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Kaon semileptonic decay form factors from $N_f = 2$ non-perturbatively $O(a)$-improved Wilson fermions

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We present first results from the QCDSF collaboration for the kaon semileptonic decay form factors at zero momentum transfer, using two flavours of non-perturbatively $O(a)$-improved Wilson quarks. A lattice determination of these form factors is of particular interest to improve the accuracy on the CKM matrix element $|V_{us}|$. Calculations are performed on lattices with lattice spacing of about 0.08 fm with different values of light and strange quark masses, which allows us to extrapolate to chiral limit. Employing double ratio techniques, we are able to get small statistical errors.

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1. Introduction

Recently high emphasis is placed on testing the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. In particular, the unitarity of the CKM matrix implies the unitarity constraint of the first row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta\,. \tag{1.1}$$

According to the PDG [1] δ is equal to $0.0008(11)$. The uncertainty is still quite substantial and has to be decreased. As $|V_{ub}|$ is much less than unity, about half of the error of δ comes from the uncertainty of $|V_{us}|$. Since the kaon semileptonic decay rate is proportional to $|V_{us}|^2|f_+(0)|^2$, this matrix element can be determined by combining experimental results for this decay rate and theoretical calculations of the vector form factor at zero momentum transfer $f_+(0)$.

The kaon semileptonic decay form factors $f_+(q^2)$ and $f_-(q^2)$ are defined as

$$\langle\pi(p')|V_\mu|K(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu\,, \tag{1.2}$$

where $q^2 = (p-p')^2$ is the momentum transfer and $V_\mu = \bar{s}\gamma_\mu u$ is the weak current. It is also convenient to introduce the scalar form factor defined as

$$f_0(q^2) = f_+(q^2)\left(1 + \frac{q^2}{M_K^2 - M_\pi^2}\xi(q^2)\right), \text{ and } \xi(q^2) = \frac{f_-(q^2)}{f_+(q^2)}\,. \tag{1.3}$$

The form factor $f_+(0)$ at zero momentum transfer was estimated by Leutwyler and Roos [2] in 1984. They obtained the value $0.961(8)$, which is still used as a reference value to extract $|V_{us}|$ from the experimental data. However, to estimate higher order terms in the chiral perturbation theory (ChPT) expansion, they used a model of the wave function of the pseudoscalar meson. Recent ChPT calculations [3] favour a slightly larger value of $f_+(0) = 0.984(12)$. Thus, it is desirable to calculate $f_+(0)$ non-perturbatively and lattice calculation provides such an opportunity.

Recently a lot of lattice groups reported calculations of $f_+(0)$ [4, 5, 6, 7, 8, 9, 10, 11]. Their results coincide with each other and are in agreement with that of Leutwyler and Roos. For a recent review see [12].

In this paper we present first preliminary results for these form factors from $N_f = 2$ flavours of light dynamical, non-perturbatively $O(a)$-improved Wilson fermions and Wilson glue.

2. Lattice setup

For our calculation we use a gauge ensemble at $\beta = 5.29$, which corresponds to a lattice spacing $a = 0.075$ fm,[1] and $\kappa_{\text{sea}} = 0.13590$, which corresponds to a pion mass $M_\pi = 0.591(2)\,\text{GeV}$. The lattice volume is $24^3 \times 48$ with spacial extent equal to $1.9\,\text{fm}$. The correlation functions have been calculated on about 800 gauge configurations using various source locations to reduce statistical noise. We identify the sea quark mass with the light quark mass and the valence quark mass with the strange quark mass using 3 values $\kappa_s = 0.13485, 0.13530$ and 0.13570. The corresponding kaon masses are $M_K = 0.780(11), 0.704(13), 0.629(14)\,\text{GeV}$.

[1] For translation into physical units the Sommer parameter $r_0 = 0.467$ fm is used.

3. Correlators

On the lattice we calculate two and three point functions, which are defined by

$$C_P(t,\vec{p}) = \sum_{\vec{x}} \langle O_{P,snk}(\vec{x},t) O^{\dagger}_{P,src}(\vec{0},0)\rangle e^{-i\vec{p}\vec{x}} \xrightarrow[t\to\infty]{} \frac{Z^*_{P,src} Z_{P,snk}}{2E_P(\vec{p})} e^{-E_P(\vec{p})t}, \quad (3.1)$$

$$C^{PQ}_{\mu}(t,t',\vec{p},\vec{p}') = \sum_{\vec{x},\vec{x}'} \langle O_{Q,snk}(\vec{x}',t') V_{\mu}(\vec{x},t) O^{\dagger}_{P,src}(\vec{0},0)\rangle e^{-i\vec{p}'(\vec{x}'-\vec{x})-i\vec{p}\vec{x}} \quad (3.2)$$

$$\xrightarrow[t,(t'-t)\to\infty]{} \frac{Z^*_{P,src} Z_{Q,snk}}{4E_P(\vec{p})E_Q(\vec{p}')Z_V} \langle Q(p')|V^{(R)}_{\mu}|P(p)\rangle e^{-E_P(\vec{p})t - E_Q(\vec{p}')(t'-t)}, \quad (3.3)$$

where P and Q denote either the K or π meson. The energy of meson P (Q) is denoted by $E_P(\vec{p})$ ($E_Q(\vec{p})$). The renormalised vector current, including the renormalisation factor Z_V, is denoted by $V^{(R)}_{\mu}$. The overlap with the physical meson states is given by $Z_{P,snk}$ and $Z_{P,src}$.

In the following we will assume the point meson sources to be inserted at time $t_{src} = 0$ (thus t_{src} is omitted in Eq. (3.3)) and point sinks are inserted at $t' = T/2$. As t' is fixed the t' dependence of all quantities is ignored in the following. For this choice of t' the three point functions (and therefore also the double ratios) are symmetric with respect to $T/2$. We make use of this property and average over both time ranges to increase the precision of calculations.

Note that smearing of meson operators is not used. This is because it leads to a momentum dependence of the overlap $Z_{P,snk}(p)$ and $Z_{P,src}(p)$. See, for example, Appendix A in Ref. [13] and the discussion below.

4. Scalar form factor at q^2_{max}

The scalar form factor $f_0(q^2)$ at $q^2 = q^2_{max} = (M_K - M_\pi)^2$ can be obtained from the double ratio of three point functions (which was originally proposed to calculate the $B \to Dl\nu$ form factor in Ref. [14]):

$$R(t) = \frac{C^{K\pi}_4(t,\vec{0},\vec{0}) C^{\pi K}_4(t,\vec{0},\vec{0})}{C^{KK}_4(t,\vec{0},\vec{0}) C^{\pi\pi}_4(t,\vec{0},\vec{0})} \xrightarrow[t\to\infty]{} \frac{(M_K+M_\pi)^2}{4M_K M_\pi} (f_0(q^2_{max}))^2 = \bar{R}. \quad (4.1)$$

Our results for $R(t)$ for three values of the strange quark masses are shown in the left panel of Fig. 1. Note that renormalisation constants and exponential factors exactly cancel in $R(t)$. We can extract $f_0(q^2_{max})$ with a statistical uncertainty $< 0.1\%$.

In the $SU(3)$ symmetric limit $\bar{R}$ is equal to unity. The deviation of $\bar{R}$ from unity depends on the physical $SU(3)$ breaking effects on $f_0(q^2_{max})$, which can be seen in the right panel of Fig. 1.

5. Interpolation to zero momentum transfer

To study the q^2 dependence of the form factor we calculate

$$F(\vec{p},\vec{p}') = \frac{f_+(q^2)}{f_0(q^2_{max})} \left(1 + \frac{E_K(\vec{p}) - E_\pi(\vec{p}')}{E_K(\vec{p}) + E_\pi(\vec{p}')} \xi(q^2)\right) \quad (5.1)$$

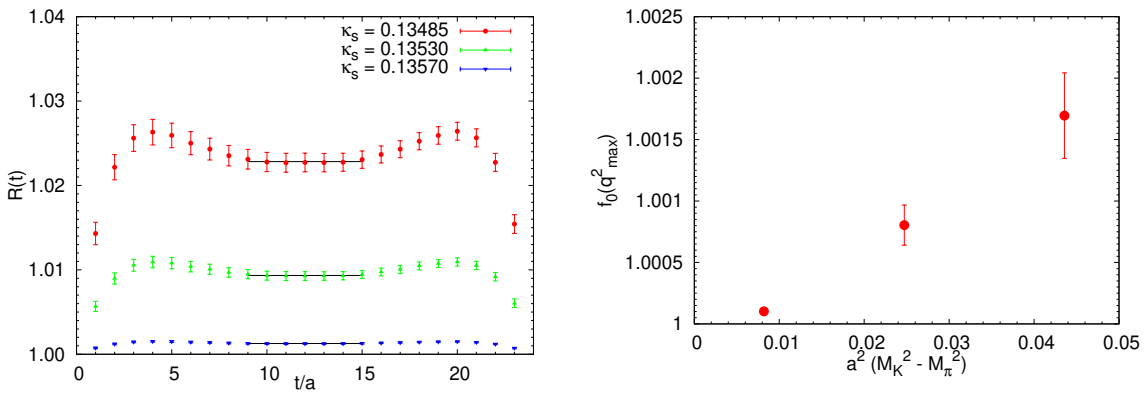

Figure 1: Left: Time dependence of the double ratio $R(t)$ (Eq. (4.1)) for three values of the strange quark masses. **Right:** Values of $f_0(q^2_{max})$ as a function of the $SU(3)$ breaking parameter $a^2\Delta M^2 = a^2(M_K^2 - M_\pi^2)$.

from the second double ratio

$$R_F(t,\vec{p},\vec{p}\,') = \frac{C_4^{K\pi}(t,\vec{p},\vec{p}\,')C^K(t,\vec{0})C^\pi(T/2-t,\vec{0})}{C_4^{K\pi}(t,\vec{0},\vec{0})C^K(t,\vec{p})C^\pi(T/2-t,\vec{p}\,')} \xrightarrow[t\to\infty]{} \frac{E_K(\vec{p})+E_\pi(\vec{p}\,')}{M_K+M_\pi}F(\vec{p},\vec{p}\,') = \bar{R}_F\,. \quad (5.2)$$

For meson point operators the overlaps cancel in (5.2). This is not true anymore if smeared meson operators are used, because their overlap with the physical state depends on the momentum. That is why we are using point sources and sinks. A systematic study of this effect will be reported elsewhere.

The double ratio $\bar{R}_F(\vec{p},\vec{p}\,')$ is symmetric under exchange of π and K and their momenta, i.e. $\bar{R}_F^{K\to\pi}(\vec{p},\vec{p}\,') = \bar{R}_F^{\pi\to K}(\vec{p}\,',\vec{p})$, which we make use of to increase the statistics at some values of q^2. In the left panel of Fig. 2 we show, as an example, the data for $R_F(t,\vec{p},\vec{p}\,')$ with $|\vec{p}\,'|=0, |\vec{p}|=1$ for $K\to\pi$ and $|\vec{p}\,'|=1, |p|=0$ for the $\pi\to K$ transition.

In order to convert $F(\vec{p},\vec{p}\,')$ to $f_+(q^2)$, we need to calculate $\xi(q^2)$. To do so, we define a third double ratio:

$$R_k(t,\vec{p},\vec{p}\,') = \frac{C_k^{K\pi}(t,\vec{p},\vec{p}\,')C_4^{KK}(t,\vec{p},\vec{p}\,')}{C_4^{K\pi}(t,\vec{p},\vec{p}\,')C_k^{KK}(t,\vec{p},\vec{p}\,')}\,,\ \ k=1,2,3\,. \quad (5.3)$$

Then $\xi(q^2)$ is given by

$$\xi(q^2) = \frac{-(\vec{p}+\vec{p}\,')_k(E_K(\vec{p})+E_K(\vec{p}\,')) + (\vec{p}+\vec{p}\,')_k(E_K(\vec{p})+E_\pi(\vec{p}\,'))\bar{R}_k(\vec{p},\vec{p}\,')}{(\vec{p}-\vec{p}\,')_k(E_K(\vec{p})+E_K(\vec{p}\,')) - (\vec{p}+\vec{p}\,')_k(E_K(\vec{p})-E_\pi(\vec{p}\,'))R_k(\vec{p},\vec{p}\,')}\,, \quad (5.4)$$

where $\bar{R}_k(\vec{p},\vec{p}\,') = \lim_{t\to\infty} R_k(t,\vec{p},\vec{p}\,')$. The double ratio $\bar{R}_k(\vec{p},\vec{p}\,')$ is exactly unity in the $SU(3)$ symmetric limit and is sensitive to $SU(3)$ breaking effects in the matrix element $C_\mu^{K\pi}$.

This ratio is the noisiest of all of them. Thus, $\xi(q^2)$ could only be extracted with a large uncertainty. The relative error turns out to be $20\% - 80\%$ for our statistics, even after we make use of the symmetry:

$$\bar{R}_k^{K\to\pi}(\vec{p},\vec{p}\,') = \bar{R}_k^{\pi\to K}(\vec{p}\,',\vec{p}) = \bar{R}_k^{\pi\to K}(-\vec{p}\,',-\vec{p})\,.$$

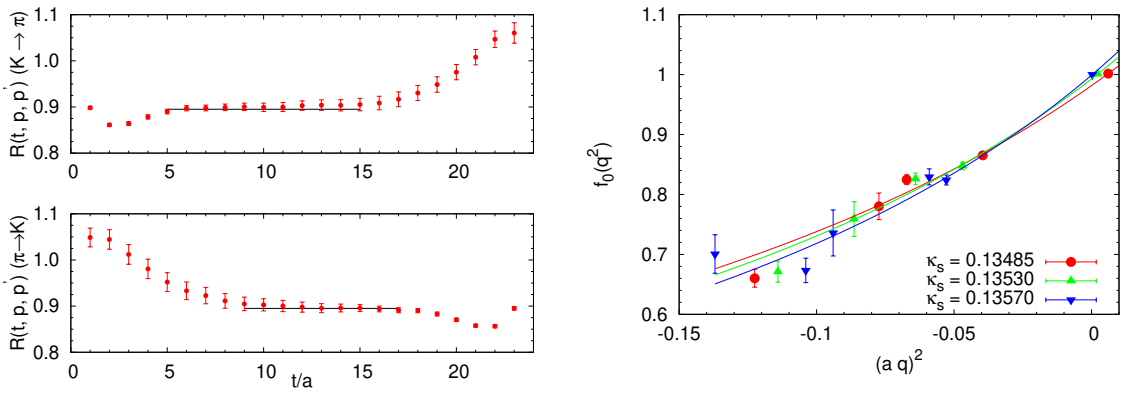

Figure 2: Left: Time dependence of double ratio $R_F(t,\vec{p},\vec{p}\,')$ for $\kappa_s = 0.13485$. **Right:** Scalar form factor $f_0(q^2)$ for different values of the strange quark mass. The solid line is the result of a monopole fit as described in the text.

From $f_0(q^2_{max})$, $F(\vec{p},\vec{p}\,')$ and $\xi(q^2)$ we can calculate the scalar form factor $f_0(q^2)$. The results are shown in the right panel of Fig. 2. To interpolate $f_0(q^2)$ to zero momentum transfer, we fit it with a monopole ansatz $f_0(q^2) = f_0(0)/(1-q^2/M^2)$. The result of the fit is also shown in the figure. In the given range of momenta the data is well described by this ansatz.

6. Chiral extrapolation

In order to calculate the physical value of $f_+(0) = f_0(0)$ we have to extrapolate our results to the physical pion and kaon masses. We make use of the results of ChPT to guide our extrapolation. In ChPT $f_+(0)$ can be expanded in terms of light pseudoscalar meson masses giving

$$f_+(0) = 1 + f_2 + f_4 + \dots \;,\; f_n = \mathcal{O}(M^{2n}_{\pi,K,\eta})\,. \tag{6.1}$$

The leading correction f_2 receives only contributions from non-local operators and can be determined unambiguously in terms of M_K, M_π and f_π (see [15]). We compute f_2 at the actual pion and kaon masses and define

$$\Delta f = f_+(0) - (1+f_2)\,, \tag{6.2}$$

which receives only contributions from local operators.

The Ademollo-Gatto theorem [16] states that Δf is proportional to $(M_K^2 - M_\pi^2)^2$. Hence, we may write

$$\Delta f = a + b(M_K^2 - M_\pi^2)^2\,, \tag{6.3}$$

to obtain $f_+(0)$ by extrapolating (6.3) to the physical point. The data points and the resulting fit are shown in the left panel of Fig. 3. We find $a \approx 0$, in agreement with the expected result $\Delta f = 0$ in the limit of flavour $SU(3)$.

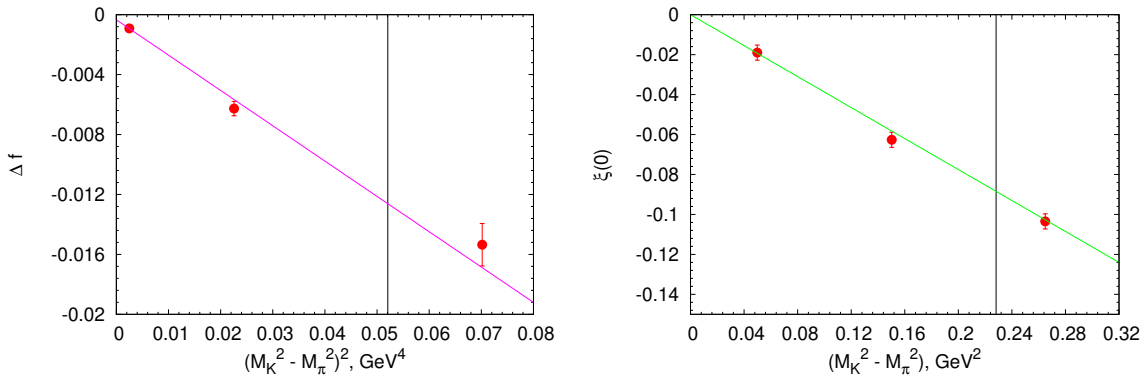

Figure 3: Interpolation of Δf **(Left)** and $\xi(0)$ **(Right)** to physical meson masses indicated by the vertical lines.

At the physical meson masses we find $\Delta f = -0.0126(15)$ [2]. This is to be compared with $\Delta f = -0.016(8)$ of Ref. [2]. Inserting the physical value of f_2, $f_2 = -0.0227$, into Δf, we obtain

$$f_+(0) = 0.9647(15)_{stat}\,. \tag{6.4}$$

We extract the CKM matrix element $|V_{us}|$ from the experimental value of $|V_{us}|f_+(0)$ averaged over all decay modes [17], $|V_{us}|f_+(0) = 0.21673(46)$. This finally gives

$$|V_{us}| = 0.2247(5)_{exp}(4)_{stat}\,. \tag{6.5}$$

To compute $\xi(0)$ at the physical meson masses, we make the ansatz $\xi(0) = c(M_K^2 - M_\pi^2)$, in accord with $\xi(0) = 0$ in the $SU(3)$ limit. In the right panel of Fig. 3 we show the data points and the resulting fit. We obtain

$$\xi(0) = -0.10(2)_{stat}\,.$$

This value is consistent with the experimental values $-0.01(6)$ from K^0_{l3} decay and $-0.125(23)$ from K^+_{l3} decay.

Note that in our analysis only statistical errors are estimated. Analysis of systematic errors which include extrapolation errors of $f_0(q^2)$ to zero momentum transfer and chiral extrapolation errors is underway.

7. Conclusions and outlook

In this work we have presented preliminary results for the kaon semileptonic decay form factors and $|V_{us}|$ from $N_f = 2$ non-perturbatively $O(a)$-improved Wilson fermions. We found

$$f_+(0) = 0.9647(15)_{stat}\ ,\ |V_{us}| = 0.2247(5)_{exp}(4)_{stat}\,, \tag{7.1}$$

[2] Here and in the following only the statistical error is quoted.

in agreement with the results of other lattice groups [4, 5, 6, 7, 8, 9, 10] as well as the estimate of Leutwyler and Roos [2], within the error bars.

The QCDSF collaboration is going to improve the accuracy of the results by performing calculations at lighter quark masses down to pion masses of about 300 MeV and using partially twisted boundary conditions [9]. In addition, we are going to study discretisation effects using gauge ensembles at different lattice spacings in the range of 0.115 fm to 0.070 fm. Furthermore, we plan to study finite size effects.

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