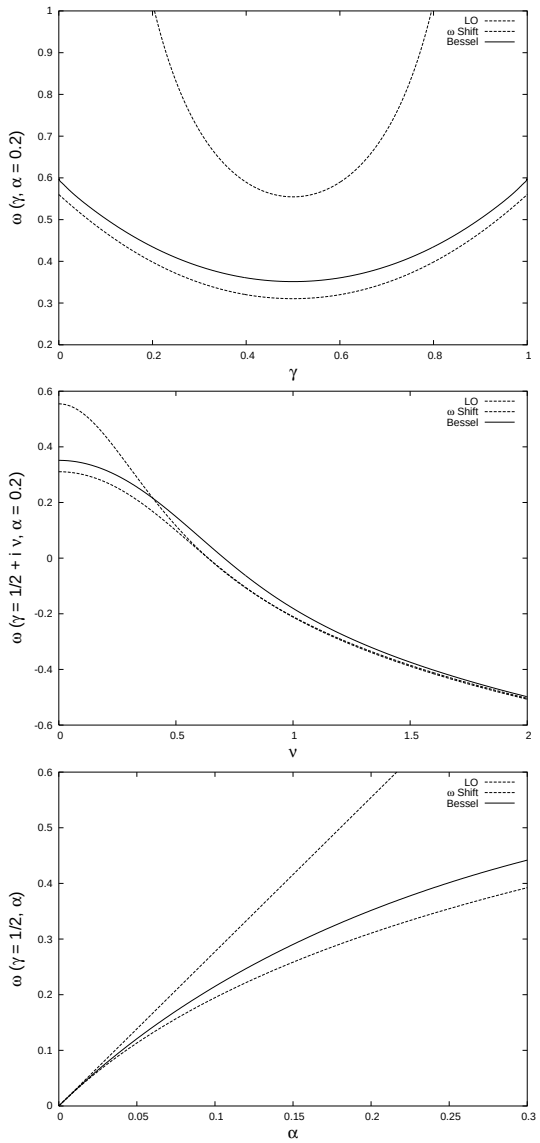

Figure 1: Behaviour of the γ–representation of the LL BFKL kernel compared to the ω–shift of Eq. (11). For a fixed coupling of $\bar{\alpha}_s = 0.2$ on the top and middle plots around the saddle point, and at the dominant region $\gamma = 1/2$ for different perturbative values of the coupling (bottom). The approximation to the shift using the Bessel function resummation as in Eq. (12) is also shown in the plots.

In those regions close to the $\gamma \simeq -m$ poles a good approximation to Eq. (15) is

$$\omega \simeq \frac{\bar{\alpha}_s\,(1+\mathrm{A}\bar{\alpha}_s)}{\gamma+m+\frac{\omega}{2}+\mathrm{B}\,\bar{\alpha}_s}, \tag{16}$$

with the straightforward solution

$$\omega = -(\gamma+m+\mathrm{B}\,\bar{\alpha}_s)+|\gamma+m+\mathrm{B}\,\bar{\alpha}_s|\left(1+\frac{2\bar{\alpha}_s\,(1+\mathrm{A}\bar{\alpha}_s)}{(\gamma+m+\mathrm{B}\,\bar{\alpha}_s)^2}\right)^{\frac{1}{2}}. \tag{17}$$

The same logic applies to the regions around the $\gamma \simeq 1+m$ poles. In the following it will be shown how, to a very good accuracy, the solution to the shift in Eq. (15) can be obtained by simply adding all the approximated solutions at the different poles plus a term related to the virtual contributions of the LL BFKL kernel, *i.e.*

$$\begin{aligned}\omega = \sum_{m=0}^{\infty}\Bigg\{&-(1+2m+2\,\mathrm{B}\,\bar{\alpha}_s)+|\gamma+m+\mathrm{B}\,\bar{\alpha}_s|\left(1+\frac{2\bar{\alpha}_s\,(1+\mathrm{A}\bar{\alpha}_s)}{(\gamma+m+\mathrm{B}\,\bar{\alpha}_s)^2}\right)^{\frac{1}{2}}\\ &+|1-\gamma+m+\mathrm{B}\,\bar{\alpha}_s|\left(1+\frac{2\bar{\alpha}_s\,(1+\mathrm{A}\bar{\alpha}_s)}{(1-\gamma+m+\mathrm{B}\,\bar{\alpha}_s)^2}\right)^{\frac{1}{2}}-\frac{2\bar{\alpha}_s\,(1+\mathrm{A}\bar{\alpha}_s)}{m+1}\Bigg\}.\end{aligned} \tag{18}$$

From now on this expression will be indicated as the "all–poles" approximation. In the region of interest, $0<\gamma<1$, Eq. (18) simplifies because

$$|\gamma+m+\mathrm{B}\,\bar{\alpha}_s| = \gamma+m+\mathrm{B}\,\bar{\alpha}_s, \tag{19}$$

$$|1-\gamma+m+\mathrm{B}\,\bar{\alpha}_s| = 1-\gamma+m+\mathrm{B}\,\bar{\alpha}_s, \tag{20}$$

and one can also introduce the all–orders expansion of the square roots:

$$\begin{aligned}\omega = \sum_{m=0}^{\infty}\Bigg\{&-\frac{2\bar{\alpha}_s\,(1+\mathrm{A}\bar{\alpha}_s)}{m+1}\\ &+\left(\sum_{n=0}^{\infty}\frac{(-1)^n(2n)!}{2^n n!(n+1)!}\frac{\left(\bar{\alpha}_s+\mathrm{A}\bar{\alpha}_s^2\right)^{n+1}}{(\gamma+m+\mathrm{B}\,\bar{\alpha}_s)^{2n+1}}+\{\gamma\to 1-\gamma\}\right)\Bigg\}.\end{aligned} \tag{21}$$

At the $\gamma=0,1$ poles this expansion generates the NLL terms:

$$\omega \simeq \frac{\bar{\alpha}_s}{\gamma}+\bar{\alpha}_s^2\left(\frac{\mathrm{A}}{\gamma}-\frac{\mathrm{B}}{\gamma^2}-\frac{1}{2\gamma^3}\right)+\{\gamma\to 1-\gamma\}. \tag{22}$$

Therefore, to match the original kernel at NLL, it is needed to set $\mathrm{A}=\mathrm{a}$ and $\mathrm{B}=-\mathrm{b}$ from Eq. (9).

To demonstrate that this expansion improves the convergence of the BFKL calculation in the same way as the original ω–shift, it is needed to include the

full NLL scale invariant kernel without double counting terms, *i.e.*

$$\begin{aligned}\omega &= \bar{\alpha}_s\chi_0(\gamma) + \bar{\alpha}_s^2\chi_1(\gamma) \qquad (23)\\ &+ \left\{\sum_{m=0}^{\infty}\left[\left(\sum_{n=0}^{\infty}\frac{(-1)^n(2n)!}{2^n n!(n+1)!}\frac{\left(\bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2\right)^{n+1}}{\left(\gamma+m-\mathrm{b}\,\bar{\alpha}_s\right)^{2n+1}}\right)\right.\right.\\ &\left.\left. -\frac{\bar{\alpha}_s}{\gamma+m} - \bar{\alpha}_s^2\left(\frac{\mathrm{a}}{\gamma+m}+\frac{\mathrm{b}}{(\gamma+m)^2}-\frac{1}{2(\gamma+m)^3}\right)\right] + \{\gamma\to 1-\gamma\}\right\}.\end{aligned}$$

In this way, as in the original ω–shift, the NLL results are untouched and only higher order terms are added to the kernel. The extra terms being resummed to all orders in $\bar{\alpha}_s$ will be important to find a closed form for the kernel with corresponding eigenvalues as in Eq. (23).

It turns out that when the matching to NLL is performed the "all–poles" result reproduces the ω–shift very accurately. This can be seen in Fig. 2 where the full ω–shift and the "all–poles" kernel are compared, together with the LL and NLL results which are also included. The fact that the imaginary part of γ at the maximum of the NLL scale invariant eigenvalue (middle plot of Fig. 2) is not zero results in the oscillations in the q_1^2/q_2^2 variable. This feature is removed when the RG–improved kernel is used, as it also happens for the "all–poles" kernel. The estimated intercept from the ω–shift is of about 0.27, while that coming from the "all–poles" kernel is 0.26 (for $\bar{\alpha}_s = 0.2$). The similarity between the "all–poles" kernel and the original ω–shift remains for different values of the coupling (see bottom plot of Fig. 2).

4 Bessel representation in transverse momentum space

The most striking feature of Eq. (23) is that the ω–space is decoupled from the γ–representation, *i.e.* they no longer mix as it occurs in the original ω–shift. This has the implication that it should be possible to find an expression for a RG–improved BFKL kernel which does not mix longitudinal with transverse degrees of freedom. To find such a kernel it is convenient to define the following function related to a simple representation of the LL kernel:

$$\begin{aligned}\Omega(\gamma) &\equiv \int_0^\infty \frac{dq^2}{|k^2-q^2|}\left(\left(\frac{q^2}{k^2}\right)^{\gamma-1-\mathrm{b}\bar{\alpha}_s\frac{|k-q|}{k-q}} - 2\frac{\min\left(k^2,q^2\right)}{k^2+q^2}\right) \qquad (24)\\ &= \int_0^1 \frac{dx}{1-x}\left(x^{\gamma-1-\bar{\alpha}_s\mathrm{b}} + x^{-\gamma-\bar{\alpha}_s\mathrm{b}} - 2\right)\\ &= 2\psi(1) - \psi(\gamma-\bar{\alpha}_s\mathrm{b}) - \psi(1-\gamma-\bar{\alpha}_s\mathrm{b})\\ &= \sum_{m=0}^{\infty}\left(\frac{1}{\gamma+m-\mathrm{b}\,\bar{\alpha}_s} + \frac{1}{1-\gamma+m-\mathrm{b}\,\bar{\alpha}_s} - \frac{2}{m+1}\right).\end{aligned}$$

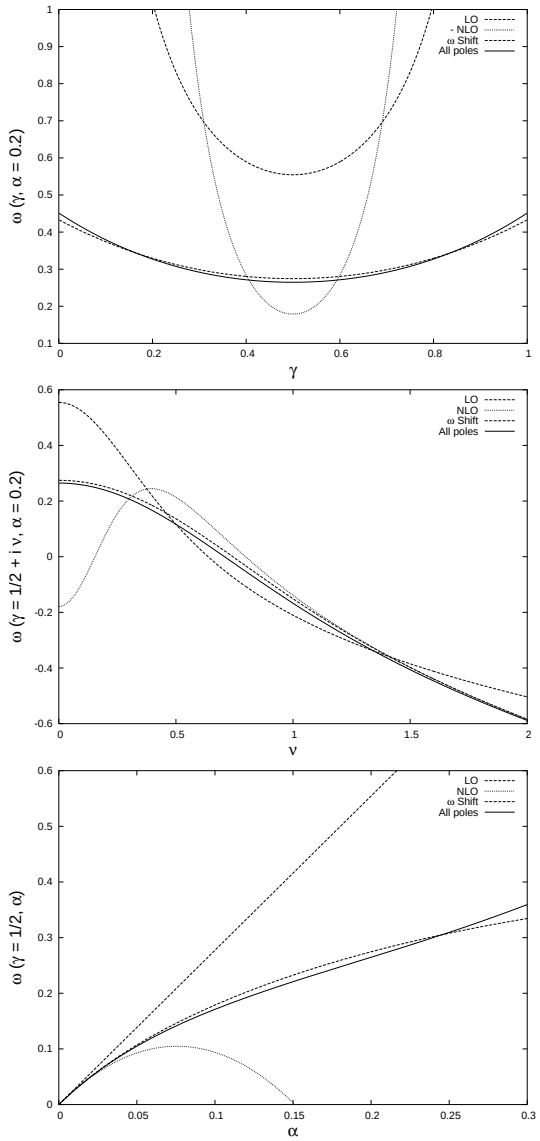

Figure 2: The γ–representation of the LL and NLL scale invariant kernels showing the instable behaviour of the last one. The RG–improved kernel by a ω–shift is also included, together with the "all–poles" approximation proposed in the text.

Keeping in mind the coefficients of the expansion in Eq. (23) the following summation can be constructed:

$$\sum_{n=1}^{\infty} \frac{(-1)^n \left(\bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2\right)^{n+1}}{2^n n!\,(n+1)!} \frac{d^{2n}}{d\gamma^{2n}} \Omega(\gamma) \quad = \tag{25}$$

$$\int_0^{\infty} \frac{dq^2}{|k^2-q^2|} \left(\frac{q^2}{k^2}\right)^{\gamma - 1 - \mathrm{b}\bar{\alpha}_s \frac{|k-q|}{k-q}} \sum_{n=1}^{\infty} \frac{(-1)^n \left(\bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2\right)^{n+1}}{2^n n!\,(n+1)!} \ln^{2n}\left(\frac{q^2}{k^2}\right)$$

$$= \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{2^n n!\,(n+1)!} \frac{\left(\bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2\right)^{n+1}}{\left(\gamma + m - \mathrm{b}\,\bar{\alpha}_s\right)^{2n+1}} + \{\gamma \to 1-\gamma\}\,.$$

In order to include the $n = 0$ case it is needed to add a term corresponding to the virtual contributions in the LL BFKL kernel, ensuring, in this way, the infrared finiteness of the final result, *i.e.*

$$\int_0^{\infty} \frac{dq^2}{|k^2-q^2|} \left(\left(\frac{q^2}{k^2}\right)^{\gamma - 1 - \mathrm{b}\bar{\alpha}_s \frac{|k-q|}{k-q}} \sum_{n=0}^{\infty} \frac{(-1)^n \left(\bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2\right)^{n+1}}{2^n n!\,(n+1)!} \ln^{2n}\left(\frac{q^2}{k^2}\right) \right.$$

$$\left. -2\left(\bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2\right) \frac{\min(k^2,q^2)}{k^2+q^2} \right) \quad = \quad -\sum_{m=0}^{\infty} \frac{2\left(\bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2\right)}{m+1}$$

$$+ \left(\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{2^n n!\,(n+1)!} \frac{\left(\bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2\right)^{n+1}}{\left(\gamma + m - \mathrm{b}\,\bar{\alpha}_s\right)^{2n+1}} + \{\gamma \to 1-\gamma\} \right). \tag{26}$$

By introducing this extra $n = 0$ piece it is then possible to resum the expansion in terms of a Bessel function and Eq. (26) is equivalent to

$$\int_0^{\infty} \frac{dq^2}{|k^2-q^2|} \left(-2\left(\bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2\right) \frac{\min(k^2,q^2)}{k^2+q^2} \right. \tag{27}$$

$$\left. + \left(\frac{q^2}{k^2}\right)^{\gamma - 1 - \mathrm{b}\bar{\alpha}_s \frac{|k-q|}{k-q}} \sqrt{\frac{2\left(\bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2\right)}{\ln^2\left(\frac{q^2}{k^2}\right)}} J_1\left(\sqrt{2\left(\bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2\right)\ln^2\left(\frac{q^2}{k^2}\right)}\right) \right).$$

In order to match this expression to the NLL BFKL formulation it is sufficient to perturbatively expand Eq. (27) up to order $\bar{\alpha}_s^2$:

$$\left(\frac{q^2}{k^2}\right)^{-\mathrm{b}\bar{\alpha}_s \frac{|k-q|}{k-q}} \sqrt{\frac{2\left(\bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2\right)}{\ln^2\left(\frac{q^2}{k^2}\right)}} J_1\left(\sqrt{2\left(\bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2\right)\ln^2\left(\frac{q^2}{k^2}\right)}\right) \tag{28}$$

$$\simeq \bar{\alpha}_s + \mathrm{a}\,\bar{\alpha}_s^2 - \frac{\bar{\alpha}_s^2}{4}\ln^2\left(\frac{q^2}{k^2}\right) - \mathrm{b}\,\bar{\alpha}_s^2 \frac{|k-q|}{k-q}\ln\left(\frac{q^2}{k^2}\right) + \ldots$$

It is important to note that this expression does not depend on the angle between the $\vec{q}$ and $\vec{k}$ two–dimensional vectors, therefore this scale invariant resummation can be directly added to the original NLL BFKL kernel without having to

angular average. The "all–poles" approximation only affects the zero conformal spin sector of the theory. This is particularly interesting for the study of final states where angular correlations are relevant. Moreover, these new higher order terms do not affect the cancellation of infrared divergences in the original kernel.

It is also noteworthy to point out how the perturbative expansion of the "all–poles" kernel naturally generates the double logarithm already present in the original BFKL kernel (see Eq. (1)). What this resummation then achieves (as in the original ω–shift) is to resum to all orders those most dominant transverse logarithms which will make the expansion compatible with renormalization group evolution in the collinear and anticollinear limits relevant in DIS–like configurations.

For the sake of clarity, the only modification needed in the full NLL kernel to introduce the "all–poles" resummation is to remove the term

$$-\frac{\bar{\alpha}_s^2}{4}\frac{1}{(\vec{q}-\vec{k})^2}\ln^2\left(\frac{q^2}{k^2}\right) \tag{29}$$

in the real emission kernel, $\mathcal{K}_r\left(\vec{q},\vec{k}\right)$, and replace it with

$$\frac{1}{(\vec{q}-\vec{k})^2}\left\{\left(\frac{q^2}{k^2}\right)^{-\mathrm{b}\bar{\alpha}_s\frac{|k-q|}{k-q}}\sqrt{\frac{2\left(\bar{\alpha}_s+\mathrm{a}\,\bar{\alpha}_s^2\right)}{\ln^2\left(\frac{q^2}{k^2}\right)}}J_1\left(\sqrt{2\left(\bar{\alpha}_s+\mathrm{a}\,\bar{\alpha}_s^2\right)\ln^2\left(\frac{q^2}{k^2}\right)}\right)\right.$$
$$\left.-\bar{\alpha}_s-\mathrm{a}\,\bar{\alpha}_s^2+\mathrm{b}\,\bar{\alpha}_s^2\frac{|k-q|}{k-q}\ln\left(\frac{q^2}{k^2}\right)\right\}. \tag{30}$$

From the asymptotic behaviour of the Bessel function, when the coupling is small and the difference between the q^2 and k^2 scales is not very large then

$$J_1\left(\sqrt{2\bar{\alpha}_s\ln^2\left(\frac{q^2}{k^2}\right)}\right) \simeq \sqrt{\frac{\bar{\alpha}_s}{2}\ln^2\left(\frac{q^2}{k^2}\right)}, \tag{31}$$

and the influence of the "all–poles" resummation is minimal. Therefore the "Regge–like" region in momentum space is not largely affected. But when the logarithm in the ratio of transverse momenta becomes larger due to a larger difference between the scales entering the real production vertex, then the asymptotic behaviour develops the oscillatory form

$$J_1\left(\sqrt{2\bar{\alpha}_s\ln^2\left(\frac{q^2}{k^2}\right)}\right) \simeq \left(\frac{2}{\pi^2\bar{\alpha}_s\ln^2\left(\frac{q^2}{k^2}\right)}\right)^{\frac{1}{4}}\cos\left(\sqrt{2\bar{\alpha}_s\ln^2\left(\frac{q^2}{k^2}\right)}-\frac{3\pi}{4}\right) \tag{32}$$

automatically compensating for the unphysical oscillations present in the original formulation of the NLL BFKL equation.

5 Change of renormalization scheme

In this section the study of the effect of changing the renormalisation point, and using the gluon–bremsstrahlung (GB) instead of the $\overline{\rm MS}$ scheme, is performed. This is equivalent to the following redefinition of the position of the Landau pole:

$$\Lambda_{\rm GB} \;=\; \Lambda_{\overline{\rm MS}} \exp\left(\mathcal{S}\frac{2N_c}{\beta_0}\right). \tag{33}$$

With this choice the coefficient for the double poles in the NLL kernel does not change while the one for the simple poles now reads

$$\mathrm{a} \;=\; \frac{\pi^2}{12} - \frac{13}{36}\frac{n_f}{N_c^3} - \frac{67}{36}. \tag{34}$$

In Fig. 3 this GB scheme is under analysis. There it is shown how again the "all–poles" kernel is a very good approximation to the ω–shift. With a $\overline{\rm MS}$ value of $\bar{\alpha}_s = 0.2$ the corresponding coupling in GB scheme is $\bar{\alpha}_s^{\rm GB} \simeq 0.23$. The value of the intercept at LL is of ~ 0.64 while the RG–improved prediction is of ~ 0.29, which coincides with that steming from the "all–poles" analysis. It is clear that the influence of the choice of renormalisation scheme will be larger when the full kernel is introduced and running coupling effects are also taken into account.

As a final remark, if running coupling effects are to be included, it is interesting to note that Eq. (1) corresponds to an angular–averaged representation of the gluon Regge trajectory and the real emission kernel. If this angular integration is not performed then the cancellation of infrared divergencies can be written as

$$\omega_0(q^2,\lambda) + \int \frac{d^2\vec{k}}{\pi k^2}\Gamma_{\rm cusp}(\bar{\alpha}_s(k^2))\theta(k^2-\lambda^2), \tag{35}$$

where the gluon Regge trajectory in this regularization reads

$$\omega_0(q^2,\lambda) \;=\; -\int_{\lambda^2}^{q^2}\frac{dk^2}{k^2}\Gamma_{\rm cusp}(\bar{\alpha}_s(k^2)) + \text{constant}. \tag{36}$$

The constant of integration can be fixed to be $\bar{\alpha}_s^2\frac{3}{2}\zeta_3$ from the first reference in [2] and

$$\Gamma_{\rm cusp}(\bar{\alpha}_s(k^2)) \;=\; \bar{\alpha}_s(k^2) + \bar{\alpha}_s^2\mathcal{S}, \tag{37}$$

is the so–called "cusp anomalous dimension" of Ref. [14]. In principle it is possible to use $\bar{\alpha}_s(k^2) = 4N_c/(\beta_0 \ln k^2/\Lambda^2)$, multiply the NLL equation by the extra logarithm and obtain the gluon Green's function as the solution to a diferential equation in γ–representation. An alternative is to solve the equation as proposed in Ref. [11].

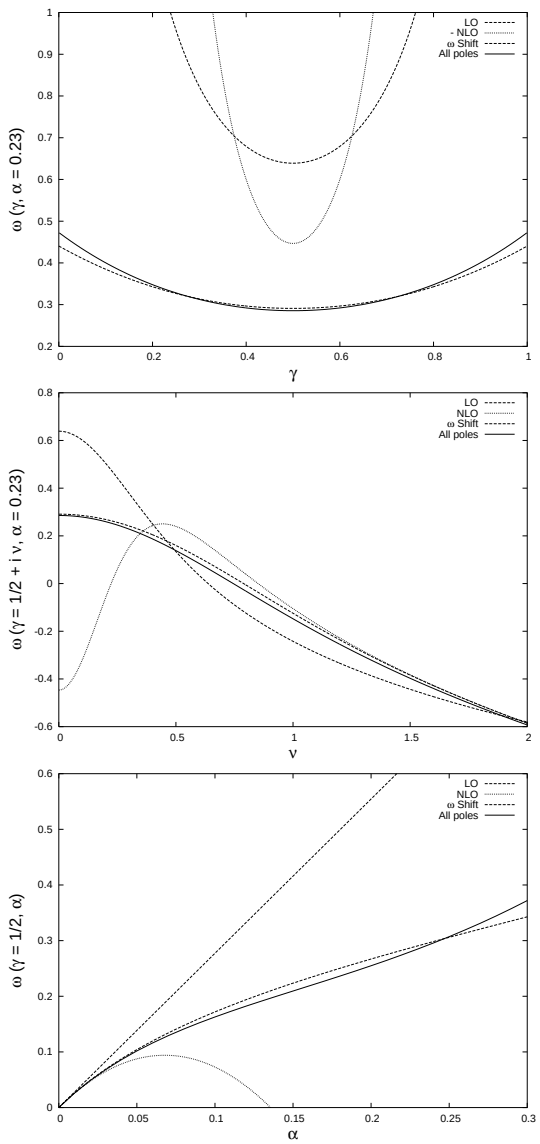

Figure 3: The γ–representation of the LL and NLL scale invariant kernels in the GB scheme. The RG–improved kernel using a ω–shift and the "all–poles" kernel are also included.

One of the targets of the present paper is to show how it is possible to introduce a RG–improved kernel which still allows for an exact solution of the NLL BFKL equation as that proposed in Ref. [11]. The solution there calculated is based on the iteration of the BFKL equation for the partial wave expansion of the Green's function. This iteration has a multiple pole structure in the complex ω–space which allows for a simple inversion back into energy space. This inversion using a Mellin transform would not be possible if an extra ω–dependence in the kernel, as the one in the ω–shift, is present. In this context the prescription proposed here in Eq. (29) and Eq. (30) allows for a straightforward implementation of the RG improvements given that it just modifies the real emission part of the NLL BFKL kernel without introducing any extra ω–dependence. Therefore the iterative procedure used in Ref. [11] remains valid. In particular, if a different choice for the argument in the running coupling is considered the method used in Ref. [11] always provides the exact solution. The study of all these effects at the level of the gluon Green's function will be performed in a coming publication.

6 Conclusions

The original analysis of Ref. [8] for the improvement of the convergence in transverse momentum space of the NLL BFKL kernel is revisited. An intrinsic feature of such renormalization group improved kernels is that the variables Mellin–conjugated of the energy and the transverse momenta, ω and γ respectively, do mix due to the so–called ω–shift. This makes the study of these schemes quite complicated. In the present work it is shown how it is possible to avoid the difficulties of the ω–shift without loosing the physical insight. This is done by approximating the solution to the original equation at each of the poles of the original kernel. The result in γ–representation is that of Eq. (23) named as the "all–poles" resummation. It turns out that the corresponding scale invariant kernel in transverse momentum space with eigenvalue as in Eq. (23) can be found and has a very simple form. Its structure in terms of a Bessel function of the first kind is shown in Eq. (30). The main feature of this kernel is that it only depends on transverse momenta and not on longitudinal degrees of freedom.

This representation of the collinear improved kernels can be implemented immediately using the iterative approach described in Ref. [11]. The advantage of this approach is that it takes into account all physical effects, including the running of the coupling, exactly. It also allows for the study of final states and angular correlations.

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