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Theoretical Interest in B-Meson Physics at the B Factories, Tevatron and the LHC*

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We review the salient features of B-meson physics, with particular emphasis on the measurements carried out at the B-factories and Tevatron, theoretical progress in understanding these measurements in the context of the standard model, and anticipation at the LHC. Topics discussed specifically are the current status of the Cabibbo-Kobayashi-Maskawa matrix, the CP-violating phases, rare radiative and semileptonic decays, and some selected non-leptonic two-body decays of the B mesons.

1. Introduction

The two B-meson factories operating at the KEK and SLAC e^+e^- storage rings have outperformed their projected luminosities and have produced a wealth of data on the process $e^+e^- \to \Upsilon(4S) \to B\bar{B}$, with subsequent weak decays of the B and $\bar{B}$ mesons. The exclusivity of the final state, the asymmetric beam energies and the large statistics collected at the $\Upsilon(4S)$ ($O(10^9)B\bar{B}$ events) have led to a number of impressive and quantitative results, which include, among other measurements, precise determination of the weak mixing (Cabibbo-Kobayashi-Maskawa CKM [1, 2]) matrix elements $|V_{cb}|$ and $|V_{ub}|$ [3], a large number of CP-violating asymmetries in exclusive decays of the B mesons [4], and rare radiative and semileptonic decays (implying flavor changing neutral current FCNC transitions) involving the B^0 and B^+ mesons and their charge conjugates [5]. Recently, Belle collaboration have published a number of interesting results on the decays of

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the B_s-meson highlighted by the first measurement of the radiative penguin decay $B_s \to \phi\gamma$ [6]. This is the SU(3)-counterpart to the much celebrated decay $B \to K^*\gamma$, first measured by CLEO and further consolidated by the BABAR and BELLE collaborations [7].

The two B-meson factories are not the only players currently active in the study of B-meson physics. In fact, b-physics is the main beneficiary of the upgrade program at the high energy proton-antiproton collider Tevatron at Fermilab. Since this upgrade, physics news from Fermilab are dominated by the achievements in the b-quark sector. We have seen very impressive and seminal results in this field by the two Fermilab experiments CDF and D0, including the landmark measurement of the B_s^0 - $\overline{B_s^0}$-mixing induced mass difference ΔM_{B_s} [8, 9] and the first result on direct CP violation in the B_s-meson sector $\mathcal{A}_{\rm CP}(B_s^0 \to K^+\pi^-)$ [10, 11]. These experiments have established (if any proof was needed) that cutting edge flavor physics is done also at hadron machines. In all likelihood, this success story of the hadron machines will be set forth at the Large Hadron Collider (LHC), commissioned to operate in 2008 at CERN. In particular, the experiment LHCb [12], dedicated to precision measurements of the physics of the entire family of B-mesons (B^0, B^+, B_s^0, B_c^+, their charged conjugated states, and excited states) and Λ_b baryons (including the entire baryon spectrum containing at least one b-quark), will greatly broaden our knowledge of both the spectroscopic and dynamical aspects of b-physics. The other two LHC experiments ATLAS and CMS will also contribute to b physics.

In this talk, I will briefly review the highlights of the measurements already accomplished, making contacts with theoretical expectations in the standard model (SM). The topics discussed are: (i) an update of the CKM matrix, with emphasis on the matrix elements in the third row and the third column of $V_{\rm CKM}$ from direct decays (which determine $|V_{cb}|, |V_{ub}|$ and $|V_{tb}|$), and from induced transitions involving the mass differences ΔM_{B_d}, ΔM_{B_s} and the electromagnetic penguin decays $b \to (s,d)\gamma$ (which determine the matrix elements $|V_{td}|$ and $|V_{ts}|$) (ii) measurements of the CP-violating phases α, β and γ from exclusive B-decays, and (iii) radiative, semileptonic and leptonic rare B-decays, a field which has received a lot of theoretical attention both in the context of the SM and in extensions of it. Much more detailed discussions of these topics can be found in the original literature cited below, in the reviews, such as the Review of Particle Properties by the Particle Data Group [13], and in the proceedings of the topical workshops on flavor physics [14].

2. Status of the CKM Matrix

The CKM matrix is written below in the Wolfenstein parameterization [15] in terms of the four parameters $A,\ \lambda,\ \ \rho,\ \ \eta$:

$$V_{\rm CKM} \equiv \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3\left(\rho-i\eta\right) \\ -\lambda(1+iA^2\lambda^4\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3\left(1-\rho-i\eta\right) & -A\lambda^2\left(1+i\lambda^2\eta\right) & 1 \end{pmatrix} . \qquad (1)$$

Anticipating precision data, a perturbatively improved version [16] of the Wolfenstein parameterization will be used below with $\bar{\rho} = \rho(1-\lambda^2/2),\ \ \bar{\eta} = \eta(1-\lambda^2/2)$.

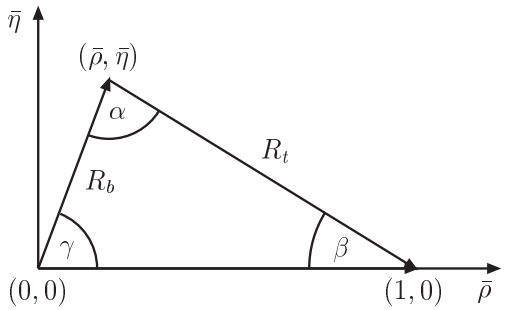

Fig. 1. The unitarity triangle with unit base in the $\bar{\rho}$ - $\bar{\eta}$ plane.

Unitarity of the CKM matrix implies six relations, of which the one resulting from the equation $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ is the principal focus of the current experiments in B-decays. This is a triangle relation in the complex plane (i.e. $\bar{\rho}$–$\bar{\eta}$ space), and the three angles of this triangle are called α, β and γ, with the BELLE convention being $\phi_1 = \beta$, $\phi_2 = \alpha$ and $\phi_3 = \gamma$ (see Fig.1). The unitarity relation in discussion can also be written as

$$R_b e^{i\gamma} + R_t e^{-i\beta} = 1\,, \qquad (2)$$

where $R_b = \left(1-\frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| = \sqrt{\bar{\rho}^2+\bar{\eta}^2}$ and $R_t = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right| = \sqrt{(1-\bar{\rho})^2+\bar{\eta}^2}$. Thus, precise determination of $|V_{cb}|$, $|V_{ub}|$ and $|V_{td}|$ and the three CP-violating phases α, β, γ is crucial in testing the CKM paradigm.

In this section, we mainly review the current status of the CKM matrix elements in the third row and the third column, namely $V_{ub}, V_{cb}, V_{td}, V_{ts}$

and V_{tb}. The other four CKM matrix elements, V_{ud}, V_{us}, V_{cd} and V_{cs} are discussed in [13, 3], from where we quote the current best measurements.

- V_{ud} is obtained from the nuclear β-decay of the (super-allowed) $O^+ \to O^+$ transitions [17, 18], yielding: $V_{ud} = 0.97377(27)$.
- V_{us} is obtained from the $K_{\ell 3}$ decays and chiral perturbation theory, yielding $V_{us} = 0.22535 \pm 0.00116$. In getting this, the Leutwyler-Roos value for the form factor $f_+(0) = 0.961 \pm 0.008$ [19] has been used. This is to be contrasted with the value, $V_{us} = \lambda = 0.2265(7)$, obtained from the unitarity condition on the first row of the CKM matrix and the input value for V_{ud} given above. They are in good agreement with each other, providing a precision test of the unitarity of the first row of $V_{\rm CKM}$.
- V_{cs} is obtained from the decay $D \to Ke\nu_e$ and the Lattice-QCD result for the $D \to K$ form factor [20], yielding $|V_{cs}| = 0.996 \pm 0.008 \pm 0.015 \pm 0.104$, where the first two errors are experimental and the third (dominant) error is from theory.
- Despite impressive progress in the measurement of the Cabibbo-suppressed D-meson decays and Lattice-QCD, the determination of V_{cd} is still dominated by the neutrino-nucleon production of the charm quark, yielding: $|V_{cd}| = 0.230 \pm 0.011$ [13]. Within errors, $|V_{cd}| = |V_{us}|$, as anticipated from eq. (1).

2.1. *Current determinations of $|V_{cb}|$ and $|V_{ub}|$*

Determinations of $|V_{cb}|$ are based on the semileptonic decay $b \to c\ell\nu_\ell$. This transition can be measured either inclusively through the process $B \to X_c\ell\nu_\ell$, where X_c is a hadronic state with a net c-quantum number, or exclusively, such as the decays $B \to (D, D^*)\ell\nu_\ell$. In either case, intimate knowledge of QCD is required to go from the partonic process to the hadronic states. The fact that $m_b \gg \Lambda_{\rm QCD}$ has led to novel applications of QCD in which heavy quark expansion (HQE) plays a central role [21] and the underlying theory is termed as HQET.

2.1.1. *$|V_{cb}|$ from the decays $B \to X_c\ \ell\ \nu_\ell$*

Concentrating first on the inclusive decays, the semileptonic decay rate can be calculated as a power series

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 + ..., \qquad (3)$$

where each Γ_i is a perturbation series in $\alpha_s(m_b)$, the QCD coupling constant at the scale m_b. Here Γ_0 is the decay width of a free b-quark, which gives the parton-model result. The coefficient of the leading power correction Γ_1 is absent [22], and the effect of the $1/m_b^2$ correction is collected in Γ_2, which can be expressed in terms of two non-perturbative parameters called λ_1 - kinetic energy of the b-quark - and λ_2 - its chromomagnetic moment. These quantities, also called μ_π^2 and μ_G^2, respectively, in the literature, are defined in terms of the following matrix elements [23, 24, 25, 26]:

$$\begin{aligned} 2M_B\lambda_1 &\equiv \langle B(v)|\bar{Q}_v(iD)^2 Q_v|B(v)\rangle, \\ 6M_B\lambda_2 &\equiv \langle B(v)|\bar{Q}_v\sigma_{\mu\nu}[iD^\mu, iD^\nu]Q_v|B(v)\rangle \ , \end{aligned} \tag{4}$$

where D_μ is the covariant derivative and heavy quark fields are characterized by the 4-velocity, v. At $\mathcal{O}(\Lambda_{\rm QCD}^3/m_b^3)$, six new matrix elements enter in Γ_3, usually denoted by $\rho_{1,2}$ and $\mathcal{T}_{1,2,3,4}$.

Data have been analyzed in the theoretical accuracy in which corrections up to $\mathcal{O}(\alpha_s^2\beta_0)$, $\mathcal{O}(\alpha_s\Lambda_{\rm QCD}/m_b)$ and $\mathcal{O}(\Lambda_{\rm QCD}^3/m_b^3)$ are taken into account [27, 28], with β_0 being the lowest order coefficient in the expansion of the QCD β-function. In addition to the other parameters, a quark mass scheme has to be specified. Bauer et al. [27] have carried out a comprehensive study of the scheme dependence using five quark mass schemes: $1S$, PS, $\overline{\rm MS}$, kinematic, and the pole mass.

To extract the value of $|V_{cb}|$ and other fit parameters, three different distributions, namely the charged lepton energy spectrum and the hadronic invariant mass spectrum in $B \to X_c\ell\bar{\nu}_\ell$, and the photon energy spectrum in $B \to X_s\gamma$ have been studied. Theoretical analyses are carried out in terms of the moments. Defining the integral $R_n(E_{\rm cut},\mu) \equiv \int_{E_{\rm cut}} dE_\ell (E_\ell - \mu)^n d\Gamma/dE_\ell$, where $E_{\rm cut}$ is a lower cut on the charged lepton energy, moments of the lepton energy spectrum are given by $\langle E_\ell^n\rangle = R_n(E_{\rm cut},0)/R_0(E_{\rm cut},0)$. For the $B \to X_c\ell\bar{\nu}_\ell$ hadronic invariant mass spectrum, the moments are defined likewise with the cutoff $E_{\rm cut}$. Analyses of the data along these lines have been presented by a large number of experiments. For a summary and references to the original literature, see HFAG [7].

The global fit of the world data in the so-called $1S$-scheme for the b-quark mass undertaken by Bauer et al. [27] leads to the following fit values for $|V_{cb}|$ and m_b^{1S}:

$$\begin{aligned} |V_{cb}| &= (41.78 \pm 0.30 \pm 0.08_{\tau_B}) \times 10^{-3}, \\ m_b^{1S} &= (4.701 \pm 0.030)\,{\rm GeV}. \end{aligned} \tag{5}$$

The BABAR collaboration have studied the dependence of the lepton and

hadron moments on the cutoff $E_{\rm cut}$ and compared their measurements with the theoretical calculation by Gambino and Uraltsev [28] using the so-called kinematic scheme for the b-quark mass $m_b^{\rm kin}(\mu)$, renormalized at the scale $\mu = 1$ GeV. Agreement between experiment and theory allows to determine the fit parameters in this scheme with the results [7]:

$$\begin{aligned} |V_{cb}| &= (41.91 \pm 0.19_{exp} \pm 0.28_{HQE} \pm 0.59_{\Gamma_{\rm sl}}) \times 10^{-3}, \\ m_b\ (1{\rm GeV}) &= (4.613 \pm 0.022_{exp} \pm 0.027_{HQE})\ {\rm GeV}, \\ m_c\ (1{\rm GeV}) &= (1.187 \pm 0.033_{exp} \pm 0.040_{HQE})\ {\rm GeV}\ . \end{aligned} \tag{6}$$

The two analyses (5) and (6) are in excellent agreement with each other. The achieved accuracy $\delta|V_{cb}|/|V_{cb}| \simeq 2\%$ is impressive, and the precision on m_b is also remarkable, $\delta m_b/m_b = O(10^{-3})$, with a similar precision obtained on the mass difference $m_b - m_c$.

2.2. $|V_{cb}|$ *from* $B \to (D, D^*)\ell\nu_\ell$ *decays*

The classic application of HQET in heavy $\to$ heavy decays is in the decay $B \to D^*\ell\nu_\ell$. The differential distribution in the variable $\omega(= v_B.v_{D^*})$, where $v_B(v_{D^*})$ is the four-velocity of the $B(D^*)$-meson, is given by

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{4\pi^3}\,|V_{cb}|^2\,m_{D^*}^3\,(m_B - m_{D^*})^2\,\left(\omega^2-1\right)^{1/2}\,\mathcal{G}(\omega)\,|\mathcal{F}(\omega)|^2\ ,$$

where $\mathcal{G}(\omega)$ is a phase space factor with $\mathcal{G}(1) = 1$, and $\mathcal{F}(\omega)$ is the Isgur–Wise (IW) function [29] with the normalization at the symmetry point $\mathcal{F}(1) = 1$. Leading $\Lambda_{\rm QCD}/m_b$ corrections in $\mathcal{F}(1)$ are absent due to Luke's theorem [30]. Theoretical issues are the precise determination of the second order power correction to $\mathcal{F}(\omega = 1)$, the slope ρ^2 and the curvature c of the IW-function:

$$\mathcal{F}(\omega) = \mathcal{F}(1)\ \left[1 - \rho^2\,(\omega-1) + c\,(\omega-1)^2 + ...\right].$$

Bounds on ρ^2 have been obtained by Bjorken [31] and Uraltsev [32], which can be combined to yield $\rho^2 > 3/4$. Likewise, bounds on the second (and higher) derivatives of the IW-function have been worked out by the Orsay group [33], yielding $c > 15/32$. Apparently, the data sets used in this analysis differ significantly from experiment to experiment, resulting in considerable dispersion in the values of $\mathcal{F}(1)|V_{cb}|$ and ρ^2 and hence in a large χ^2 of the combined fit, summarized by HFAG [7]:

$$\begin{aligned} \mathcal{F}(1)|V_{cb}| &= (35.89 \pm 0.56) \times 10^{-3}\ , \\ \rho^2 &= 1.23 \pm 0.05 \quad (\chi^2 = 37.8/17; {\rm CL} = 0.026). \end{aligned} \tag{7}$$

To convert this into a value of $|V_{cb}|$, we need to know $\mathcal{F}(1)$. In terms of the perturbative (QED and QCD) and non-perturbative (leading δ_{1/m^2} and sub-leading δ_{1/m^3}) corrections, $\mathcal{F}(1)$ can be expressed as follows:

$$\mathcal{F}(1) = \eta_A \left[1 + \delta_{1/m^2} + \delta_{1/m^3}\right] \ , \tag{8}$$

where η_A is the perturbative renormalization of the IW-function, known in the meanwhile to three loops [34]. One- and two-loop corrections yield $\eta_A \simeq 0.933$ and the $O(\alpha_s^3)$ contribution amounts to $\eta_A^{(3)} = -0.005$. A Lattice-QCD calculation in the quenched approximation yields [35] $\mathcal{F}(1) = 0.919^{+0.030}_{-0.035}$, which is now being reevaluated with dynamical quarks. Taking into account this theoretical input, the value quoted at the Lepton-Photon-2007 Symposium is [3]:

$$|V_{cb}|_{B\to D^*\ell\nu_\ell} = (39.1 \pm 0.65_{\rm exp} \pm 1.4_{\rm theo}) \times 10^{-3}. \tag{9}$$

The resulting value of $|V_{cb}|$ is in excellent agreement with the ones given in (5) and (6) obtained from the inclusive decays.

2.3. $|V_{ub}|$ *from the decays* $B \to X_u\ell\nu_\ell$

HQET techniques allow to calculate the inclusive decay rate $B \to X_u\ell\nu_\ell$ rather accurately. However, the experimental problem in measuring this transition lies in the huge background from the dominant decays $B \to X_c\ell\nu_\ell$ which can be brought under control only through severe cuts on the kinematics. For example, these cuts are imposed on the lepton energy, demanding $E_\ell > (m_B^2 - m_D^2)/2m_B$, and/or the momentum transfer to the lepton pair q^2 restricting it below a threshold value $q^2 < q^2_{\rm max}$, and/or the hadron mass recoiling against the leptons, which is required to satisfy $m_X < m_D$. With these cuts, the phase space of the decay $B \to X_u\ell\nu_\ell$ is greatly reduced. A bigger problem is encountered in the end-point region (also called the shape function region), where the leading power correction is no longer $1/m_b^2$ but rather $1/m_b\Lambda_{\rm QCD}$, slowing the convergence of the expansion. Moreover, in the region of energetic leptons with low invariant mass hadronic states, the differential rate is sensitive to the details of the shape function $f(k_+)$ [36], where $k_+ = k_0 + k_3$ with $k^\mu \sim O(\Lambda_{\rm QCD})$.

The need to know $f(k_+)$ can be circumvented to a large extent by doing a combined analysis of the data on $B \to X_u\ell\nu_\ell$ and $B \to X_s\gamma$. Using the operator product expansion (OPE) to calculate the photon energy spectrum in the inclusive decay $B \to X_s\gamma$, the leading terms in the spectrum (neglecting the bremsstrahlung corrections) can be re-summed into a shape

function [37]:

$$\frac{d\Gamma_s}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts}V_{tb}^*|^2 |C_7^{\rm eff}|^2 f(1-x) \ , \tag{10}$$

where $x = \frac{2E_\gamma}{m_b}$, and $C_7^{\rm eff}$ is an effective Wilson coefficient, characterizing the strength of the electromagnetic dipole operator. In the leading order, E_ℓ- and M_{X_u}-spectra in $B \to X_u \ell \nu_\ell$ are also governed by $f(x)$. Thus, $f(x)$ can be measured in $B \to X_s \gamma$ and used in the analysis of data in $B \to X_u \ell \nu_\ell$.

Following this argument, a useful relation emerges [38, 39, 40]

$$|\frac{V_{ub}}{V_{tb}V_{ts}^*}| = \left(\frac{3\alpha}{\pi} |C_7^{\rm eff}|^2 \frac{\Gamma_u(E_c)}{\Gamma_s(E_c)} \right)^{\frac{1}{2}} (1 + \delta(E_c)) \ , \tag{11}$$

where

$$\Gamma_u(E_c) \equiv \int_{E_c}^{m_B/2} dE_\ell \frac{d\Gamma_u}{dE_\ell} \ ,$$
$$\Gamma_s(E_c) \equiv \frac{2}{m_b} \int_{E_c}^{m_B/2} dE_\gamma (E_\gamma - E_c) \frac{d\Gamma_s}{dE_\gamma} \ , \tag{12}$$

and $\delta(E_c)$ incorporates the sub-leading terms in $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$, which can only be modeled at present. In addition, there are perturbative corrections to the spectra and in the relation (11) [37, 41, 38].

The other strategy is to extend the measurements of the inclusive decay $B \to X_u \ell^+ \nu_\ell$ to the kinematic regions which are reachable by the decay $B \to X_c \ell^+ \nu_\ell$, thus enlarging the region where the light-cone momentum component satisfies $P_+ \equiv E_X - |\vec{P}_X| \gg \Lambda_{\rm QCD}$, obviating the need to know the shape function. Both these methods have been used in the determination of $|V_{ub}|$, summarized below.

• Determination of $|V_{ub}|$ using the combined cuts on the variables m_X and q^2 following the suggestion by Bauer, Ligeti and Luke (BLL) [42]. Taking the HQE parameter input from the analysis of the decay $B \to X_c \ell \nu_\ell$ and $B \to X_s \gamma$, HFAG [7] quotes an average value $|V_{ub}|({\rm BLL}) = (4.83 \pm 0.24 \pm 0.37) \times 10^{-3}$, using the b-quark mass $m_b(1{\rm S}) = (4.70 \pm 0.03)$ GeV in the so-called $1S$-scheme.

• Determination of $|V_{ub}|$ from a fully differential decay rate for $B \to X_u \ell \nu_\ell$ based on the soft collinear effective theory (SCET) techniques [43, 44, 45, 46], hereinafter called the Bosch-Lange-Neubert-Paz (BLNP) approach [47]. The three independent kinematic variables are chosen to be: $P_\ell = M_B - 2E_\ell$, $P_- = E_X + |\vec{P}_X|$, and $P_+ = E_X - |\vec{P}_X|$, where $P_\pm$ are the light-cone components of the hadronic final-state momentum along the jet direction, E_X is the jet energy, $\vec{P}_X$ is the jet momentum, and E_ℓ is

the charged-lepton energy. In terms of these variables, the triple differential distribution is:

$$\frac{d^3\Gamma}{dP_+ dP_- dP_\ell} = \frac{G_F^2 |V_{ub}|^2}{16\pi^2}(M_B - P_+)\left\{(P_- + P_+)(M_B - P_- + P_\ell - P_+)F_1 \right. \\ \left. +(M_B - P_-)(P_- - P_+)F_2 + (P_- - P_\ell)(P_\ell - P_+)F_3\right\}. \quad (13)$$

The "structure functions" F_i can be expressed as product of the hard (perturbatively calculable) coefficient and a jet function, which are convoluted with the soft light-cone distribution functions, the shape functions of the B meson. SCET allows to separate the two scales here, namely $\mu_h \sim m_b$ and $\mu_i \sim \sqrt{m_b \Lambda_{\rm QCD}}$ and enables to sum large logarithms involving the two scales μ_h and μ_i. The dependence on the subleading shape functions is studied by taking several models. Fixing the HQE parameters (in the so-called shape function scheme) to the values $m_b({\rm SF}) = (4.63 \pm 0.06)$ GeV, $\mu_\pi^2({\rm SF}) = (0.18 \pm 0.06)$ GeV^2, and the exponential form for the shape function, HFAG [7] quotes $|V_{ub}| = (4.31 \pm 0.17 \pm 0.35) \times 10^{-3}$ in the BLNP approach.

• Determination of $|V_{ub}|$ using the so-called Dressed Gluon Exponentiation (DGE) advocated by Andersen and Gardi [48]. The basic assumption of this approach is that properly defined quark distribution in an on-shell heavy quark provides a good approximation to the distribution in the meson. The problematic small hadronic mass M_{X_u} region in the decay $B \to X_u \ell^+ \nu_\ell$ is characterized by a large hierarchy in the ratio $p_j^+/p_j^- \ll 1$ involving the partonic light-cone coordinates $p_j^+ \le p_j^- \le m_b$. Defining the moments n with respect to the powers of $1 - p_j^+/p_j^-$, the region of small p_j^+ is probed by high moments $n \to \infty$, giving rise to the Sudakov logarithms $\ln n$. Infrared sensitivity appears in the moment-space Sudakov exponents through infrared renormalons, leading to the divergence of the (higher order perturbative) series in the Sudakov exponent. However, the leading infrared renormalon ambiguity cancels exactly against the pole-mass renormalon ambiguity, which enters as a kinematic factor in the differential decay width, making the on-shell perturbative calculation directly applicable for the phenomenology. This approach, applied to the data yields [7] $|V_{ub}| = (4.34 \pm 0.16 \pm 0.25) \times 10^{-3}$ for the input value $\overline{m_b}(\bar{m}_b) = (4.20 \pm 0.07)$ GeV.

Thus, the three theoretical approaches used in the determination of $|V_{ub}|$ from the inclusive decays $B \to X_u \ell \nu_\ell$ give very consistent values.

2.4. $|V_{ub}|$ *from exclusive decays*

$|V_{ub}|$ has also been determined from the exclusive decays $B \to \pi\ell\nu_\ell$. Theoretical accuracy is limited by the imprecise knowledge of the form factors. A number of theoretical techniques has been used to determine them. These include, among others, Light-cone QCD sum rules [49], Quenched- and Unquenched-Lattice QCD simulations [50, 51, 52]. HFAG quotes the following extracted values of $|V_{ub}|$ using the full q^2-range for the form factors (in units of 10^{-3}) [7]:
$|V_{ub}| = 3.43 \pm 0.10^{+0.67}_{-0.42}$ (LCSR: Ball-Zwicky[49]);
$|V_{ub}| = 3.17 \pm 0.10^{+0.77}_{-0.48}$ (Lattice: HPQCD[50]);
$|V_{ub}| = 3.82 \pm 0.12^{+0.88}_{-0.52}$ (Lattice: FNAL[51]);
$|V_{ub}| = 3.61 \pm 0.11^{+1.11}_{-0.57}$ (Lattice: APE[52]).

These values of $|V_{ub}|$ from the exclusive decays (typically 3.5×10^{-3}) are smaller than the ones extracted from the inclusive measurements (typically 4.3×10^{-3}). This mismatch is a mild irritant, waiting for its resolution through progress in Lattice-QCD and more B-factory data.

To reduce the form-factor related uncertainties in extracting $|V_{ub}|$ from exclusive decays $B \to (\pi, \rho)\ell\nu_\ell$, input from the rare B-decays $B \to (K, K^*)\ell^+\ell^-$ and HQET may be helpful. A proposal along these lines is the so-called Grinstein's double ratio which would determine $|V_{ub}|/|V_{tb}V_{ts}^*|$ from the end-point region of exclusive rare B-meson decays [53]. To carry out this program one has to measure four distributions in the decays $B \to \rho\ell\nu_\ell$, $B \to K^*\ell^+\ell^-$, and $D \to (\rho, K^*)\ell\nu_\ell$. With the help of this data and HQET, the ratio of the CKM factors $|V_{ub}|/|V_{tb}V_{ts}^*|$ can be determined through the double ratio

$$\frac{\Gamma(\bar{B} \to \rho\ell\nu)}{\Gamma(\bar{B} \to K^*\ell^+\ell^-)} \frac{\Gamma(D \to K^*\ell\nu)}{\Gamma(D \to \rho\ell\nu)}. \tag{14}$$

At the B factories, one expects enough data on these decays to allow a 10% determination of $|V_{ub}|$ from exclusive decays.

3. Status of the Third Row of $V_{\rm CKM}$

FCNC transitions $b \to s$ and $b \to d$ (as well as $s \to d$) give information on the third row of the CKM matrix $V_{\rm CKM}$ and allow to search for physics beyond the SM. In the SM these transitions are generally dominated by the (virtual) top quark contributions giving rise to the dependence on the matrix elements $|V_{tb}^*V_{ts}|$ (for $b \to s$ transitions) and $|V_{tb}^*V_{td}|$ (for $b \to d$ transitions). Of these, only the matrix element $|V_{tb}|$ has been measured by a tree process $t \to Wb$ at the Tevatron through the ratio

$$R_{tb} \equiv \frac{\mathcal{B}(t \to Wb)}{\mathcal{B}(t \to Wq)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2}\,. \tag{15}$$

The current measurements yield [54]: $R_{tb} = 1.12^{+0.21}_{-0.19}(\mathrm{stat})^{+0.17}_{-0.13}(syst)$, yielding $R_{tb} > 0.61$ (at 95% C.L.), which in turn gives $|V_{tb}| > 0.78$ at 95% C.L..

Precision on the direct measurement of $|V_{tb}|$ is rather modest (unitarity gives $|V_{tb}| \simeq 0.9992$.), which will be greatly improved, in particular, at a Linear Collider [55], but also at the LHC. The corresponding measurements of $|V_{ts}|$ and $|V_{td}|$ from the tree processes are not on the cards. These matrix elements are determined by (loop) induced processes discussed below.

3.1. *Status of* $|V_{td}|$

The current best measurement of $|V_{td}|$ comes from ΔM_{B_d}, the mass difference between the two mass eigenstates of the B^0_d - $\overline{B^0_d}$ complex. This has been measured in a number of experiments and is known to an accuracy of $\sim 1\%$; the current world average is [7] $\Delta M_{B_d} = 0.507 \pm 0.005$ (ps)$^{-1}$.

In the SM, ΔM_{B_d} and its counterpart ΔM_{B_s}, the mass difference in the B^0_s - $\overline{B^0_s}$ system, are calculated by box diagrams, dominated by the Wt loop. Since $(M_W, m_t) \gg m_b$, ΔM_{B_d} is governed by the short-distance physics. The expression for ΔM_{B_d} taking into account the perturbative-QCD corrections reads as follows [56]

$$\Delta M_{B_d} = \frac{G_F^2}{6\pi^2}\,\hat{\eta}_B\,|V_{td}V_{tb}^*|^2\,M_{B_d}\,(f_{B_d}^2\hat{B}_{B_d})\,M_W^2\,S_0(x_t)\,. \tag{16}$$

The quantity $\hat{\eta}_B$ is the next-to-leading log (NLL) perturbative QCD renormalization of the matrix element of the $(|\Delta B| = 2, \Delta Q = 0)$ four-quark operator, whose value is $\hat{\eta}_B = 0.55 \pm 0.01$ [57]; $x_t = m_t^2/M_W^2$ and $S_0(x_t) = x_t f_2(x_t)$ is an Inami-Lim function [58], with

$$f_2(x) = \frac{1}{4} + \frac{9}{4}\frac{1}{(1-x)} - \frac{3}{2}\frac{1}{(1-x)^2} - \frac{3}{2}\frac{x^2\ln x}{(1-x)^3}\,. \tag{17}$$

The quantity $f_{B_d}^2\hat{B}_{B_d}$ enters through the hadronic matrix element of the four-quark box operator, defined as:

$$\langle \bar{B}^0_q|(\bar{b}\gamma_\mu(1-\gamma_5)q)^2|B^0_q\rangle \equiv \frac{8}{3}f_{B_q}^2 B_{B_q} M_{B_q}^2\,, \tag{18}$$

with $B_q = B_d$ or B_s. With ΔM_{B_d} and $\hat{\eta}_B$ known to a high accuracy, and the current value of the top quark mass, defined in the $\overline{\mathrm{MS}}$ scheme, $\bar{m}_t(m_t) = (162.3 \pm 2.2)$ GeV, leading to $S_0(x_t) = 2.29(5)$, i.e., $\delta S_0(x_t)/S_0(x_t) \simeq 2.5\%$,

the combined error from these sources is a few percent. This is completely negligible in comparison with the current theoretical uncertainty on the matrix element $f_{B_d}\sqrt{\hat{B}_{B_d}}$. For example, $O(\alpha_s)$-improved calculations in the QCD sum rule approach yield $f_{B_d} = (210\pm19)$ MeV [59] and $f_{B_d} = (206\pm20)$ MeV [60], whereas $\overline{B}_{B_d}$ in the $\overline{\rm MS}$ scheme in this approach is estimated as [61] $\overline{B}_{B_d} = 1$ to within 10%, yielding for the renormalization group invariant quantity $\hat{B}_{B_d} \simeq 1.46$, and an accuracy of about $\pm15\%$ on $f_{B_d}\sqrt{\hat{B}_{B_d}}$. Lattice calculations for $f_{B_d}\sqrt{\hat{B}_{B_d}}$ are uncertain due to the chiral extrapolation. Taking this into account, the current unquenched lattice QCD calculations from the JLQCD and HPQCD Collaborations yield [62, 63]

$$f_{B_d}\sqrt{\hat{B}_{B_d}} = (244 \pm 11 \pm 24)\ \text{MeV}\,, \tag{19}$$

where the first error is statistical, and the second combines the uncertainty from chiral extrapolation and other systematic errors. Using $|V_{tb}| = 1$, this yields [64]

$$|V_{td}| = (7.4 \pm 0.8) \times 10^{-3}\,. \tag{20}$$

3.2. *Present status of* $|V_{ts}|$

The quantity which currently provides the best determination of $|V_{ts}|$ is ΔM_{B_s}, measured by D0 and CDF, with the CDF measurement being more precise $\Delta M_{B_s} = (17.77 \pm 0.10 \pm 0.07)$ (ps)$^{-1}$ [11]. The expression for ΔM_{B_s} in the SM can be obtained from the one for ΔM_{B_d} (16) by the replacements: $M_{B_d} \to M_{B_s}$, $(f^2_{B_d}\hat{B}_{B_d}) \to (f^2_{B_s}\hat{B}_{B_s})$, and, most importantly, $V_{td} \to V_{ts}$. Using the recent Lattice-QCD result $f_{B_s}\sqrt{\hat{B}_{B_s}} = (281 \pm 21)$ MeV from the HPQCD collaboration [65] allows to determine $|V_{ts}|$ from ΔM_{B_s} with a precision of about 10%, comparable to the precision on $|V_{td}|$ in eq. (20). This is not competitive with the indirect estimate of this matrix element from the CKM unitarity, which yields $V_{ts} \simeq -V_{cb} = -4.1(1) \times 10^{-2}$.

The compatibility of the SM with the measured value of ΔM_{B_s} is usually tested by taking the value of V_{ts} from the unitarity fits, the measured value of m_t, and the Lattice-QCD value for $f_{B_s}\sqrt{\hat{B}_{B_s}}$. Following this reasoning, the Lattice-HPQCD collaboration estimates $\Delta M_{B_s}(\text{SM}) = (20.3\pm3.0\pm0.8)$ (ps)$^{-1}$ [65]. The corresponding estimate from the UTfit is $\Delta M_{B_s}(\text{SM}) = (20.9 \pm 2.6)$ (ps)$^{-1}$ [66] and the CKMfitter yields $\Delta M_{B_s}(\text{SM}) = (21.7^{+5.9}_{-4.2})$ (ps)$^{-1}$ [67]. Thus, the experimental measurement of ΔM_{B_s} is typically about

1σ below the SM estimates, with $\Delta M_s(\text{expt})/\Delta M_s(\text{SM}) = 0.88 \pm 0.13$ [65], 0.85 ± 0.10 [66] and 0.88 ± 0.20 [67].

Possible implications of the CDF measurement of ΔM_{B_s} have been studied in several papers [68, 69, 70, 71, 66]. Perhaps, it is to the point to mention here that a value of $\Delta M_{B_s} \simeq 18$ (ps)$^{-1}$ was also hinted by the LEP data with the central value being 17.7 (ps)$^{-1}$. Anticipating this, the consequences of an eventual measurement of ΔM_{B_s} around this value were worked out some eight years ago [72] for the parameters of the SM and the minimal flavor-violating supersymmetry [73]. It was emphasized that a measurement of ΔM_{B_s} around this value would confirm the SM. The CDF measurement of ΔM_{B_s} is very precise, and our knowledge of the CKM parameters and the non-perturbative quantities has in the meanwhile also improved, as discussed in this report. However, the bottom line remains the same, namely that the SM has passed this crucial test comfortably.

The ratio of the mass differences $\Delta M_{B_d}/\Delta M_{B_s}$, now measured very precisely, can be used to constrain the CKM ratio $|V_{td}|/|V_{ts}|$ using the SM relation [74]:

$$\frac{\Delta M_{B_s}}{\Delta M_{B_d}} = \xi \, \frac{M_{B_s}}{M_{B_d}} \, \frac{|V_{tb}^* V_{ts}|^2}{|V_{tb}^* V_{td}|^2} \,, \tag{21}$$

where $\xi \equiv f_{B_s}\sqrt{\hat{B}_{B_s}}/f_{B_d}\sqrt{\hat{B}_{B_d}}$. Theoretical uncertainty in ξ in the Lattice QCD approach is arguably smaller compared to the one in $f_{B_s}\sqrt{\hat{B}_{B_s}}$, as in the SU(3) limit $\xi = 1$, and the uncertainty is actually in the SU(3)-breaking corrections. Current estimate in the unquenched lattice calculations of ξ is [64] $\xi = 1.21^{+0.047}_{-0.035}$, which yields [11]

$$|V_{td}/V_{ts}| = 0.2060 \pm 0.0007(\text{exp})^{+0.008}_{-0.006}(\text{th}). \tag{22}$$

This is by far the best measurement of this CKM ratio, and it provides a non-trivial constraint on the allowed profile of the unitarity triangle. Combining eqs. (22) and (20) yields $|V_{ts}| = (36 \pm 4) \times 10^{-3}$ with the error dominated by theory. This completes our review of the CKM matrix elements V_{ij}.

4. Radiative and Semileptonic Rare B Decays

Two inclusive rare B-decays of current experimental interest are $B \to X_s\gamma$ and $B \to X_s l^+ l^-$, where X_s is any charmless hadronic state with the strangeness quantum number $s = 1$. They probe the SM in the electroweak $b \to s$ penguin sector. The CKM-suppressed decays $B \to X_d\gamma$

and $B \to X_d l^+ l^-$ are difficult to measure due to low rates and formidable backgrounds. Instead, the search for $B \to X_d\gamma$ decay has been carried out in the exclusive decay modes $B \to (\rho,\omega)\gamma$. Combined with the decay $B \to K^*\gamma$, these decays provide constraints on the CKM parameters. The CKM-suppressed decays $B \to (\pi,\rho,\omega)\ell^+\ell^-$ have not yet been measured. We review some of these rare B-decays in the context of the SM.

4.1. $B \to X_s\gamma$: SM vs. Experiments

The effective Lagrangian for the decays $B \to X_s\gamma$ obtained by integrating out the top quark and the heavy electroweak bosons reads as follows in the SM:

$$\mathcal{L}_{\rm eff} \;=\; \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i \;. \tag{23}$$

In writing this, unitarity of the CKM matrix has been used and the term proportional to the small matrix elements $V_{us}^* V_{ub}$ has been ignored. The complete list of operators and their Wilson coefficients in the NNLO approximation evaluated at the scale $\mu = m_b$ can be seen elsewhere [75].

The dominant four-quark operators Q_1 and Q_2 are

$$Q_1 = (\bar{s}\, c)_{V-A}\, (\bar{c}\, b)_{V-A}, \qquad Q_2 = (\bar{s}_i c_j)_{V-A}\, (\bar{c}_j b_i)_{V-A}, \tag{24}$$

and the electromagnetic and chromomagnetic penguin operators Q_7 and Q_8 are

$$Q_7 = -\frac{e\,\overline{m}_b(\mu)}{8\pi^2}\, (\bar{s}\,\sigma^{\mu\nu}\,[1+\gamma_5]\,b) F_{\mu\nu}\,, \qquad Q_8 = -\frac{g\,\overline{m}_b(\mu)}{8\pi^2}\, (\bar{s}\,\sigma^{\mu\nu}\,[1+\gamma_5]\,T^a\,b) G^a_{\mu\nu}. \tag{25}$$

The factor $\overline{m_b}(\mu)$ is the $\overline{\rm MS}$ mass of the b quark. QCD-improved calculations in the effective theory require three steps (for a review see [76]):
(i) Matching $C_i(\mu_0)$ ($\mu_0 \sim M_W, m_t$): They have been calculated up to three loops [77, 78]. The three-loop matching is found to have less than 2% effect on $\mathcal{B}(B \to X_s\gamma)$ [78].
(ii) Operator mixing: This involves calculation of the anomalous dimension matrix, which is expanded in $\alpha_s(\mu)$. The anomalous dimensions up to $\alpha_s^2(\mu)$ are known [79] since a decade, and the $\alpha_s^3(\mu)$ calculations have been completed recently in a series of papers [80, 81, 82].
(iii) Matrix elements $\langle O_i\rangle(\mu_b)$ ($\mu_b \sim m_b$): The first two terms in the expansion in $\alpha_s(\mu_b)$ are known since long [83]. Exact results to $\mathcal{O}(\alpha_s^2)$ were obtained for Q_7 in [84, 85] and for Q_8 in [86]. For Q_1 the virtual corrections

at $\mathcal{O}(\alpha_s)$ were calculated in [83, 87, 88], but those at $\mathcal{O}(\alpha_s^2)$ are known only in the large-β_0 limit [89]. A calculation that goes beyond this approximation by employing an interpolation in the charm quark mass m_c was reported in [90], and has been used in estimating the NNLO branching fraction for the inclusive decay $B \to X_s\gamma$ [91]. Finally, one has to add the Bremsstrahlung contribution $b \to s\gamma g$ to get the complete decay rate, which in $O(\alpha_s)$ was done in [92] and in $O(\alpha_s^2)$ in [93].

In the $\overline{\rm MS}$ scheme, the NNLO branching ratio for $E_\gamma > 1.6$ GeV is calculated as [91]:

$$\mathcal{B}(B \to X_s\gamma)_{\rm SM} = (3.15 \pm 0.23) \times 10^{-4} \; . \tag{26}$$

This amounts to a theoretical precision of about 8%, comparable to the current experimental precision [7]

$$\mathcal{B}(B \to X_s\gamma)_{\rm Expt.} = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4} \; . \tag{27}$$

A comparison of the two shows that the SM estimate is in agreement with data though the SM central value lies below the experiment by about 1σ. This allows for speculations about a beyond-the-SM contribution interfering constructively with the SM amplitude. A case in point is a 2Higgs doublet model (2HDM); the preferred value is $m_{H^+} \simeq 650$ GeV with a 95% C.L. lower bound $m_{H^+} > 295$ GeV. However, more conservatively, the proximity of $\mathcal{B}(B \to X_s\gamma)$ in the SM and experiment puts bounds on the parameters characterizing new physics. This has been worked out, together with other constraints, in the context of supersymmetry [94].

The current (NNLO) theoretical precision on $\mathcal{B}(B \to X_s\gamma)$ has also been investigated in the context of SCET using a multi-scale OPE involving three low energy scales: m_b, $\sqrt{m_b\Delta}$ and $\Delta = m_b - 2E_0$, where E_0 is the lower cut on the photon energy. Large logarithms associated with these scales are summed at NLL order. The sensitivity to the scale Δ introduces additional uncertainties. Taking this into account, Becher and Neubert [95] estimate $\mathcal{B}(B \to X_s\gamma) = (2.98 \pm 0.26) \times 10^{-4}$, which increases the departure of the SM from data to about 1.4σ.

4.2. $B \to X_s\ell^+\ell^-$: SM vs. Experiments

To study the decays $B \to X_s\ell^+\ell^-$, one has to extend the operator basis in the effective Lagrangian (23) by adding two semileptonic operators [96]:

$$Q_9 = \frac{e^2}{16\pi^2}(\bar{s}\,\gamma_\mu\,b)_{V-A}\,(\bar{\ell}\,\gamma^\mu\,\ell) \; , \qquad Q_{10} = \frac{e^2}{16\pi^2}(\bar{s}\,\gamma_\mu\,b)_{V-A}\,(\bar{\ell}\,\gamma^\mu\gamma_5\,\ell) \; .$$

The corresponding Wilson coefficients $C_9(\mu)$ and $C_{10}(\mu)$ have the following perturbative expansion:

$$C_9 = \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + ...$$
$$C_{10} = C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + ... \ . \tag{28}$$

After an expansion in α_s, the term $C_9^{(-1)}(\mu)$ reproduces the dominant part of the electroweak logarithms that originate from photonic penguins with charm quark loops:

$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + O(\alpha_s) \ , \tag{29}$$

leading to $\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) \simeq 2$. With $C_9^{(0)}(m_b) \simeq 2.2$, one needs to calculate in the NNLO accuracy. The NNLO calculation of the decay $B \to X_s l^+ l^-$ corresponds to the NLO calculation of $B \to X_s \gamma$, as far as the number of loops in the diagrams is concerned.

The process $B \to X_s \ell^+ \ell^-$ differs greatly from the radiative decay $B \to X_s \gamma$ as far as non-perturbative contributions are concerned. The largest effect in $B \to X_s \ell^+ \ell^-$ from the intermediate $c\bar{c}$ states comes from the resonances J/ψ, ψ' and ψ'' decaying to $\ell^+ \ell^-$, which can be either modeled, for example, as done by Krüger and Sehgal [97] using dispersion relations and data on $\sigma_(e^+ e^- \to c\bar{c} \to$ hadrons), or else experimental cuts are imposed on q^2 to remove the resonant regions and the short-distance contribution is extrapolated through these cuts. Then, there are factorizable $1/m_c$ and $1/m_b$ power corrections, similar to those in $B \to X_s \gamma$, which can be calculated using the OPE and HQET. As is the case for $B \to X_s \gamma$ and $B \to X_u \ell \nu_\ell$, there are no $1/m_b$ corrections. The $O(1/m_b^2)$ corrections in this framework were calculated first in [98] and corrected in [99]. The $O(1/m_b^3)$ corrections were calculated in [100]. The $1/m_c$ factorizable power corrections were calculated in [101].

Including the leading power corrections in $1/m_b$ and $1/m_c$ and taking into account various parametric uncertainties, the branching ratios for the decays $B \to X_s \ell^+ \ell^-$ in NNLO are [102]:

$$\mathcal{B}(B \to X_s e^+ e^-)_{\rm SM} \simeq \mathcal{B}(B \to X_s \mu^+ \mu^-)_{\rm SM} = (4.2 \pm 0.7) \times 10^{-6} \ , \tag{30}$$

where a dilepton invariant mass cut, $m_{\ell\ell} > 0.2$ GeV, has been assumed for comparison with data given below. These estimates make use of the NNLO calculation by Asatryan et al. [103], restricted to $\hat{s} \equiv q^2/m_b^2 < 0.25$. The

spectrum for $\hat{s} > 0.25$ has been obtained from the NLO calculations using the scale $\mu_b \simeq m_b/2$, as this choice of the scale reduces the NNLO contributions. Subsequent NNLO calculations covered the entire dilepton mass spectrum and are numerically in agreement with this procedure, yielding $\mathcal{B}(B \to X_s \mu^+ \mu^-)_{\rm SM} = (4.6 \pm 0.8) \times 10^{-6}$ [104, 105] . The difference in the central values in these results and (30) is of parametric origin.

The BABAR and BELLE collaborations have measured the invariant dilepton and hadron mass spectra in $B \to X_s \ell^+ \ell^-$. Using the SM-based calculations to extrapolate through the cut-regions, the current averages of the branching ratios are [7]:

$$\begin{aligned} \mathcal{B}(B \to X_s e^+ e^-) &= (4.7 \pm 1.3) \times 10^{-6}, \\ \mathcal{B}(B \to X_s \mu^+ \mu^-) &= (4.3^{+1.3}_{-1.2}) \times 10^{-6}, \\ \mathcal{B}(B \to X_s \ell^+ \ell^-) &= (4.5^{+1.03}_{-1.01}) \times 10^{-6}. \end{aligned} \tag{31}$$

Thus, within the current experimental accuracy, which is typically 25%, data and the SM agree with each other in the $b \to s$ electroweak penguins. The low q^2-region (say, $q^2 < 8$ GeV^2), which allows the most precise comparison with the SM, suffers both from the statistics and a cut on the invariant hadronic mass recoiling against the dilepton. A cut $m_X > 2GeV$ and $m_X > 1,8$ GeV have been used by the BELLE and BABAR collaborations, respectively. The effects of these cuts have been studied in the Fermi-motion model [106], which has been used in the experimental analysis of the data so far. Subsequently, the $B \to X_s \ell^+ \ell^-$ rate with an m_X cut in the low-q^2 region has been calculated using the $B \to X_s \gamma$ shape function [107]. This work, whose impact on the analysis of the $B \to X_s \ell^+ \ell^-$ data has yet to be studied, reduces some of the theoretical errors in the SM estimates given in eq. (30). In the same vein, it has also been recently argued [108] that the non-perturbative uncertainties in the large-q^2 region ($q^2 \geq 14$ GeV^2) can be significantly reduced by normalizing the partial decay width of $B \to X_s \ell^+ \ell^-$ with the corresponding partial width of the decay $B \to X_u \ell \nu_\ell$. With more data from the B factories, these theoretical developments will enable a more precise test of the SM in the $B \to X_s \ell^+ \ell^-$ decays.

The measurements (27) and (31) provide valuable constraints on beyond-the-SM physics scenarios. Following the earlier analysis to determine the Wilson coefficients in $b \to s$ transitions [109, 102, 110], it has been recently argued [111] that data now disfavor solutions in which the coefficient $C_7^{\rm eff}$ is similar in magnitude but opposite in sign to the SM coefficient.

Exclusive decays $B \to (K, K^*) \ell^+ \ell^-$ ($\ell^\pm = e^\pm, \mu^\pm$) have also been mea-

sured by the BABAR and BELLE collaborations, and the current world averages of the branching ratios are [7]:

$$\begin{aligned}
\mathcal{B}(B \to K\ell^+\ell^-) &= (3.9 \pm 0.6) \times 10^{-7}, \\
\mathcal{B}(B \to K^* e^+ e^-) &= (11.3^{+2.8}_{-2.6}) \times 10^{-7} , \\
\mathcal{B}(B \to K^* \mu^+ \mu^-) &= (10.3^{+2.6}_{-2.3}) \times 10^{-7} , \qquad (32) \\
\mathcal{B}(B \to K^* \ell^+ \ell^-) &= (9.4^{+1.7}_{-1.6}) \times 10^{-7} . \qquad (33)
\end{aligned}$$

They are also in agreement with the SM-based estimates of the same. A calculation based on the light cone QCD sum rules for the form factors [112] yields [102]: $\mathcal{B}(B \to K\ell^+\ell^-) = (3.5 \pm 1.2) \times 10^{-7}$, $\mathcal{B}(B \to K^* e^+ e^-) = (15.8 \pm 4.9) \times 10^{-7}$, and $\mathcal{B}(B \to K^* \mu^+ \mu^-) = (11.9 \pm 3.9) \times 10^{-7}$ with the errors dominated by uncertainties on the form factors. In the future, these errors can be reduced by using the data on $B \to (\pi, \rho)\ell\nu_\ell$ to determine the $B \to (\pi, \rho)$ form factors. This information can be combined with estimates of the SU(3)-symmetry breaking to determine the $B \to (K, K^*)$ form factors, enabling to predict the FCNC decay rates and spectra more precisely. For the low invariant mass of the dileptons, say $q^2 < 8\ \mathrm{GeV}^2$, the SCET framework can be employed to reduce the number of form factors and improve the perturbative aspects of these decays.

The Forward-Backward (FB) asymmetry in the decay $B \to X_s\ell^+\ell^-$, defined as [112]

$$\bar{\mathcal{A}}_{\rm FB}(q^2) = \frac{1}{d\mathcal{B}(B \to X_s\ell^+\ell^-)/dq^2} \int_{-1}^{1} d\cos\theta_\ell \; \frac{d^2\mathcal{B}(B \to X_s\ell^+\ell^-)}{dq^2 \; d\cos\theta_\ell} \mathrm{sgn}(\cos\theta_\ell) \; ,$$

provides additional constraints on the Wilson coefficients. In particular, the location of the zero-point of this asymmetry (called below q_0^2) is a precision tests of the SM. In NNLO, one has the following predictions for the inclusive decays $B \to X_s\ell^+\ell^-$: $q_0^2 = (3.90 \pm 0.25)\ \mathrm{GeV}^2$ [$(3.76 \pm 0.22_{\rm theory} \pm 0.24_{m_b})\ \mathrm{GeV}^2$], obtained by Ghinculov et al. [113] [Asatrian et al. [114]].

In the SM (and its extensions in which the operator basis remains unchanged), the FB-asymmetry in $B \to K\ell^+\ell^-$ is zero and in $B \to K^*\ell^+\ell^-$ it depends on the decay form factors. Model-dependent studies yield small form factor-related uncertainties in the zero-point of the asymmetry $\hat{s}_0 = q_0^2/m_B^2$ [115]. HQET provides a symmetry argument why the uncertainty in $\hat{s}_0$ can be expected to be small which is determined by [116] $C_9^{eff}(\hat{s}_0) = -\frac{2m_b}{M_B \hat{s}_0} C_7^{eff}$. However, $O(\alpha_s)$ corrections to the HQET-symmetry relations lead to substantial change in the profile of the FB-asymmetry function as well as a significant shift in $\hat{s}_0$ [117, 118]. They have been worked out for $B \to K^*\ell^+\ell^-$ using SCET [119]. Restricting ourselves

to the kinematic region where the light K^* meson moves fast and can be viewed approximately as a collinear particle, a factorization formula for the decay amplitude of $B \to K^* \ell^+ \ell^-$, to leading power in $1/m_b$, has been derived in SCET [119]. This coincides *formally* with the formula obtained earlier by Beneke et al. [117], using the QCD factorization approach [120, 121], but is valid to all orders of α_s:

$$\langle K_a^* \ell^+ \ell^- | H_{eff} | B \rangle = T_a^I(q^2) \zeta_a(q^2) + \\ + \sum_{\pm} \int_0^\infty \frac{d\omega}{\omega} \phi_\pm^B(\omega) \int_0^1 du \ \phi_{K^*}^a(u) T_{a,\pm}^{II}(\omega, u, q^2) \ , \tag{34}$$

where $a = \|, \perp$ denotes the polarization of the K^* meson. The functions T_a^I and $T_{a,\pm}^{II}$ are perturbatively calculable. $\zeta_a(q^2)$ are the soft form factors defined in SCET while $\phi_\pm^B(\omega)$ and $\phi_{K^*}^a(u)$ are the light-cone distribution amplitudes (LCDAs) for the B and K^* mesons, respectively. In particular, the location of the zero of the forward-backward asymmetry in $B \to K^* \ell^+ \ell^-$, q_0^2, can be predicted more precisely in SCET due to the improved theoretical precision on the scale dependence of q_0^2.

Including the order α_s corrections, the analysis in [119] estimates the zero-point of the FB asymmetry to be

$$q_0^2 = (4.07^{+0.16}_{-0.13}) \ \text{GeV}^2 \ , \tag{35}$$

of which the scale-related uncertainty is $\Delta(q_0^2)_{\text{scale}} = ^{+0.08}_{-0.05}$ GeV^2 for the range $m_b/2 \le \mu_h \le 2m_b$ together with the jet function scale $\mu_l = \sqrt{\mu_h \times 0.5 \ \text{GeV}}$. This is to be compared with the result given in [118], also obtained in the absence of $1/m_b$ corrections: $q_0^2 = (4.39^{+0.38}_{-0.35})$ GeV^2. Of this the largest single uncertainty (about ± 0.25 GeV^2) is attributed to the scale dependence. The difference in the estimates of the scale dependence of q_0^2 in [119] and [118] is both due to the incorporation of the SCET logarithmic resummation (done in [119]) and the different (scheme-dependent) definitions of the effective form factors for the SCET currents used in these references. Power corrections in $1/m_b$ are probably comparable to the $O(\alpha_s)$ corrections, as argued in [118]. So far, q_0^2 has not been measured experimentally.

BELLE has published the first measurements [5, 122] of the forward-backward asymmetry (FBA) [112]. The best-fit results by BELLE for the Wilson coefficient ratios for negative value of C_7, $\frac{C_9}{C_7} = -15.3^{+3.4}_{-4.8} \pm 1.1$ and $\frac{C_{10}}{C_7} = 10.3^{+5.2}_{-3.5} \pm 1.8$, are consistent with the SM values $C_9/C_7 \simeq -13.7$ and $C_{10}/C_7 \simeq +14.9$, evaluated in the NLO approximation. However, for the positive value of C_7, the measurements lead to $\frac{C_9}{C_7} = -16.3^{+3.7}_{-3.7} \pm 1.1$ and $\frac{C_{10}}{C_7} = +11.1^{+6.0}_{-3.9} \pm 1.8$ and the two solutions are of comparable significance.

With more data at the current B factories, and yet more anticipated at the LHC, these measurements are expected to become very precise, providing a precision test of the SM in the flavor sector.

4.3. $B \to V\gamma$: SM vs. Experiments

The decays $B \to V\gamma$ ($V = K^*, \rho, \omega$) have been calculated in the NLO approximation using the effective Lagrangian given in (23) and its analogue for $b \to d$ transitions. Two dynamical approaches, namely the QCD Factorization [120] and pQCD [123] have been employed to establish factorization of the radiative decay amplitudes in the heavy-quark limit. We illustrate the QCD-F method, where this factorization is worked out for the $B \to V\gamma$ [124, 125, 118, 126, 127, 128, 129] (see [130, 131] for phenomenological updates in NLO, and [132, 133] for the alternative "perturbative QCD" approach). In particular, the matrix element of a given operator in the effective weak Hamiltonian can be written in the form

$$\langle V\gamma\,|Q_i|\,\bar{B}\rangle = F^{B\to V_\perp}\,T_i^{\rm I} + \int d\omega\,du\,\phi_+^B(\omega)\,\phi_\perp^V(u)\,T_i^{\rm II}(\omega,u)\,. \qquad (36)$$

The non-perturbative effects are contained in $F^{B\to V_\perp}$, the $B \to V$ transition form factor at $q^2 = 0$, and in ϕ_+^B and $\phi_\perp^V$, the leading-twist LCDAs of the B- and V-mesons. The hard-scattering kernels $T_i^{\rm I}$ and $T_i^{\rm II}$ include only short-distance effects and are calculable in perturbation theory. Contributions to the kernel $T^{\rm I}$ are closely related to the virtual corrections to the inclusive decay rate, and are referred to as vertex corrections. Those to the kernel $T^{\rm II}$ are related to parton exchange with the light quark in the B-meson, a mechanism commonly referred to as hard spectator scattering. It is expected that the factorization formula is valid up to corrections of $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$.

The derivation of the factorization formula from a two-step matching procedure in SCET has provided additional insight into its structure. The technical details for $B \to V\gamma$ in NLO have been provided in [134, 135]. In the SCET approach the factorization formula is written as

$$\langle V\gamma\,|Q_i|\,\bar{B}\rangle = \Delta_i C^A \zeta_{V_\perp} + \frac{\sqrt{m_B}F f_{V_\perp}}{4}\int d\omega\,du\,\phi_+^B(\omega)\,\phi_\perp^V(u)\,t_i^{\rm II}(\omega,u)\,, \qquad (37)$$

where F and $f_{V_\perp}$ are meson decay constants. The SCET form factor $\zeta_{V_\perp}$ is related to the QCD form factor through perturbative and power corrections [117, 136, 137, 138, 139, 140, 141]. In SCET the perturbative hard-scattering kernels are the matching coefficients $\Delta_i C^A$ and $t_i^{\rm II}$. They are known completely to next-to-leading order (NLO) ($\mathcal{O}(\alpha_s)$) in renormalization-group

(RG) improved perturbation theory [135]. Recently, important steps towards a complete analysis at the next-to-next-to-leading order (NNLO) in $B \to V\gamma$ decays have been derived in [142] by obtaining full results for the hard-scattering kernels for the dipole operators Q_7 and Q_8, and partial results for Q_1, valid in the large-β_0 limit and neglecting NNLO corrections from spectator scattering. In addition, this work provides the virtual corrections to this order for the $B \to V\gamma$ decays, as they can not be obtained from the published calculations for the inclusive decay $B \to X_s\gamma$, discussed previously.

In SCET the hard-scattering kernel $t_i^{\rm II}$ for a given operator is subfactorized into the convolution of a hard-coefficient function with a universal jet function, in the form

$$t_i^{\rm II}(u,\omega) = \int_0^1 d\tau \Delta_i C^{B1}(\tau) j_\perp(\tau,u,\omega) \equiv \Delta_i C^{B1} \star j_\perp . \qquad (38)$$

The hard coefficients $\Delta_i C^{B1}$ contain physics at the hard scale m_b, while the jet function $j_\perp$ contains physics at the hard-collinear scale $\sqrt{m_b\Lambda}$. The hard coefficient is identified in a first step of matching QCD $\to$ SCET$_{\rm I}$, and the jet function in a second step of matching SCET$_{\rm I}$ $\to$ SCET$_{\rm II}$. Details in NLO have been worked out for $B \to V\gamma$ in [134, 135].

The effective field-theory techniques are crucial for providing a field-theoretical definition of the objects in (37), and for resumming large perturbative logarithms of the ratio $m_b/\Lambda_{\rm QCD}$ in $t_i^{\rm II}$. In the effective-theory approach resummation is carried out by solving the renormalization-group equations for the matching coefficients $\Delta_i C^{B1}$. Since these coefficients enter the factorization formula in a convolution with the jet function $j_\perp$, their anomalous dimension is a distribution in the variables τ and u. The evolution equations must be solved before performing the convolution with $j_\perp$. Therefore, resummation is not possible in the original QCD factorization formula (36), where the hard-scattering kernels $T_i^{\rm II}$ are obtained only after this convolution has been carried out.

Using the SCET framework, the branching ratios in the NNLO are as follows [142]:

$$\begin{aligned}
\mathcal{B}_{\rm NNLO}(B^0 \to K^{*0}\gamma) &\simeq (4.3 \pm 1.4) \times 10^{-5} , \\
\mathcal{B}_{\rm NNLO}(B^\pm \to K^{*\pm}\gamma) &\simeq (4.6 \pm 1.4) \times 10^{-5} , \\
\mathcal{B}_{\rm NNLO}(B_s^0 \to \phi\gamma) &\simeq (4.3 \pm 1.4) \times 10^{-5} .
\end{aligned}$$

It should be noted that, very much like the $B \to X_s\gamma$ case, the complete NNLO calculations for the virtual corrections to the matrix element of

the operator O_1 in $B \to K^*\gamma$ are not yet at hand. In addition, the hard spectator corrections from this operator are calculated only in NLO. In the NNLO branching ratios quoted above, the errors are increased to take these missing pieces into account.

The above theoretical branching ratios, compared with the current experimental measurements [6, 7], yield the following results [142]:

$$\frac{\mathcal{B}_{\rm NNLO}(B^0 \to K^{*0}\gamma)}{\mathcal{B}_{\rm Expt}(B^0 \to K^{*0}\gamma)} = 1.1 \pm 0.35 \pm 0.06,$$
$$\frac{\mathcal{B}_{\rm NNLO}(B^\pm \to K^{*\pm}\gamma)}{\mathcal{B}_{\rm Expt}(B^\pm \to K^{*\pm}\gamma)} = 1.1 \pm 0.35 \pm 0.07,$$
$$\frac{\mathcal{B}_{\rm NNLO}(B_s^0 \to \phi\gamma)}{\mathcal{B}_{\rm Expt}(B_s^0 \to \phi\gamma)} = 0.8 \pm 0.2 \pm 0.3\ .$$

The decays $B \to (\rho,\omega)\gamma$ involve in addition to the (short-distance) penguin amplitude also significant long-distance contributions, in particular in the decays $B^\pm \to \rho^\pm\gamma$. In the factorization approximation, typical Annihilation-to-Penguin amplitude ratio is estimated as [143]: $\epsilon_{\rm A}(\rho^\pm\gamma) = 0.30 \pm 0.07$. $O(\alpha_s)$ corrections to the annihilation amplitude in $B \to \rho\gamma$ are not known; also the proof of factorization of this amplitude is still not at hand. The annihilation contribution to the decays $B^0 \to \rho^0\gamma$ and $B^0 \to \omega\gamma$ is expected to be suppressed (relative to the corresponding amplitude in $B^\pm \to \rho^\pm\gamma$) due to the electric charges ($Q_d/Q_u = -1/2$) and the color factors, and the corresponding A/P ratio for these decays is estimated as $\epsilon_{\rm A}(\rho^0\gamma) \simeq -\epsilon_{\rm A}(\omega\gamma) \simeq 0.05$.

The decay amplitudes for $B \to (\rho,\omega)\gamma$ depend on the CKM matrix elements $V_{td}^*V_{tb}$ (from the penguin diagrams) and $V_{ub}^*V_{ud}$ (from the annihilation diagrams). Hence, these decays provide potentially very powerful constraints on the CKM parameters, $\bar\rho$ and $\bar\eta$. Since a large number of observables can be measured in these decays, such as the individual branching ratios for $B^\pm \to \rho^\pm\gamma$ and $B^0 \to (\rho^0,\omega)\gamma$, isospin- and SU(3)-violating asymmetries in the decay rates, and direct and time-dependent CP asymmetries, they have been studied theoretically in a number of papers [124, 125, 127, 128, 130, 131, 144]. Experimentally, a beginning has been made in the measurements of the $b \to d\gamma$ transition through the measurements of the branching ratios for $B \to (\rho,\omega)\gamma$, reported by BABAR and BELLE. Current measurements are not very precise, as can be seen from the current world averages [7] (in units of 10^{-6}): $\mathcal{B}(B^\pm \to \rho^\pm\gamma) = 0.96 \pm 0.23$, $\mathcal{B}(B^0 \to \rho^0\gamma) = 0.77 \pm 0.14$ and $\mathcal{B}(B^0 \to \omega\gamma) = 0.41 \pm 0.15$. In addition, theoretical estimates suffer from large hadronic uncertainties, dominated

by the imprecise knowledge of the form factors. Hence, the resulting constraints on the CKM parameters are not very quantitative. Theoretical uncertainties are greatly reduced in the ratios of the branching ratios involving the decays $B \to (\rho,\omega)\gamma$ and $B \to K^*\gamma$. Calling the ratios of the branching ratios $R^{\pm}(\rho\gamma/K^*\gamma)$ and $R^0(\rho\gamma/K^*\gamma)$, for the decays of the $B^{\pm}$ and B^0 mesons, respectively, one has [124]

$$\begin{aligned} R^{\pm}(\rho\gamma/K^*\gamma) &= \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(M_B^2 - M_\rho^2)^3}{(M_B^2 - M_{K^*}^2)^3}\ \zeta^2(1 + \Delta R^{\pm}(\epsilon_{\rm A}^{\pm},\bar\rho,\bar\eta))\ , \\ R^{0}(\rho\gamma/K^*\gamma) &= \frac{1}{2}\left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(M_B^2 - M_\rho^2)^3}{(M_B^2 - M_{K^*}^2)^3}\ \zeta^2(1 + \Delta R^{0}(\epsilon_{\rm A}^{0},\bar\rho,\bar\eta))\ , \end{aligned} \tag{39}$$

where $\zeta = T_1^{\rho}(0)/T_1^{K^*}(0)$, with $T_1^{\rho}(0)$ and $T_1^{K^*}(0)$ being the transition form factors evaluated at $q^2 = 0$ in the decays $B \to \rho\gamma$ and $B \to K^*\gamma$, respectively. The functions $\Delta R^{\pm}(\epsilon_{\rm A}^{\pm},\bar\rho,\bar\eta)$ and $\Delta R^{0}(\epsilon_{\rm A}^{0},\bar\rho,\bar\eta)$, appearing on the r.h.s. of the above equations encode both the $O(\alpha_s)$ contribution to the penguin amplitudes and annihilation contributions, with the latter estimated so far only in the lowest order. They have a non-trivial dependence on the CKM parameters $\bar\rho$ and $\bar\eta$ [124, 125]. Theoretical uncertainty in the evaluation of the ratios $R^{\pm}(\rho\gamma/K^*\gamma)$ and $R^0(\rho\gamma/K^*\gamma)$ is dominated by the error on the quantity ζ, and to some extent also by the errors on the parameters $\epsilon_A^{\pm}$ and ϵ_A^0, characterizing the annihilation/penguin ratios. In the SU(3) limit $\zeta = 1$; SU(3)-breaking corrections have been calculated in several approaches, including the QCD sum rules and Lattice QCD. With the current values for the ratios $R^{\pm}(\rho\gamma/K^*\gamma) = 0.032 \pm 0.008$, $R^0(\rho\gamma/K^*\gamma) = 0.039 \pm 0.007$ and $R^0(\omega\gamma/K^*\gamma) = 0.021 \pm 0.007$, the current world average of $|V_{td}/V_{ts}|$ from the ratio of $B \to (\rho,\omega)\gamma$ and $B \to K^*\gamma$ is [145]:

$$|V_{td}/V_{ts}| = 0.194^{+0.015}_{-0.014}(\text{exp}) \pm 0.014(\text{th})\ , \tag{40}$$

where the Light-cone QCD sum rules [131] have been used to estimate the hadronic input quantities. This determination is compatible with the one from the mass difference ratio $\Delta M_{B_s}/\Delta M_{B_d}$ given in eq. (22), but less precise.

5. $B \to M_1 M_2$ Decays

Exclusive non-leptonic decays are the hardest nuts to crack in the theory of B-decays. Basically, there are four different theoretical approaches to calculate and/or parameterize the hadronic matrix elements in $B \to M_1 M_2$ decays:

(1) SU(2)/SU(3) symmetries and phenomenological Ansaetze [146, 147, 148, 149]

(2) Dynamical approaches based on perturbative QCD, such as the QCD Factorization [120] and the competing pQCD approach [123].

(3) Charming Penguins [150] using the renormalization group invariant topological approach of Buras and Silvestrini [151].

(4) Soft Collinear Effective Theory (SCET) [43, 44, 45, 46], which we have already discussed in the context of radiative and semileptonic decays.

These approaches will be discussed on the example of the $B \to \pi\pi$ and $B \to K\pi$ decays for which now there exist enough data to extract the underlying dynamical parameters. Prior to this, however, we discuss the measurements of the angle β (or ϕ_1) from the experiments at the B-factories.

5.1. *Interplay of Mixing and Decays of B^0- and $\bar{B}^0$-Mesons to CP Eigenstates*

We start with the discussion of the transition $b \to c\bar{c}s$, which is dominated by the tree topology. The time-dependent CP asymmetry in the decays $B^0 \to f$ and $\bar{B}^0 \to f$, where f is a CP eigenstate, such as $J/\psi K_s$ and $J/\psi K_L$, is defined as:

$$\mathcal{A}_f(t) = \frac{\Gamma[\bar{B}^0(t) \to f] - \Gamma[B^0(t) \to f]}{\Gamma[\bar{B}^0(t) \to f] + \Gamma[B^0(t) \to f]}. \tag{41}$$

The time evolution of the two flavor eigenstates B^0 and $\bar{B}^0$ is determined by (2×2) Hermitian matrices M and Γ. The physical states (with definite masses and lifetimes) are the linear combinations of B^0 and $\bar{B}^0$, with $|B_d(L,H)\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle$, dependent on two complex parameters p and q. Defining the decay amplitudes $A(f) \equiv \langle f|H|B^0\rangle$ and $\bar{A}(f) \equiv \langle f|H|\bar{B}^0\rangle$ of the B^0- and $\bar{B}^0$-mesons into the final state f, the time-dependent CP asymmetry is determined by the quantity λ_f, involving the interplay of mixing and decay amplitudes:

$$\lambda_f = \frac{q}{p}\,\rho(f), \qquad \rho(f) = \frac{\bar{A}(f)}{A(f)}. \tag{42}$$

For the B_d^0 - $\bar{B}_d^0$ mixing, the ratio q/p involves the phase β (or ϕ_1), which is one of the angles of the unitarity triangle:

$$\frac{q}{p} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} = \mathrm{e}^{-2i\phi_{\mathrm{mixing}}} = \mathrm{e}^{-2i\beta}. \tag{43}$$

The time dependent CP asymmetry (41) is then expressed as

$$\mathcal{A}_f(t) = C_f \cos(\Delta M_{B_d} t) + S_f \sin(\Delta M_{B_d} t), \tag{44}$$

where $\Delta M_{B_d} = (0.507 \pm 0.005)$ ps^{-1} is the mass difference between the heavy and light B_d^0-meson mass eigenstates and the difference in the decay widths $\Delta\Gamma_{B_d}$ has been neglected. The quantities C_f and S_f, called the direct and mixing-induced CP asymmetries, respectively, are defined in terms of the complex variable λ_f as follows:

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \qquad S_f = \frac{2\mathrm{Im}\lambda_f}{1 + |\lambda_f|^2}. \tag{45}$$

If the decays $B^0 \to f$ and $\bar{B}^0 \to f$ are dominated by a *single* amplitude, the ratio $\rho(f) = \eta_f \, \mathrm{e}^{-2i\phi_{\mathrm{decay}}}$, where $\eta_f = \pm 1$ is the CP parity of the state f, is a pure phase factor and the asymmetries (45) reduce to the expressions:

$$C_f = 0, \qquad S_f = -\eta_f \sin 2(\phi_{\mathrm{mixing}} + \phi_{\mathrm{decay}}). \tag{46}$$

The decays $B^0/\bar{B}^0 \to J/\psi K_s$, $J/\psi K_L$, and a number of related final states with f being $\psi(2S)K_s$, $\eta_c K_s$, $\chi_{c1} K_s$, and $J/\psi K^{*0}(K^{*0} \to K_s\pi^0)$ belong to the category of *gold plated* decays [152]. In all these modes, the direct CP asymmetry C_f, to a very high accuracy, vanishes, and the quantity S_f, the mixing-induced CP asymmetry, measures $\sin(2\beta)$. Averaging over all the decay channels, the results of the BABAR and BELLE measurements are as follows [4]:

$$C = 0.049 \pm 0.022 \pm 0.017, \quad S = 0.714 \pm 0.032 \pm 0.018, \quad \text{[BABAR]} \tag{47}$$

$$C = -0.019 \pm 0.025, \quad S = 0.651 \pm 0.034. \quad \text{[BELLE]} \tag{48}$$

In the BABAR result, the first error is statistical and the second is systematic, while in the BELLE data both the errors have been combined. The current world average for $S_f = \sin(2\beta)$ for the quark transition $b \to c\bar{c}s$ is [7]:

$$\sin(2\beta) = 0.681 \pm 0.025, \tag{49}$$

where the data from LEP and Tevatron have also been included. Restricting β in the range $0 \le \beta \le \pi/2$, two possible values can be extracted $\beta = (21.5 \pm 1.0)^\circ$ and $\beta = (68.5 \pm 1.0)^\circ$. The two-fold ambiguity has now been resolved by several $\cos(2\beta)$ measurements, involving the Dalitz analysis of the decay modes $B^0 \to D^0_{3-\mathrm{body}} h^0$, $B^0 \to K_s\pi^+\pi^-$, $B^0 \to K_s K^+ K^-$, and the older results on $B^0 \to J/\psi K^{*0}$, leading to the determination $\beta = (21.5 \pm 1.0)^\circ$.

The *direct* measurement of $\sin(2\beta)$ in eq.(49) is to be compared with the *indirect* estimate of the same, obtained from the fits of the CKM unitarity

triangle (UT). For this, the UTfit collaboration [66] quotes $\sin(2\beta) = 0.739 \pm 0.044$, obtained from the sides of the UT alone, and $\sin(2\beta) = 0.736 \pm 0.042$, by including also the CP-violating quantity ϵ_K in the K-decays. The results from the CKMfitter group [67] are similar. Thus, SM passes this test comfortably.

Another key test of the SM in the flavor sector is to compare the CP-violating quantities S_f and C_f involving the penguin-topology dominated quark transitions $b \to s\bar{s}s$ and $b \to sd\bar{d}$ with the ones from the transition $b \to c\bar{c}s$, dominated by the tree topology and discussed quantitatively above. The point here is that penguin amplitudes may receive contributions from New Physics. For example, new phases, present generically in supersymmetric theories, may reveal themselves, leading to $S_{f=c\bar{c}s} \neq S_{f=s\bar{s}s;sd\bar{d}}$. Examples of the final states induced by the transition $b \to s\bar{s}s$ are $(\phi, \eta, \eta', K\bar{K})K_s$, and the ones induced by the transition $b \to sd\bar{d}$ are $K_s^0(\pi^0, \rho^0, \omega)$ ($B^0 \to \eta' K_s$ receives contributions from both the transitions). The current measurements of $S_f = -\eta_f \sin(2\beta^{\rm eff})$ from the penguin-dominated decays are [7]: $S_{\phi K^0} = 0.39 \pm 0.17$, $S_{\eta' K^0} = 0.61 \pm 0.07$, $S_{K_s K_s K_s} = 0.58 \pm 0.20$, $S_{\pi^0 K_s} = 0.38 \pm 0.19$, $S_{\rho^0 K_s} = 0.61^{+0.25}_{-0.27}$, $S_{\omega K_s} = 0.48 \pm 0.24$, $S_{f_0 K^0} = 0.84 \pm 0.07$, $S_{K^+K^-K^0} = 0.73 \pm 0.10$, and $S_{\pi^0\pi^0 K_s} = -0.52 \pm 0.41$. These measurements are not as precise as the ones from the $b \to c\bar{c}s$ decays due to the much smaller branching ratios (typically 10^{-5}) compared to the decay $B^0 \to J/\psi K_s$. Also, they involve more than one decay topologies. However, within (large) errors, the values of $\sin(2\beta^{\rm eff})$ from the penguin-dominated transitions are consistent with the value of $\sin(2\beta)$ from the tree-dominated transition given above in eq.(49), with the possible exception of $S_{\phi K^0}$, which deviates by about 2σ, and the poorly measured odd-man out $S_{\pi^0\pi^0 K_s}$. It seems that the fog on S_f in the penguin-dominated decays from the initial epoch of the B-factory experiments has largely evaporated, and the emerging contours of CP asymmetries in these decays are very much the same as predicted by the SM.

5.2. *$B \to \pi\pi$: SM vs. Experiments*

The determination of the phase α is based on the branching ratios and CP asymmetries in the quark transition $b \to u\bar{u}d$. They metamorphise in the decays $B \to \pi\pi$, $B \to \rho\pi$ and $B \to \rho\rho$, apart from other final states. We concentrate here on the decay $B \to \pi\pi$, which has received a lot of theoretical attention.

There are three dominant topologies in the $B \to \pi\pi$ decays termed as Tree (T), Penguin (P) and Color-suppressed (C). In addition, there are

several other subdominant topologies which will be neglected in the discussion below. Parametrization of the T, P, and C amplitudes is convention-dependent. In the Gronau-Rosner c-convention [149], these amplitudes can be represented as

$$\begin{aligned}
\sqrt{2}\,A^{+0} &= -|T|\,\mathrm{e}^{i\delta_T}\,\mathrm{e}^{i\gamma}\left[1+|C/T|\,\mathrm{e}^{i\Delta}\right]\ , \\
A^{+-} &= -|T|\,\mathrm{e}^{i\delta_T}\left[\mathrm{e}^{i\gamma}+|P/T|\,\mathrm{e}^{i\delta}\right]\ , \\
\sqrt{2}\,A^{00} &= -|T|\,\mathrm{e}^{i\delta_T}\left[|C/T|\,\mathrm{e}^{i\Delta}\,\mathrm{e}^{i\gamma}-|P/T|\,\mathrm{e}^{i\delta}\right].
\end{aligned} \tag{50}$$

The charged-conjugate amplitudes $\bar{A}^{ij}$ differ by the replacement $\gamma \to -\gamma$. The amplitudes (50) and the charged-conjugate ones obey the isospin relations:

$$A^{+0} = \frac{1}{\sqrt{2}}\,A^{+-} + A^{00}, \qquad \bar{A}^{-0} = \frac{1}{\sqrt{2}}\,\bar{A}^{+-} + \bar{A}^{00}. \tag{51}$$

There are 5 dynamical parameters $|T|$, $r \equiv |P/T|$, δ, $|C/T|$, Δ, with $\delta_T = 0$ assumed for the overall phase. Thus, the weak phase γ can be extracted together with other quantities if the complete set of experimental data on $B \to \pi\pi$ decays is available.

Table 1. Branching ratios (in units of 10^{-6}) and CP asymmetries in the $B \to \pi\pi$ decays

$\mathcal{B}(B^+ \to \pi^+\pi^0) = 5.59^{+0.41}_{-0.40}$	$A_{\rm CP}(\pi^+\pi^0) = 0.06 \pm 0.05$
$\mathcal{B}(B^0 \to \pi^0\pi^0) = 1.31 \pm 0.21$	$A_{\rm CP}(\pi^0\pi^0) = 0.48^{+0.32}_{-0.31}$
$\mathcal{B}(B^0 \to \pi^+\pi^-) = 5.16 \pm 0.22$	$C_{\rm CP}(\pi^+\pi^-) = -0.38 \pm 0.07$
	$S_{\rm CP}(\pi^+\pi^-) = -0.61 \pm 0.08$

The experimental branching ratios and the direct CP asymmetries $A_{\rm CP}(\pi^0\pi^0)$ and $C_{\rm CP}(\pi^+\pi^-)$, as well as the value of the coefficient $S_{\rm CP}(\pi^+\pi^-)$ in time-dependent CP asymmetry, presented in Table 1, have been fitted to determine the various parameters (the direct CP asymmetry $A_{\rm CP}(\pi^+\pi^0)$ is not relevant for this analysis but can be important in determining the size of electroweak contribution in the decays considered). An updated analysis by Parkhomenko based on the paper [153] yields the following values for the hadronic parameters:

$$\begin{aligned}
|P/T| &= 0.473^{+0.060}_{-0.055}, \qquad \delta = (-40.2^{+6.8}_{-4.7})^\circ\ , \\
|C/T| &= 0.966^{+0.058}_{-0.061}, \qquad \Delta = (-56.3^{+8.4}_{-7.9})^\circ,
\end{aligned} \tag{52}$$

and for the CKM unitarity triangle angle γ (or equivalently α)

$$\gamma = \left(65.9^{+3.0}_{-3.2}\right)^\circ , \qquad \alpha = \pi - \beta - \gamma = \left(92.6^{+3.4}_{-3.2}\right)^\circ . \tag{53}$$

Similar fits based on their data have been performed by the BABAR and BELLE collaborations resulting in slightly larger values: $\alpha = (96^{+10}_{-6})^\circ$ (BABAR) and $\alpha = (97 \pm 11)^\circ$ (BELLE). The overall fits performed by the CKM-Fitter and UT-Fit groups prefer slightly smaller values, yielding: $\alpha = (90.7^{+4.5}_{-2.9})^\circ$ [67] and $\alpha = (88.7 \pm 6.2)^\circ$ [66], respectively. All the above estimates are in good agreement with each other within the quoted errors, stating that the data on $B \to \pi\pi$ (as well as the other decay modes $B \to \rho\pi$ and $B \to \rho\rho$) are in agreement with the indirect estimate of the phase α from the unitarity triangle. The strong phases δ and Δ in Eq.(52) come out rather large. In particular, they are much larger than the predictions of the QCD-F approach [120], with pQCD [123] in better agreement with data, but neither of these approaches provides a good fit of the entire $B \to \pi\pi$ data.

Data on $B \to \pi\pi$ decays are in agreement with the phenomenological approach of the so-called charming penguins [154], and with the SCET-based analyses by Bauer et al. [155, 156] which also attributes a dominant role to the charming penguin amplitude. However, a proof of the factorization of the charming penguin amplitude in the SCET approach remains to be provided. In addition, SCET makes a number of predictions in the $B \to \pi\pi$ sector, such as the branching ratio $\mathcal{B}(B^0 \to \pi^0\pi^0)$: [155]

$$\mathcal{B}(B^0 \to \pi^0\pi^0)\Big|_{\gamma=64^\circ} = (1.3 \pm 0.6) \times 10^{-6} . \tag{54}$$

In contrast, predictions of the QCD-F and pQCD approaches are rather similar: $\mathcal{B}(B^0 \to \pi^0\pi^0) \sim 0.3 \times 10^{-6}$, in substantial disagreement with the data.

5.3. *Present bounds on the phase γ from B decays*

The classic method for determining the phase γ (or ϕ_3) [157, 158, 159, 160] involves the interference of the tree amplitudes $b \to uW^- \to u\bar{c}s$ leading to $B^- \to D^0 K^-$ and $b \to cW^- \to c\bar{u}s$ leading to $B^- \to \overline{D^0} K^-$. These amplitudes can interfere if D^0 and $\overline{D^0}$ decay into a common hadronic final state. Noting that the CP$= \pm 1$ eigenstates $D^0_\pm$ are linear combinations of the D^0 and $\overline{D^0}$ states: $D^0_\pm = (D^0 \pm \overline{D^0})/\sqrt{2}$, both branches lead to the same final states $B^- \to D^0_\pm K^-$. So, the condition of CP interferometry is fulfilled. The decays $B^- \to D^0_\pm K^-$ are described by the amplitudes:

$$A(B^- \to D^0_\pm K^-) = \frac{1}{\sqrt{2}} \left[A(B^- \to D^0 K^-) \pm A(B^- \to \overline{D^0} K^-) \right] . \tag{55}$$

Since, the weak phase of the $b \to u$ transition is γ but the $b \to c$ transition has no phase, a measurement of the CP asymmetry through the interference of these two amplitudes yields γ. The four equations that will be used to extract γ are:

$$R_\pm \equiv \frac{\mathcal{B}(B^- \to D^0_\pm K^-) + \mathcal{B}(B^+ \to D^0_\pm K^+)}{\mathcal{B}(B^- \to D^0 K^-) + \mathcal{B}(B^+ \to D^0 K^+)} = 1 + r^2_{\rm DK} \pm 2 r_{\rm DK} \cos\delta_{\rm DK} \cos\gamma \,,$$
$$A_\pm \equiv \frac{\mathcal{B}(B^- \to D^0_\pm K^-) - \mathcal{B}(B^+ \to D^0_\pm K^+)}{\mathcal{B}(B^- \to D^0_\pm K^-) + \mathcal{B}(B^+ \to D^0_\pm K^+)} = \frac{\pm 2 r_{\rm DK} \sin\delta_{\rm DK} \sin\gamma}{1 + r^2_{\rm DK} \pm 2 r_{\rm DK} \cos\delta_{\rm DK} \cos\gamma} \,. \quad (56)$$

Here, $r_{\rm DK}$ is the ratio of the two tree amplitudes [161] $r_{\rm DK} \equiv |T_1/T_2| \sim (0.1 - 0.2)$, with T_1 and T_2 being the CKM suppressed ($b \to u$) and CKM allowed ($b \to c$) amplitudes, respectively, and $\delta_{\rm DK}$ is the relative strong phase between them. The construction of the final states involves flavor and CP-tagging of the various D^0 states, which can be done, for example, through the decays $D^0_+ \to \pi^+\pi^-$, $D^0_- \to K_S\pi^0$, and $D^0 \to K^-\pi^+$. With three unknowns $(r_{\rm DK}, \delta_{\rm DK}, \gamma)$, but four quantities which will be measured, $R_\pm$ and $A_\pm$, one has, in principle, an over constrained system.

Experimentally, the quantities $R_\pm$ are measured through the ratios:

$$R(K/\pi) \equiv \frac{\mathcal{B}(B^- \to D^0 K^-)}{\mathcal{B}(B^- \to D^0 \pi^-)} \,, \qquad R(K/\pi)_\pm \equiv \frac{\mathcal{B}(B^\pm \to D^0_\pm K^\pm)}{\mathcal{B}(B^\pm \to D^0_\pm \pi^\pm)} \,. \quad (57)$$

With all three quantities $R(K/\pi)$ and $R(K/\pi)_\pm$ measured, one can determine $R_\pm = R(K/\pi)_\pm / R(K/\pi)$. More useful decay modes to construct the $B \to DK$ triangle can be added to reduce the statistical errors. Along these lines, Atwood and Soni [162] have advocated to also include the decays of the vector states in the analysis, such as $B^- \to K^{*-}D^0$, $B^- \to K^-D^{*0}$, and $B^- \to K^{*-}D^{*0}$, making use of the $D^{*0} \to D^0\gamma$ and $D^{*0} \to D^0\pi^0$ modes.

Present measurements in the $B \to DK$ and $B \to D\pi$ decays by the BABAR and BELLE collaborations yielding $R_\pm$ and $A_\pm$ for the $D_{\rm CP}K^-$ mode are summarized by HFAG [7]:

$$R_+ = 1.09 \pm 0.09 \,, \qquad A_+ = 0.26 \pm 0.08 \;\; [\text{BELLE, BABAR}] \,,$$
$$R_- = 0.90 \pm 0.10 \,, \qquad A_- = -0.16 \pm 0.09 \;\; [\text{BELLE}] \,.$$

The corresponding quantities for the $D^*_{\rm CP}K^-$ and $D_{\rm CP}K^{*-}$ are also given by HFAG [7].

A modification of the Gronau-London-Wyler (GLW) method described above has been suggested by Atwood, Dunietz and Soni (ADS), where $B^- \to D^0K^-$ with $D^0 \to K^+\pi^-$ (or similar) and the charge conjugate

decays are implied. BABAR and BELLE use the following definitions for the quantities called $A_{\rm ADS}$ and $R_{\rm ADS}$, (the decay modes $B^- \to D^0 K^-$ followed by $D^0 \to K^+\pi^-$ are used to exemplify the method)

$$\begin{aligned} R_{\rm ADS} &\equiv \frac{\mathcal{B}(B^- \to [K^+\pi^-]_D K^-) + \mathcal{B}(B^+ \to [K^-\pi^+]_D K^+)}{\mathcal{B}(B^- \to [K^-\pi^+]_D K^-) + \mathcal{B}(B^+ \to [K^+\pi^-]_D K^+)}\,, \\ A_{\rm ADS} &\equiv \frac{\mathcal{B}(B^- \to [K^+\pi^-]_D K^-) - \mathcal{B}(B^+ \to [K^-\pi^+]_D K^+)}{\mathcal{B}(B^- \to [K^+\pi^-]_D K^-) + \mathcal{B}(B^+ \to [K^-\pi^+]_D K^+)}\,. \end{aligned} \tag{58}$$

The current measurements of these observables are summarized by HFAG [7]. In the analysis of data, usually the GLW and ADS methods are combined and a χ^2-fit is done to determine the profile of the phase γ.

A variant of the $B \to DK$ method of measuring γ is to use the decays $B^\pm \to DK^\pm$ followed by multi-body decays of the D-meson, such as $D^0 \to K_S\pi^-\pi^+$, $D^0 \to K_S K^- K^+$ and $D^0 \to K_S\pi^-\pi^+\pi^0$, in which a binned Dalitz plot analysis of the decays $D^0/\overline{D^0} \to K_S\pi^-\pi^+$ was proposed [160, 163]. Assuming no CP asymmetry in D^0 decays, the amplitude of the $B^+ \to D^0 K^+ \to (K_S\pi^+\pi^-)K^+$ can be written as

$$M_+ = f(m_+^2, m_-^2) + r_{DK}\,\mathrm{e}^{i(\gamma+\delta_{DK})} f(m_-^2, m_+^2)\,, \tag{59}$$

where m_+^2 and m_-^2 are the squared invariant masses of the $K_S\pi^+$ and $K_S\pi^-$ combinations in the D^0 decay, and f is the complex amplitude of the decay $D^0 \to K_S\pi^+\pi^-$. The quantities r_{DK} and δ_{DK} are the relative magnitudes and strong phases of the two amplitudes, already discussed earlier. The amplitude for the charge conjugate B^- decay is

$$M_- = f(m_-^2, m_+^2) + r_{DK}\,\mathrm{e}^{i(-\gamma+\delta_{DK})} f(m_+^2, m_-^2)\,. \tag{60}$$

Once the functional form of f is fixed by a choice of a model for $D^0 \to K_S\pi^+\pi^-$ decay, the Dalitz distribution for B^+ and B^- decays can be fitted simultaneously by the expressions for M_+ and M_-, with r_{DK}, δ_{DK} and γ (or ϕ_3) as free parameters. The model-dependence could be removed by a binned Dalitz distribution [163]. This is usually called the GGSZ method, and has been used to determine γ. The combined fit of both of these methods by CKMfitter [67] yields $\gamma = (76.8^{+30.4}_{-31.5})^\circ$, to be compared with their overall fit from the CKM unitarity $\gamma = (67.6^{+2.8}_{-4.5})^\circ$. The corresponding fit by the UTfit group [66] yields $\gamma = (67 \pm 7)^\circ$. Thus, we see that within the current experimental error of the *direct* measurements, which is quite large, also the phase γ is compatible with its *indirect* estimates in the SM. The experimental precision will greatly improve at the LHC, in particular, by using 2-body B_s-decays.

5.4. $B \to K\pi$: SM vs. Experiments

We now discuss the decays $B \to K\pi$. First, we note that the direct CP-asymmetry in the $B \to K\pi$ decays has now been measured by the BABAR, BELLE and CDF collaborations:

$$A_{\rm CP}(\pi^+K^-) = \begin{cases} (-10.7 \pm 1.8^{+0.7}_{-0.4})\% & \text{[BABAR]}, \\ (-9.3 \pm 1.8 \pm 0.8)\% & \text{[BELLE]}, \\ (-8.6 \pm 2.3 \pm 0.9)\% & \text{[CDF]}, \end{cases} \tag{61}$$

to be compared with the predictions of the two factorization-based approaches: $A_{\rm CP}(\pi^+K^-) = (-12.9 \div -21.9)\%$[pQCD] [123] and $A_{\rm CP}(\pi^+K^-) = (-5.4 \div +13.6)\%[{\rm QCD-F}]$ [120], with the latter falling short of a satisfactory description of data.

The charged and neutral $B \to \pi K$ decays have received a lot of theoretical attention. In particular, many ratios involving these decays have been proposed to test the SM [165, 166, 167, 168] and extract useful bounds on the angle γ, starting from the Fleischer-Mannel bound [165]:

$$\sin^2\gamma \le R \equiv \frac{\tau_{B^+}}{\tau_{B^0_d}} \frac{\mathcal{B}(B^0_d \to \pi^-K^+) + \mathcal{B}(\bar{B}^0_d \to \pi^+K^-)}{\mathcal{B}(B^+ \to \pi^+K^0) + \mathcal{B}(B^- \to \pi^-\bar{K}^0)} \,. \tag{62}$$

The current experimental average $R = 0.899 \pm 0.049$ allows to put a bound: $\gamma < 92^\circ$ (at 95% C.L.). This is in agreement with the determination of γ from the $B \to \pi\pi$ and $B \to D^{(*)}K^{(*)}$ decays given earlier and the indirect unitarity constraints. Thus, both R and $A_{\rm CP}(\pi^+K^-)$ are in agreement with the SM. The same is the situation with the Lipkin sum rule [167]:

$$R_L \equiv 2\,\frac{\Gamma(B^+ \to K^+\pi^0) + \Gamma(B^0 \to K^0\pi^0)}{\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^0 \to K^+\pi^-)} = 1 + \mathcal{O}(\frac{P_{\rm EW}+T}{P})^2 \,, \tag{63}$$

implying significant electroweak penguin contribution in case R_L deviates significantly from unit. With the current experimental average $R_L = 1.071 \pm 0.049$, this is obviously not the case. This leaves then the two other ratios R_c and R_n involving the $B \to \pi K$ decays of $B^\pm$ and B^0 mesons:

$$R_c \equiv 2\,\frac{\mathcal{B}(B^\pm \to \pi^0K^\pm)}{\mathcal{B}(B^\pm \to \pi^\pm K^0)} \,, \qquad R_n \equiv \frac{1}{2}\,\frac{\mathcal{B}(B^0_d \to \pi^\mp K^\pm)}{\mathcal{B}(B^0_d \to \pi^0K^0)} \,. \tag{64}$$

Their experimental values $R_c = 1.117 \pm 0.071$ and $R_n = 0.980 \pm 0.067$ are to be compared with the current SM-based estimates [164] $R_c = 1.14 \pm 0.05$ and $R_n = 1.11^{+0.04}_{-0.05}$. This implies $R_c({\rm SM}) - R_c({\rm Exp}) = 0.02 \pm 0.09$ and $R_n({\rm SM}) - R_n({\rm Exp}) = 0.13 \pm 0.08$. Possible deviations from the SM, if confirmed, would imply new physics, advocated in this context, in particular,

by Yoshikawa [169], Beneke and Neubert [170] and Buras et al. [164]. However, as of now, one has to conclude that SM is in agreement with the measurements of both R_c and R_n.

Finally, a bound on $\mathcal{B}(B^0 \to K^0\overline{K^0})$ based on $SU(3)$ and $B \to \pi\pi$ data, obtained recently by Fleischer and Recksiegel [171], yielding $\mathcal{B}(B^0 \to K^0\bar{K}^0) < 1.5\times10^{-6}$ is well satisfied by the current world average [7] $\mathcal{B}(B^0 \to K^0\bar{K}^0) = (0.96^{+0.21}_{-0.19}) \times 10^{-6}$.

6. B_s^0 Physics: Eldorado for the Tevatron and the LHC

The main goal of b physics at the hadron colliders Tevatron and the LHC is to chart out the physics of the B_s^0 and $B_c^\pm$ mesons and of the b-baryons. The current information on the spectroscopic and decay characteristics of these hadrons is still very much in the offing, though clearly the two Tevatron experiments have made some incisive inroads in these otherwise uncharted territories. Despite the overwhelming performance of the B factory experiments, there still remain a few landmark measurements to be carried out involving B_d^0 and $B^\pm$ mesons. These include, precise measurements of the CP asymmetries in the penguin-dominated exclusive decays, quantitative determinations of the Wilson coefficients in the effective theory for weak decays (C_7, C_8, C_9, C_{10}), which will be made possible by the precise measurements of the radiative and semileptonic decays $B \to (X_s, K^*)\gamma$ and $B \to (X_s, K, K^*)\ell^+\ell^-$. It is challenging to measure the inclusive decays at the LHC, but certainly exclusive decays will be well measured.

In this section, a brief list of some selected b physics topics to be studied at the LHC is given and discussed.

- B_s^0 - $\overline{B_s^0}$ Mixing.

Apart from the precise measurement of $\Delta M_{B_s} = (17.77 \pm 0.10 \pm 0.07)$ $(\text{ps})^{-1}$ by the CDF collaboration, there are two other quantities still to be measured in this complex: Lifetime difference $\Delta\Gamma_{B_s}$ and the phase ϕ_s. These quantities have been calculated to a high precision in the SM [172]. A recent update of this work yields [71]

$$\frac{\Delta\Gamma_{B_s}}{\Delta M_{B_s}} = (49.7 \pm 9.4) \times 10^{-4}; \quad \Delta\Gamma_{B_s} = (0.096 \pm 0.039)(\text{ps})^{-1},$$
$$\phi_s = (4.2 \pm 1.4) \times 10^{-3} = 0.24^\circ \pm 0.08^\circ \ . \tag{65}$$

The current measurements of these quantities from the D0 collaboration are [173]

$$\Delta\Gamma_{B_s} = (0.12^{+0.08}_{-0.10} \pm 0.02)(\text{ps})^{-1} \text{ (assuming } \phi_s = 0) \ , \tag{66}$$

and

$$\Delta\Gamma_{B_s} = (0.17 \pm 0.09 \pm 0.02)(\text{ps})^{-1} \; ; \phi_s = -0.79 \pm 0.56^{+0.14}_{-0.01} \; . \qquad (67)$$

The corresponding measurement (assuming $\phi_s = 0$) for $\Delta\Gamma_{B_s}$ from CDF is [174] $\Delta\Gamma_{B_s} = (0.076^{+0.059}_{-0.063} \pm 0.006)$ (ps)$^{-1}$, where the first error is statistical and the second systematic. At the LHCb [12], one anticipates a statistical sensitivity of $\sigma(\sin\phi_s) \sim 0.031$ and $\sigma(\Delta\Gamma_s/\Gamma_s) \sim 0.011$, assuming an integrated luminosity of 2 (fb)$^{-1}$ and using the decay $B_s \to J/\psi\phi$. This sensitivity will be improved by accumulating more data and adding the CP modes $B_s \to J/\psi\phi$ and the pure CP modes $B_s \to J/\psi\eta$ and $B_s \to J/\psi\eta'$. The ATLAS and CMS sensitivities on ϕ_s are expected to be somewhat worse by typically a factor 2. Thus, experiments at the LHC will be able to test the SM estimates for both the quantities $\Delta\Gamma_{B_s}$ and ϕ_s.

- Precise measurement of the phase γ.

At the LHC one can combine the methods of determining the phase γ already in use at the B factories involving the decays $B^+ \to D^{(*)}K^+$ and $B^0 \to DK^{(*)}$, with the decay $B_s^0 \to D_s K$. In addition, one can use the U-spin symmetry arguments advocated, in particular, by Fleischer [175], to combine data from the decays $B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ to constrain γ. An educated guess [176] is that a sensitivity $\sigma(\gamma) \simeq 4^\circ$ with 2 (fb)$^{-1}$ of data is reachable at the LHCb, improving to $\sigma(\gamma) \simeq 2.4^\circ$ with 10 (fb)$^{-1}$. This will result in an order of magnitude improvement over the current precision on this phase. Modest improvements are also anticipated for the other two phases α and β at the LHC.

- Leptonic decay $B_s^0 \to \mu^+\mu^-$

New and improved upper limits have been presented by the CDF [177] and D0 [178] collaborations for the decays $B_s^0 \to \mu^+\mu^-$ and $B_d^0 \to \mu^+\mu^-$. They are as follows (at 95% C.L.)

$$\begin{aligned} &\mathcal{B}(B_s^0 \to \mu^+\mu^-) < 9.3\ [5.8] \times 10^{-8} \quad \text{D0[CDF]} \; , \\ &\mathcal{B}(B_d^0 \to \mu^+\mu^-) < 1.8 \times 10^{-8} \quad \text{[CDF]}. \end{aligned} \qquad (68)$$

The CDF and DO upper limits have been combined to yield [5] $\mathcal{B}(B_s^0 \to \mu^+\mu^-) < 4.5 \times 10^{-8}$, to be compared with the SM predictions [179] $\mathcal{B}(B_s^0 \to \mu^+\mu^-) = 3.4 \times 10^{-9}$ and $\mathcal{B}(B_d^0 \to \mu^+\mu^-) = 1.0 \times 10^{-10}$ with $\pm 15\%$ theoretical uncertainty. Hence, currently there is no sensitivity for the SM decay rate. However, as the leptonic branching ratios probe the Higgs sector in beyond-the-SM scenarios, such as supersymmetry, and they

depend sensitively on $\tan\beta$, the Tevatron upper limit on $\mathcal{B}(B_s^0 \to \mu^+\mu^-)$ probes the large $\tan\beta$ parameter space, though the precise constraints are model dependent [180, 181]. At the LHC, the two main collider experiments ATLAS and CMS will reach the SM sensitivity, certainly with the higher LHC luminosity, $L_{\rm LHC} = 10^{34}\ cm^{-2}\ {\rm s}^{-1}$, as the decay $B_s^0 \to \mu^+\mu^-$ remains triggerable with the high luminosity.

- Charmless non-leptonic $B_s \to h_1 h_2$ decays.

The experimental program to study non-leptonic decays $B_s^0 \to h_1 h_2$ has started (here $h_{1,2}$ stand for charmless light vector or pseudoscalar mesons) with first measurements for the branching ratios $B_s^0 \to K^+\pi^-$ and $B_s^0 \to K^+K^-$ made available recently by the CDF collaboration [9, 10]. Remarkably, the first direct CP asymmetry involving the decay $B_s^0 \to K^+\pi^-$ and its CP conjugate mode reported by CDF is found to be large, with $A_{\rm CP}(\overline{B_s^0} \to K^+\pi^-) = (39 \pm 15 \pm 8)\%$. This large CP asymmetry was predicted by Lipkin [182] based on SU(3) symmetry arguments. This already tests various dynamical models, such as QCDF [183], SCET [184] and pQCD [185]. With the ongoing b-physics program at the Tevatron, but, in particular, with the onset of the LHC, we expect a wealth of data involving the decays of the hitherto less studied B_s^0 meson. The charmless $B_s^0 \to h_1 h_2$ decays are also important for the CP asymmetry studies and the determination of the inner angles of the unitarity triangle. As already stated, a number of charmless decays $B_s^0 \to h_1 h_2$ can be related to the $B_d^0 \to h_1 h_2$ decays using SU(3) (or U-spin) symmetry, and hence data on these decays can be combined to test the underlying standard model and search for physics beyond the SM under less (dynamical) model-dependent conditions. Anticipating the experimental developments, many studies have been devoted to the interesting charmless $B_s^0 \to h_1 h_2$ decays, waiting to be tested at the LHC.

7. Summary and Outlook

Summarizing, dedicated experiments carried out over several decades combined with progress in theoretical techniques embedded in QCD have enabled a precise determination of the CKM matrix elements. The knowledge of the third row and the third column of $V_{\rm CKM}$ has come from b-physics, which we discussed at length in this review. Of these, precise determination of V_{cb} and V_{ub} required good control over the perturbative and non-perturbative aspects of QCD. The current precision on the direct determination of V_{tb} from the decay $t \to bW$ is limited by statistics and this

will vastly improve from the top quark studies at the LHC and later at the ILC. The determination of V_{ts} and V_{td} require not only precise knowledge of QCD in b decays but implicitly also the assumption of the CKM unitarity, as they are determined from the loop-induced $b \to s$ and $b \to d$ transitions. Their current best measurement is through the mass differences ΔM_{B_s} and ΔM_{B_d}, and the precision on these matrix elements (typically 10%) is completely dominated by theory. A complementary determination of V_{td} and V_{ts} is also at hand from the radiative penguin transitions $b \to (d, s)\gamma$ and the exclusive decays $B \to (K^*, \rho, \omega)\gamma$, but the current precision is limited by both experimental statistics and non-perturbative aspects of QCD, such as the transition form factors in exclusive decays. This surely will improve over the next several years.

Experiments have also firmly established the phenomenon of CP violation in the K and B meson sectors. The various CP asymmetries in these decays are found compatible with each other and, with some help from QCD, have a consistent interpretation in terms of the single complex phase of the CKM matrix. Again, in principle, there is ample room also for beyond-the-SM weak phases, which would lead to very different patterns of CP asymmetries in the tree-dominated versus loop-dominated transitions. This has not been borne out by experiments at the B factories. While the current data is not equivocal on all the decay channels, and the dynamical aspects of not all the measured B-meson decays are quantitatively understood, the experimental case for the extra weak-phases is rather weak. Ongoing experiments at the B factories are expected to significantly reduce the errors on the quantities S_f and C_f in penguin-dominated decays to settle the issue of new weak phases in B decays.

From the foregoing one has to tentatively conclude that the CKM paradigm is now firmly established as the dominant mechanism of flavor transitions in the quark sector. Whether future experiments, such as at the LHC and (Super) B factories, will force us to modify this paradigm remains to be seen. We expect on theoretical grounds that there is New Physics, probably just around the corner, to solve the outstanding issue of the gauge hierarchy. The resolution of this problem together with the unification of the gauge couplings and the search of viable candidate(s) for dark matter requires a TeV scale New Physics. Assuming that supersymmetry is the most viable candidate for the impending New Physics to be discovered by experiments at the LHC, the central issue in the LHC era would be to pin down the underlying flavor aspects of this theory. However, if the current experimental trend is any indicator, then very likely the New Physics will

be of the minimal flavor violating type, or something akin to it. Verifying this and, more importantly, quantifying the subdominant flavor structures in supersymmetry (or its alternative) is the next task to which theory and experiment have to gear themselves up.

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