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Status of Deeply Inelastic Parton Distributions

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Abstract
A brief review on the status of unpolarized parton densities and the determination of the QCD scale $\Lambda_{\rm QCD}$ from deep-inelastic scattering data is presented.

1 Introduction

Deeply inelastic lepton–nucleon scattering provides a clean way to extract the parton densities of the nucleons together with the QCD scale $\Lambda_{\rm QCD}$. The exact determination of the parton densities is decisive for the understanding of the scattering cross sections at hadron colliders as LHC [1]. The main goal of the investigation is the measurement of the leading twist distributions. In the large x region higher twist effects are measurable as well [2,3]. [1] During the last years the determination of moments of parton distribution functions [5] and the QCD scale [6,7] within lattice-QCD calculations became more and more precise. A comparison of these results and the measurement of the corresponding quantities from precision data using higher order perturbation theory will provide highly non-trivial test of Quantum Chromodynamics. On the perturbative side, the running of $\alpha_s(Q^2)$ is known to 4–loop orders [8] while the anomalous dimensions and the massless Wilson coefficients were calculated to 3–loop order [9, 10]. The heavy flavor Wilson coefficients are known to 2–loop order only [11–13]. A first coefficient contributing at 3–loop order was calculated recently [14]. Due to this the QCD analysis of deeply inelastic structure functions in $l^{\pm}N$ scattering may be performed for flavor non–singlet combinations to $O(\alpha_s^3)$ and to a very good approximation even to $O(\alpha_s^4)$, cf. [3]. In the flavor singlet case, strictly speaking, the analysis cannot be performed to 3-loop order, since the corresponding heavy flavor Wilson coefficients are not known yet. It can be performed in an approximation to 3–loop order, describing the heavy flavor contributions to 2–loop order, which induces a remaining theoretical error. In the present paper we concentrate on the case of unpolarized deep-inelastic scattering. A recent overview on the status of polarized parton densities was given in [15]. The paper is organized as follows. In section 2 we summarize main aspects of QCD analyzes and discuss recent progress in measuring unpolarized parton distribution functions. Section 3 summarizes determinations of $\Lambda_{\rm QCD}$ in deeply inelastic scattering and in Section 4 we discuss future perspectives.

[1] Similarly, one may hope to find higher twist effects in the region of small x in the future [4].

2 QCD Analysis of Unpolarized Structure Functions

In case of light–cone dominance the deeply inelastic structure functions at twist–2 are described by a Mellin convolution of the bare parton densities and the hard scattering cross sections, which are both infinite, but are renormalized to finite parton densities and Wilson coefficients by absorbing the ultraviolet singularities of the latter into the former :

$$\begin{aligned} F_j(x,Q^2) &= \hat{f}_i(x,\mu^2) \otimes \sigma_j^i \left(\alpha_s, \frac{Q^2}{\mu^2}, x\right) \qquad (1)\\ &\qquad \uparrow \text{bare pdf} \qquad \uparrow \text{sub} - \text{system cross} - \text{sect.} \\ &= \underbrace{\hat{f}_i(x,\mu^2) \otimes \Gamma_k^i \left(\alpha_s(R^2), \frac{M^2}{\mu^2}, \frac{M^2}{R^2}\right)}_{\text{finite pdf}\equiv f_k} \otimes \underbrace{C_j^k \left(\alpha_s(R^2), \frac{Q^2}{\mu^2}, \frac{M^2}{R^2}, x\right)}_{\text{finite Wilson coefficient}} \end{aligned}$$

The scale evolution of the structure functions is described by the Symanzik-Callan equations for the ultraviolet singularities [16], and likewise for the renormalized parton densities and Wilson coefficients,

$$\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} - 2\gamma_\psi(g)\right] F_i(N) = 0 \qquad (2)$$

$$\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} + \gamma_\kappa^N(g) - 2\gamma_\psi(g)\right] f_k(N) = 0 \qquad (3)$$

$$\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} - \gamma_\kappa^N(g)\right] C_j^k(N) = 0\,. \qquad (4)$$

Here the Wilson coefficients contain as well the heavy quark degrees of freedom, while the parton distributions can only be defined for strictly massless partons in the respective kinematic region, i.e. for collinear particles. Clearly for $Q^2 \lessapprox m_H^2$ heavy quarks cannot be treated as partons. It is known for long [17] that the heavy quark contributions have quite different scaling violations if compared to light partons for a very large range in Q^2.

The solution of the evolution equations is easiest being performed in Mellin space. Here the corresponding evolution equations can be solved to all orders in the coupling constant analytically, cf. e.g. [18]. The solution has to be continued analytically from even values of the Mellin moment $N \to N\,\epsilon\,\mathbf{C}$. This requires the continuation of harmonic sums [19] representing the higher order anomalous dimensions and light flavor Wilson coefficients [20] and that of heavy flavor Wilson coefficients [12]. At every loop order and expansion depth in the dimensional regularization parameter ε a uniform maximal number of basis elements is needed to construct the respective single–scale quantities. To 3–loop orders 14 basic Mellin transforms are sufficient [21]. The structure of this representation is characterized by meromorphic functions in the complex N plane,

the perturbative part of it obeys nested recursions $z \to z-1$ and can be constructed analytically starting from the respective asymptotic representation in the region $|z| \to \infty$, cf. [22]. The expression for the structure functions used in the χ^2-minimization can be easily obtained by a single fast numeric contour integral around the singularities of the problem. To keep the evolution code fast all relations expressing the evolution kernels can be stored in large arrays during the initialization of the code, while in the minimization procedure only the parameters of the parton distribution functions are varied along with $\Lambda_{\rm QCD}$. The procedure can be systematically generalized including resummations, e.g. in the small–x region [18]. These effects, however, were found to be non-dominant in the region of HERA data. Initially large effects are likely canceled by sub-leading terms almost completely, as being the case for all quantities calculated in fixed orders up to $O(\alpha_s^3)$. Usually three sub-leading terms (series) are required to obtain the correct result, cf. [18]. We will therefore not include effects of this kind in the present analysis, see [23] for a survey. Other recent analyzes also find only small small x effects [24] in the evolution of $F_2(x,Q^2)$ in the region $x \gtrapprox 10^{-4}$ currently probed at HERA.

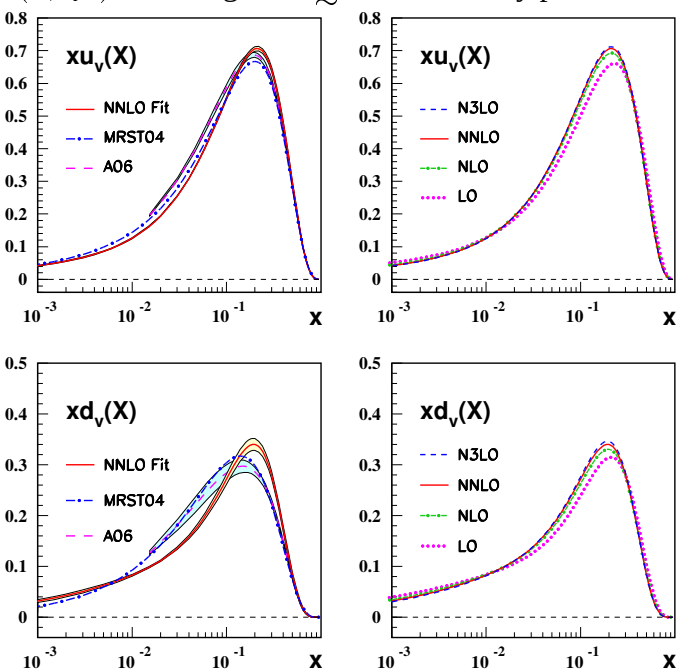

Figure 1: The NNLO valence quark distributions, [3], compared to other analyzes and perturbative stability of the fit comparing different higher order corrections.

A flavor non–singlet analysis of the deep-inelastic world data was carried out recently in [3]. This analysis primarily aimed on measuring $\alpha_s(M_Z^2)$ widely free of gluonic effects. Due to the fact that the $O(\alpha_s^3)$ Wilson coefficients dominate the scaling violations at the 4–loop level and the effect of the splitting function is rather minor only, as estimated by a Padé-approximation, the analysis is effectively of 4–loop order. We accounted for a $\pm$ 100% error in the estimated 4-loop anomalous dimension. Comparison

with the second moment of the non-singlet 4–loop anomalous dimension [25] showed agreement within better than 20 % well confirming our error treatment. In Figure 1 the fit results are shown for the valence quarks and compared to other analyzes [26, 27] (left figures). The right figures show the convergence of the analysis from leading order (LO) to 4-loop order (NNNLO).

In Ref. [3] also a model-independent extraction of higher twist-contributions in the large x region was performed. Here it is essential to describe the leading twist contributions as accurately as possible, since the leading twist Wilson coefficients are large in the large x region.

The light sea quark densities are known at lower precision if compared to that of the valence quarks. Here still more data are required. The distribution $x(\overline{u} - \overline{d})(x, Q^2)$ can be obtained from Drell–Yan data. In a recent analysis [27] improved sea quark distributions $x(\overline{u} \pm \overline{d})$ were obtained, see Figure 2a. The 3–loop corrections lower the theory error to the level of the experimental accuracy. A recent determination of the strange quark density was performed by the CTEQ collaboration [28], see Figure 2b. This distribution is about half the value of that of the up and down sea quarks.

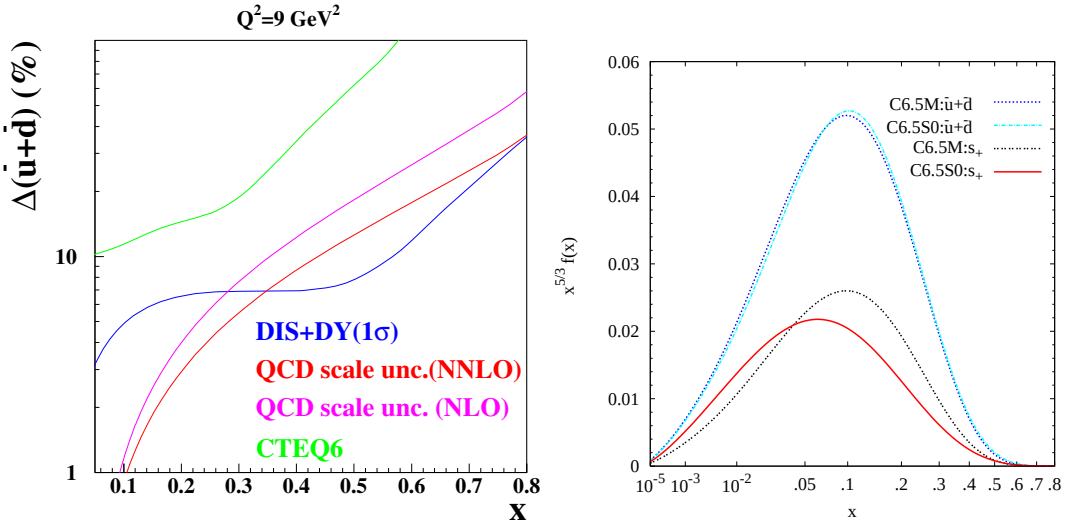

Figure 2: Uncertainty of $x(\overline{u} + \overline{d})$ distribution [27] (left). The light flavor distributions for $Q^2 = 1.69\mathrm{GeV}^2$, Ref. [28].

The correct determination of the gluon density is of central importance since many scattering processes at LHC are gluon induced. The gluon distribution is rapidly growing as $x \to 0$ with rising values of Q^2. This expectation is confirmed by different analyzes [27, 29, 30]. As an example we show the results of the recent analysis [29] in Figure 3a, where a rising behaviour is found down to scales of $Q^2 = 2\ \mathrm{GeV}^2$. In contrast to this MSTW [31] find a gluon distribution which is turning to lower values in the region $x \approx 10^{-3}$ for scales $Q^2 = 5\ \mathrm{GeV}^2$ and lower, contrary to the results found in [27,29,30]. The value of $\alpha_s(M_Z^2)$ in [31] $0.1191 \pm 0.002 \pm 0.003$ comes out larger than that in

[3, 27, 29], $\alpha_s(M_Z^2) = 0.1142 \pm 0.0021; 0.1128 \pm 0.0015; 0.112$. In [30] a determination of α_s is not undertaken, since the different data sets used in the fit bear too different systematics to allow this, which was outlined in [32] in detail. The analysis in [26] differs from that in [27] due to the inclusion of jet data from Tevatron, which are known to require a larger value of $\alpha_s(M_Z^2)$.

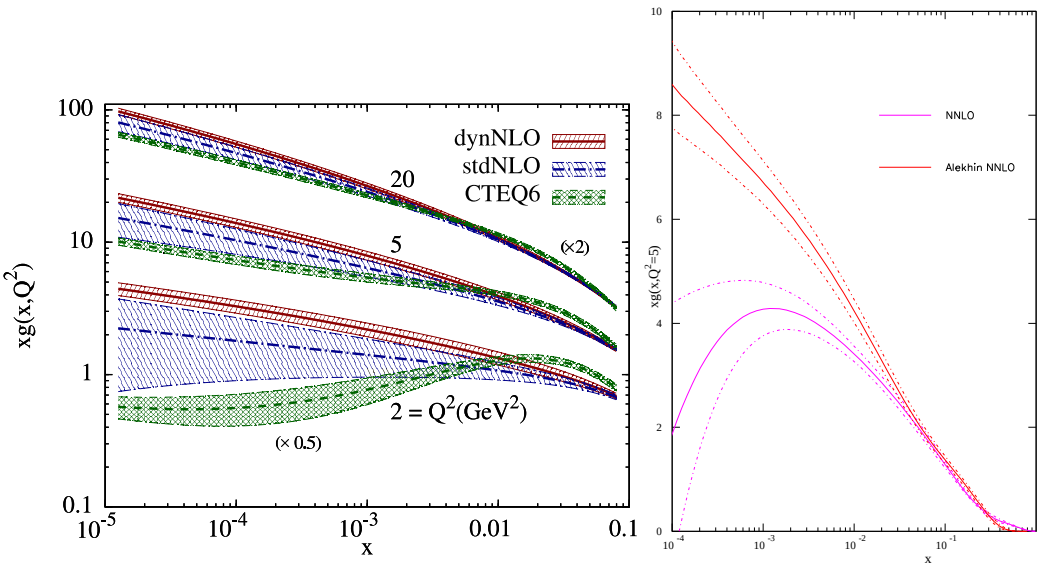

Figure 3 : Gluon momentum distribution at NLO [29, 30] (left) and at NNLO [26, 27] (right).

The measurement of $F_L(x, Q^2)$ can help to clarify this question. A recent analysis [33] shows very good agreement with the current measurements [34], which are partly still preliminary. The question of the correct value of the gluon distribution function should be clarified soon.

3 $\Lambda_{\rm QCD}$ and $\alpha_s(M_Z^2)$

A summary on different measurements of $\alpha_s(M_Z^2)$ from $l^{\pm}N$ scattering data in NLO, NNLO, and NNNLO is given in Figure 4, see also [35]. Present analyzes are carried out at the 3-loop level based on the anomalous dimensions [9] and Wilson coefficients [36]. If the analysis is restricted to deeply inelastic data the values of $\alpha_s(M_Z^2)$ come out somewhat lower as the world average [37]. The convergence of the perturbative extraction of $\alpha_s(M_Z^2)$ out of the deeply–inelastic world data [3] is illustrated comparing the central values from NLO to NNNLO :

$$\alpha_s(M_Z^2) = 0.1148 \rightarrow 0.1134 \rightarrow 0.1142 \pm 0.0021. \tag{5}$$

The change from the N^{2}LO to the N^{3}LO value is found deeply inside the current experimental error. The N^{3}LO value corresponds to

$$\Lambda_{\rm QCD}^{\overline{\rm MS},{\rm N_f}=4} = 234 \ \pm 26 \ {\rm MeV}. \tag{6}$$

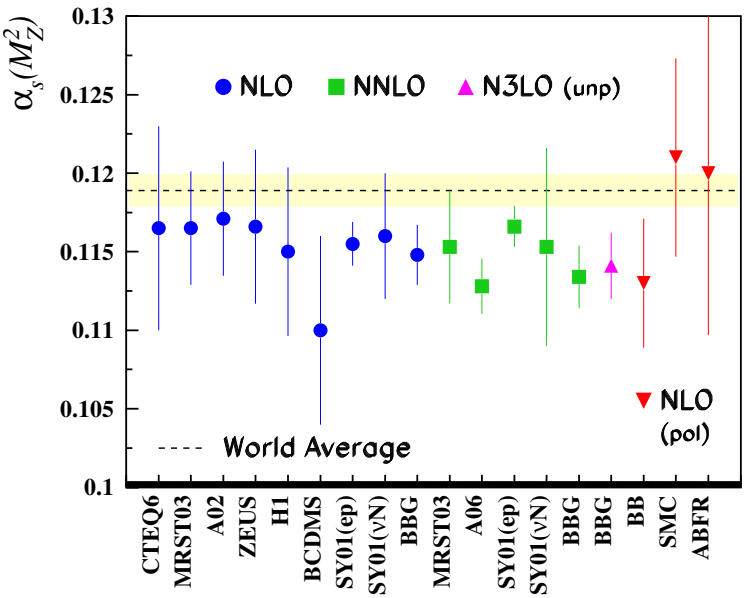

Figure 4: Summary of $\alpha_s(M_Z^2)$ measurements in deep-inelastic unpolarized and polarized $l^{\pm}N$ scattering: from NLO to NNNLO, Ref. [3].

$\Lambda_{\rm QCD}^{\overline{\rm MS}}$ was measured also in two recent lattice simulations based on two active flavors ($N_f = 2$). These investigations paid special attention to non-perturbative renormalization and kept the systematic errors as small as possible.

$$\Lambda_{N_f=2}^{\rm latt} = 245 \pm 16 \pm 16 \text{ MeV } [6], \qquad \Lambda_{N_f=2}^{\rm latt} = 261 \pm 17 \pm 26 \text{ MeV } [7]\,. \tag{7}$$

A direct comparison with the case $N_f = 4$ in the above data analyzes is not yet possible. However, the difference between the earlier $N_f = 0$ and the present result in $\Lambda_{\rm QCD}$ amounts to $O(10 \text{ MeV})$ only. We have to wait and see what is obtained for $N_f = 4$ in coming analyzes.

4 Future Perspectives

Most of the data taken at HERA still have to be analyzed to extract the final data of $F_{2,L}(x,Q^2)$, $F_2^{Q\overline{Q}}(x,Q^2)$, and other structure functions. The analysis of these measurements will be mandatory for the final precision determination of the parton distribution functions in the small x region, in particular for the gluon and sea quark distribution functions. Important informations on the large x behaviour of the valence quark densities will be obtained from JLAB [38]. Currently our knowledge of the individual light flavor sea quark distributions is still rather limited. Here, the measurement of the Drell-Yan process and $W^{\pm}$ and Z–production at LHC will add in significant further information. That far signs of non-linear gluon evolution were not found in deeply inelastic scattering, unlike suggested by earlier theoretical expectations [4]. As the scale at which these effects come into operation cannot be determined pertubatively one has to search for

those effects at still smaller values of x using suitable scattering cross sections at LHC in the near future. After the completion of the HERA programme still inclusive measurements at much higher luminosity are required to determine some of the parton densities at higher precision. For a more detailed measurement of the sea quark distributions deuteron targets are required at high luminosity [39]. Here a programme like foreseen for the EIC [40] can contribute essentially. The flavor contents of the sea-distribution can be analysed in great detail at high luminosity neutrino factories operating at higher energies [41]. The results of both these facilities will be instrumental to explore distributions, which are more difficult to access as the polarized distribution functions, the transversity distribution, as well as the twist–3 and higher twist correlation functions to perform further rather non-trivial tests of QCD also in this area. Various of these observables can be accessed at high precision in lattice calculations in the near future. In this way ab-initio predictions at the one side can be compared to precision data analyzed within perturbation theory to higher orders on the other side. It is therefore highly desirable, that these facilities [40, 41] are built in the future.

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