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Calculation of the nucleon axial charge in lattice QCD

D.B. Renner[1], R.G. Edwards[2], G. Fleming[3], Ph. Hägler[4], J.W. Negele[5], K. Orginos[2,6], A.V. Pochinsky[5], D.G. Richards[2] and W. Schroers[7]

[1] University of Arizona, Department of Physics, 1118 E 4th St, Tucson AZ 85721
[2] Thomas Jefferson National Accelerator Facility, Newport News, VA 23606
[3] Sloane Physics Laboratory, Yale University, New Haven, CT 06520
[4] Institut für Theoretische Physik, TU München, D-85747 Garching, Germany
[5] Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139
[6] Department of Physics, College of William and Mary, Williamsburg VA 23187
[7] John von Neumann-Institut für Computing NIC/DESY, D-15738 Zeuthen, Germany

Abstract. Protons and neutrons have a rich structure in terms of their constituents, the quarks and gluons. Understanding this structure requires solving Quantum Chromodynamics (QCD). However QCD is extremely complicated, so we must numerically solve the equations of QCD using a method known as lattice QCD. Here we describe a typical lattice QCD calculation by examining our recent computation of the nucleon axial charge.

1. Introduction

Quantum Chromodynamics is the theory of the Strong Force of nature. The theory dictates how quarks, the fundamental components of ordinary matter, interact by exchanging particles known as gluons. The interactions between quarks and gluons are sufficiently strong to bind them into bound states such as protons, neutrons, and pions. This phenomenon not only gives these bound states a rich and detailed structure but also makes pencil-and-paper calculations of many properties of QCD impossible. Consequently we numerically solve the equations of QCD by using a discrete space-time lattice. We use this method, known as lattice QCD, to calculate the properties of protons and neutrons, collectively known as nucleons, and to ask how nature constructs such objects from their quark and gluon constituents. Here we describe a typical calculation, the calculation of the nucleon axial charge, g_A, to illustrate our effort to determine the quark and gluon structure of the nucleon.

1.1. Quantum Chromodynamics

Quantum Chromodynamics is the theory within the Standard Model of particle physics that explains the Strong Force between the quarks and gluons. There are six quarks named up, down, strange, charm, bottom, and top. The fundamental interactions among the quarks and gluons are conceptually quite simple but often very complicated in practice. In Figure 1 we show the three fundamental interactions between quarks and gluons in QCD. All processes in QCD are built by combining these three basic interactions in all possible ways (Figure 7).

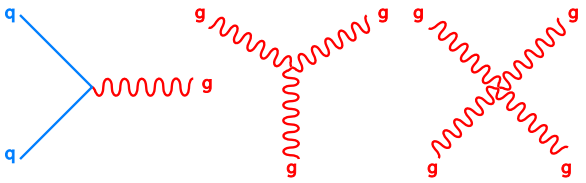

Figure 1. Fundamental quark and gluon interactions of QCD

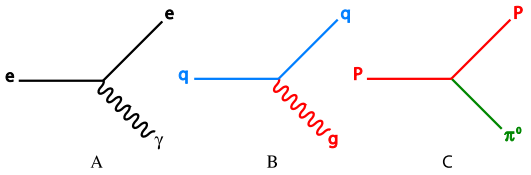

Figure 2. Examples of charges in QED (A), QCD (B) and χPT (C)

1.2. Nucleon Axial Charge

There are many kinds of charges in particle physics. Electrons have an electric charge to which photons couple (Figure 2-A). Quarks have a color charge to which gluons couple (Figure 2-B). Protons have an axial charge to which pions couple (Figure 2-C). The electric and color charges are fundamental properties of elementary particles and have prescribed values, however the axial charge of the proton is a consequence of its quark and gluon structure and hence must be measured experimentally or calculated theoretically.

2. Nucleon Axial Charge

The nucleon axial charge, g_A, is a particularly illuminating property of the nucleon. It reveals how the up and down quark intrinsic spin contribute to the spin of the proton and neutron. It is a basic ingredient in the expressions that determine how the neutron decays. It is a measure of the extent to which spontaneous breaking of chiral symmetry impacts the structure of the proton and neutron. And it determines how nucleon properties vary as you change the quark masses in QCD. The current experimental measurement is $g_A = 1.2695 \pm 0.0029$ [1]. Our lattice QCD calculation gives $g_A = 1.212 \pm 0.084$ [2] which has a 7% fractional error and agrees with the experimental determination.

2.1. Proton Spin Physics

Both elementary and composite particles have intrinsic angular momentum known as spin. The rotational properties of three dimensional space dictate that the proton must have precisely a spin of 1/2. Additionally the constituents of the proton, quarks and gluons, must have exactly a spin of 1/2 and 1 respectively. However the spin of the quarks and gluons may be orientated in any direction. Furthermore they may also move within the proton giving rise to additional orbital angular momentum. Thus the proton spin has four sources: intrinsic quark and gluon spin as well as quark and gluon orbital angular momentum (Figure 3), and these four contributions must somehow conspire to sum to precisely 1/2. Of these four contributions, the axial charge determines the difference of the intrinsic up and down quark contributions to the nucleon spin: $g_A = (0.75) - (-0.52) = 1.2695 \pm 0.0029$ [1, 3].

2.2. Neutron Beta Decay

There is no experimental evidence that the proton ever decays. However the neutron does decay, and it does so with a mean lifetime of nearly 15 minutes. The mechanism underlying neutron decay is the conversion of a down quark inside of a neutron into an up quark. This process is mediated by the Weak Force and is a fundamental interaction in the Standard Model. The result is that a neutron becomes a proton while emitting two other particles: an electron and electron anti-neutrino. The rate of neutron decay is determined by several parameters of the Standard Model as well as the nucleon axial charge.

Neutron beta decay impacts a range of physical phenomena from astrophysics, to nuclear physics, to hadronic physics, and down to particle physics. In astrophysics, beta decay of the

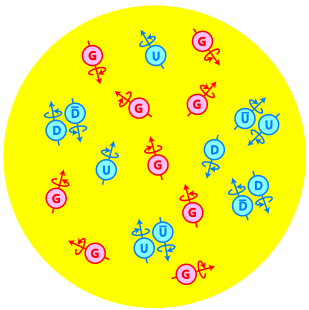

Figure 3. The nucleon is a composite object whose spin arises from the combination of quark and gluon spin as well as quark and gluon orbital motion.

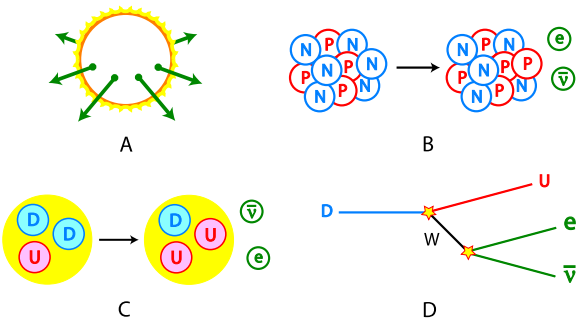

Figure 4. These pictures illustrate the relevance of the nucleon axial charge to astrophysics (A), nuclear physics (B), hadronic physics (C) and particle physics (D).

neutron, $n \to p e \bar{\nu}$, and electron capture on the proton, $e p \to n \nu$, play an important role in the emission of neutrinos from stars (Figure 4-A). In nuclear physics, nuclear beta decay occurs when a neutron in the nucleus undergoes beta decay to a proton (Figure 4-B). In hadronic physics, the axial charge can be determined from the beta decay of the neutron (Figure 4-C). And in particle physics, the axial form factor of the nucleon can be measured in neutrino-nucleon scattering which probes the weak interactions of the up and down quarks (Figure 4-D).

2.3. Spontaneous Chiral Symmetry Breaking

Chiral symmetry is a fundamental symmetry of the quark and gluon interactions in the limit of zero quark masses. Quarks with their spin aligned to their direction of motion, called right-handed, have gluon interactions identical to quarks with their spin anti-aligned, called left-handed. However the vacuum of QCD breaks this symmetry. Pairs of right-handed quarks and left-handed anti-quarks or left-handed quarks and right-handed anti-quarks condense and fill the vacuum with quark-anti-quark pairs (Figure 5) that break chiral symmetry. An elementary argument demonstrates that the axial charge of the nucleon would vanish in the limit of zero quark masses if this symmetry breaking did not occur, indicating the intimate connection between g_A and the spontaneous breaking of chiral symmetry in QCD.

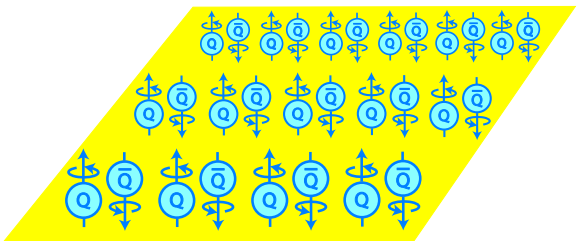

Figure 5. The QCD vacuum is filled with a condensate of quark-anti-quark pairs that gives the axial charge a non-zero value.

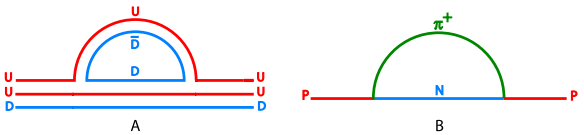

Figure 6. Interactions in QCD involving quarks and gluons (A) are replaced by equivalent interactions of pions and nucleons (B) in chiral perturbation theory. The axial charge determines the strength of these pion and nucleon interactions.

2.4. Chiral Perturbation Theory

Chiral perturbation theory describes the physics of QCD with light quark masses. It does so by replacing the quark and gluon interactions by a set of pion and nucleon interactions (Figure 6)

chosen in precisely the correct way to reproduce the low energy predictions of QCD. Chiral perturbation theory is particularly useful because it predicts the quark mass dependence of lattice QCD calculations (Figure 9). In particular the axial charge effects how rapidly the properties of the nucleon vary with the quark masses.

3. Lattice QCD

Numerical calculations are performed in lattice QCD by replacing continuous space-time with a discrete lattice. In this way lattice calculations reduce to the evaluation of very high dimensional integrals. State-of-the-art lattice QCD computations use lattices as large as $48^3 \times 144$ with 4×8 degrees of freedom per lattice point which results in just slightly more than half a billion integration variables. Such enormous integrals are evaluated using Monte Carlo methods. The dominant limitation of this method is the time required to invert large, poorly conditioned matrices that account for the effects of quark-anti-quark pairs in the vacuum. This problem becomes increasingly more severe with lighter quark masses, hence lattice calculations are performed with quark masses heavier than those in nature and the results are extrapolated to the physical point.

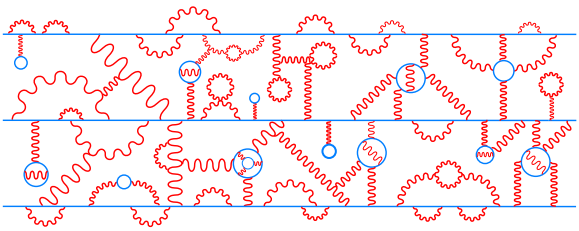

Figure 7. A nucleon consists of three net quarks which persist in time from the left to the right but are accompanied by arbitrarily many gluons and quark-anti-quark pairs.

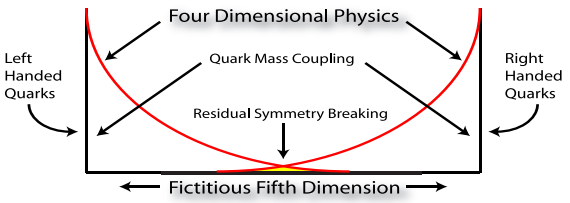

Figure 8. Domain wall fermions use a fictitious fifth dimension to construct a formulation of quarks in lattice QCD with an exact chiral symmetry even on the lattice.

3.1. Domain Wall Fermions

There are many ways to place quarks on a space-time lattice. In fact, a broad class of lattice theories all give rise to the same QCD predictions in the limit of vanishing lattice spacing. However there is a "no-go" theorem stating under general assumptions that no *simple* four dimensional representation of quarks can have the correct chiral symmetries of QCD. Domain wall fermions were invented to elude this obstacle by using a fictitious fifth dimension (Figure 8). The four dimensional physics is exponentially bound to the edges of the fifth dimension. All artifacts of the five dimensional theory vanish and an exact chiral symmetry emerges as the extent of the fifth dimension is made larger and larger.

3.2. Our Latest Lattice Results

We use the gluon configurations from the MILC collaboration [4] for the evaluation of the lattice QCD integrals. Additionally we use domain wall fermions to represent the quarks inside the nucleon. Our current calculation is done at five quark masses, equivalently pion masses, at one volume and additionally we have repeated the lightest calculation at a larger volume to check for finite size errors (Figure 9). We then match the lattice results to chiral perturbation theory in order to extrapolate to both the zero quark mass and infinite volume limits. The final result for our computation of the axial charge is shown in Figure 9 for which the lattice computation of $g_A = 1.212 \pm 0.084$ [2] agrees to within the errors with the experimental determination of $g_A = 1.2695 \pm 0.0029$ [1].

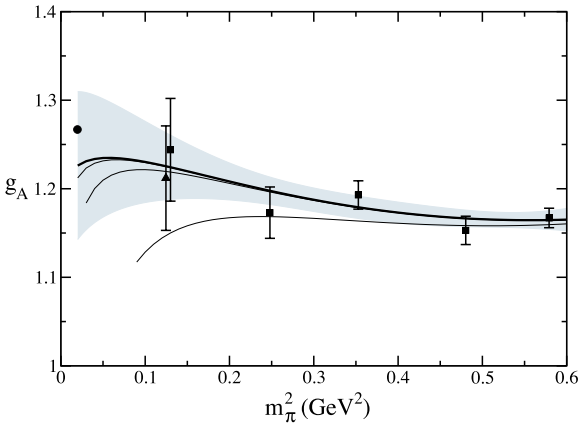

Figure 9. Our recent lattice QCD results for g_A [2]

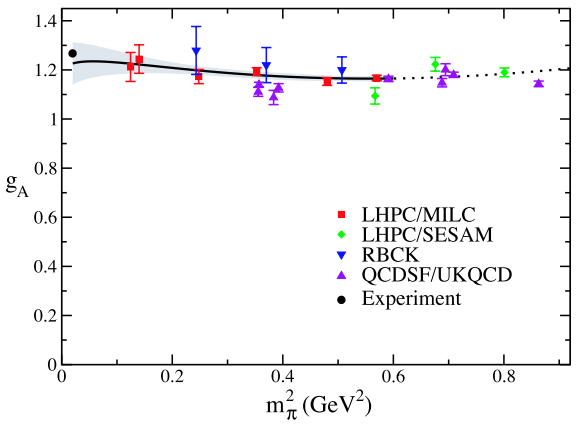

Figure 10. World's collection of lattice QCD results for g_A [2, 5, 6, 7]

3.3. World's Lattice Results

There have been several lattice QCD calculations of the nucleon axial charge, however, only four calculations in the world have included the effects of quark-anti-quark pairs in the vacuum. We performed the first such calculation [7], and we have since extended the calculation as described here [2]. The dominant source of error in these calculations is the extrapolation to the physical quark masses, and as Figure 10 shows, our method has allowed us to calculate the axial charge with lighter quark masses than any other group.

4. Conclusions

The calculation of the nucleon axial charge described here represents a significant advance in our ability to compute properties of the proton and neutron from lattice QCD. Current calculations are focused on reducing the error at the lightest quark masses in Figure 9, extending our calculations to lighter masses still, and additionally calculating with a smaller lattice spacing. Continued improvement in our calculation of the axial charge will bolster the strength behind our computation of nucleon observables that can not be measured experimentally but yet reveal a great deal about the quark and gluon structure of the nucleon.

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