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# STRINGS FOR QUANTUMCHROMODYNAMICS

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During the last decade, intriguing dualities between gauge and string theory have been found and explored. They provide a novel window on strongly coupled gauge physics, including QCD-like models. Based on a short historical review of modern string theory, we shall explain how so-called AdS/CFT dualities emerged at the end of the 1990s. Some of their concrete implications and remarkable recent progress are then illustrated for the simplest example, namely the multicolor limit of $\mathcal{N} = 4$ SYM theory in four dimensions. We end with a few comments on existing extensions to more realistic models and applications, in particular to the sQGP. This text is meant as a non-technical introduction to gauge/string dualities for (particle) physicists.

*Keywords*: String theory; Gauge theory; Strong/weak coupling duality

## 1. Introduction and early history

String theory today is mostly perceived as a theory of Planck scale physics, offering one promising path towards a unification of all interactions. But when it was first born around 1970, it was meant to model strong interactions, in particular the large number of resonances that were observed in laboratory experiments. It is widely known that these early applications of fundamental strings to GeV scale physics failed rather miserably. Therefore, it may seem a bit surprising to observe the large number of recent meetings devoted to connections of strings and Quantumchromodynamics (QCD) or even to see string theory being highlighted in talks on QCD and Heavy Ion collisions at this conference (see in particular the contributions of G. Marchesini[1] and X.N. Wang).

The aim of this lecture is to explain how such a turn could occur and to give some idea of what we can expect from it in the future. To this end, we shall retrace the history of string theory, starting from the early attempts to describe strong interactions, then passing through more modern developments in the 1980s and 1990s until we reach the discovery of intriguing novel dualities between string and gauge theory that became known as AdS/CFT correspondence.

During the 1960s, physicists found an enormous number of strongly interacting hadrons. The longer the searches were pursued, the higher became the spins $J$ and masses $m$ of the observed resonances. In addition, a curious linear relation $J = \alpha_0 + \alpha' m^2$ emerged which could be characterized by the so-called Regge slope $\alpha'$ and intercept $\alpha_0$. In the absence of any other theoretical explanation, string theory seemed to provide an exciting perspective on these findings. Namely, it was shown that simple (open) string theories in flat space[2,3,4] naturally lead to a scattering amplitudes that had been proposed by Veneziano[5]

$$\mathcal{A}(s,t) \;=\; \frac{\Gamma(-1-\alpha' s)\Gamma(-1-\alpha' t)}{\Gamma(-2-\alpha'(s+t))} \qquad (1)$$

where $s, t$ are the Mandelstam invariants of the scattering process. While $s$ parametrizes the center an mass energy, $t$ is related to the scattering angle of the event. From the pole structure of $\Gamma$ functions it is easy to deduce the following expansion of $\mathcal{A}$ at small $s$,

$$\mathcal{A}(s,t) \;\sim\; -\sum_J \frac{P^J(s)}{\alpha' t - J + 1}\,. \qquad \text{(small s)}$$

Here, $P^J$ is a polynomial of degree $J$. Hence, $\mathcal{A}$ does indeed encode the exchange of reso-

nances which lie on a Regge trajectory $m^2 = (J-1)/\alpha'$. This success of string theory is not too difficult to understand. String modes in flat space are harmonic oscillators and it is well known from basic quantum mechanics that these possess a linear spectrum with a distance between the spectral lines that is determined by the tension $T_s$ of the string. If we choose the latter to be $T_s \sim 1/\alpha'$ then we may identify hadronic resonances with vibrational modes of a string (provided we are willing to close an eye on the first resonance with $J = 0$ which is tachyonic).

Obviously, the formula (1) must not be restricted to small center of mass energies. It can also be evaluated e.g. for fixed angle scattering at large $s$. Using once more some simple properties of the $\Gamma$ function one can derive

$$\mathcal{A}(s,t) \;\sim\; f(\theta)^{-1-\alpha' s} \qquad \text{(large s)}$$

where $f$ is some function of the center of mass scattering angle $\theta$ whose precise form is not relevant for us. The result shows that fixed angle scattering amplitudes predicted by flat space string theory fall of exponentially with the energy $\sqrt{s}$. Unfortunately for early string theory, this is not at all what is found in experiments which display much harder high energy cross sections. The failure of string theory to produce the correct high energy features of scattering experiments is once more easy to understand: strings are extended objects and as such they do not interact in a single point but rather in an extended region of space-time. Consequently, their scattering amplitudes are rather soft at high energies (small distances), at least compared to point particles. In this sense, experiments clearly favored a point particle description of strongly interacting physics over fundamental GeV scale strings.

As we all know, a highly successful point particle model for strong interactions, known as Quantumchromodynamics, was established only a few years later. It belongs to the class of gauge theories that have ruled our description of nature for several decades now. Due to its asymptotic freedom, high energy QCD is amenable to perturbative treatment. On the other hand, low energy (large distance) physics is strongly coupled and therefore remains difficult to address. Even though the problem to understand e.g. confinement remains unsolved, QCD has at least never made any predictions that could be clearly falsified in a simple laboratory experiment, in contrast to what we have reviewed about early string theory. So, in spite of its intriguing success with hadronic resonances, string theory retracted from the area of strong interactions, it even disappeared from physics for more than a decade before re-emerging as a quantum theory of gravity.

Our discussion throughout the last few paragraphs did provide a very simple explanation for the failure of early string theory, showing that it was linked directly to the strings' extended nature. This might make it difficult to believe that string theory could ever make it back into strong interactions. But the early attempts were based on the implicit assumption that the relevant strings were moving in the same 4-dimensional space-time as the gauge theory objects (possibly with some additional compact dimensions). At the time, there was neither any reason nor sufficient technical ability to think of any other scenario. But as string theory was developed, the assumption appeared less and less natural until it was eventually understood that many gauge theories do admit a dual description that involves strings moving in curved 5-dimensional backgrounds.

## 2. A Sketch of String Theory

In order to prepare for such insights, we need to review the development of string theory throughout the 1980s and 90s. We shall begin with a brief sketch of the relation between

closed strings and gravity, then discuss the so-called branes along with their open string excitations. The latter bring in gauge theories and thereby shall enable us to argue for an intriguing novel relation between closed strings and gauge theory.

### 2.1. *Closed Strings and Gravity*

As we mentioned before, superstring theory re-emerged in the 80s after it had been realized that it provided a natural and consistent host for gravitons[6]. In order to be a bit more specific, we shall consider closed strings propagating in some background geometry $X$. It is widely known that superstrings require $X$ to be 10-dimensional, so that contact with 4-dimensional physics is often made by rolling extra directions up on small circles, or through more general compactifications.

Strings possess infinitely many vibrational modes which we can think of as an infinite tower of massless and massive particles propagating on $X$. The mass spectrum of the theory is linear, with the separation that is parametrized by the tension $T_s \sim 1/\alpha'$ or, equivalently, by the length $l_s = \sqrt{\alpha'}$ of the string. As strings propagate through $X$, they can interact by joining and splitting. A simple such process for a one-loop contribution to the $2 \to 2$ scattering of closed strings is depicted in figure 1. Let us observe that any such diagram, no matter how many external legs and loops it has, may be cut into 3-vertices. Consequently, all interactions between strings are controlled by a single coupling constant $g_s$ that comes with the 3-vertex.

String theory possesses a consistent set of rules and elaborate computational tools to calculate scattering amplitudes. These produce formulas of the form (1). It is of particular interest to study their low energy properties. When $E \ll l_s^{-1}$, vibrational modes cannot be excited and all we see are massless point-like objects. One may ask whether these behave like any of the particles we know. The answer is widely known: at low energies, massless closed string modes scatter like gravitons and a bunch of other particles that form the particle content of 10-dimensional supergravity theories. This observation is fundamental for string theory's advance into quantum gravity, which came after failed attempts to develop perturbative quantum gravity had led to the conclusion that Einstein's theory is unlikely to be a fundamental theory of gravity. Similarly to Fermi's theory of weak interactions, it should rather be considered as an effective low energy theory that must be deformed at high energies in order to be consistent with the principles of quantum physics. String theory imposed itself as the most promising candidate for a fundamental theory of gravity.

### 2.2. *Solitonic and D-Branes*

For a moment, let us turn our attention to (super-)gravity theories. We are all familiar with the Schwarzschild solution of Einstein's theory of gravity. It describes a black hole in our 4-dimensional world, i.e. a heavy object which is localized somewhere in space. Similar solutions certainly exist for the supergravity equations of motion. The massive (and charged) objects they describe may but need not be point-like localized in the 9-dimensional space. In fact, explicit solutions are known[7] in which the mass density is localized along p-dimensional surfaces with $p = 1$ corresponding to strings, $p = 2$ to membranes etc. Such solutions were named

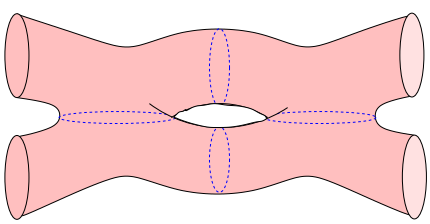

Fig. 1. Any string amplitude can be decomposed into 3-vertices.

black p-branes. Like ordinary black holes, however, most of these objects decay. But there exist certain extremal solutions, also known as solitonic p-branes, that are stable.

Now let us recall from the previous subsection that supergravity emerges as a low energy description of closed string theory. Consequently, if supergravity contains massive $p+1$ dimensional objects, the same should be true for closed string theory. One may therefore begin to wonder about the role p-branes could play in string theory. In order to gain some insight, let us suppose that a brane has been placed into the 9-dimensional space of our string background. Since it is heavy and charged, it will interact with the closed string modes in this background. In supergravity, we would describe this interaction through the exchange of gravitons or other particles mediating the relevant interaction. In string theory, a similar picture is possible only that now the interaction is mediated by exchange of closed strings as shown on the left hand side of figure 2. But the figure suggests another way to think about the very same process. In order to allow for an unbiased view, we have re-drawn the interaction process on the right hand side of figure 2. What we see now is an infalling closed string that seems to open up when it hits the brane. For a brief period, an excited state is formed in which an open string propagates with both its ends remaining attached to the brane. Finally, this state decays again by emitting a closed string. Hence, we found two very different ways to think about exactly the same process. One of them involves an excited state of the p-brane in which an open string travels along the p+1 dimensional world-volume. In order for such a state to exist, branes in string theory must be objects on which open strings can end. This is indeed the defining feature of so-called D(irichlet)p-branes in string theory[8].

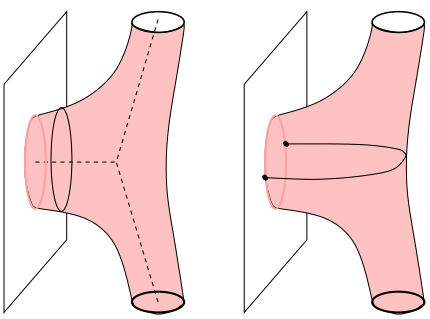

Fig. 2. There are two ways to think about the interaction between closed strings and brane.

### 2.3. *D-Branes and Gauge Theory*

In the previous subsection we argued that D-brane excitations can be thought of as open strings whose endpoints move within the p-dimensional space of a brane. Therefore, branes provide us with a second set of light objects, namely the vibrational modes of open strings. One can ask again whether the massless open string modes behave like any of the known particles. The answer is known for a long time: When $E \ll l_s^{-1}$, massless open string modes scatter like gauge bosons or certain types of matter.

In order to obtain non-abelian gauge theories it is necessary to consider clusters of branes. It is a remarkable fact of supergravity that special clusters can give rise to stable configurations. This is true in particular for

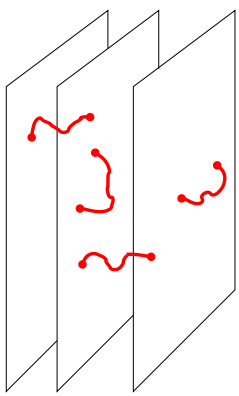

Fig. 3. Open strings can stretch between any pair of branes in a stack. For bookkeeping purposes we introduce the color indices $a, b$.

a stack of $N$ parallel branes. Let us number the member branes of such a cluster or stack by indices $a, b = 1, \ldots, N$. Open strings must have their end-points moving along one of these $N$ branes (see figure 3). Since an open string has two ends, modes of an open string carry a pair $a, b$ of 'color' indices. Hence, massless open string modes on a stack of $N$ parallel branes can be arranged in a $N \times N$ matrix, just as the components of a $U(N)$ Yang-Mills field. In addition to non-abelian gauge bosons, various matter multiplets can emerge from open strings. The precise matter content of the resulting low energy theories depends much on the brane configuration under consideration and we shall not make the attempt to describe it in any more detail.

It is worth rehashing how $p+1$ dimensional gauge theories have entered the stage through the back door. When we began this short cartoon of string theory, closed strings (and therefore gravitons) were all we had. Then be convinced ourselves that the theory contains additional heavy $p+1$ dimensional D-branes. Their excitations brought open strings into the picture and thereby another set of light degrees of freedom, including non-abelian gauge bosons. Let us stress once more that the latter do not propagate in the 10-dimensional space-time but rather on the $p+1$-dimensional brane worlds. The dimension $p+1$ can take various values one of them being $p+1=4$! Our sketch of modern string theory has now brought us to the mid 1990s. At this point we have gathered all the ingredients that are necessary to discover a novel set a of equivalences between gauge and string theory.

## 3. AdS/CFT Correspondence

We have reached the main part of this lecture in which we will motivate and describe the celebrated AdS/CFT correspondence[9]. Special attention will be paid to the simplest example of such a duality between 4-dimensional $\mathcal{N}=4$ Super Yang-Mills (SYM) theory and closed strings on $AdS_5 \times S^5$. This will enable us to outline the formulation and the use of such dualities.

### 3.1. *String/Gauge Dualities*

The main origin of the novel dualities is not too difficult to grasp if we cleverly combine what we have seen in the previous part. To this end, let us suppose that we have placed two branes in our 10-dimensional background and that they are separated by some distance $\Delta y$. Since all branes are massive and charged objects, they will interact with each other. In supergravity, we would understand this interaction as an exchange of particles, such as gravitons etc. Our branes, however, are objects in string theory and hence there exists an infinite tower of vibrational closed string modes modes that mediate the interaction between them. A tree level exchange is shown on the left hand side of figure 4. But as in our previous discussion, there exists another way to think of exactly the same process in terms of open strings. This is visualized on the right hand side of figure 4. There, the interaction appears to originate from pair creation/annihilation of open string modes with one end on each of the branes. In string theory, these two prescriptions of the interaction give exactly the same final result for the force between the two branes.

A closer look reveals that the equivalence of our two computational schemes, one in terms of closed strings the other in terms of open strings, is surprisingly non-trivial. Suppose, for example, that the distance $\Delta y$ between the branes is very large. Then the closed string modes have to propagate very far in order to get from one brane to the other. Consequently, contributions from massive string modes may be neglected and it is sufficient to focus on massless closed string modes, i.e. on the particles found in

10-dimensional supergravity. In the other regime in which the separation between the two branes becomes of the order of the string length $l_s$, such an approximation cannot give the right answer. Instead, the full tower of closed string modes must be taken into account. In other words, when $\Delta y \sim l_s$ the supergravity approximation breaks down and we have to carry out a full string theory computation. From the point of view of open strings, the situation is reversed. When the branes are far apart, pair created open strings only propagate briefly before they annihilate again and hence the entire infinite tower of open string modes contributes to this computation. In the opposite regime where $\Delta y \sim l_s$, however, the interaction may be approximated by restricting to massless open string modes, i.e. all we need to perform is some gauge theory computation.

As simple as these comments on figure 4 may seem, they lead to a remarkable conclusion: In the regime $\Delta y \sim l_s$, some calculation performed in the gauge theory on the world volume of our branes should lead to the same result as a full fledged string theory calculation for closed strings propagating in the 10-dimensional background. There are a few aspects of this relation that deserve to be stressed. In fact, we observe that it

- does neither preserve character nor difficulty of the computation,
- relates diagrams involving a different number of loops,
- relates two theories in different dimensions, i.e. it is holographic.

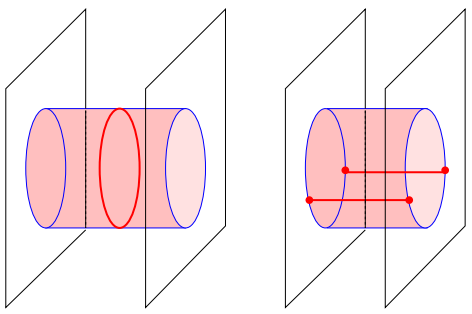

Fig. 4. There are two ways to think about the interaction between two branes. One involves opens strings, the other is mediated by closed string modes.

These three features emerge clearly from our analysis. The first point is obvious. In fact, the two computations are so different that they would usually not be performed by members of the same scientific community. Furthermore, in our example, we related a gauge theory one-loop amplitude to a tree level diagram of closed string theory, i.e. we showed that classical string theory encodes information on quantum gauge theory and vice versa. Finally, the gauge theory degrees of freedom are bound to the $p$+1-dimensional world-volume of our branes whereas closed strings can propagate freely in 9+1 dimensions. We shall see these three features re-emerge in the concrete incarnations of the gauge/string theory dualities we are about to discuss.

### 3.2. $\mathcal{N} = 4$ *SYM theory & AdS*$_5$

In the previous subsection we argued that string theory should be able to produce fascinating novel relations between gauge and string theory. But the picture was a bit too general to fully appreciate the powerful implication of our discussion. In order to be more specific, let us focus on the most studied example.

It arises from a stack of $N$ parallel D3 branes that are placed in a flat 10-dimensional (type IIB) superstring background[9]. According to our general discussion, low energy excitations on such a brane configuration are described be some 3+1-dimensional gauge theory. The theory in question turns out to be an $\mathcal{N} = 4$ Super Yang-Mills theory. In addition to the SU(N) gauge bosons, this model possesses six scalar fields and a bunch of fermions. Admittedly, except for being 4-dimensional, this is not the most realistic model of our world. Not only

does it possess the wrong matter content, it also is an example of a conformal field theory (CFT), i.e. it looks exactly the same on all length scales, in sharp contrast to e.g. QCD. In particular, the $\mathcal{N} = 4$ SYM quantum theory has no confining phase. But for the moment we only intend to explain some general ideas and so we defer such concerns to the next section.

Gauge/string duality claims that, in the limit of large number $N$ of colors, $\mathcal{N} = 4$ SYM theory is dual to a theory of closed strings which propagate in the curved near-horizon geometry of our stack or D3 branes. The latter can be shown to split into the product of a compact 5-sphere $S^5$ and a non-compact 5-dimensional Anti-deSitter (AdS) space $AdS_5$. Everybody knows that $S^5$ consists of all points in 6-dimensional Euclidean space that possess the same distance $R$ from the origin. $AdS_5$ can be constructed in the same way only that the 6-dimensional Euclidean space is now replaced by a space with 2 time-like and 4 space-like coordinates,

$$y_1^2 + y_2^2 + y_3^2 + y_4^2 - y_5^2 - y_6^2 = R^2 \ . \quad (AdS_5)$$

The resulting 5-dimensional non-compact space includes one coordinate $r$ that measures the radial distance from the stack of branes in addition to the branes' four world-volume coordinates.

The assertion of the duality is that computations in $\mathcal{N} = 4$ SYM theory and in string theory on $AdS_5 \times S^5$ give identical results! Of course, gauge physicists and string theorists need to use an extensive dictionary in order to compare the outcome of their respective computations. We shall only discuss a few entries of this dictionary here in order to give an idea of how this works.

To begin with, let us talk about the parameters in the two theories. On the gauge theory side, there are two of them, namely the number $N$ of colors and the Yang-Mills coupling $g_{YM}$. In the context of large $N$ limits, it is more appropriate to work with the so-called 't Hooft coupling $\lambda = g_{YM} N^2$ instead of $g_{YM}$. On the string theory side, we have the string coupling $g_s$ and the radius $R/l_s$ of the $AdS_5$ space measured in units of the string length $l_s$. We are prepared now to state the first entry in the AdS/CFT dictionary which relates the two sets of parameters as follows

$$\lambda = (R/l_s)^4 \quad , \quad N = \lambda g_s^{-1} \quad . \quad (2)$$

Gauge theory computations are perturbative in $\lambda$ and hence get mapped onto the extremely stringy regime in which the curvature radius $R$ of $AdS_5$ is of the order of the string length $l_s$. Furthermore, the comparison with perturbative string theory results requires the string coupling $g_s$ to be small and hence a large number of colors. We shall discuss this further in the next subsection.

Let us turn to the second entry of the AdS/CFT dictionary. When dealing with gauge theories, we are interested in gauge invariant fields or operators, such as the stress-energy tensor, the trace of the field strength etc. According to the AdS/CFT dictionary, such operators correspond to the modes of closed strings moving in $AdS_5 \times S^5$. One example of this map between gauge theory operators and string modes involves the stress energy tensor of the gauge theory which gets mapped to the massless graviton of the closed string theory.

Listing gauge invariant operators in terms of closed string modes may not seem such a big deal at first, until it is realized that this map preserves additional data. We have mentioned above that $\mathcal{N} = 4$ SYM theory looks the same on all scales. Hence, re-scalings can always be undone by a field re-definition. In this way, every field is assigned its length dimension $\Delta$. In most cases, the latter receives quantum corrections, i.e. it is given by the classical dimension $\Delta_0$ of the field plus a quantum contribution, the so-called anomalous dimension $\delta = \Delta - \Delta_0$. With gauge invariant oper-

ators on the SYM side being in one-to-one correspondence to closed string modes, one may now wonder whether it is possible to determine anomalous dimensions from string theory. AdS/CFT duality suggests that this is the case and that the relevant quantity to compute is the mass of the associated string mode. Let us test this quickly for the only example we can treat without any effort: The stress energy tensor of the gauge theory has a vanishing anomalous dimension. This matches the fact that the graviton mode of the closed string is massless. Even without carrying our discussion of the dictionary through to any of its many further entries[9], we hope to have shown that the novel gauge/string dualities are not only non-trivial but also quite concrete.

### 3.3. *Solution of the AdS/CFT*

In principle, the option to compute e.g. anomalous dimensions from string theory opens very exciting new avenues. But there is one issue with putting it to use right away: At this point, string theory on $AdS_5$ has not been solved! In particular, we are not able to write down its mass spectrum. In fact, while it is easy to determine the spectrum of vibrational modes for strings in flat space, solving the same problem for strings in the curved $AdS_5$ space is a very hard technical challenge. A helpful analogy is provided by the spectrum of the Laplace operator. Finding its eigenvalues on a torus required no work at all since eigenfunction are simply plane waves. The same problem on a sphere or any other curved space is significantly harder.

After this bad news there is a bit of good news too. String theory on $AdS_5$ is solvable and there exists a certain amount of technology already that deals with somewhat similar problems in lower dimensions[11,12,13]. Much more work will go into further developing the existing methods of so-called integrable models, into the investigation of their symmetries and various limiting regimes, before we can hope to extract the desired information. But there is a growing number of collaborations that have made this one of their prime tasks.

In the meantime, the AdS/CFT correspondence is far from being useless. Let us note first that there exits a limit in which we can estimate string theory quantities by their supergravity approximations. As usual, this requires $l_s$ to be small or, equivalently, $R/l_s$ to be large. Hence, according to eq. (2), supergravity computations encode information on strongly coupled gauge theory. This is certainly very exciting, but it requires some faith into the correspondence, at least when applied to gauge theory quantities that supersymmetry does not protect from receiving quantum corrections. We shall come back to some concrete supergravity predictions later on.

During the last years, a much more advanced approach was developed that, for the first time, interpolates non-trivially between the regime of perturbative gauge theory on one side and supergravity on the other. It is based on the idea that, even before being able to quantize string theory in $AdS_5$, we can determine its spectrum in a semiclassical approximation when some quantum numbers become large. To be a bit more specific, let us consider the anomalous dimension $\delta = \delta(\lambda)$ of twist-2 operators in the limit of large spin $S$ which can be shown to take the form,

$$\delta(\lambda) \;=\; f(\lambda)\,\log S + \ldots \qquad (3)$$

where the $\ldots$ stand for lower order terms in the spin $S$. The universal scaling function $f(\lambda)$ multiplying the leading $\log S$ term is also known as the cusp anomalous dimension. Its behavior at large $\lambda$ was first calculated from string theory a few years ago[14,15]. In a beautiful development initiated by Minahan and Zarembo[16], the function $f(\lambda)$ has been determined recently by Beisert, Eden and Staudacher[17]. Their formula correctly

reproduces highly non-trivial gauge theory results up to four (!) loops[19,18,20] and then interpolates all the way to large $\lambda$ where it matches the strong coupling predictions[21]. To date, this is certainly the most impressive demonstration of the AdS/CFT correspondence. It required the use of highly non-trivial technologies borrowed from integrable systems, in particular the so-called Bethe-Ansatz that was introduced to solve problems in statistical physics[22].

## 4. Extensions and Applications

In this final section we would like to briefly touch upon some of the extensions and applications that go beyond the simple example we have discussed so extensively above. This is an extremely active field right now and therefore we are not be able to do it any justice. In particular, we apologize to many authors of original contributions for (almost) systematically avoiding references.

### 4.1. *More Realistic Models*

As we have pointed out before, the $\mathcal{N} = 4$ SYM theory we have used in the last section to illustrate the AdS/CFT correspondence is far away from anything that resembles nature. In order for gauge/string dualities to be of practical use, one needs to construct examples in which the gauge theory has more realistic features. These certainly include broken scale invariance, and in particular confining models, (partially) broken super-symmetry, the inclusion of finite temperature, flavor, chiral symmetry etc. The search strategy is rather clear: Our first example was obtained by placing D3-branes in a 10-dimensional flat background. In order to find new pairs of dual theories we can modify both the background and the brane configurations we start with. In this way, all the properties we listed above have been realized in one way or another.

To address confinement, we should understand in some more detail why string theory in $AdS_5 \times S^5$ cannot produce a confining phase. Confinement is normally detected through a term in the quark anti-quark potential that grows linearly with the separation. On the dual string theory side we think of the quark anti-quark pair as being located at $r = \infty$. The two gauge theory particles sit at the ends of a string which hangs deeply into the fifth dimension of $AdS_5$, pulled by the gravitational attraction of the branes at $r = 0$. For our discussion of confinement it is a crucial observation that the stack of D3 branes produces a gravitational red-shift factor that vanishes at $r = 0$, causing the string's effective tension to approach zero in the vicinity of the branes. Hence, stretching a string along the brane at $r = 0$ costs no energy. This implies that there is no linear term in the quark anti-quark potential and hence there is no confinement. Consequently, we may think of a confining background as one that is been capped off at some finite $r = r_{min}$ close to the branes' location so the the red-shift remains non-zero. Several concrete and basic constructions of confining models[23,24,25] have been explored.

Studying gauge theories with less supersymmetry is possible if we start our construction with a less supersymmetric background and/or brane configuration. Similarly, we can heat the dual pair of theories to finite temperature by placing a black hole into the 5-dimensional geometry. Its temperature is felt in both string and gauge theory. For lack of space we can neither go into any more detail nor continue listing further QCD-like features and their string theoretic implementation. The interested reader can find a more satisfactory account of early constructions in a nice review of Klebanov[26]. More recent developments in this lively field run under keywords such as holographic QCD or AdS/QCD.

### 4.2. *AdS/CFT and sQGP*

As described in the talk of X.N. Wang, there is evidence that heavy ion collisions at RHIC produce a strongly coupled quark gluon plasma (sQGP). From the point of view of string theory such a discovery has some appeal. In fact, as we have stressed several times before, the AdS/CFT correspondence provides new leverage for advancement into strongly coupled gauge physics and hence a sQCP could serve as an almost ideal laboratory. Many characteristics of the plasma are discussed in a rapidly growing number of publications on AdS/CFT and sQCD, including shear viscosity, jet quenching etc. We are not able to cover even a small fraction of these and limit ourselves to comments on some early observations.

The QGP produced in heavy ion collision behaves like a liquid that may be treated using concepts and methods of hydrodynamics. In particular, by Kubo's formula, its viscosity $\eta$ is related to correlations of the gauge theory's stress energy tensor through,

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt d^3x e^{i\omega t} \langle [T(t,x), T(0,0)] \rangle .$$

The right hand side of this equation should be evaluated at the temperature of the plasma and for some finite gauge theory coupling $\lambda$. Since the stress energy has spin $S = 2$, a string theory evaluation of Kubo's formula can only be performed in the supergravity limit, i.e. at infinite 't Hooft coupling $\lambda$. For the background of D3 branes, the result is[27]

$$(\eta/\varsigma)^{\mathcal{N}=4}_{\lambda=\infty} \;=\; 1/4\pi \quad . \qquad (4)$$

Here, $\varsigma$ denotes the entropy density of the plasma. Even though the computation applies to the large color limit of $\mathcal{N} = 4$ SYM at infinite coupling, rather than to usual QCD, the results underestimates the observed values merely by a factor of two. This seems particularly remarkable when compared to perturbative gauge theory results which make predictions for small coupling that are one order of magnitude higher than observations. More realistic string theory models at finite coupling have been argued to give numerical values for $\eta/\varsigma$ that are bounded from below by the result (4).

### 4.3. *High Energy scattering*

We do not want to conclude this introduction to the AdS/CFT duality without explaining how the novel gauge string dualities manage to circumvent what seemed like a 'no-go' theorem in the introduction. There we understood that the extended nature of strings, as desirable as it is when modeling hadronic resonances, causes cross sections to fall off way too fast at high energies. The resolution is directly linked to the fact that strings in the AdS/CFT correspondence possess a fifth dimension to propagate in, namely the direction that parametrizes the distance $r$ from the stack of branes. Consider an observer at infinity who is searching for string modes at some given energy $E$, much smaller than the string tension $T_s$. If the strings are far away from the brane, the energy $E$ is not sufficient to excite the strings' vibrational modes and all the observer sees are point-like objects with the usual hard high energy scattering amplitudes. Strings closer to the branes, however, appear red-shifted through the branes' gravitational field. In other words, they possess an effective tension $T_s^{eff}(r)$ that depends on $r$ and can become so small that vibrational modes can be excited with the energy $E$. The concrete from of the gravitational red-shift in the D3 brane geometry implies that

$$T_s^{eff}(r) \;=\; (r/R)^2 \; T_s \quad . \qquad (5)$$

Through this effect, our observer at $r = \infty$ is able to see a tower of vibrational modes that are associated with closed strings near $r = 0$. Gauge theory amplitudes at small $s$ and $t > 0$ receive their dominant contribution from the region near the branes' location

and hence display the usual Regge behavior with a Regge slope given by

$$\alpha'_{eff} \;=\; \alpha' \, (R/r_{min})^2 \quad .$$

The length $r_{min}$ was introduced in subsection 4.1 in the context of our discussion of confining theories. For large center of mass energy $s$ and $t < 0$, however, amplitudes are dominated by string scattering processes near $r = \infty$ and hence they possess the hard features of particle models[28], in agreement with observation. Thereby we have explained how the existence of a non-trivial fifth dimension for strings does indeed overcome longstanding problems for the application of string theory to strong interactions/gauge physics.

This brings us to the end of our short introduction to gauge/string dualities. We have seen how they emerge from the modern picture of string theory and were able to glimpse at some of their powerful and concrete implications. Admittedly, much further work is necessary, in particular to elevate our understanding of the relevant string theories beyond supergravity and semiclassical approximations and, of course, to get closer to studying real QCD. On the other hand, the path seems very promising.

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