Two-loop Heavy Fermion Corrections to Bhabha Scattering

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We derive the two-loop corrections to Bhabha scattering from heavy fermions in the limit $m_e^2 << m_f^2 << s, t, u$. These $n_f = 2$ contributions arise from self-energies, vertices and box topologies. The numerical effects are estimated for small and large angle scattering at 10, 91, 500 GeV and are at the level of per mille or less. The corrections for $m_f^2 \sim s, t, u$ and those due to hadronic insertions remain to be studied by another technique.

1 Introduction

The Bhabha cross-section has to be determined with an NNLO accuracy.[a] The production of W^+W^- or of fermion pairs, including wide angle Bhabha scattering, at the ILC is anticipated with 10^6 events, and at the GigaZ (or MegaW) option with rates being up to two orders of magnitude higher. Further, it is planned to measure the luminosity with small angle Bhabha scattering. Thus, the cross-section for Bhabha scattering has to be predicted to better than few permille, preferrably at the 10^{-4} level; see the talk [3]. The kinematics is compatible with $m_e^2 << s, t, u$, so that the NNLO virtual corrections may be determined in the limit of vanishing electron mass, even for very small scattering angles.

Existing Monte Carlo packages don't cover these corrections completely, see e.g. the review [4] and the reports on small angle Bhabha scattering [5–9], as well as the talks [10–13]. The Monte Carlo programs use small but non-vanishing electron and photon masses as infrared regulators.

Quite recently, the photonic two-loop corrections for massive Bhabha scattering have been determined in a series of papers [14–16], using the analytically known result for the massless case [17]. Together with NNLO contributions from diagrams with electron loop insertions [18–21], the $n_f = 1$ two-loop Bhabha cross-section evaluation was completed [22–24].

An additional class of diagrams with one more scale, the $n_f = 2$ contributions, with heavy fermion loops was calculated quite recently with two different methods; by a direct

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[a] A link to the slides of this contribution is [1]. Additional material may be found at the webpage [2].

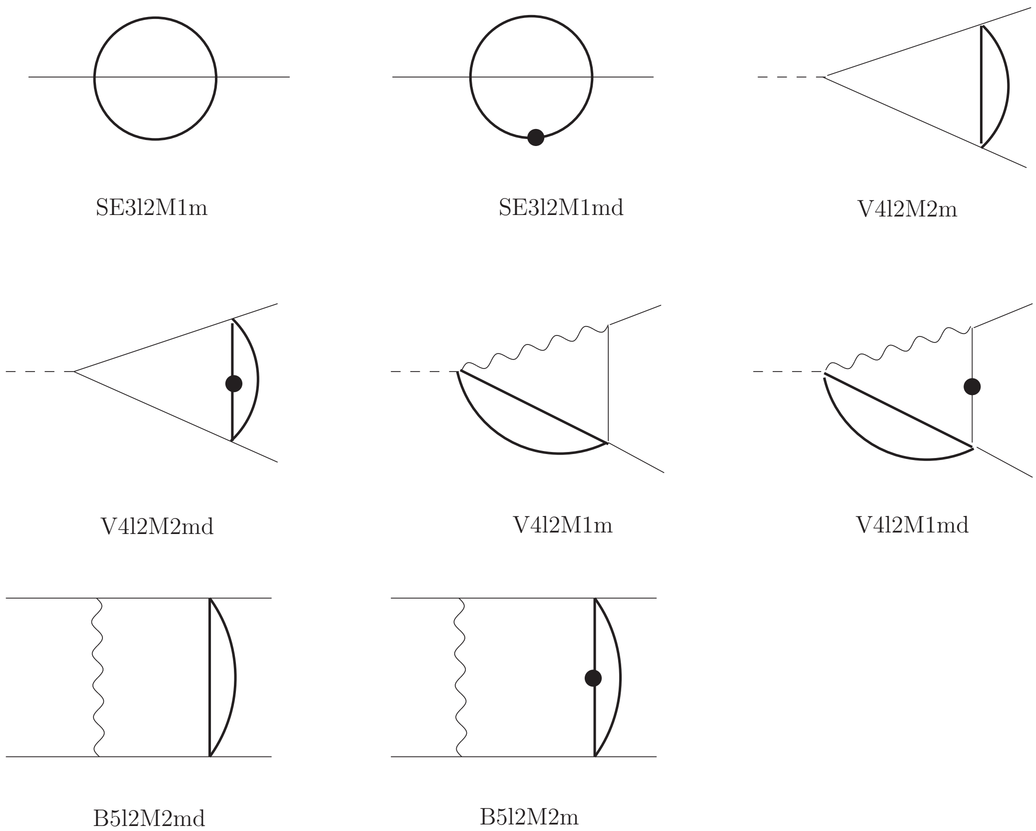

Figure 1: The $n_f = 2$ master integrals

Feynman diagram calculation in [25] and by relating massless and massive diagrams in [26] (see for the method also [27]). In this contribution, we report on our determination of the $n_f = 2$ contributions by evaluation of Feynman diagrams with Mellin-Barnes representations of master integrals and their subsequent expansion first in m_e^2/s, and then in m_f^2/s, at $s \sim t \sim u$.

2 Master integrals

The eight master integrals for the $n_f = 2$ Feynman diagrams for massive Bhabha scattering have been identified in [21] with the Laporta algorithm [28], using the package `IdSolver`. They are shown in Figure 1. These diagrams depend on three different scales: $s/m_e^2, t/m_e^2, M^2/m_e^2$. We evaluated them with the Mellin-Barnes technique (with the algorithm introduced in [29] and realized in the Mathematica packages `ambre` [30]) and `MB` [31], expanding in two steps in the mass scales, $m_e^2 << M^2 << s, t, u$ [25, 32]. Let us reproduce here the two double box masters ($m_e = m$):

$$\texttt{B5l2M2m[x,y]} \quad = \quad \frac{m^{-4\epsilon}}{x} \Big\{ \frac{1}{\epsilon^2} L_m(x) + \frac{1}{\epsilon} \Big(-\zeta_2 + 2L_m(x) + \frac{1}{2} L_m^2(x) + L_m(x) L_m(y) \Big)$$

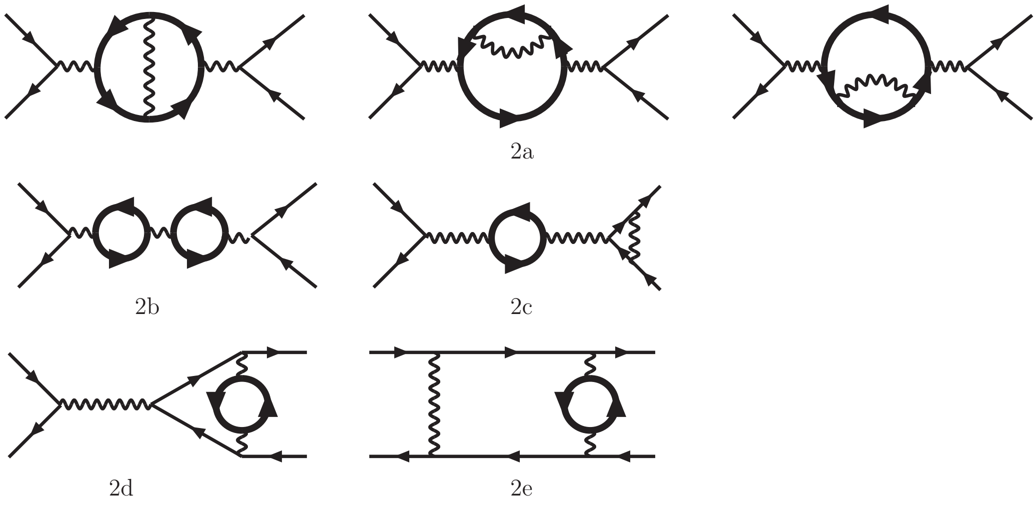

Figure 2: Two-loop diagrams with heavy fermion insertions

$$
\begin{aligned}
&- \quad 2\zeta_2 - 2\zeta_3 + 4L_m(x) + L_m^2(x) + \frac{1}{3}L_m^3(x) - 4\zeta_2 L_m(y) \\
&+ \quad 2L_m(x)L_m(y) + L_m(x)L_m^2(y) - \frac{1}{6}L_m^3(y) \\
&- \quad \Big(3\zeta_2 + \frac{1}{2}L_m^2(x) - L_m(x)L_m(y) + \frac{1}{2}L_m^2(y)\Big)\ln\Big(1+\frac{y}{x}\Big) \\
&- \quad \Big(L_m(x) - L_m(y)\Big)\mathrm{Li}_2\Big(-\frac{y}{x}\Big) + \mathrm{Li}_3\Big(-\frac{y}{x}\Big)\Big\}, \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
\texttt{B5l2M2md[x,y]} \quad = \quad & \frac{m^{-4\epsilon}}{xy}\Big\{\frac{1}{\epsilon}\Big[-L_m(x)L_m(y) + L_m(x)L(R)\Big] - 2\zeta_3 + \zeta_2 L_m(x) + 4\zeta_2 L_m(y) \\
- \quad & 2L_m(x)L_m^2(y) + \frac{1}{6}L_m^3(y) - 2\zeta_2 L(R) + 2L_m(x)L_m(y)L(R) - \frac{1}{6}L^3(R) \\
+ \quad & \Big(3\zeta_2 + \frac{1}{2}L_m^2(x) - L_m(x)L_m(y) + \frac{1}{2}L_m^2(y)\Big)\ln\Big(1+\frac{y}{x}\Big) \\
+ \quad & \Big(L_m(x) - L_m(y)\Big)\mathrm{Li}_2\Big(-\frac{y}{x}\Big) - \mathrm{Li}_3\Big(-\frac{y}{x}\Big)\Big\}. \qquad (2)
\end{aligned}
$$

We use $L(R) = \ln(m^2/M^2)$, $L_m(x) = \ln(-m^2/x)$, and $L_M(x) = \ln(-M^2/x)$.

3 Cross-sections

The two-loop diagrams to be evaluated are shown in Figure 2. Their interference with the Born diagrams has to be combined with loop-by-loop corrections and soft real bremsstrahlung in order to get an infrared finite cross-section:

$$
\frac{d\sigma^{\rm NNLO}}{d\Omega} + \frac{d\sigma_\gamma^{\rm NLO}}{d\Omega} \quad = \quad \frac{d\sigma^{\rm NNLO,e}}{d\Omega} + \sum_{f\neq e} Q_f^2 \frac{d\sigma^{\rm NNLO,f^2}}{d\Omega} + \sum_{f\neq e} Q_f^4 \frac{d\sigma^{\rm NNLO,f^4}}{d\Omega}
$$

$$+ \sum_{f_1,f_2\neq e} Q_{f_1}^2 Q_{f_2}^2 \frac{d\sigma^{\mathrm{NNLO,2f}}}{d\Omega}. \qquad (3)$$

The most complicated part is due to the double box diagrams, it is contained in $d\sigma^{\mathrm{NNLO,f^2}}/d\Omega$:

$$\frac{d\sigma^{\mathrm{NNLO,f^2}}}{d\Omega} = \frac{\alpha^2}{s}\left\{\sigma_1^{\mathrm{NNLO,f^2}} + \sigma_2^{\mathrm{NNLO,f^2}}\ \ln\left(\frac{2\omega}{\sqrt{s}}\right)\right\}$$

The virtual part of the contribution is (with $x = -t/s$):

$$\begin{aligned}
\sigma_1^{\mathrm{NNLO,f^2}} &= \frac{\left(1-x+x^2\right)^2}{3x^2}\Big\{-\frac{1}{3}\Big[\ln^3\left(\frac{s}{m_e^2}\right) + \ln^3\left(R_f\right)\Big] + \ln^2\left(\frac{s}{m_e^2}\right)\Big[\frac{55}{6} - \ln\left(R_f\right)\\
&+ \ln\left(1-x\right) - \ln\left(x\right)\Big] + \ln\left(\frac{s}{m_e^2}\right)\Big[-\frac{589}{18} + \frac{37}{3}\ln\left(R_f\right) - \ln^2\left(R_f\right)\\
&- 2\ln\left(R_f\right)\Big(\ln\left(x\right) - \ln\left(1-x\right)\Big) - 8\mathrm{Li}_2\left(x\right)\Big] + \frac{4795}{108} - \frac{409}{18}\ln\left(R_f\right) + \frac{19}{6}\ln^2\left(R_f\right)\\
&- \ln^2\left(R_f\right)\Big(\ln\left(x\right) - \ln\left(1-x\right)\Big) - 8\ln\left(R_f\right)\mathrm{Li}_2\left(x\right) + \frac{40}{3}\mathrm{Li}_2\left(x\right)\Big\}\\
&+ \ln\left(\frac{s}{m_e^2}\right)\Big[\zeta_2\Big(-\frac{2}{3x^2} + \frac{4}{3x} + \frac{11}{2} - \frac{23}{3}x + \frac{16}{3}x^2\Big) + \ln^2\left(x\right)\Big(-\frac{1}{3x^2} + \frac{17}{12x}\\
&- \frac{5}{4} - \frac{x}{12} + \frac{2}{3}x^2\Big) + \ln^2\left(1-x\right)\Big(-\frac{2}{3x^2} + \frac{11}{6x} - \frac{5}{2} + \frac{11}{6}x - \frac{2}{3}x^2\Big)\\
&+ \ln\left(x\right)\ln\left(1-x\right)\Big(\frac{2}{3x^2} - \frac{4}{3x} - \frac{1}{2} + \frac{5}{3}x - \frac{4}{3}x^2\Big) + \ln\left(x\right)\Big(\frac{55}{9x^2} - \frac{83}{9x} + \frac{65}{6}\\
&- \frac{85}{18}x + \frac{10}{9}x^2\Big) + \frac{1}{3}\ln\left(1-x\right)\Big(-\frac{10}{3x^2} + \frac{31}{6x} - 10 + \frac{31}{6}x - \frac{10}{3}x^2\Big)\Big]\\
&+ \frac{1}{3}\ln^3\left(x\right)\Big(-\frac{1}{3x^2} + \frac{31}{12x} - \frac{11}{6} - \frac{x}{6} + \frac{x^2}{3}\Big) + \frac{1}{3}\ln^3\left(1-x\right)\Big(-\frac{1}{3x^2} + \frac{1}{x}\\
&- \frac{4}{3} + x - \frac{x^2}{3}\Big) + \ln^2\left(x\right)\ln\left(1-x\right)\Big(-\frac{1}{3x^2} + \frac{1}{3x} - \frac{4}{3} + x - \frac{x^2}{3}\Big)\\
&+ \frac{1}{3}\ln\left(x\right)\ln^2\left(1-x\right)\Big(-\frac{1}{x^2} + \frac{2}{x} - \frac{7}{4} + \frac{x}{2}\Big) + \ln^2\left(x\right)\Big[\frac{55}{18x^2} - \frac{46}{9x}\\
&+ \frac{14}{3} - \frac{4}{9}x - \frac{10}{9}x^2 + \ln\left(R_f\right)\Big(-\frac{1}{3x^2} + \frac{17}{12x} - \frac{5}{4} - \frac{x}{12} + \frac{2}{3}x^2\Big)\Big]\\
&+ \ln^2\left(1-x\right)\Big[\frac{10}{9x^2} - \frac{29}{9x} + \frac{9}{2} - \frac{29}{9}x + \frac{10}{9}x^2 + \ln\left(R_f\right)\Big(-\frac{2}{3x^2} + \frac{11}{6x}\\
&- \frac{5}{2} + \frac{11}{6}x - \frac{2}{3}x^2\Big)\Big] + \ln\left(x\right)\ln\left(1-x\right)\Big[-\frac{10}{9x^2} + \frac{37}{18x} + \frac{1}{2} - \frac{25}{9}x\\
&+ \frac{20}{9}x^2 + \ln\left(R_f\right)\Big(\frac{2}{3x^2} - \frac{4}{3x} - \frac{1}{2} + \frac{5}{3}x - \frac{4}{3}x^2\Big)\Big] + \ln\left(x\right)\Big[-\frac{589}{54x^2} + \frac{1753}{108x}\\
&- \frac{701}{36} + \frac{925}{108}x - \frac{56}{27}x^2 + \mathrm{Li}_2\left(x\right)\Big(-\frac{4}{x^2} + \frac{19}{3x} - 7 + 3x - \frac{2}{3}x^2\Big)\\
&+ \ln\left(R_f\right)\Big(\frac{37}{9x^2} - \frac{56}{9x} + \frac{47}{6} - \frac{67}{18}x + \frac{10}{9}x^2\Big) + \zeta_2\Big(-\frac{2}{3x^2} + \frac{4}{x} - \frac{1}{6}\\
&- \frac{10}{3}x + 2x^2\Big)\Big] + \ln\left(1-x\right)\Big[\frac{56}{27x^2} - \frac{161}{54x} + \frac{56}{9} - \frac{161}{54}x + \frac{56}{27}x^2\\
&+ \ln\left(R_f\right)\Big(-\frac{10}{9x^2} + \frac{31}{18x} - \frac{10}{3} + \frac{31}{18}x - \frac{10}{9}x^2\Big) + \zeta_2\Big(-\frac{2}{x^2} + \frac{20}{3x} - \frac{32}{3} + \frac{20}{3}x\\
&- 2x^2\Big)\Big] + \mathrm{Li}_3\left(x\right)\Big(\frac{4}{3x^2} - \frac{7}{3x} + 3 - \frac{5}{3}x + \frac{2}{3}x^2\Big) + \frac{2}{3}S_{1,2}\left(x\right)\Big(-\frac{1}{x^2} + \frac{1}{x}
\end{aligned}$$

$$
- \quad x + x^2\Big) + \zeta_2\Big[\frac{19}{9x^2} - \frac{13}{18x} - \frac{43}{3} + \frac{311}{18}x - \frac{98}{9}x^2 + \ln(R_f)\Big(-\frac{2}{3x^2} + \frac{4}{3x}
$$

$$
+ \quad \frac{11}{2} - \frac{23}{3}x + \frac{16}{3}x^2\Big)\Big] + \zeta_3\Big(-\frac{4}{3x^2} + \frac{3}{x} - 5 + \frac{11}{3}x - 2x^2\Big). \tag{4}
$$

4 Numerical results

The numerical results are summarized in Table 1 for small angle scattering and in Table 2 for large angle scattering. The net $n_f = 2$ corrections are small compared to the photonic

$d\sigma$ / $d\Omega$ [nb] | $\sqrt{s}$ [GeV]	10	91	500
QED Born	440873	5323.91	176.349
full Born	440875	5331.5	176.283
NNLO (e)	-1397.35	-35.8374	-1.88151
NNLO (e + μ)	-1394.74	-43.1888	-2.41643
NNLO (e + μ +τ)			-2.55179
NNLO photonic	9564.09	251.661	12.7943

Table 1: Numerical values for the NNLO corrections to the differential cross section. Results are expressed in nanobarns for a scattering angle $\theta = 3°$. Empty entries are related to cases where the high-energy approximation cannot be applied. For comparison, we show also the QED and the full electroweak Born cross sections.

$d\sigma$ / $d\Omega$ [nb] | $\sqrt{s}$ [GeV]	10	91	500
QED Born	0.466409	0.00563228	0.000186564
full Born	0.468499	0.127292	0.0000854731
NNLO (e)	-0.00453987	-0.0000919387	-4.28105 $\cdot 10^{-6}$
NNLO (e + μ)	-0.00570942	-0.000122796	-5.90469 $\cdot 10^{-6}$
NNLO (e + μ +τ)	-0.00586082	-0.000135449	-6.7059 $\cdot 10^{-6}$
NNLO (e + μ +τ + t)			-6.6927 $\cdot 10^{-6}$
NNLO photonic	0.0358755	0.000655126	0.0000284063

Table 2: Same as Table 1, but for a scattering angle $\theta = 90°$.

corrections or to those with an electron loop ($n_f = 1$ corrections). Nevertheless, they reach the level of few permille in certain kinematical regions, and for an accuracy of 10^{-4} one definitely has to take them into account. Thus, at several of the ILC instances, they will be needed. We have not combined them with unresolved real fermion pair emission, which might diminish the numerical effects further due to a compensation of the leading logarithmic terms.

Note added. After this conference, a longer write-up of the material presented here was published [25]. Later, the dispersion technique was applied to the $n_f = 2$ contributions with semi-analytical predictions for the cross-sections, and relaxing the scale conditions applied here, to $m_e^2 << M^2, s, t, u$ [33]. Another approach was applied to these $n_f = 2$ contributions in [34].

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