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BCFT and Ribbon Graphs as tools for open/closed string dualities

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Abstract

In the framework of simplicial models, we construct and we fully characterize a scalar boundary conformal field theory on a triangulated Riemann surface. The results are analysed from a string theory perspective as tools to deal with open/closed string dualities.

Simplicial techniques have being subject of renewed attention since it was clarified that they can play a role in the worldsheet description of open/closed string dualities [1]. In this connection, the hypothesis that the link between SYM gauge and closed string worldsheet dynamics could be explained through the combinatorial data associated to Strebel differentials, suggests that we are undercovering some deep discrete foundation of string dualities, which however is still far from being understood.

Aiming to give further insights in this topic, we decided to explore the connection between combinatorial data and string dualities from a more general standpoint. We considered a geometrical set-up in which ribbon graphs are dual to triangulations with localized curvature defects. These provide a natural order parameter which allows to map a N-punctured closed Riemann surface

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into an open one with N boundary components. An example of such a mapping has been given, in an hyperbolic setting, in [2]. A different construction was obtained describing a random Regge triangulation (RRT) [3] as the uniformization of an open Riemann surface M_∂ with a set of N annuli, $\Delta^*_{\varepsilon(p)}$, $p = 1, \ldots, N$ each of which is defined in the neighborhood of the p-th vertex of the original triangulation. Each annulus is endowed with a correspondent Euclidean cylindrical metric and, via a conformal mapping can be equivalently interpreted as a cylinder of *finite height*. The decorated Riemann surface is subsequently constructed glueing the above local uniformizations along the pattern defined by the ribbon graph baricentrically dual to the parent triangulation.

This geometrical setup, which trades the localised curvature degrees of freedom of the parent triangulation into modular data of the new discrete surface, is simpler than that analysed in [2], but it can be dynamically coupled with matter field theory. In [4] we showed that this leads to the definition of a new kinematical background in which it could be possible to investigate dynamical processes typical of open/closed string dualities.

1 Boundary Insertion Operators

The geometry we dealt with is characterized by N cylinders of finite heights, which can be interpreted as open string worldsheet, connected through their inner boundary to a ribbon graph. Hence, the latter is the natural *locus* where N copies of a given Boundary Conformal Field Theory, each defined on a single cylindrical end, interact.

The quantization of a BCFT on a cylindrical domain is a delicate issue relying on the knowledge of the data associated to the correspondent *bulk* CFT. This is uniquely characterized by two copies of a chiral algebra, $\mathcal{W}$ and $\overline{\mathcal{W}}$. Their generators, respectively W^a_n and $\overline{W}^a_n$, are the Laurent's modes of the holomorphic and antiholomorphic chiral fields of the theory, which we will call $W^a(\zeta)$ and $\overline{W}^a(\overline{\zeta})$ respectively. The extension of such a bulk CFT to a BCFT on $\Delta^*_{\varepsilon(p)}$ consists in the choice of a boundary condition $A(p)$ on $\partial\Delta^*_{\varepsilon(p)}$. This is usually done by specifying a glueing automorphism, $\Omega_{A(p)}$, which, relating the holomorphic and anti-holomorphic chiral fields on the boundary itself, avoids flux of informations through it. This process allows to define, out of $W^a(\zeta)$ and $\overline{W}^a(\overline{\zeta})$, a single chiral field, $\mathbb{W}_{\Omega(p)}(\zeta)$. This is continuous on the full complex plane and it contains the information about the boundary condition. Its Laurent modes close a single copy of the chiral algebra, whose irreducible representations define the Hilbert space of states of the boundary theory.

In the framework dual to a RRT, each (p, q)-edge of the ribbon graph is connected to two cylindrical ends, $\Delta^*_{\varepsilon(p)}$ and $\Delta^*_{\varepsilon(q)}$. Hence, its oriented boundaries are decorated by two different boundary conditions, say $A(p)$ and $B(q)$ respectively. In this picture, we do not have a jump between two boundary conditions taking place in a precise boundary's point. Therefore, we cannot apply standard BCFT rules which, assuming the existence of a vacuum state not invariant under translations, allow a boundary condition to change along a single boundary component.

In order to mediate between *adjacent* boundary conditions, we have thus introduced a different process which, restoring the flux of information trough the boundaries connected to the ribbon graph, allows to describe the interaction

of the different BCFTs on the ribbon graph. In the limit when the thickness of the graph goes to zero, we introduced, for each pair (p,q) of adjacent BCFTs, a further glueing automorphism $\Omega_{A(p)B(q)}$. This deforms $\mathbb{W}_{\Omega_{A(p)}}$ into $\mathbb{W}_{\Omega B(q)}$ continuously on the graph's edge and viceversa. This ultimately means that we are associating to each (p,q) edge of the ribbon graph a unique copy of the chiral algebra. Since the latter encodes the information about both the boundary conditions applied on the adjacent polytopes, it is natural to choose the highest weight operators of its irreducible representation as objects which mediates between adjacent boundary conditions. We called this new class of fields Boundary Insertion Operators.

2 BIO in the rational limit

BIOs as described in last section are purely formal objects. However we characterized explicitly their algebraic and analytic structure in particular limits of the BCFT. We considered a D-dimensional bosonic field theory, $X^\alpha : M_\partial \to \mathcal{T}$, $\alpha = 1, \dots, D$, where the geometry of the target space $\mathcal{T}$ is encoded, in the k-th cylindrical end, by the background matrix $E(k) = G(k) + B(k)$. We focused on a flat toroidal background, where the D directions are compactified and $E(k)$'s components are X-independent. Within this framework, each cylindrical end is an open-string worldsheet whose outer boundary can lay on a stack of N_c D-branes. While, on the one hand, the latters allow to decorate each open string with a suitable assignment of Chan-Paton factors, on the other hand they act as source of gauge fields as well. In the case of static brane and constant gauge field strength, the gauge dynamic can be encoded by the toroidal background via the identification $F_{\alpha\beta} = \frac{1}{4\pi} B_{\alpha\beta}$. This translates the problem of coupling the model with a dynamic gauge field into picking up a special point in the toroidal compactifications moduli space such that only the component of the Kalb-Ramond field along the directions parallel to the brane are non zero.

Moreover, if we break $U(N_c) \to U(1)^{N_c}$, and we fix the metric and the antisymmetric field in function of the Cartan matrix of a semi-simple simply laced Lie algebra of total rank D, $\mathfrak{g}_D$, we are choosing those points in the moduli space which are fixed under the action of the generalized T-duality group. The resultant theory of D compactified bosons turns out to be rational, since it is quantum equivalent to the $\hat{\mathfrak{g}}_{k=1}$-WZW model, where $\hat{\mathfrak{g}}_{k=1}$ is the affine extension of $\mathfrak{g}_D$ at level $k = 1$.

In this connection, we showed that we can parameterize the entire set of boundary conditions we can apply on the boundary of $\Delta^*_{\varepsilon(k)}$ by the pair $[||\hat{\omega}_I\rangle\rangle, \Gamma(p)]$. The first element of the pair, $||\hat{\omega}_I\rangle\rangle$, is a Cardy boundary state of the WZW model. It is uniquely associated to the $\hat{I}$-th level one irreps. of $\hat{\mathfrak{g}}_{k=1}$. The second element, $\Gamma(p)$, is an element of the quotient of the universal covering group of $\mathfrak{g}_D$ by its centre, $\Gamma(p) \in \frac{G_D}{B(G_D)}$. This parameterization allowed to give a description of the model as a deformation of the $\hat{\mathfrak{g}}_{k=1}$-WZW model described *à la Cardy* by means of a precise boundary action $S_\Gamma = \int du \Gamma_a J^a(u)$. In the last formula, coefficients Γ_a are defined via a suitable immersion of $\Gamma(p)$ into the universal covering group G_D. This description led to the characterization of the glueing automorphism associated to the (p,q)-edge of the ribbon graph, $\Omega_{[\hat{\jmath}_2\Gamma_2](q)[\hat{\jmath}_1\Gamma_1](p)}$, by means of the fusion coefficient of the WZW model, $\mathcal{N}^{\hat{\jmath}_2(q)}_{\hat{\jmath}_1(p)\hat{I}(p,q)}$, and by a defor-

mation induced by the rotation $\Gamma(p,q) = \Gamma_2(q)\Gamma_1(p)^{-1}$. As a consequence, we wrote the components of Boundary Insertion Operators in the rational limit of the conformal theory as $\psi^{[\hat{J}_2,\Gamma_2](q)\,[\hat{J}_1,\Gamma_1](p)}_{[\hat{I},\,m](p,q)} = \sum_{n=0}^{\dim|\hat{I}|} R^{\hat{I}(p,q)}_{m\,n(p,q)}(\Gamma_2{\Gamma_1}^{-1})\,\psi^{\hat{J}_2(q)\,\hat{J}_1(p)}_{[\hat{I},\,n](p,q)}$, where $\psi^{\hat{J}(p)\,\hat{J}(q)}_{\hat{J}(p,q)}(x(p,q)) = \mathcal{N}^{\hat{J}(q)}_{\hat{J}(p)\,\hat{I}(p,q)}\,\psi_{\hat{I}(p,q)}(x(p,q))$ and where $R^{\hat{I}(p,q)}_{m\,n(p,q)}$ is the action of $\Gamma_2{\Gamma_1}^{-1}$ in a given representation. In this case, BIOs algebra is set in terms of the unperturbed WZW model fusion rules. This property is particularly manifest in the algebraic form for the expansion coefficients of the OPE between $\psi^{[\hat{J}_1,\Gamma_1](p)\,[\hat{J}_3,\Gamma_3](r)}_{\hat{I}_1(r,p)}$ and $\psi^{[\hat{J}_3,\Gamma_3](r)\,[\hat{J}_2,\Gamma_2](q)}_{\hat{I}_2(q,r)}$ in terms of $\psi^{[\hat{J}_1,\Gamma_1](p)\,[\hat{J}_2,\Gamma_2](q)}_{\hat{I}_3(q,p)}$, namely $\mathcal{C}^{\hat{J}_1(p)\,\hat{J}_3(r)\,\hat{J}_2(q)}_{\hat{I}_1(r,p)\,\hat{I}_2(q,r)\,\hat{I}_3(q,p)}$, whose labellings are left untouched by the deformations. The variable connectivity of the ribbon graph, which has been inherited by the original triangulation, allowed to identify the above coefficients with the $\hat{\mathfrak{g}}$-WZW model fusion matrices [5, 6, 7], namely the quantum 6-j symbols $\mathcal{C}^{\hat{J}_1(p)\,\hat{J}_2(q)\,\hat{J}_3(s)}_{\hat{I}_1(q,p)\,\hat{I}_2(s,q)\,\hat{I}_3(s,p)} = \left\{ \begin{smallmatrix} \hat{I}_1(q,p) & \hat{J}_1(p) & \hat{J}_2(q) \\ \hat{J}_3(s) & \hat{I}_2(s,q) & \hat{I}_3(s,p) \end{smallmatrix} \right\}_{Q\,=\,e^{\frac{2\pi i}{1+h^\vee}}}$ [8].

3 Discussion and Conclusions

The last formula provides an explicit expression for the formal rules describing the interplay among BCFT defined on the adjacent cylinders which build the discrete surface M_∂. In this sense, it completes the description of the dynamical coupling between the discrete geometric set-up developed in [3] and a matter field theory. Out of this, in [4] we wrote a full amplitude on a fixed geometry specified by a choice for the ribbon graph and for a set of localized curvature assignations.

Our aim for the future is to generalize this scenario to the non-Abelian case, in which the $U(N_c)$ symmetry carried by the D-Branes is not broken. In this connection, the open string interpretation would provide naturally a non-trivial gauge colouring of the ribbon graph. This ultimately leads to the definition of a genuine 't Hooft diagram and, hence, of new kinematical background in which it would be possible to investigate dynamical processes proper of open/closed string dualities.

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