COSMOLOGICAL ACCELERATION: DARK ENERGY OR MODIFIED GRAVITY?

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(Dated: June 12, 2006)

The recently observed accelerating cosmological expansion or "dark energy" may be either a negative pressure constituent in General Relativity (Dark Energy) or modified gravity (Dark Gravity) without any constituent Dark Energy. Low- or high-curvature modifications of Einstein gravity are distinguished by the spacetime (Ricci) curvature of their vacua. If constituent Dark Energy does not exist, so that our universe is now dominated by pressure-free matter, Einstein gravity must be modified at low- curvature, becoming asymptotically de Sitter.

The dynamics of either kind of "dark energy" cannot be derived from the homogeneous expansion history alone, but requires also observing the growth of inhomogeneities. Present and projected observations are all consistent with a small fine-tuned cosmological constant, but also allow nearly static Dark Energy or gravity modified at cosmological scales. The growth of cosmological fluctuations will potentially distinguish between static and "dynamic "dark energy". But, cosmologically distinguishing dynamic Dark Energy from Dark Gravity will require a weak lensing shear survey more ambitious than any now projected. Dvali-Gabadadze-Porrati low-curvature modifications of Einstein gravity may also be detected in refined observations in the solar system or in isolated galaxy clusters.

We review local and cosmological tests of General Relativity and modified gravity. Dark Energy is epicyclic in character, requires fine-tuning to explain why its energy density is just now comparable to ordinary matter density, and cannot be detected in the laboratory or solar system. This, along with braneworld theories, now motivate searching for Dark Gravity on solar system, galaxy cluster and cosmological scales.

arXiv:astro-ph/0605198 v2 12 Jun 2006

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I. INTRODUCTION: COSMOLOGICAL SYMMETRY VS. DYNAMICS

The greatest mystery in cosmology is the "dark energy" source of the late (redshift $z \lesssim 1/2$) cosmological acceleration. This "dark energy" may be static or dynamic and either an additional negative-pressure matter constituent within General Relativity (Dark Energy), or a modification of General Relativity (Dark Gravity (Gu and Hwang, 2002)). This review rests on the observed global homogeneity and isotropy of the universe (Robertson-Walker cosmology RW), and will emphasize the difference between Robertson-Walker *kinematics* and *dynamics*. We will recall when RW symmetry could determine the cosmodynamics: To explain the observed present cosmological acceleration without constituent Dark Energy, Einstein dynamics must be modified at low spacetime (Ricci) curvature.

Because the homogeneous expansion history $H(z)$ of the global universe measures only kinematic variables, it cannot fix the underlying dynamics: cosmographic measurements of the late accelerating universe, are consistent with either a *static* cosmological constant or a *dynamic* "dark energy", which itself may be constituent Dark Energy or modified gravity (Dark Gravity). The Concordance Model ΛCDM, with a small *static* Dark Energy or cosmological constant Λ, gives a good fit to all present observations, but also allows a moderately dynamic "dark energy" (Spergel et al., 2006). We will review how static and dynamic "dark energy" and how dynamic Dark Energy and Dark Gravity could be distinguished theoretically and by future cosmological, local or intermediate scale observations.

Whether Dark Energy or Dark Gravity, "dark energy" has two distinct dynamical effects: it alters the homogeneous expansion history $H(z)$ and it suppresses the growth of density fluctuations δ at large cosmological scale $a(t)$. Because the growth function $g(z) \equiv \delta/a$ depends on both these effects, large angular scale CMB temperature anisotropies, the late-time growth of large scale structure, and refined weak lensing observations potentially distinguish static from dynamic "dark energy" and Dark Energy from Dark Gravity (Section II).

In any metric theory of gravitation, the material stress-energy sources $T_{\mu\nu}$ determines the spacetime (Ricci) curvature tensor $R_{\mu\nu}$ or Einstein curvature tensor $G_{\mu\nu} \equiv R_{\mu\nu} - g_{\mu\nu}R/2$. Before considering dynamical alternatives, we recall how the asymptotic spacetime curvature, at vanishing matter density, can constrain any RW dynamics, without assuming Einstein gravity: Birkhoff's Theorem is a *geometric* theorem, holding in *any* spherically symmetric metric geometry whose vacuum has vanishing spacetime (Ricci) scalar curvature (C. Callan and Peebles, 1965; Peebles, 1980). Applied to any spatially homogeneous RW universe, Birkhoff's Theorem asserts that local Newtonian gravity fixes the global dynamics of any matter-dominated universe whose vacuum is Ricci-flat (C. Callan and Peebles, 1965; McCrea and Milne, 1934; Milne, 1934; Weinberg, 1972). The Ricci curvature of the vacuum, or cosmological constant, distinguishes high- from low-curvature alternatives to Einstein's original gravity theory (Section III.A). High-curvature modifications (such as (Arkani-Hamad et al., 1998; Randall and Sundrum, 1999; Binutray et al., 2000)) require sub-millimeter corrections to Newton's inverse-square gravity; low-curvature modifications (such as ΛCDM, Dvali-Gabadadze-Porrati (DGP) (Dvali et al., 2000; Deffayet, 2001) theories) can preserve Newtonian gravity locally, but must be asymptotically dominated by a cosmological constant. Without Dark Energy, our accelerating universe is matter-dominated, and Einstein gravity needs low-curvature modification.

Section III also emphasizes that contrived (epicyclic) dynamic Dark Energy can explain the present acceleration, but still cannot explain the Cosmic Coincidence ("Why so small now?"), without fine tuning or anthropic reasoning. This will lead us, in Section V, to consider the Dark Gravity dynamical alternatives to Dark Energy.

Section IV reviews how Dark Energy or Dark Gravity dynamics determines the adiabatic and the effective sound speeds, which govern the growth of fluctuations. To illustrate how different dynamics and effective sound speeds can underly the same equation of state, we compare canonical (quintessence) and non-canonical (k-essence) scalar field descriptions of Chaplygin gas Dark Energy .

For a truly static cosmological constant, we revert to Einstein's original definition as an intrinsic *geometric* classical parameter. By disconnecting the cosmological constant from energy-momentum sources, we side-step the mysteries of why quantum vacuum fluctuations apparently do not gravitate and why the present matter density is rougly equal to the present "dark energy" density. We review present local and cosmological constraints on General Relativity, before proceeding to prospective cosmological, solar system and isolated galaxy cluster tests for modified gravity. We emphasize the difficulties coming tests of relativistic cosmology face: The next decade may distinguish static from dynamic "dark energy", but will still not distinguish constituent Dark Energy from Dark Gravity (Ishak et al., 2005). Besides cosmological tests, low-curvature modifications of Einstein gravity may yet be tested in the solar system (anomalous orbital precession, increasing Astronomical unit) or in any isolated rich cluster of galaxies (Lue and Starkman, 2003; Iorio, 2005a,b) (Section V).

In conclusion, cosmological scale modifications of classical Einstein gravity are less contrived than fine-tuned Dark Energy and arise naturally in braneworld cosmology. By making intrinsic curvature the source of cosmological acceleration, Dark Gravity avoids an additional epicyclic matter constituent, may unify early and late inflation, and may be refuted by laboratory, solar system, or galaxy cluster (Section VI).

TABLE I Kinematic observables for any RW geometry, in terms of Hubble expansion rate $H \equiv \dot{a}/a$.

Description	Definition
Hubble horizon	$1/\mathcal{H} \equiv 1/aH = d\eta/dN$
bulk expansion	$da^3/a^3 = 3dN = 3Hdt = 3\mathcal{H}d\eta$
conformal time since big bang	$\eta(z) \equiv \int_0^t dt'/a(t') = \int_z^\infty dz'/H(z')$
proper motion distance back to redshift z	$d_M(z) = c\int_0^z dz'/H(z') = c(\eta_0 - \eta(z))$
spacetime curvature	$R \equiv 6(\dot{H} + 2H^2) = 6H^2(1+q)$
acceleration	$\ddot{a}/a = H^2 + \dot{H}, \qquad q \equiv \ddot{a}/aH^2 = 1 - dH^{-1}/dt = -d(1/\mathcal{H})/d\eta \equiv 1 - \epsilon_H$
"slow-roll" parameter	$\epsilon_H \equiv dH^{-1}/dt = -d\ln H/dN$
overall "equation of state	$w(z) = d\ln H^2/(1+z)^3/3d\ln(1+z) = -1 + 2\epsilon_H/3$
cosmological jerk	$\dddot{a}/a = H^3 + 3H\dot{H} + \ddot{H}, \qquad j \equiv \dddot{a}/aH^3 = 1 + 3\dot{H}/H^2 + \ddot{H}/H^3$

II. EXPANSION HISTORY H(Z) IN ROBERTSON-WALKER COSMOLOGIES

Our universe is apparently homogeneous and isotropic (Robertson-Walker) in the large. These Robertson-Walker cosmologies are four-dimensional conformally-flat generalizations of General Relativity, in which the spacetime (Ricci) curvature R and the Einstein scalar curvature $G = 3(k/a^2 + H^2)$ are determined by the matter density ρ, according to the gravitational field equations. The homogeneous expansion of our flat Robertson-Walker universe is described by the kinematic (geometric) observables in Table I, wherein the cosmological scale $a(t) = 1/(1+z)$ and the number of e-folds $N \equiv \ln a$, so that $dN = -d\ln(1+z) = Hdt = \mathcal{H}d\eta$. Overhead dots denote derivatives with respect to cosmic time t, so that the conformal Hubble expansion rate $\mathcal{H} \equiv \dot{a}$, the Hubble expansion rate $H \equiv \dot{a}/a$, and the Hubble time $H^{-1} = d\, d_M/cdz$ is the derivative of the conformal time $d_M(z)/c$ back to redshift z. Subscripts 0 denote present values, so that $H_0 = 73 \pm 3\ km/sec/Mpc$, $H_0^{-1} = 13.4 \pm 0.6\ Gyr$, $cH_0^{-1} = 4.11 \pm 0.17\ Gpc$ (Spergel et al., 2006).

By measuring the evolution of the mean curvature of the background, cosmography maps the homogeneously expanding universe, without reference to dynamics or sources of curvature. However, we will see in Section III.C that the asymptotic Ricci curvature or vacuum Ricci curvature $R_\infty \equiv 4\varkappa^2\rho_{DE}(a=\infty)$ does constrain the Robertson-Walker cosmodynamics, distinguishing high- and low-curvature modifications of Einstein gravity. In Einstein gravity,

$$G = \varkappa^2\rho/3, \varkappa^2 \equiv 8\pi G_N \equiv 1/M_P^2,$$

in terms of Newton's constant G_N and the reduced Planck mass M_P. In Einstein's original field equations, $H(t)$ is the only degree of freedom, only the tensor components of the metric $g_{\mu\nu}$ are propagating, and (absent a cosmological constant) the asymptotic or empty space scalar curvature $R_\infty = 0$. When Einstein gravity is modified, $\dot{H} \equiv dH/dt$ and $\ddot{H} \equiv d^2H/dt^2$, or the cosmological acceleration $q(t)$ and jerk $j(t)$ become additional degrees of freedom, describable by scalar or vector gravitational fields.

Conformal flatness means that light propagates in Robertson-Walker cosmologies as in Minkowski space. This directly implies a Hubble expansion in cosmological scale $a(t)$, an expansion history $H(z)$, different cosmological distances, and other kinematic quantitities listed in Table I.

A. Kinematics: Distances to Supernovae, Luminous Red Galaxies, Last Scattering Surface

The CMB shift and the first baryon acoustic peak are standard rulers measuring proper motion distances

$$d_M(z) \equiv \int_0^z dz'/H(z')$$

back to the last scattering surface at redshift $z_r = 1089$ and to luminous red galaxies typically at redshift $z_1 = 0.35$, by observing the CMB shift parameter $S \equiv \sqrt{\Omega_{m0}}H_0 d_M(z_r) = 1.716 \pm 0.062$ (Spergel et al., 2006))and the first baryon acoustic peak $A \equiv \sqrt{\Omega_{m0}}H_0[d_M^2(z_1)/z_1^2 H(z_1)]^{1/3} = 0.469$ (Fairbairn and Goobar, 2005; Eisenstein et al., 2005). Calibrated supernovae Ia are standard candles at low redshift $z < 1.7$, whose observed flux=absolute luminosity/$4\pi d_L(z)^2$, measures their luminosity distance $d_L(z) = (1+z)d_M(z)$ (Perlmutter et al., 1999; Riess et al., 2004; Astier et al., 2005).

These cosmological distances then map the evolution history $H(z)$, but only after differentiation with respect to redshift. Differentiating the Gold Sample (Riess et al., 2004) supernova luminosity distances $d_L(z)$, Tegmark *et al*

(Wang and Tegmark, 2005) obtained Figure 1, a plot of $h(z) \equiv H(z)/100$. A second differentiation of the observed distances gives the overall "equation of state" $w(z) \equiv \gamma(z) - 1 \equiv d\ln H^2/(1+z)^3/3d\ln(1+z)$, decreasing from 0 in the matter-dominated epoch, to $-2/3$ at present, and apparently tending towards -1 in the far future (Tegmark, 2002; Wang and Tegmark, 2004). Determining the overall effective "equation of state" thus requires two numerical differentiations of the sparse, noisy primary data on cosmological distances.

As will be discussed in Section III.A below, the late accelerating expansion is conventionally described by an effective mixture of General Relativity ideal fluids, now consisting of pressure-free matter (baryons+CDM) and "dark energy" defined by $\rho_{DE} \equiv 3M_P^2H^2 - \rho_m$ and $\gamma_{DE} \equiv -d\ln\rho_{DE}/3dN$, so that the overall stiffness and "equation of state"

$$\gamma \equiv -d\ln H^2/3dN = \gamma_m\Omega_m + \gamma_{DE}\Omega_{DE}, \qquad \Omega_{DE} \equiv 1 - \Omega_m$$
$$w = w_m\Omega_m + w_{DE}\Omega_{DE} = w_{DE}(1-\Omega_m).$$

If Dark Energy exists as a matter constituent in General Relativity, then w_{DE} is its "equation of state". Otherwise, $w_{DE}(1-\Omega_m) = d\ln H^2/(1+z)^3/3d\ln(1+z)$ measures the Dark Gravity modification to the Einstein-Friedmann equation.

Because $\Omega_{m0} \sim 1/3$, the effective "dark energy equation of state" is now $w_{DE0} \sim -1$, so that the "dark energy", whether Dark Energy or Dark Gravity, is now static or quasi-static, nearly a cosmological constant $\Lambda \equiv \varkappa^2\rho_{DE} \approx 2H_0^{-2}$. Indeed, the recent three-year WMAP data, with narrowed constraints on $\Omega_m h^2 = 0.126 \pm 0.009$, $\Omega_m = 0.234 \pm 0.035$, $n_s = 0.961 \pm 0.017$, expansion age $t_0 = 13.73^{+0.13}_{-0.17}\,Gyr$, large scale structure (Tegmark et al., 2004) and supernova data, makes $w_{DE0} = -0.926^{+0.051}_{-0.075}$ (Spergel et al., 2006), consistent with the Concordance Model ΛCDM and severely limiting how dynamic any "dark energy" can now be.

B. Expansion History Does Not Determine Cosmodynamics

The expansion history $H(z)$ constrains but does not determine the cosmodynamics. Most simply, in Einstein's field equations

$$G_{\mu\nu} = \varkappa^2 T_{\mu\nu}/3, \qquad \varkappa^2 \equiv 8\pi G_N \equiv 1/M_P^2,$$

whose time-time component is the Einstein-Friedmann equation

$$k/a^2 + H^2 = \varkappa^2\rho/3,$$

the dynamics and expansion history cannot distinguish between a cosmological constant (static Dark Gravity) $-\Lambda g_{\mu\nu}$ added to the left side and a constant vacuum energy density (static Dark Energy) $\rho_{vac} = \Lambda g_{\mu\nu}/\varkappa^2$ on the right side. This familiar Dark Energy/Dark Gravity degeneracy persists when both are dynamic. Thus, Table II presents five two-parameter fits to the combined Supernova Legacy Survey (Astier et al., 2005), baryon acoustic peak and CMB observations (Maartens and Majerotto, 2006). The first and best fit is to the Concordance Model

$$(H/H_0)^2 = \Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0} + \Omega_{K0}, \qquad \Omega_{m0} + \Omega_{\Lambda 0} + \Omega_{K0} \equiv 1.$$

The next two fits are to spatially flat Dark Energy models, in which the Einstein-Friedmann equation is

$$(H/H_0)^2 = \Omega_{m0}(1+z)^3 + \Omega_{X0}(1+z)^{3(1+\overline{w_{DE}})}, \qquad \Omega_{m0} + \Omega_{X0} \equiv 1,$$

and the "equation of state" is $w_{DE} = const$ or $w_0 + w_a z/(1+z)$, with past-average $\overline{w_{DE}} \equiv (1/N)\int_0^N w_{DE}(N')\,dN' = w_{DE} = w_0 - w_a z\ln(1+z)/(1+z)$ (Linder, 2005). The last two fits (Astier et al., 2005; Maartens and Majerotto, 2006) are to the original spatially curved Dvali-Gabadadze-Porrati (DGP) Dark Gravity model (Dvali et al., 2000; Deffayet, 2001; Bento et al., 2006)

$$(H(z)/H_0)^2 = [1/2\beta + \sqrt{(1/2\beta)^2 + \Omega_{m0}(1+z)^3}]^2 + \Omega_{K0}(1+z)^2, \quad \Omega_{m0} + \Omega_{K0} + \sqrt{1-\Omega_{K0}}/\beta \equiv 1,$$

and to a generalized flat DGP model

$$(H/H_0)^2 = \Omega_{m0}(1+z)^3 + (1-\Omega_{m0})(H/H_0)^\alpha,$$

fitted only to SNLS+BAO and assuming the prior $\Omega_{m0} = 0.27^{+0.06}_{-0.04}$ (Fairbairn and Goobar, 2005; Alam and Sahni, 2005). The spatial curvature Ω_{K0} is set equal to zero in the (Fairbairn and Goobar, 2005; Astier et al., 2005; Linder, 2005) models and emerges as practically zero in the (Maartens and Majerotto, 2006) models. The dynamical Dark

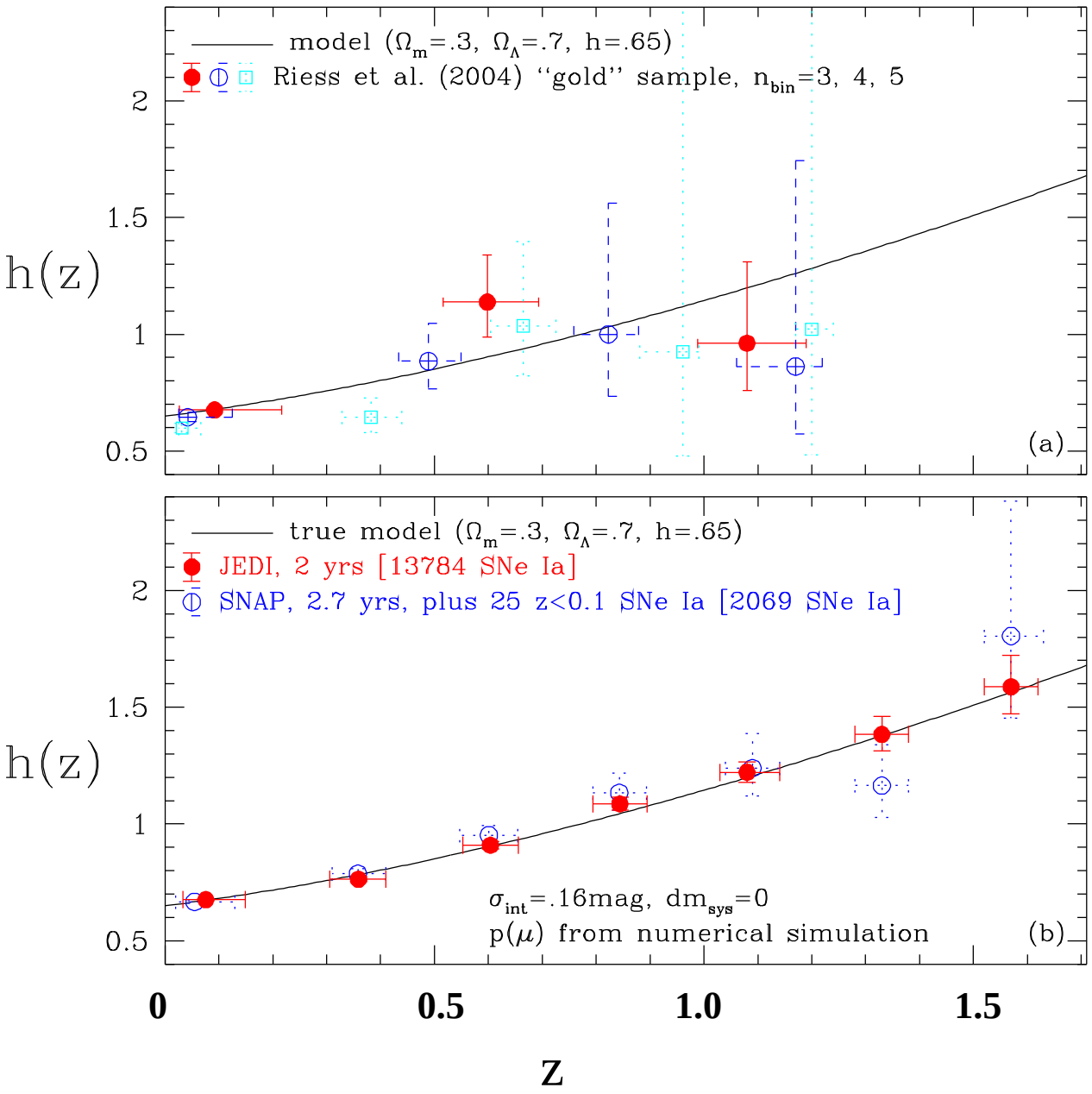

FIG. 1 The cosmic expansion history (dimensionless Hubble parameter h(z)=H(z)/100) from the Riess *et al.* (2004) Gold sample (top panel) and from simulated future data (bottom panel) for the NASA/JDEM mission concepts JEDI (solid points) and SNAP (dotted points) (Wang and Tegmark, 2005). The existing Gold sample data and the simulated future JEDI are both consistent with ΛCDM (solid curve) (from (Wang and Tegmark, 2005)).

TABLE II Model Fits to the Observed Expansion History: Two parameter fits to the joint SNLS, BAO and CMB shift data by the Concordance Model, two different dynamical Dark Energy, and two different Dark Gravity cosmological models.

Model	Ω_K	Ω_{m0}	other parameters fitted
Concordance Model (ΛCDM) (Maartens and Majerotto, 2006)	-0.0050	0.265	$\Omega_\Lambda = 0.740$
flat constant w_{DE} (Astier et al., 2005)	0	0.271 ± 0.021	$w_{DE} = -1.023 \pm 0.087$
flat $w_{DE}(z) = w_0 + w_a z/(1+z)$ (Linder, 2005)	0	0.260	$w_0 = -0.78$, $w_a = 0.32$
original Dvali-Gabadadze-Porrati (Maartens and Majerotto, 2006)	-0.0297	0.260	$\beta \equiv H_0 r_c = 1.39$
generalized flat DGP (Fairbairn and Goobar, 2005)	0	$0.27^{+0.06}_{-0.04}$	$\alpha = -0.17^{+0.87}_{-0.63}$

Energy and Dark Gravity expansion histories are equivalent under the substitution $(1+z)^{3(1+\overline{w_{DE}})} \leftrightarrow (H/H_0)^{\alpha}$, with an average $\overline{w_{DE}} = -1 + \alpha/2 = -1.08^{+0.44}_{-0.32}$ (Dvali and Turner, 2003). The Gold SN data (Riess et al., 2004) would have yielded slightly poorer fits than the SNLS data we have used.

Because the supernovae are observed only at low redshifts and the CMB first acoustic peak and the luminous red galaxies at recombination redshifts $z_r = 1089$ and $z_1 = 0.35$, other smooth parameterizations could fit the past data equally well. In any case, because the cosmological distances involve two integrations over $w(z)$, they all smear out information on the "equation of state" (Moar et al., 2001). This requires smooth parametrization of the "equation of state" and binning of the sparse, noisy data (Wang and Tegmark, 2004), and justifies using no more than two parameters for present and next decade observations (Linder and Huterer, 2005; Caldwell and Linder, 2005). (In retrospect, because observations constrain the directly observable $H^2(z)$ and the "dark energy" density better than its derivative, the "equation of state", it might have been better to parameterize the past average $\overline{w_{DE}}(z)$, rather than $w_{DE}(z)$ (Wang and Freese, 2004)).

The observed evolutionary history is already somewhat better fitted by the static ΛCDM model (Maartens and Majerotto, 2006) than by any dynamical Dark Energy or Dark Gravity model in Table II, and the latest WMAP data $w_{DE0} = -0.926^{+0.051}_{-0.075}$ (Spergel et al., 2006), even more severely limits how dynamic any "dark energy" can now be. Nevertheless, the uncertainties still allow some late-time evolution of $w(z)$. Our purpose will now be to discriminate among these nearly static alternatives by observing the fluctuation *growth factor* on Hubble horizon scales. Figure 2, from (Tegmark, 2002), shows the ranges of redshift and conformal length scales over which such spacetime fluctuations are likely to be measured cosmologically, in the next few years.

III. CLASSIFICATION OF ROBERTSON-WALKER COSMOLOGIES BY THEIR VACUUM SYMMETRY

A. Homogeneous Evolution Conventionally Described by General Relativity Perfect Fluids

By General Relativity, we understand Einstein's original field equations without cosmological constant, before introducing the cosmological constant on the left (geometric) side of his original field equations, changing the field equations from Einstein to Einstein-Lemaitre and the asymptotic Ricci curvature from flat to curved. This introduces into the classical action a very small energy scale $\Lambda \sim H_0^2 \lll M_P^2$. By avoiding identifying the cosmological constant with any right-side matter stress-energy content, this classical approach distinguishes static from dynamic "dark energy" and avoids considering the two cosmological constant problems, why quantum vacuum energies apparently do not gravitate and why the present matter density $\rho_{m0} \sim M_P^2\ \Lambda$. (Replacing the Einstein Lagrangian R by $R - 2\Lambda$ is equivalent to unimodular gravity (Buchmuller and Dragon, 1988; Unruh, 1989) in which the Einstein-Hilbert action is varied holding $\sqrt{g} = -1$: Instead of appearing in the Hilbert-Einstein action, Λ then enters as an undetermined Lagrange multiplier.)

Although Robertson-Walker cosmology does not assume General Relativity, its expansion history may be expressed in terms of an equivalent perfect fluid defined by $\rho \equiv 3M_P^2H^2 \equiv \rho_m + \rho_{DE}$ and

overall equivalent pressure: $P/c^2 \equiv -M_P^2(3H^2 + 2\dot{H}) \equiv P_m + P_{DE}$

overall "equation of state": $w \equiv P/\rho c^2 = -(1 + 2\dot{H}/3H^2) \equiv -1 + 2\epsilon_H/3 = w_m\Omega_m + w_{DE}\Omega_{DE}$

overall equivalent fluid stiffness: $\gamma(z) \equiv 1 + w = -2\dot{H}/3H^2 \equiv 2\epsilon_H/3 = \gamma_m\Omega_m + \gamma_{DE}\Omega_{DE}$

overall equivalent enthalpy density: $\rho + P/c^2 \equiv -2M_P^2\dot{H} = -d\rho/3Hdt = \rho_m + P_m - d\rho_{DE}/3dN.$

So defined, ρ_{DE} is either constituent Dark Energy or a Dark Gravity addition to the Einstein-Friedmann equation that is non-linear in H^2. For ordinary non-relataivistic matter $\rho_m \sim a^{-3}, \gamma_m = 1, P_m = 0 = w_m$. Thus, ever since matter dominated over radiation, the overall "equation of state" $w = w_{DE}(1 - \Omega_{m0})$.

Our universe apparently evolved from a high-curvature de Sitter (early inflationary) universe $P = -\rho = const.$ Assuming the Weak Energy Condition $w \geq -1$, so that no phantom matter or cosmic rip intervenes, it will expand monotonically $\dot{H} \leq 0$, towards a different (late inflationary) low-curvature de Sitter universe. The deceleration $-q(t)$ decreased from 1, when radiation dominated, and is still decreasing, along with the Hubble expansion rate and Ricci curvature.

Deceleration changed over to acceleration when the Hubble horizon changed from expanding to shrinking with conformal time, when the "slow-roll" parameter $\epsilon_H \equiv dH^{-1}/dt = -d\ln H/dN$ fell below 1, and w fell below $-1/3$. The acceleration has already increased to present values $q_0 \approx 0.52$, $w_0 \approx -0.74$, but because $\epsilon_{H0} \approx 0.4$, this recent inflation is still far from truly slow-rolling. The jerk $j(t)$ increases monotonically from a minimum $j_{min} = -1/8$ when $w = -1/2$, towards $j \to 1$, as the universe asymptotes towards a terminal de Sitter universe, with low constant

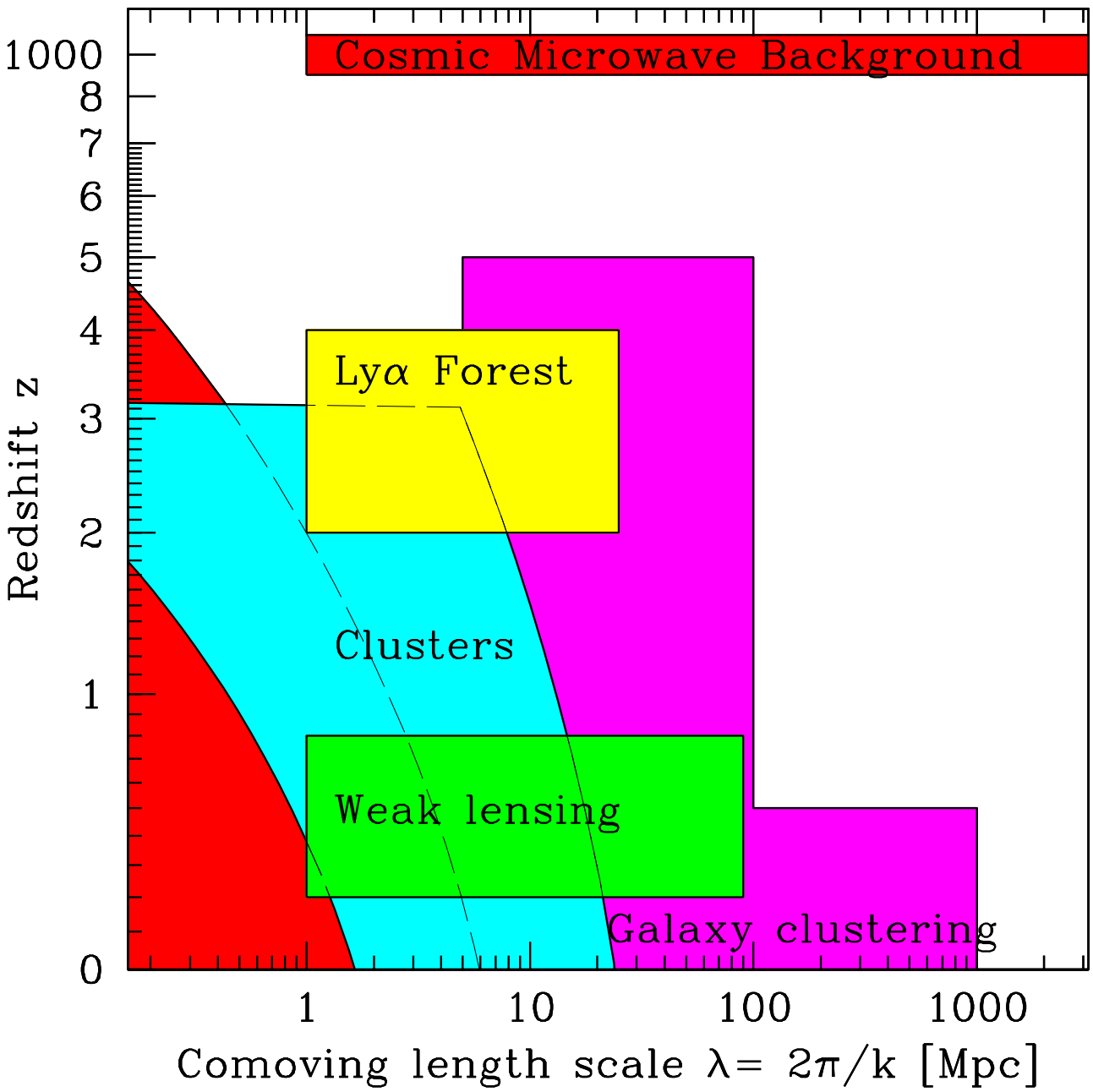

FIG. 2 Shaded regions show ranges of scale and redshift over which various cosmological observations are likely to measure spacetime fluctuations over the next few years. The lower left region, delimited by the dashed line, is the non-linear regime where rms density fluctuations exceed unity in the ΛCDM model (from (Tegmark, 2002)).

curvature $R_\infty = 12H_\infty^2 \sim 4H_0^2$. Such a de Sitter attractor explains the growth of "dark energy", but not why it *now* approximates the energy density of ordinary matter.

During three long epochs listed in Table III, the universe is dominated by a single barotropic phase with a constant equation of state w and diminishing Ricci scalar curvature $R(t)$, in which the scale $a(t) \sim t^{2/3(1+w)}$, $H = 2/3(1+w)t$ and the acceleration and jerk are fixed at $q = -(1+3w)/2$, $j = 1+9w(1+w)/2$. When these perfect fluids are mixed or when cosmological scalar fields appear, the "equation of state" $w(z)$ and the jerk $j(z) = 1+9w(1+w)/2-3dw/2dN$ change, the composite fluid is imperfect and supports entropic perturbations.

TABLE III Kinematics and Ricci curvature for barotropic phases with power-law growth $a \sim t^{2/3(1+w)}$, $a \sim \exp(Ht)$.

w	a(t)	H(t)	q(t)	j(t)	$R(t)$	Model Universe
1/3	$t^{1/2}$	1/2t	-1	3	0	radiation
0	$t^{2/3}$	2/3t	-1/2	1	$3H^2$	non-relativistic matter (Einstein-de Sitter)
-1/3	t	1/t	0	0	$6H^2$	curvature energy (constant conformal expansion$\mathcal{H}$, Milne
-1	$\exp Ht$	const	1	1	$12H^2$	empty de Sitter

B. Dark Energy Requires Fine Tuning to Explain the Cosmic Coincidence

Dark Energy is reviewed in (Padmanabhan, 2003). If it exists, Dark Energy is usually attributed to an additional ultra-light scalar field ϕ, so that $p = p(\rho, \phi)$ is adiabatic only when the scalar field is frozen or tracks the background. Defining $X \equiv \partial_\mu \phi \partial^\mu \phi$, the scalar field is canonical (*quintessence*) when its kinetic energy is $X/2$, non-canonical (*k-essence*) when the kinetic energy is non-linear in X. Canonical quintessence is driven by a slow-rolling potential and can track the background matter, making $dw/dz > 0$. k-essence is driven by a non-canonical kinetic energy and can be arranged to switch from tracking during radiation dominance over towards a dominating cosmological constant, making $dw/dz < 0$ recently. Of course, this different dynamics gives quintessence and k-essence different clustering properties.

The present "dark energy" density $\rho_{DE0} \sim 2H_0^2 M_P^2 \lll M_P^4$. While ultra-light dynamic Dark Energy can evolve down to this very small value, it does not explain the Cosmic Coincidence, "Why so small now?", why $\rho_{DE0} \sim 2\rho_{m0}$, without the extreme fine-tuning it was invoked to avoid. Canonical and non-canonical Dark Energy ultimately require fine-tuning: quintessence, in order to explain the Cosmic Coincidence; k-essence, in order to initiate the transition towards a cosmological constant after radiation dominance ends.

C. Without Dark Energy, Our Universe Must Be Asymptotically de Sitter

Birkhoff's Theorem (C. Callan and Peebles, 1965; Peebles, 1980): In any locally isotropic (spherically symmetric) system whose vacuum is Ricci-flat ($R_{\mu\nu}$=0), the vacuum metric must be Schwarzshild:

$$g_{tt} = g_{rr}^{-1} = 1 - 2G_N M(r)/r,$$

where $M(r)$ is the mass interior to r. For any small spherical shell in empty space, the Newtonian potential must vanish inside, and decrease as $1/r$ outside. Birkhoff's Theorem is a *geometric* theorem, which generalizes Newton's iron sphere theorem (C. Callan and Peebles, 1965; Peebles, 1980) from Newtonian gravity to Einstein gravity or any high-curvature modification of Einstein gravity.

Application to any Robertson-Walker Cosmology (C. Callan and Peebles, 1965; Weinberg, 1972): Applied to a homogeneous universe with matter density $\rho(a)$, $M(r) = 4\pi\rho(a)r^3/3$, Birkhoff's Theorem has remarkable *dynamical* consequences. In a homogeneous expanding universe, a small comoving shell lying at $r(t) = \lambda_* a(t)$, encloses a mass $M(r) = \lambda_*^3 4\pi\rho(a)a^3/3$, and has constant Newtonian energy

$$\dot{r}^2/2 - G_N M(r)/r = \lambda_*^2[\dot{a}^2/2 + \varkappa^2\rho(a)a^2/6].$$

Using Birkhoff's Theorem, Milne and McCrae (McCrea and Milne, 1934; Milne, 1934) derived the global Friedmann equation

$$\dot{a}^2 - \varkappa^2\rho a^2/3 = const, \qquad k/a^2 + H^2 = \varkappa^2\rho/3$$

for any pressure-free universe, without assuming Einstein's field equations. If Dark Energy does not exist and the vacuum is Ricci flat, Birkhofff's Theorem would make our presently pressure-free universe *decelerate* according to this Friedmann expansion equation.

Classification of Dark Gravity Theories: If there is no Dark Energy, the Ricci curvature of the vacuum distinguishes high-curvature from low-curvature modifications of General Relativity: Robertson-Walker universes whose vacua remain Ricci-flat in four dimensions (e.g. Arkani-Hamed *et al* (Arkani-Hamad et al., 1998), Randall-Sundrum (Randall and Sundrum, 1999), Binutray (Binutray et al., 2000)) can only modify Einstein gravity in the ultra-violet. Robertson-Walker universes which maintain Einstein gravity locally can only modify Einstein gravity cosmologically (e.g. ΛCDM, self-accelerated DGP). If there is no Dark Energy, our accelerating universe is now dominated by

pressure-free matter, so that the Friedmann-Robertson-Walker equation must be modified at *low* Ricci curvature, by introducing a cosmological constant.

When Einstein gravity is modified: If the universe is asymptotically Ricci-curved, the modified Friedmann equation

$$k/a^2 + H^2 = \varkappa^2 f(\rho)/3,$$

maintains Einstein gravity at high density $f(\rho) \to \rho$, but crosses over to de Sitter $\to const \equiv M_P^2\ \Lambda$ at scales r_c approaching the Hubble horizon $H(z)^{-1}$. About any isolated source of mass M and Schwarzchild radius $r_S \equiv 2G_N M$, Einstein gravity remains a good approximation only for distances $r < r_*$ up to the Vainstein scale $r_\star \equiv (r_S r_c^2)^{1/3} \sim (H_0 r_S)^{1/3} H_0^{-1} \ll H_0^{-1}$ at which $2G_N M/r_* \equiv r_S/r_* = H^2(a) r_*^2 = r_*^2/r_c^2$. These are the promising low-curvature modification of Einstein gravity that asymptote to de Sitter, to be discussed in Section V.B.

IV. DYNAMICS DETERMINES THE GROWTH OF FLUCTUATIONS

Allowing inhomogeneities breaks translational invariance, leading to Goldstone mode sound waves that lower the CMB angular power spectrum at large scales (low multipoles l) and leads to growth of large scale structure. In a mixture of cosmological fluids or dynamic scalar fields, the equation of state is generally not adiabatic: fluctuations propagate in the conformal Newtonian gauge with an *effective sound speed* $c_s^2 = P_{,X}/\rho_{,X} = w - dw/3(1+w)dN)$, generally different from the adiabatic sound speed $c_a^2 = \partial P/\partial\rho = \dot{P}/\dot{\rho}$. For canonical scalar field quintessence, $c_s^2 = 1$, but for non-canonical k-essence, c_s can vary rapidly, nearly vanishing near the radiation/matter cross-over, where $w(z)$ is changing.

Because the entropic pressure fluctuations are proportional to $(1+w)(c_s^2 - c_a^2)$, they are insensitive to the effective sound speed in the quasi-static limit $w(z) \sim -1$. This minimizes the differences between static and dynamic "dark energy" and between any dynamical Dark Energy and Dark Gravity, making their fluctuation growth factors hard to distinguish, in present and in next-generation experiments (Section V.C).

With the same equation of state and adiabatic sound speed, different dynamics generally leads to different effective sound speeds. This equation of state degeneracy is illustrated by the toy Chaplygin Gas, whose adiabatic equation of state $P = -A/\rho$ and sound speed $c_a^2 = -w(z)$ can be derived from adiabatic fluid dynamics, from a non-canonical Born-Infeld scalar field, or from a canonical tracking scalar field with potential $V(\phi) = (\sqrt{A}/2)[\cosh 3\phi + 1/\cosh 3\phi]$. If derived from a perfect fluid or from the Born-Infeld Lagrangian $\mathcal{L}_\phi = -V_0\sqrt{1+\varkappa^2 X}$, with non-canonical $\rho = V_0/\sqrt{1-\varkappa^2 X},\ P = -V_0\sqrt{1-\varkappa^2 X},\ w = -1 + \varkappa^2 X$, the perturbations are adiabatic, and the effective sound speed $c_s^2 = -w(z) = c_a^2$. But, if derived from the canonical scalar field with potential $V(\phi)$, entropic perturbations make the effective sounds speed $c_s^2 = 1$. Agreeing in the static limit, these three theories give the same adiabatic sound speed $c_a^2 = -w(z)$. Differing dynamically, they show different effective sound speeds, $c_s^2 = -w(z),\ -w(z),\ 1$.

This toy model illustrates how the adiabatic sound speed depends only on the equation of state, but the effective sound speed and growth of fluctuations depend on dynamics. All three Chaplygin models can fit the kinematic observations of the expansion history, but the Born-Infeld model fails dynamically by providing insufficient power in the observed large scale mass spectrum (Amendola et al., 2003). This failure can be remedied by generalizing the equation of state to $P = -A/\rho^\alpha$ with $\alpha \sim 0$, so that this generalized Chaplygin gas is nearly indistinguishable from a cosmological constant (Amendola et al., 2003; Bento et al., 2005; Zhu, 2004).

V. DARK GRAVITY: DYNAMICAL MODIFICATIONS OF GENERAL RELATIVITY

Because Dark Energy is contrived, requires fine tuning and apparently cannot be tested in the laboratory or solar system, we now turn to Dark Gravity as the alternative source of cosmological acceleration. This Dark Gravity alternative arises naturally in braneworld theories, naturally incorporates a classical extremely low spacetime intrinsic curvature, and may unify "dark energy" and dark matter, and possibly early and late inflation. We will see how Dark Gravity, besides being tested cosmologically, can also be tested in the solar system, Galaxy or galaxy clusters.

A. Present Local and Cosmological Tests of General Relativity

General Relativity is a rigid metric structure incorporating general covariance (co-ordinate reparametrization invariance), the Equivalence Principle, and the local validity of Newtonian gravity with constant G_N, in the weak field and non-relativistic limits. General covariance implies four local matter conservations laws (Bianchi identities). The Einstein-Hilbert action is linear in Ricci curvature, so that the Einstein field equations are second order, the two tensorial (graviton) degrees of freedom are dynamic, but the scalar and vector $g_{\mu\nu}$ degrees of freedom are constrained

to be non-propagating. Quantum gravity has always motivated high-curvature (Planck scale) modifications of General Relativity. Now, the surprising discovery of the accelerating universe motivates extreme low-curvature (cosmological scale) modifications of General Relativity.

General Relativity differs from Newtonian cosmology only by pressure or relativistic velocity effects, which are tested in the solar system, in gravitational lensing of light, in the primordial abundance of light elements, in the dynamical age, and in the large angular scale CMB and late-time mass power spectrum. Therefore, in order of linear scale, modifications of General Relativity must be sought in:

- laboratory violations of the Equivalence Principle (Eötvos experiments) and solar system tests (Damour et al., 1990) (lunar ranging, deflection of light, anomalous orbital precessions of the planets, Moon (Lue and Starkman, 2003; Lue, 2005; Gabadadze and Iglesias, 2005), secular increase in the Astronomical Unit (Iorio, 2005a))
- galaxy and galaxy cluster number counts (Iorio, 2005b)
- gravitational weak lensing
- cosmological variation of Newton's G_N and other "constants"
- the suppression of fluctuation growth on large scales or at late times.

B. Classification of Modified Gravity Theories by Spacetime Curvature of Their Vacuua

As already noted, in General Relativity only the tensor degrees of freedom in the metric are propagating and the homogeneous RW evolution depends only on $H(z)$. If the Einstein-Hilbert action is modified, additional scalar and vector degrees of freedom will appear, and the evolution will also depend on $\dot{H}, \ddot{H}$, which can be represented by scalar or vector gravitational fields. The basic distinction between high- and low-curvature modifications of General Relativity depends on the spacetime (Ricci) curvature of their vacua. It is simplest to begin by considering four-dimensional metrical deformations of General Relativity, which are often inspired by string-theory or M-theory (Damour and Polyakov, 1994a,b) or appear as projections of higher-dimensional theories.

1. Extra Degrees of Freedom in Four-Dimensional Gravity

- Scalar-tensor gravity, the simplest and best-motivated extension of General Relativity (Fujii and Maeda, 2003; Capozziello et al., 2005): In the original Jordon frame, a scalar gravitational field proportional to time-varying $1/G_N$, is linearly coupled to the Ricci scalar R. After a conformal transformation to the Einstein frame, the scalar gravitational field is minimally coupled to gravity, non-minimally coupled to matter. In the Einstein frame, the gravitational field equations look like Einstein's, but the matter field is coupled to the scalar gravitational field as strongly as to the tensor gravitational field, so that test particles do not move along geodesics of the Einstein metric. Test particles move along geodesics of the original Jordon metric, so that the Weak Equivalence Principle holds.

  Scalar-tensor theories modify Einstein gravity at all scales and must be fine-tuned, in order to satisfy observational constraints. Nucleosynthesis and solar system constraints severely restrict any scalar field component, rendering any Dark Gravity effects on the CMB or $H(z)$ evolution imperceptible (Bertotti et al., 2003; Catena et al., 2004; Capozziello et al., 2006a).

- higher-order metric $f(R)$ theories: Stability of the equations of motion allows the Lagrangian to depend only on R, and only trivially on other curvature invariants, $P \equiv R_{\mu\nu}R^{\mu\nu}$, $Q \equiv R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ (Ostrogradski, 1850) or derivatives of any curvature scalar (Woodard, 2006). These $f(R)$ theories are equivalent to scalar tensor theories with vanishing Brans-Dicke parameter $\omega_{BD} = 0$ (Teyssandier and Tourrenc, 1983; Olmo, 2005; Capozziello et al., 2006b).

  The simplest low-curvature modification (Carroll et al., 2004, 2005; Nojiri and Odintsov, 2006), replacing the Einstein Lagrangian density by $R-\mu^4/R$, leads to accelerated expansion at low-curvature $R \leq \mu^2 \sim H_0^2$, but has negative kinetic energies and is tachyonically unstable. This instability would be tolerable in empty space, but would be vastly and unacceptably amplified inside matter (Dolgov and Kawasaki, 2003), and phenomenologically unacceptable outside matter (Soussa and Woodard, 2004). These $f(R)$ theories, like their scalar-tensor gravity equivalents, can be fine-tuned to avoid these potential instabilities and satisfy supernova and solar system constraints (Woodard, 2006; Soussa and Woodard, 2004; Nojiri and Odintsov, 2004, 2003), but not cosmological constraints (Amendola et al., 2006).

- TeVeS (relativistic MOND theory): Adding an additional vector gravitational field, could explain galactic rotation curves and the Tully-Fisher relation, without invoking dark matter, and possibly unify dark matter and "dark energy" (Bekenstein, 2004). Because gravitons and matter have different metric couplings, TeVeS predicts that gravitons should travel on geodesics different from photon and neutrino geodesics, with hugely different arrival times from supernova pulses. It also predicts insufficient power in the third CBR acoustic peak (Skordis et al., 2006). In any case, now that WMAP data requires dark matter (Spergel et al., 2006), the motivation for TeVeS disappears.

2. Extra Dimensional Modifications of Einstein Gravity

In extra dimensional braneworld theories, scalar fields appear naturally as dilatons and modify Einstein gravity at high- curvature, by brane warping (Randall and Sundrum, 1999; Binutray et al., 2000), or at low- curvature, by brane leakage of gravity (Dvali et al., 2000). If quantized, these theories encounter serious theoretical problems (ghosts, catastrophic ultra-violet instabilities, strong coupling problems). Until these problems can be overcome, these theories can only be regarded as effective field theories, incorporating an extremely low *infra-red* scale at low spacetime curvature. This suggests an infra-red/ultra-violet connection, since effective field theories ordinarily incorporate *ultra-violet* parameters.

In the original DGP model (Dvali et al., 2000; Deffayet et al., 2002a,b), the brane's finite stiffness leads to an effective modified Friedmann equation,

$$H^2 + k/a^2 - H/r_c = H^2 + k/a^2 - H H_0/\beta = \varkappa^2\rho/3$$
$$(H(z)/H_0)^2 = [1/2\beta + \sqrt{(1/2\beta)^2 + \Omega_{m0}(1+z)^3}]^2 + \Omega_{K0}(1+z)^2, \quad 1 \equiv \Omega_{m0} + \Omega_{K0} + \sqrt{1-\Omega_{K0}}/\beta$$

on the four-dimensional brane by inducing an additional curvature term H/r_c at the cosmological scale $\beta \equiv H_0 r_c = 1.39$, $r_c = \beta/H_0 \equiv H_\infty^{-1} \sim 5.7\ Gpc$. This modified Friedmann equation interpolates between Einstein's pressure-free universe at large redshifts, and the empty de Sitter universe with constant Hubble expansion $H_\infty \equiv 1/r_c = H_0/\beta$, in the asymptotic future. The universe began its late acceleration at $z_{acc} = (2\Omega_{m0}/\beta^2)^{1/3} - 1 \sim 0.58$. This is the original DGP model fit on the fourth line of Table I, which turns out to be spatially practically flat.

In flat 3-space, the modified Friedmann equation has the self-accelerating solution

$$H = \sqrt{\varkappa^2\rho/3 + (1/2r_c)^2} + 1/2r_c$$
$$H(z)/H_0 = 1/2\beta + \sqrt{(1/2\beta)^2 + \Omega_{m0}(1+z)^3}, \qquad 1 \equiv \Omega_{m0} + 1/\beta,$$

Because this self-accelerating solution has a Ricci-curved vacuum, Birkhoff's Theorem does not apply on the 4D brane, and Einstein gravity still holds at the shortest distances. However, about any isolated condensation of Schwarzchild radius $r_S \equiv 2G_N M/c^2$, the self-accelerating metric

$$g_{tt} = 1 - r_S/2r + \sqrt{r_S^2 r/2r_\star^3}, \qquad g_{rr}^{-1} = 1 + r_S/2r - \sqrt{r_S^2 r/8r_\star^3}, \qquad r \lesssim r_\star,$$

and Einstein gravity already breaks down at the Vainshtein scale (Dvali et al., 2003) defined by

$$r_\star \equiv (r_S r_c^2)^{1/3} \sim (H_0 r_S)^{1/3} H_0^{-1} \ll H_0^{-1}.$$

This scale, surprisingly intermediate between r_S and H_0^{-1}, is also where the growth of fluctuations changes from Einstein gravity to linearized DGP or scalar-tensor Brans-Dicke growth, with an effective Newton's constant slowly decreasing by no more than a factor two (Lue et al., 2004).

The original flat DGP model can be generalized (Dvali and Turner, 2003) to

$$H^2 - H^\alpha r_c^{\alpha-2} = \varkappa^2\rho/3, \qquad 1/\beta \equiv (1-\Omega_{m0})^{1/(2-\alpha)}, \qquad 1 = \Omega_{m0} + \beta^{\alpha-2},$$

which is equivalent to a "dark energy" $\rho_{DE} \equiv 3M_P^2 H^2 - \rho = 3M_P^2 H^\alpha r_c^{\alpha-2}$, $w_{DE} = -1 + \alpha/2$. This generalization reduces to the original flat DGP form for $\alpha = 1$, but otherwise interpolates between the ΛCDM model for $\alpha = 0$, $\beta = 1.18$ and the Einstein-de Sitter model for $\alpha = 2$, $\beta = \infty$. For small α, it describes a slowly varying cosmological constant. Excluding the CMB shift data, the joint SNLS-BAO data fits this nearly static Dark Gravity model fit with $\alpha = -0.17^{+0.87}_{-0.63}$, $\beta = 1.16$ or $r_c = \beta/H_0 \sim 8\ Gpc$ (Fairbairn and Goobar, 2005). This is the generalized flat DGP model on the last line of Table I.

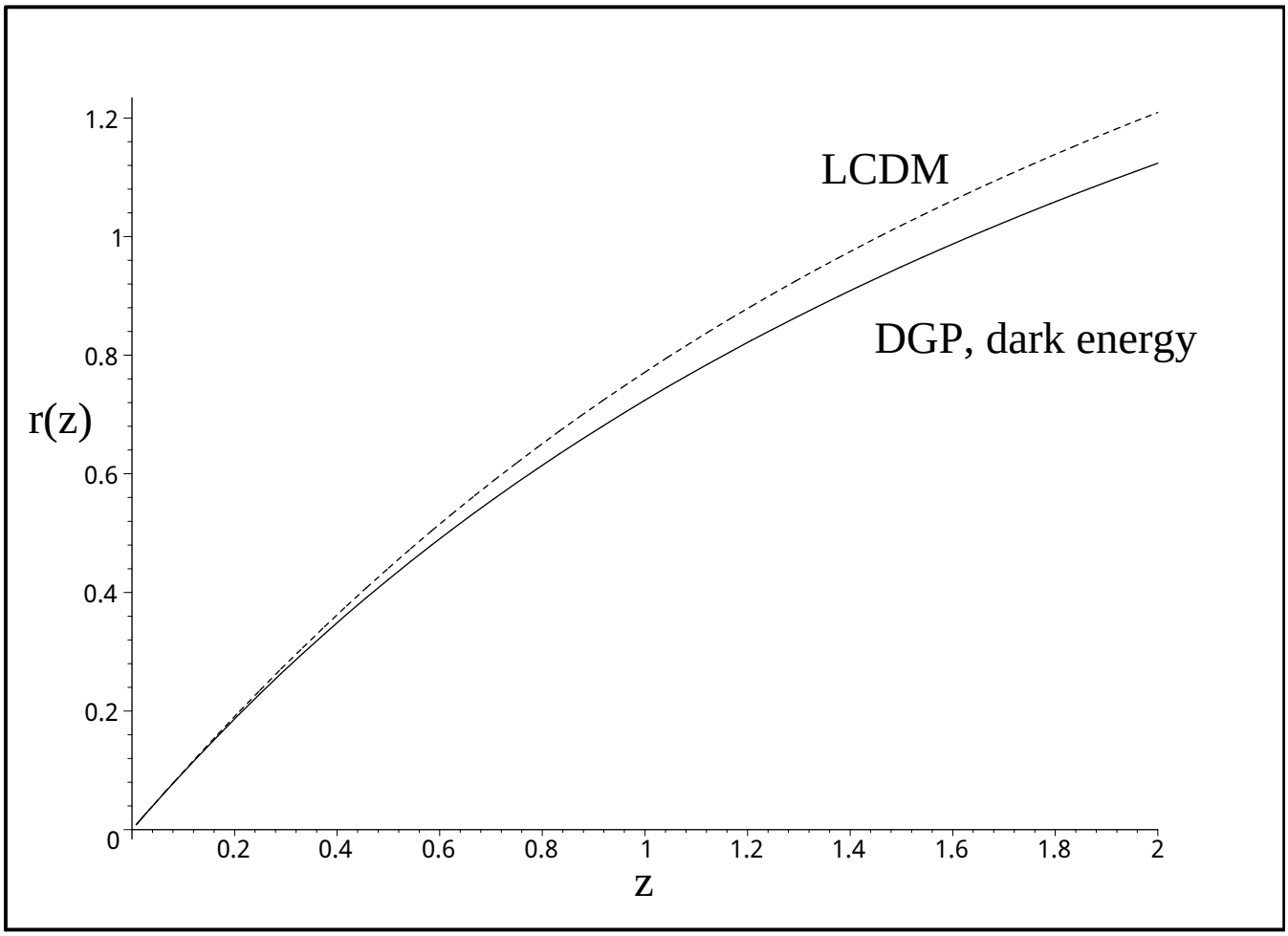

FIG. 3 The proper motion or conformal distance $r(z) = c\int_0^z dz'/H(z')$ back to redshift z, calculated for flat ΛCDM (dashed) and DGP Dark Gravity (solid) models with present matter fraction $\Omega_{m0} = 0.3$. The DGP Dark Gravity model is also mimicked by a Dark Energy model with $w(a) = -0.78 + 0.32z/(1+z)$. Between static and dynamic "dark energy", the differences in distances and in expansion history $H(z) = cdz/r(z)$ are small, but the differences in growth factor will be larger in FIG. 4. (from (Koyama and Maartens, 2005)).

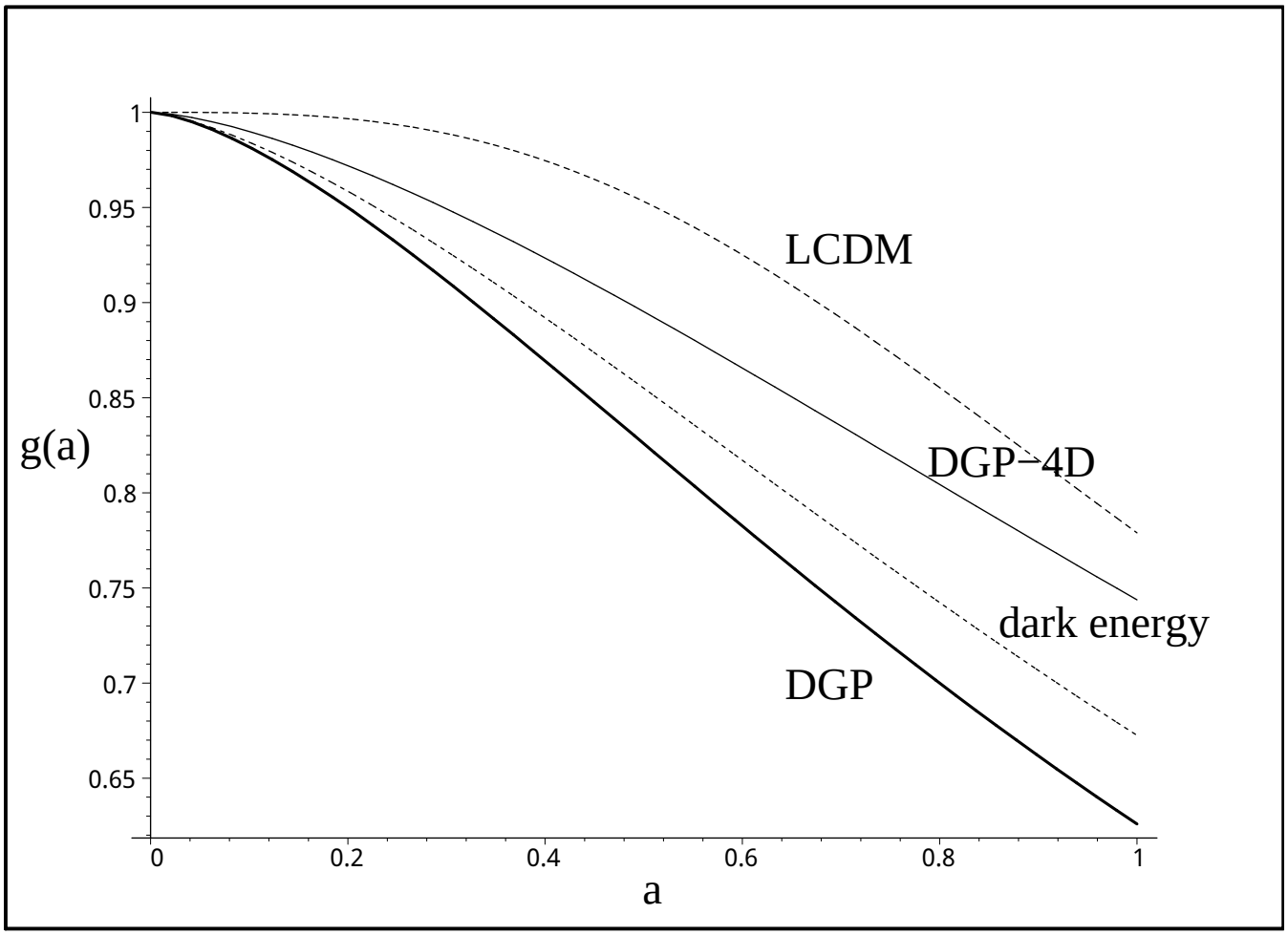

FIG. 4 The linear growth history $g(a) \equiv \delta/a$ for flat ΛCDM (long dashed), DGP Dark Gravity (thick solid) and Dark Energy models in Fig. 3. Because the DGP model Newton's "constant" weakens with time, it shows a little more growth suppression than that in the mimicking Dark Energy model. DGP-4D (thin solid) shows the incorrect result obtained by neglecting perturbations of the DGP 5D Weyl tensor. The 5D Weyl tensor perturbations make dynamical Dark Gravity (DGP) and Dark Energy hard to resolve, but both dynamical models are distinguishable from static Dark Energy. (from (Koyama and Maartens, 2005))

C. Prospective Tests of Modified Gravity

Prospective cosmological observations: Figure 3 compares the recent expansion histories Koyama and Maartens (Koyama and Maartens, 2005) calculated in the static flat ΛCDM model with the dynamic flat DGP Dark Gravity model $\beta = 1.41$, chosen to fit $\Omega_{m0} = 0.3$, and with its Dark Energy mimic. The degeneracy in these three models is lifted by their different linear growth factors plotted in Figure 4: DGP Dark Gravity always suppresses growth a little more than Dark Energy does, but substantially more than the Concordance Model. The present normalized linear growth factors $g(a) = \delta/a = 0.61, 0.68.0.80$ for DGP Dark Gravity, Dark Energy, ΛCDM respectively will lead to slightly different large-scale CMB and gravitational weak lensing (cosmic shear) convergence effects.

These linear growth factors have been calculated (Koyama and Maartens, 2005) on subhorizon scales and agree with (Ishak et al., 2005; Lue and Starkman, 2003), but are not yet reliable on superhorizon scales, where gravity leakage is most important. Unfortunately, the large-scale CMB angular power spectrum is obscured by cosmic variance and by foreground effects of nearby structures. Nevertheless, the linear growth factors already suggest that next-generation observations may distinguish static from dynamic "dark energy", but will be unable to distinguish Dark Energy from DGP Dark Gravity. To do so will require a newer, more ambitious weak lensing shear survey (Ishak et al., 2005).

Prospective solar system and galaxy cluster observations: The modifications to Einstein gravity at the Vainstein intermediate scale r_* may also be tested in next-generation solar system measurements of anomalous precessions of planetary or lunar orbits (Lue and Starkman, 2003; Dvali et al., 2003) or of a secular increase in the Astronomical Unit (Iorio, 2005b). These Vainstein scale modifications may also be observable in precision tests about other isolated stars ($r_S \sim 3\ km,\ r_\star \sim 280\ pc$) or about isolated spherical galaxy clusters $r_S \sim 100\ pc,\ r_\star \sim 28\ Mpc$) (Iorio, 2005a).

VI. CONCLUSIONS: ΛCDM, FINE-TUNED DARK ENERGY, OR MODIFIED GRAVITY

We have reviewed present and prospective observations of "dark energy" as the source of the observed late cosmological acceleration, in order to emphasize the differences between kinematical and dynamical observations, between static and dynamic "dark energy", and between Dark Energy and Dark Gravity. We conclude that

- Cosmological acceleration is explicable by either a small fine-tuned cosmological constant or by "dark energy", which is now at most moderately dynamic. This "dark energy", if dynamic, is either an additional, ultra-light matter constituent within General Relativity, or a low-curvature modification of Einstein's field equations.
- The best and simplest fit to the expansion history, static "dark energy" (ΛCDM), interprets the cosmological constant as a classical intrinsic geometric Ricci curvature, rather than as vacuum energy. This side-steps the cosmological constant problems, why quantum vacuum fluctuations apparently do not gravitate and why the current matter density is roughly that of "dark energy".
- The homogeneous expansion history can also be fitted by moderately dynamical "dark energy". Only observing the large-scale inhomogeneity growth rate will distinguish between dynamic and static "dark energy".
- Projected cosmological observations of the growth factor in the large-scale angular power spectrum, mass power spectrum, or gravitational weak lensing convergence should distinguish static from dynamic "dark energy", but not Dark Energy from Dark Gravity.

Originally invoked to explain late cosmological acceleration within General Relativity, quintessence and k-essence Dark Energy fail to explain the Cosmological Coincidence "Why Dark Energy appears now?", without fine-tuning or anthropic reasoning. Low- curvature modifications of Einstein gravity, such as DGP, are conceptually less contrived than finely-tuned Dark Energy, and arise naturally in braneworld theories. They explain cosmological acceleration as a natural consequence of geometry, may unify early and late inflation,, and may even be tested by refined solar system or galaxy observations.

Acknowledgments

I thank Richard Woodard (University of Florida), for helpful discussions of Ostrogradski's Theorem and $f(\mathcal{R})$ theories, and Dallas Kennedy (MathWorks) and Roy Maartens (Portsmouth), for critical comments.

References

J.-A. Gu and W.-Y. Hwang, Phys. Rev. D **65**, 024003 (2002).
D. Spergel et al., arXiv:astro p. 0603449 (2006), tables 2, 9.
R. D. C. Callan and P. Peebles, Am. J. Phys. **33**, 105 (1965).
P. Peebles, *The Large-Scale Structure of the Universe* (Princeton University Press, 1980), section 3.
W. McCrea and E. Milne, Quart. J. Math. (Oxford) **5**, 73 (1934).
E. Milne, Quart. J. Math. (Oxford) **5**, 64 (1934).
S. Weinberg, *Gravitation and Cosmology: Principles and applications of the general theory of relativity* (Wiley, New York, 1972).
N. Arkani-Hamad, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998).
L. Randall and R. Sundrum, Phys. Lett B **83**, 4690 (1999).
P. Binutray, C. Deffayet, and D. Langlois, Nucl. Phys. B **565**, 269 (2000).
G. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B **484**, 112 (2000), [DGP].
C. Deffayet, Phys. Lett. B **502**, 199 (2001), arXiv:hep-ph/0010186.
M. Ishak, A. Upadhye, and D. Spergel, arXiv:astro-ph p. 0507184 (2005).
A. Lue and G. Starkman, Phys. Rev. D **67**, 064002 (2003), arXiv:astro-ph/0212083.
L. Iorio, arXiv:gr-qc (2005a).
L. Iorio, J. of Cosmology and Astrop. Physics **9**, 6 (2005b).
M. Fairbairn and A. Goobar, arXiv:astro-ph p. 0511029 (2005).
D. Eisenstein et al., Astrophys. J. **633**, 560 (2005), [SDSS Luminous Red Galaxy Survey].
S. Perlmutter et al., Astrophys. J. **517**, 565 (1999), [Supernova Cosmology Project].
A. Riess et al., Astrophys. J **607**, 665 (2004), [Supernova Search Team Gold Sample].
P. Astier et al., arXiv:astro-ph p. 0510447 (2005), [Supernova Legacy Survey SNLS].
Y. Wang and M. Tegmark, Phys. Rev. D **71**, 1035103 (2005).
M. Tegmark, Science **290**, 1427 (2002), arXiv:astro-ph/0411039.
Y. Wang and M. Tegmark, Phys. Rev. Lett. **92**, 241302 (2004).
M. Tegmark et al., Astrophys. J. **606**, 702 (2004), [SDSS Collaboration].
R. Maartens and E. Majerotto, arXiv:astro-ph p. 0603353 (2006).
E. Linder, Phys. Rev. D **72**, 043529 (2005), arXiv:astro-ph/0507263.
M. Bento, O. Bertolami, M. Reboucas, and N. Santos, arXiv:astro-ph (2006).
U. Alam and V. Sahni, arXiv:astro-ph p. 0511473 (2005).
G. Dvali and M. Turner, arXiv:astro-ph p. 0301510 (2003).
L. Moar, R. Brustein, and P. Steinhardt, Phys. Rev. Lett. (2001).
E. Linder and D. Huterer, Phys. Rev. D **72**, 043509 (2005).
R. Caldwell and E. Linder, Phys. Rev. Lett. **95**, 141301 (2005).
Y. Wang and K. Freese, arXiv:astro-ph p. 0402208 (2004).
W. Buchmuller and N. Dragon, Phys. Lett. B **207**, 292 (1988).
W. Unruh, Phys. Rev. D **40**, 1048 (1989).
T. Padmanabhan, Phys. Rept. **380**, 235 (2003).
L. Amendola, F. Finelli, C. Burigana, and D. Carturan, JCAP **309**, 5 (2003), arXiv:astro-ph/0304325.
M. Bento, O. Bertolami, N. Santos, and A. Sen, Phys. Rev. D (2005).
Z.-H. Zhu, Astronomy and Astrophysics (2004), arXiv:astro-ph/0411039.
T. Damour, G. Gibbons, and C. Grunlach, Phys. Rev. Lett. **64**, 123 (1990).
A. Lue, Phys. Rep. (2005), arXiv:astro-ph/0510068.
G. Gabadadze and A. Iglesias, arXiv:astro-ph p. 0508201 (2005).
T. Damour and A. Polyakov, Nucl. Phys. B (1994a).
T. Damour and A. Polyakov, Gen. Relativ. Gravit. (1994b).
Y. Fujii and K.-I. Maeda, *The Scalar-Tensor Theory of Gravitation* (Cambridge University Press, Cambridge, England, 2003).
S. Capozziello, V. Cardone, and A. Troisi, Phys. Rev. D **71**, 043503 (2005).
B. Bertotti, L. Das, and P. Tortora, Nature **425**, 374 (2003).
R. Catena, N. Fornengo, A. Masiero, M. Pietroni, and F. Rosati, arXiv:astro-ph p. 0406152 (2004).
S. Capozziello, V. Cardone, and A. Troisi, arXiv:astro-ph p. 0602349 (2006a).
M. Ostrogradski, Mem. Ac. St. Petersbourg **VI 4**, 385 (1850).
R. Woodard, arXiv:astro-ph (2006).
P. Teyssandier and P. Tourrenc, J. Math. Phys. (1983).
G. Olmo, Phys. Rev. D **72**, 083505 (2005).
S. Capozziello, S. Nojiri, and S. Odintsov, arXiv:hep-th (2006b).
S. Carroll, V. Duvvuri, M. Trodden, and M. Turner, Phys. Rev. D **70**, 0435208 (2004).
S. Carroll, A. D. Felice, V. Duvvuri, D. Easson, M. Trodden, and M. Turner, arXiv:astro-ph p. 0410031 (2005).
S. Nojiri and S. Odintsov, arXiv:hep-th p. 0601213 (2006).
A. Dolgov and M. Kawasaki, Phys. Lett. B **573**, 1 (2003).
M. Soussa and R. Woodard, Gen. Relativ. Gravit. **36**, 855 (2004).

S. Nojiri and S. Odintsov, Gen. Relativ. Gravit. **36**, 1765 (2004).
S. Nojiri and S. Odintsov, Phys. Rev. D **68**, 123512 (2003).
L. Amendola, D. Polarski, and S. Tsujikawa, arXiv:astro-ph (2006).
J. Bekenstein, Phys. Rev. D **70**, 083509 (2004).
C. Skordis, D. Mota, P. Ferreira, and C. Boehm, Phys.Rev.Lett. **96**, 011301 (2006), arXiv:astro-ph/0505519.
C. Deffayet, G. Dvali, and G. Gabadadze, Phys. Rev. D **65**, 044023 (2002a).
C. Deffayet et al., Phys. Rev. D **66**, 024019 (2002b).
G. Dvali, A. Gruzinov, and M. Zaldarriaga, Phys. Rev. D **68**, 024012 (2003).
A. Lue, R. Scoccimaro, and G. Starkman, Phys. Rev. D **69**, 124015 (2004).
K. Koyama and R. Maartens, arXiv:astro-ph p. 0511634 (2005).