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Nucleon form factors on the lattice with light dynamical fermions

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The electromagnetic form factors provide important insight into the internal structure of the nucleon and continue to be of major interest for experiment and phenomenology. For an intermediate range of momenta the form factors can be calculated on the lattice. However, the reliability of the results is limited by systematic errors mostly due to the required extrapolation to physical quark masses. Chiral effective field theories predict a rather strong quark mass dependence in a range which was yet inaccessible for lattice simulations. We give an update on recent results from the QCDSF collaboration [1, 2] using gauge configurations with dynamical $N_f = 2$, non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions at pion masses as low as 350 MeV.

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1. Introduction

The proton and the neutron are the building blocks of atomic nuclei and therefore the most important particles subject to the strong interaction. Understanding their structure is therefore of strong interest. For several decades they have been studied in detail and it came as a great surprise at the beginning of this millennium that experiments at Jefferson Laboratory deviated from the prevalent theoretical understanding [3], see also [4] for recent reviews. The previous picture was based on the perturbative behavior of the partons at asymptotically high energies. A resolution of this mystery using non-perturbative techniques is therefore in demand. Another puzzling feature shows up when computing the size of the nucleon — expressed by its mean-squared radius, $\langle r^2 \rangle$ — within the framework of chiral perturbation theory. In this scheme the size diverges as the pion mass, m_π, decreases toward zero. Finding the correct behavior again calls for the application of model-independent non-perturbative techniques. Lattice QCD provides such a description with the merit of being free from model assumptions beyond QCD.

A further advantage of lattice QCD is the ability to vary the parameters like N_c, N_f, and m_q. This allows for the validity of specific model assumptions to be tested and is therefore of importance also for the investigation of models of the strong interaction, see e.g. [5]. Furthermore, in certain cases experimental data is very hard to extract. Generalized parton distributions [6] depend on several parameters and their extraction from experiment relies on the applicability of QCD factorization, see e.g. [7] for a discussion at this conference. On the lattice, on the other hand, these observables can be extracted without such difficulties and this has led to important insights [8].

On the downside, lattice simulations so far are limited to quark masses above those in Nature. Decreasing the quark mass comes at the expense of vastly increased demands in computer time. Despite tremendous progress in recent years, simulating at the light quark masses that Nature has chosen is still prohibitively expensive. To meet this challenge, progress is required both in machine development and in finding more efficient algorithms. To this day, even the most advanced computations only reach quark masses corresponding to pion masses of about 300 MeV and above.

To investigate hadron structure, three very different techniques are being employed on the lattice today: (a) Ongoing simulations with Wilson-type quarks down to smaller quark masses by exploiting more efficient algorithms and increased computer power [9]. (b) Starting simulations with the Ginsparg-Wilson formulation like domain-wall fermions [10] or overlap fermions [11]. This approach has several advantages over Wilson fermions, for a recent review consult [12]. It is, however, more costly at the currently accessible quark masses. At heavy quark masses these simulations are about 30 to 100 times more expensive. It is furthermore computationally demanding since the entire parameter space has to be explored again. (c) Using a hybrid-action approach with different discretizations for the sea- and valence-quarks [13]. The latter approach constitutes a compromise between low quark masses and performance at the expense of conceptual uncertainties. The theory breaks unitarity at finite lattice spacing which complicates the discussion of the continuum limit. In practice, rooted staggered quarks are being used for the sea quarks and it is not yet resolved if going to the continuum limit commutes with taking the square root, see e.g. [14] for recent reviews. Furthermore, matching sea- and valence quarks is prescription-dependent and some prescriptions may give rise to additional $\mathcal{O}(a^2)$ artifacts [15]. Expressions for finite-a chiral

extrapolations are known for several interesting cases, but not all observables [16].

This paper presents recent results from the QCDSF collaboration on the structure of the nucleon using two flavors of dynamical Wilson-Clover fermions. This approach corresponds to the choice (a) above. The merit is that it extends the existing data sets acquired over the past decade using full QCD simulations. The parameter space is well understood and surprises like unforeseen phase transitions are absent. The questions addressed are the scaling behavior of the ratio of $F_2(Q^2)/F_1(Q^2)$ at accessible values of the momentum transfer, $Q^2 \equiv -q^2 = -t$, and the mean charge radii of the corresponding form factors, $\langle r_1^2 \rangle$ and $\langle r_2^2 \rangle$, together with the anomalous magnetic moment, κ. To set the scale, we have set the Sommer parameter to a value of $r_0 = 0.467$ fm. With this choice, the lattice spacings range from $a = 0.07 \ldots 0.11$ fm and the pion masses cover a range of $m_\pi = 349 \ldots 1170$ MeV. We find that residual artifacts are small compared to the statistical errors, so we analyze the data from different lattice spacings together. The spatial volumes volumes vary between $(1.4)^3 \ldots (2.6)^3$ fm^3. It is evident that the parameter space is quite large and we address the question under which circumstances chiral extrapolations can be attempted.

In this calculation we have restricted ourselves to the case of full QCD, i.e., the sea- and valence quark masses are identical. The extraction of the matrix elements has been discussed in detail in previous publications [17]. The renormalization has been done non-perturbatively by requiring that the form factor $F_1(Q^2 = 0) = 1$ measures the electric charge of the proton.

2. Numerical results

2.1 The ratio of spin-flip to spin-non-flip form factors

As has been mentioned in the introduction, the ratio of spin-flip to spin-non-flip form factors, $F_2(Q^2)/F_1(Q^2)$, for sufficiently large values of Q^2 was one of the key investigations in the recent years. It was found that the experimental data is compatible with both

$$\begin{aligned} \frac{\sqrt{Q^2} F_2(Q^2)}{F_1(Q^2)} &\to \text{const}\,, \\ \frac{Q^2}{\log^2 Q^2} \frac{F_2(Q^2)}{F_1(Q^2)} &\to \text{const}\,. \end{aligned} \tag{2.1}$$

We have tested this behavior using our lattice data at the accessible values of Q^2. We find that the lattice data exhibits the same behavior already at values of $Q^2 > 1 \ldots 2$ GeV^2. The results from three different lattice spacings are displayed in Fig. 1 for the first ratio in Eq. (2.1). The result for the second ratio looks similar is not displayed separately. The working points correspond to a pion mass of $m_\pi \approx 600$ MeV. The behavior of the lattice data is qualitatively consistent with the phenomenological findings [18].

2.2 Charge radii and the anomalous magnetic moment

Next, we discuss the charge radii, $\langle r_1^2 \rangle$ and $\langle r_2^2 \rangle$, of F_1 and F_2 and the anomalous magnetic moment, κ. In this work we restrict ourselves to the isovector case $p - n = u - d$ since then the disconnected contributions cancel and a comparison with experiment is free from any residual systematic errors other than chiral, infinite volume and continuum extrapolations.

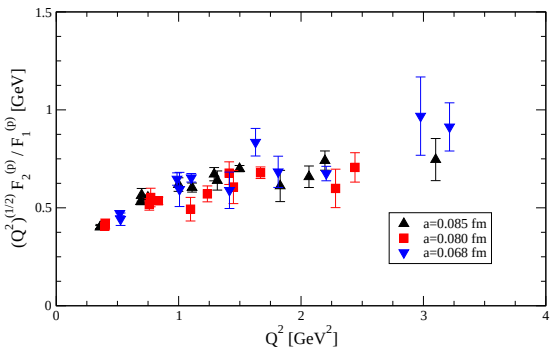

Figure 1: Momentum dependence of the form factor ratio. Results from three different lattice spacings at a pion mass $m_\pi \approx 600$ MeV are shown.

In order to parametrize the Q^2 dependence of the form factors, we have adopted dipole and tripole-type fits [1]. We have also attempted to fit the form factors using a p-pole form with a free parameter p

$$F_i(Q^2) = \frac{F_i(0)}{(1+Q^2/M^2)^p}\,. \tag{2.2}$$

However, as has been shown in [1], we are unable to measure the parameter p to sufficient accuracy to distinguish between dipole ($p=2$) and tripole ($p=3$) fits. In order to clearly determine the optimal fitting form from lattice data alone a larger range of Q^2 values is necessary. So we have to use additional phenomenological input to specify our fitting formulae and perform consistency checks.

In this work, we use the following fitting formulae:

$$\begin{aligned} F(Q^2) &= \frac{F(0)}{(1+Q^2/M^2)^2}\,, \qquad \text{for } F_1^{u-d}(Q^2)\,, \\ F(Q^2) &= \frac{F(0)}{(1+Q^2/M^2)^3}\,, \qquad \text{for } F_2^{u-d}(Q^2)\,. \end{aligned} \tag{2.3}$$

In forthcoming publications [19, 20] we will report on different fit ansätze. In this work, however, we will just employ the dipole- and tripole-fits.

From these expressions, we can extract $\langle r_1^2\rangle$, $\langle r_2^2\rangle$, and κ by expanding

$$F_i(Q^2) = F_i(0)\left(1 - \frac{1}{6}\langle r_i^2\rangle Q^2 + \mathcal{O}(Q^4)\right)\,, \quad \langle r_i^2\rangle = \frac{6p}{M^2}\,, \quad F_2(0) = 1+\kappa\,. \tag{2.4}$$

To compare them with experiment, we need to perform a chiral extrapolation. To this end, we discuss the formulae for the small scale expansion (SSE) given in [17]. Different chiral expansions have been supplied in [21].

When investigating the quantity $\langle r_1^2\rangle$, we fix the appearing parameters to phenomenologically reasonable values. Note that the radius diverges as m_π approaches zero, implying that the nucleon

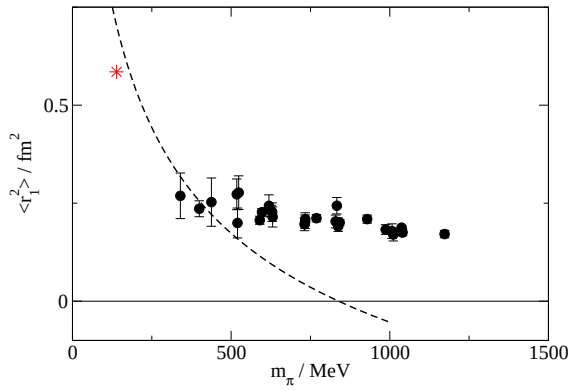

Figure 2: Physical value — indicated by the star — of $\langle r_1^2 \rangle$ in the isovector case together with the lattice data and a chiral fit obtained from the SSE expression quoted in [17].

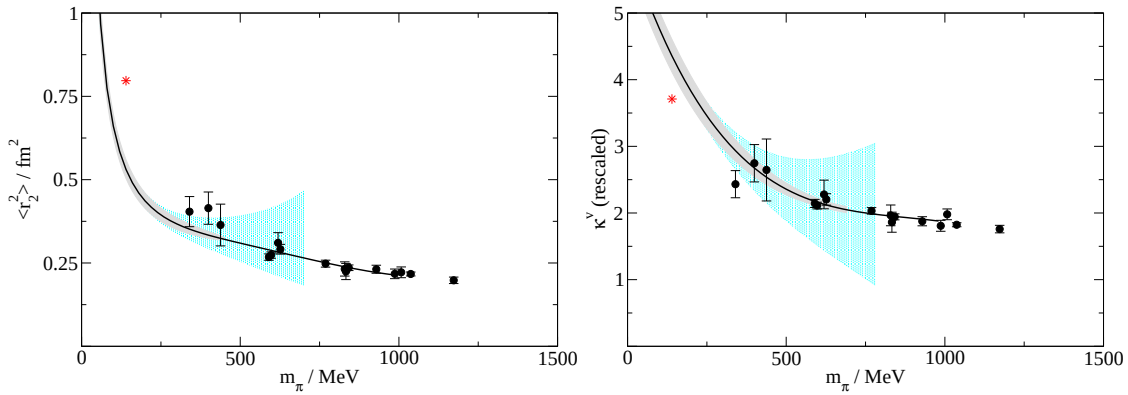

Figure 3: Combined fit to $\langle r_2^2 \rangle$ (left) and κ (right). The fits as obtained from the SSE expressions from [17] are shown together with the experimental results denoted by stars. The error bands are statistical (shaded) and systematical (dotted). κ has been rescaled by the lattice nucleon mass as described in [17].

develops a larger and larger pion cloud as the quark mass goes to zero. In the chiral limit the nucleon would be infinitely large. On the other hand, the expression for $\langle r_1^2 \rangle$ vanishes already at a finite value of m_π and becomes negative beyond. This behavior is unphysical. The resulting curve is displayed in Fig. 2 together with the experimental data point and our lattice results. The lattice data describes a nucleon which is still smaller than the experimental nucleon. Even at pion masses as low as 350 MeV there is no sign of a dramatic increase in size. The currently accessible pion mass is still beyond the threshold of an expected sharp increase in size.

When turning to the radius $\langle r_2^2 \rangle$ and the anomalous magnetic moment κ, we find that the expansion of r_2 explicitly depends on κ. Hence, we can perform a joint fit of both quantities and fix the parameters occurring in the expansion. Altogether, κ is taken to depend on three free parameters and r_2 on the same three plus an additional constant. The results of the combined fits are displayed in Fig. 3. From these fits we conclude that the resulting values of $\langle r_2^2 \rangle$ and κ are roughly compatible with the experimental values at the physical pion mass. On the other hand, the range of applicability of the chiral expansions does not seem to be as large as one might have hoped. Further study of chiral expansions is necessary to understand why these observables are

described better than $\langle r_1^2 \rangle$.

3. Summary

We have computed the behavior of the ratio $F_2(Q^2)/F_1(Q^2)$ of the proton for different momentum combinations. We have also obtained the charge radii $\langle r_1^2 \rangle$ and $\langle r_2^2 \rangle$, and the anomalous magnetic moment, κ, of the nucleon for the isovector combination $p-n=u-d$. Our calculation uses two flavors of dynamical Wilson-Clover fermions and covers a large range of parameter values down to pion masses of 350 MeV. We find that the first quantity is in qualitative agreement with the recent spin-transfer experiments conducted at Jefferson Lab. Furthermore, the chiral expansion together with the lattice data is consistent with experimental values for the radius $\langle r_2^2 \rangle$ and the anomalous magnetic moment κ.

As of today lattice simulations are established as a reliable tool for revealing the qualitative behavior of the structure of nuclear matter. To perform a similar matching quantitatively from first principles without additional model assumptions, however, will require more progress both in lattice technology and in our understanding of chiral expansions. Nonetheless, we are confident that already by the end of this decade quantitatively reliable predictions from first principles will be available.

Acknowledgments

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