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PENTAGON DIAGRAMS OF BHABHA SCATTERING*

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We report on tensor reduction of five point integrals needed for the evaluation of loop-by-loop corrections to Bhabha scattering. As an example we demonstrate the calculation of the rank two tensor integral with cancellation of the spurious Gram determinant in the denominator. The reduction scheme is worked out for arbitrary five point processes.

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1. Introduction

Evaluation of the Bhabha cross-section with two-loop accuracy [1–4] would be incomplete without taking into account contributions from one loop diagrams with photon emission from internal electron lines. So far bremsstrahlung from external legs (lowest order) has been taken into account [5] and implemented in [6–8].

The general form of the five point tensor integrals in the diagrams of Fig. 1 is:

$$I_5^{\{1,q^\mu,q^\mu q^\nu \ldots\}} = e^{\epsilon\gamma_E} \int \frac{d^d q}{i\pi^{d/2}} \frac{\{1, q^\mu, q^\mu q^\nu \ldots\}}{c_1 c_2 c_3 c_4 c_5}, \tag{1.1}$$

where $c_i = (q + q_i)^2 - m_i^2, \quad i = 1, \ldots, 5$. We also make the (arbitrary) choice $q_5 = 0$.

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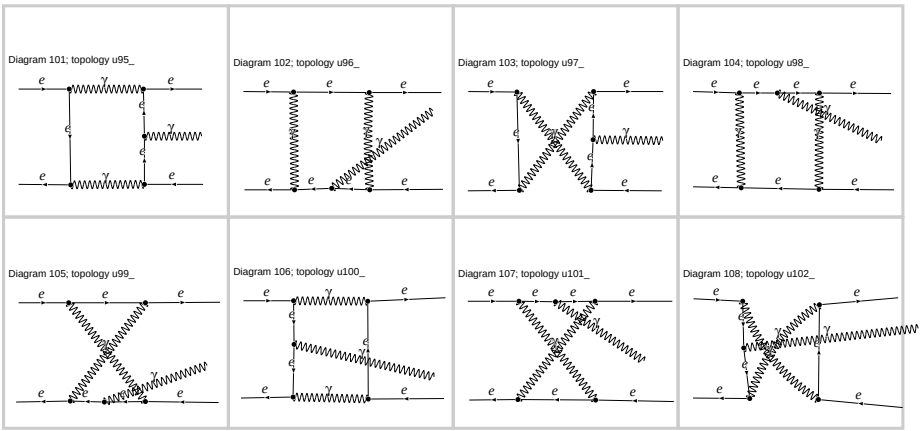

Fig. 1. Eight diagrams at one loop level obtained using DIANA [11].

Massive five point functions tend to be unstable numerically in certain kinematic domains due to the appearance of an inverse Gram determinant. Calculating the tensor integrals directly, e.g. by using Mellin-Barnes (MB-) representations, we have to cope with five dimensional MB-integrals (before expansion in epsilon). After ϵ expansion, five dimensional MB representation still remains at ϵ^0 (e.g. for second order tensor). The MB-representations are derived with the aid of the Mathematica packages `AMBRE` and `MB` [9, 10]. They are hard to evaluate both numerically (and in fact presently the `MB` package works in Euclidean region only) and analytically. A reduction of tensor five point functions to scalar four and three point functions solves that problem.

2. Reduction of five point functions

We give examples of reduction for the scalar and second rank tensor five point function where the spurious Gram determinant in denominators is avoided (see also [12]).

To make a reduction one can follow the standard Passarino-Veltman [13] reduction scheme, or, as we will do, follow a scheme based on [14–16]. For a diagram with internal lines $1 \ldots n$ we can introduce the so called "Modified

Cayley Determinant":

$$()_5 = \begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & Y_{11} & Y_{12} & \dots & Y_{1n} \\ 1 & Y_{12} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & Y_{1n} & Y_{2n} & \dots & Y_{nn} \end{vmatrix}, \tag{2.1}$$

with $Y_{ij} = -(q_i - q_j)^2 + m_i^2 + m_j^2$. By cutting from $()_5$ rows $j_1, j_2, \dots$ and columns $k_1, k_2, \dots$ we get so-called "Minors". The sign of a "Signed Minor",

$$\begin{pmatrix} j_1 & j_2 & \dots \\ k_1 & k_2 & \dots \end{pmatrix}_n, \tag{2.2}$$

is determined by the sum of indices of excluded rows and columns and by taking into account the appropriate signatures of the permutations, taken separately from excluded rows and columns (important when cancelling more than one row and one column), e.g. signatures of permutations for $\binom{12}{54}$ is: $+$ for rows and $-$ for columns. In this way:

$$\binom{0}{0}_n = \begin{vmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{12} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{1n} & Y_{2n} & \dots & Y_{nn} \end{vmatrix}. \tag{2.3}$$

Using "signed minors" one can write tensor reduction formulas in a compact and elegant way.

We start from the simplest case of a scalar five point function. As starting point we use the recursion relation [15]:

$$(d - \sum_{i=1}^{n} \nu_i + 1) G_{n-1} I_n^{(d+2)} = \left[2\Delta_n + \sum_{k=1}^{n} \left(\frac{\partial \Delta_n}{\partial m_k^2} \right) \mathbf{k}^- \right] I_n^{(d)}, \tag{2.4}$$

where

$$\Delta_n = \binom{0}{0}_n, \quad G_{n-1} = -\sum_{k=1}^{n} \left(\frac{\partial \Delta_n}{\partial m_k^2} \right) = 2()_n. \tag{2.5}$$

The operator $\mathbf{k}^-$ decreases the k-th propagator power by one. Because in our case powers of propagators are all equal to one, and we are interested in the case with $n = 5$, the above relation changes into:

$$(d-4)()_5 I_5^{(d+2)} = \binom{0}{0}_5 I_5 - \sum_{k=1}^{5} \binom{0}{k}_5 I_4^k. \tag{2.6}$$

In the limit $d \to 4$ we get as final reduction formula for the scalar five point function:

$$I_5 = \frac{1}{\binom{0}{0}_5} \sum_{k=1}^{5} \binom{0}{k}_5 I_4^k, \tag{2.7}$$

where I_4^k is the scalar four point integral obtained by cancelling the k-th line in the five point function diagram.
For the vector integral we refer to [15]. The second rank tensor integral can be written in the form:

$$I_5^{\mu\,\nu} = \sum_{i,j=1}^{4} q_i^\mu q_j^\nu n_{ij} I_{5,ij}^{[d+]^2} - \frac{1}{2} g^{\mu\nu} I_5^{[d+]}. \tag{2.8}$$

Here $[d+]^l = 4 + 2l - 2\epsilon$, and $n_{ij} = 1 + \delta_{ij}$. We start from the following formulas [15]:

$$n_{ij} I_{5,ij}^{[d+]^2} = -\frac{\binom{0}{j}_5}{()_5} I_{5,i}^{[d+]} + \sum_{s=1, s\neq i}^{5} \frac{\binom{s}{j}_5}{()_5} I_{4,i}^{[d+],s} + \frac{\binom{i}{j}_5}{()_5} I_5^{[d+]}, \tag{2.9}$$

$$g^{\mu\,\nu} = 2 \sum_{i,j=1}^{4} \frac{\binom{i}{j}_5}{()_5} q_i^\mu q_j^\nu, \tag{2.10}$$

$$I_{4,i}^{[d+],s} = -\frac{\binom{0s}{is}_5}{\binom{s}{s}_5} I_4^s + \sum_{t=1, t\neq s}^{5} \frac{\binom{ts}{is}_5}{\binom{s}{s}_5} I_3^{st}. \tag{2.11}$$

Using all the above contributions we end up with:

$$I_5^{\mu\nu} = \sum_{i,j=1}^{4} q_i^\mu q_j^\nu I_{5,ij}, \tag{2.12}$$

where

$$I_{5,ij} = \frac{1}{()_5} \left\{ -\frac{\binom{0}{j}_5}{\binom{0}{0}_5} \sum_{s=1}^{5} \binom{0i}{0s}_5 I_4^s - \sum_{s=1}^{5} \frac{\binom{s}{j}_5 \binom{0s}{is}_5}{\binom{s}{s}_5} I_4^s + \sum_{s,t=1}^{5} \frac{\binom{s}{j}_5 \binom{ts}{is}_5}{\binom{s}{s}_5} I_3^{st} \right\}. \tag{2.13}$$

However, we can avoid the determinant $()_5$ in the denominator. We begin with the following structure:

$$I_5^{\mu\nu} = [I_5^{\mu\nu} - E_{00} g^{\mu\nu}] + E_{00} g^{\mu\nu} \tag{2.14}$$

The problem of finding an appropriate ansatz for E_{00} has been solved in [16]:

$$E_{00} = \frac{1}{\binom{0}{0}} \sum_{s=1}^{5} \binom{0}{s}_5 D^s_{00}, \tag{2.15}$$

where D^s_{00} is the $g^{\mu\nu}$ term of the tensor four point function. The complete result for E_{00} is:

$$E_{00} = -\frac{1}{2}\frac{1}{\binom{0}{0}_5} \sum_{s=1}^{5} \frac{\binom{0}{s}_5}{\binom{s}{s}_5} \left[\binom{0s}{0s}_5 I^s_4 - \sum_{t=1}^{5} \binom{0s}{ts}_5 I^{st}_3 \right]. \tag{2.16}$$

To demonstrate in details how $()_5$ is cancelled in the square bracket in (2.14), we have to consider four and three point functions separately by analyzing the coefficients of I^s_4 and I^{st}_3 in (2.9), subtracting the $g^{\mu\nu}$ term according to (2.10) and (2.16). For I^s_4 we have

$$\begin{aligned}
&\frac{1}{()_5 \binom{0}{0}_5 \binom{s}{s}_5} \times \\
&\left\{ -\binom{0}{j}_5 \binom{0s}{0i}_5 \binom{s}{s}_5 - \binom{s}{j}_5 \binom{0s}{is}_5 \binom{0}{0}_5 + \binom{0}{s}_5 \binom{0s}{0s}_5 \binom{i}{j}_5 \right\}^s_{ij} \\
&\equiv \frac{1}{\binom{0}{0}_5 \binom{s}{s}_5} X^s_{ij},
\end{aligned} \tag{2.17}$$

i.e. we have to show that indeed $()_5$ cancels and we have to give an explicit expression for X^s_{ij}.

A useful property of X^s_{ij} is its symmetry w.r.t. the indices i and j for fixed s. Obviously the third term in the curly bracket of (2.17) is symmetric since we consider a symmetric determinant. The symmetry of the first two terms means

$$\begin{aligned}
&\binom{s}{s}_5 \left[\binom{0}{i}_5 \binom{0j}{0s}_5 - \binom{0}{j}_5 \binom{0i}{0s}_5 \right] \\
+ &\binom{0}{0}_5 \left[\binom{s}{i}_5 \binom{0s}{js}_5 - \binom{s}{j}_5 \binom{0s}{is}_5 \right] = 0.
\end{aligned} \tag{2.18}$$

The first square bracket of (2.18) can be evaluated using (A.13) of [14], i.e.

$$\binom{0}{j}_5 \binom{0i}{0s}_5 = -\binom{0}{0}_5 \binom{0s}{ij}_5 + \binom{0}{i}_5 \binom{0j}{0s}_5 \tag{2.19}$$

and (2.18) then results in:

$$\binom{s}{i}_5\binom{0s}{js}_5+\binom{s}{j}_5\binom{0s}{si}_5+\binom{s}{s}_5\binom{0s}{ij}_5=0. \tag{2.20}$$

This is proved by multiplication with $()_5$ and using (A.8) of [14] with $r=2$, i.e.

$$\binom{il}{jk}_5()_5=\binom{i}{j}_5\binom{l}{k}_5-\binom{i}{k}_5\binom{l}{j}_5. \tag{2.21}$$

Inserting this, products of three factors of the form $\binom{i}{k}_5$ cancel pairwise, q.e.d. .

Further, the following relations (A.11) and (A.12) of [14] are important, i.e.

$$\sum_{i=1}^{n}\binom{0}{i}_5=()_5,\qquad \sum_{i=1}^{n}\binom{j}{i}_5=0,\ (j\neq 0). \tag{2.22}$$

As simplest case we now immediately obtain from (2.17) $\{\cdots\}^s_{ss}=0$, i.e. $X^s_{ss}=0$. Applying (2.22) to (2.17), we see:

$$\sum_{j=1}^{5}\{\cdots\}^s_{ij}=-()_5\binom{0s}{0i}_5\binom{s}{s}_5 \tag{2.23}$$

and due to the symmetry in i and j we also have:

$$\sum_{i=1}^{5}\{\cdots\}^s_{ij}=-()_5\binom{0s}{0j}_5\binom{s}{s}_5, \tag{2.24}$$

which gives us a hint of how X^s_{ij} might look, namely due to (2.23) it should contain a term $-\binom{0s}{0i}_5\binom{0s}{js}_5$. A further contribution, summed over, must vanish. Due to $X^s_{ss}=0$ it must contain a factor $\binom{0j}{si}_5$. The second factor of this further contribution can only depend on s and has been determined by explicit calculation to be $\binom{0s}{0s}_5$. Thus we conclude

$$X^s_{ji}=X^s_{ij}=-\binom{0s}{0i}_5\binom{0s}{js}_5+\binom{0j}{si}_5\binom{0s}{0s}_5. \tag{2.25}$$

For I_3^{st} a slight generalisation yields the coefficient

$$X^{st}_{ij}=-\binom{0s}{0j}_5\binom{ts}{is}_5+\binom{0i}{sj}_5\binom{ts}{0s}_5, \tag{2.26}$$

with $X^s_{ij} = X^{s,t=0}_{ij}$ and the final result is:

$$I_5^{\mu\nu} = \sum_{i,j=1}^{4} q_i^\mu q_j^\nu I_{5,ij} + g^{\mu\nu} E_{00}, \tag{2.27}$$

where

$$I_{5,ij} = \frac{1}{\binom{0}{0}_5} \sum_{s=1}^{5} \frac{1}{\binom{s}{s}_5} \Big(X^{s0}_{ij} I^s_4 - \sum_{t=1}^{5} X^{st}_{ij} I^{st}_3 \Big). \tag{2.28}$$

X^{st}_{ij} is defined in (2.26) and E_{00} in 2.16. I^{st}_3 is the scalar three point function obtained by cutting the s and t lines in the five point diagram.

3. Automatization

We have developed a Mathematica package which reduces general tensor five point functions up to rank three. Using `LoopTools` [17] we have performed numerical cross-checks which ensured us about the correctness of the package. We have also made independent checks with sector decomposition and the Mellin-Barnes method. Some examples are given in [18].

In summary, we have developed tools to deal with five point Bhabha amplitudes. The next step is to implement it into the complete numerical calculation.

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