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Higher Twist Contributions to the Structure Functions $F_2^p(x,Q^2)$ and $F_2^d(x,Q^2)$ at Large x at Higher Orders

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Abstract

The higher twist contributions to the deeply inelastic structure functions $F_2^p(x,Q^2)$ and $F_2^d(x,Q^2)$ for larger values of the Bjorken variable x are extracted extrapolating the twist–2 contributions measured in the large W^2 region to the region $4\,\mathrm{GeV}^2 \leq W^2 \leq 12.5\,\mathrm{GeV}^2$ applying target mass corrections. We compare the results for the NLO, NNLO and N^{3}LO analyzes and include also the large x at N^{4}LO to the Wilson coefficients. A gradual lowering of the higher twist contributions going from NLO to N^{4}LO is observed, which stresses the importance of higher order corrections.

Deeply inelastic structure functions contain higher twist corrections [1] both in the region of large and small values of the Bjorken variable x [2–5]. While the leading twist sector both for unpolarized and polarized deeply inelastic scattering is well explored within perturbative Quantum Chromodynamics (QCD) up to the level of 3–loop, resp. 2–loop, corrections [6,7], very little is known on the scaling violations of dynamical next-to-leading twist correlation functions and the associated Wilson coefficients [1], even on the leading order level. [1] In many experimental and phenomenological analyzes, cf. [2, 3], higher twist contributions are parameterized by an 'Ansatz' [2], which is fitted accordingly. Within QCD this ad-hoc treatment usually cannot be justified, performing at the same time a higher order analysis for the leading twist terms. Since neither the corresponding higher twist anomalous dimensions nor Wilson coefficients were calculated, the data analysis has to be limited in the first place to the kinematic domain in which higher twist terms can be safely disregarded.

In the case of flavor non-singlet combinations of structure functions this is widely the case in the region $Q^2 \geq 4\,\mathrm{GeV}^2, W^2 \geq 12.5\,\mathrm{GeV}^2$, as detailed analyzes of the large W^2 region show, cf. [12]. In this region one may perform a three-loop QCD analysis, which requires the $O(\alpha_s^2)$ Wilson coefficients [14] and the 3–loop anomalous dimensions [6]. The analysis can even be extended effectively to 4–loop order, since the dominant contribution there is implied by the 3–loop Wilson coefficient [7], parameterizing the yet unknown 4–loop anomalous dimension with a $\pm$ 100 % error added to an estimate of this quantity formed as Padé-approximation out of the lower order terms. A comparison with the 2nd moment of the 4–loop anomalous dimension calculated in [15] showed [12] that the agreement is better than 20 %, which underlines that the above approximation may be possible. We limit the QCD–analysis of the twist–2 contributions to this representation since neither $\alpha_s(\mu^2)$ nor the splitting and coefficient functions are known beyond this level.

The evolution equations are solved in Mellin-N space. The non–singlet structure function at the starting scale of the evolution, Q_0^2, is given by

$$F_2^{p,d;\mathrm{NS}}(N,Q^2) = \sum_{k=0}^{\infty} a_s^{k-1}(Q^2) C_{k-1}^{\mathrm{NS}}(N) f_2^{p,d;\mathrm{NS}}(N,Q^2) \;, \tag{1}$$

with $C_k^{\mathrm{NS}}(N)$ the expansion terms of the non–singlet Wilson coefficient with $C_0(N) = 1$, $a_s(Q^2) = \alpha_s(Q^2)/(4\pi)$ and $f_2^{p,d;NS}(N,Q^2)$ the corresponding combination of quark distributions, cf. [12]. Here we identify both the renormalization and factorization scale with Q^2. Beyond $O(a_s^3)$ dominant large x contributions to the Wilson coefficient were calculated in [13].

The evolution equation for the quark densities to 4–loop order reads :

$$\begin{aligned}
f_2^{p,d;\mathrm{NS}}(N,Q^2) &= f_2^{p,d;\mathrm{NS}}(N,Q_0^2) \left(\frac{a}{a_0}\right)^{-\hat{P}_0(N)/\beta_0} \left\{ 1 - \frac{1}{\beta_0}(a-a_0)\left[\hat{P}_1^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_0(N)\right] \right. \\
&- \frac{1}{2\beta_0}\left(a^2-a_0^2\right)\left[\hat{P}_2^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_1^+(N) + \left(\frac{\beta_1^2-\beta_0\beta_2}{\beta_0^2}\right)\hat{P}_0(N)\right] \\
&\left. + \frac{1}{2\beta_0^2}(a-a_0)^2\left(\hat{P}_1^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_0(N)\right)^2 \right.
\end{aligned}$$

[1] In the small x region saturation corrections, see [4, 5], are considered which also belong to the class of higher twist corrections, applying different formalisms. An important question to be addressed here concerns sub-leading corrections, which may be large, cf. [8]. This has been confirmed in a fixed order calculation to three loops in [9]. Recent investigations of the small-x behaviour of deeply inelastic structure functions account for these effects [10, 11].

$$
-\frac{1}{3\beta_0}\left(a^3-a_0^3\right)\left[\hat{P}_3^+(N)-\frac{\beta_1}{\beta_0}\hat{P}_2^+(N)+\left(\frac{\beta_1^2-\beta_0\beta_2}{\beta_0^2}\right)\hat{P}_1^+(N)\right.
$$
$$
\left.+\left(\frac{\beta_1^3}{\beta_0^3}-2\frac{\beta_1\beta_2}{\beta_0^2}+\frac{\beta_3}{\beta_0}\right)\hat{P}_0(N)\right]\frac{(a-a_0)\left(a_0^2-a^2\right)}{2\beta_0^2}\left(\hat{P}_1^+(N)-\frac{\beta_1}{\beta_0}\hat{P}_0(N)\right)
$$
$$
\times\left[\hat{P}_2(N)-\frac{\beta_1}{\beta_0}\hat{P}_1(N)-\left(\frac{\beta_1^2-\beta_0\beta_2}{\beta_0^2}\right)\hat{P}_0(N)\right]
$$
$$
\left.-\frac{(a-a_0)^3}{6\beta_0^3}\left(\hat{P}_1^+(N)-\frac{\beta_1}{\beta_0}\hat{P}_0(N)\right)^3\right\}\,. \tag{2}
$$

Here $\hat{P}_k^+(N)$ denotes the $(k+1)$–loop anomalous dimension and β_k are the expansion coefficients of the QCD β-function, cf. [16], with

$$
\frac{da(\mu^2)}{d\ln(\mu^2)} = -\sum_{k=0}^{\infty}\beta_k a^{k+2}(\mu^2)\,. \tag{3}
$$

Eq. (3) is solved perturbatively to 4–loop order in the $\overline{\rm MS}$ scheme [17] observing the flavor matching conditions for the renormalization scale μ at the thresholds $m_c = 1.5\,\text{GeV}$ and $m_b = 4.5\,\text{GeV}$, respectively, to be able to compare to other measurements of $\alpha_s(M_Z^2)$, resp. $\Lambda_{\rm QCD}^{\rm N_f}$. To perform the data analysis the expression for the structure functions, (1), is transformed back to x–space by a numeric contour integral around the singularities of the problem in the complex N-plane.

In the analysis mass corrections have to be accounted for. These are the target mass [18] and heavy flavor corrections [19–21]. While the former are significant, the latter ones amount only 1-2 % at NLO and are expected to be even smaller in the yet unknown higher orders[2] in the flavor non-singlet case. In the evolution equations (1,2) the anomalous dimensions and coefficient functions are represented in N-space [6, 7, 20, 24, 25]. Here we applied simplifications due to algebraic [26] and structural relations [27] between harmonic sums. Under the conditions mentioned above we perform the twist–2 analysis from leading order (LO) to 4–loop order (N^{3}LO) fitting the non-singlet $F_2^{p,d}(x,Q^2)$ world data, cf. [12], using `MINUIT` [28]. We then extrapolate the results to the region $4\,\text{GeV}^2 \le W^2 \le 12.5\,\text{GeV}^2$ and determine effective higher twist coefficients $C_{\rm HT}(x,Q^2)$ given by

$$
F_2^{\rm exp}(x,Q^2) = F_2^{\rm tw2}(x,Q^2)\cdot\left[\frac{O_{\rm TMC}\left[F_2^{\rm tw2}(x,Q^2)\right]}{F_2^{\rm tw2}(x,Q^2)}+\frac{C_{\rm HT}(x,Q^2)}{Q^2[1\,\text{GeV}^2]}\right]\,. \tag{4}
$$

Here $O_{\rm TMC}[\ ]$ denotes the operator of target mass corrections.

QCD corrections beyond N^{3}LO are known in form of the dominant large-x contributions to the QCD–Wilson coefficients [13]. Since these corrections do quantitatively only apply in the range of large x we do not use them in the twist-2 QCD–fit, because here the data are mainly situated at lower values of x where beyond 4–loop order other contributions to the Wilson coefficients, which are not calculated yet, are as important. Furthermore, the 4–loop anomalous dimensions were not calculated yet. The leading large x contributions are given in terms of harmonic sums of the type $S_{1,1,\ldots,1}(N)$ which obey a determinant representation [29] in single

[2]First contributions which are relevant for the $O(\alpha_s^3)$ heavy flavor contributions to $F_2(x,Q^2)$ were calculated in [22] for the region $Q^2 \gg m^2$. Under this kinematic condition the corresponding corrections to $F_L(x,Q^2)$ were calculated in [23].

harmonic sums $S_l(N)$, i.e. they are polynomials of single harmonic sums.[3] One may calculate these sums recursively, [29]. The n-fold sum reduces to the $(n-k)$-fold sums by

$$S_{1,\ldots,1}(N) = \frac{1}{n}\left[S_n(N) + S_1(N)S_{n-1}(N) + S_{1,1}(N)S_{n-2}(N) + \ldots\right] \ . \tag{5}$$

To obtain the large x behaviour we retain the terms

$$S_1(N) \ \propto \ \ln(N) + \gamma_E \tag{6}$$
$$S_l(N) \ \propto \ \zeta_l, \quad l \geq 2 \ , \tag{7}$$

as $|N| \to \infty$, with γ_E the Euler–Mascheroni number, and ζ_l the Riemann ζ-function at integer values. We agree with the numerical parameterization given in Table 1, [10a]. We take into account the N^{4}LO terms in this approximation, which are added to the twist–2 fit results in N^{3}LO extrapolating to the region of lower values of W^2.

The effective higher twist distribution functions $C_{\rm HT}^{p,d}(x)$ extracted are shown in Figures 1 and 2 from NLO to N^{4}LO. Here we averaged over the values in Q^2 within the x–bins. [4] The leading twist terms are those given in [12], with the values of $\Lambda^{(4)}_{\rm QCD} = 265 \pm 27, 226 \pm 25, 234 \pm 26$ MeV, resp. in NLO, NNLO, and N^{3}LO. Both for the proton and deuteron data $C_{\rm HT}(x)$ grows towards large values of x, and takes values ~ 1 around $x = 0.6$. The inclusion of higher order corrections reduces $C_{\rm HT}(x)$ to lower values with a gradually smaller difference order by order. Yet for the highest bins, $x \geq 0.8$, the effect of the large x resummation terms is important. Earlier higher twist analyzes [3] limited to the next-to-leading order corrections are thus corrected by factors of 2 and larger at large x to lower values. In the present analysis we limited the investigation to the inclusion of the large x terms in N^{4}LO which are still in the vicinity of a nearly complete QCD analysis as outlined above. The present description is likely to be final for values of $x \leq 0.8$. Beyond this range there are only few data. More data in this interesting region would be welcome and can be obtained at planned high-luminosity colliders such as EIC [31]. Data from measurements at JLAB cannot be included into the present analysis, since the kinematic region covered currently averages over the resonances applying duality, which may lead to different and probably lower higher twist contributions, cf. [32].

In the present analysis we extracted the large x dynamical higher twist contributions to the structure functions in a model-independent way. It would be interesting to compare moments of the term $C_{\rm HT}(x)$ to lattice results, which allow to simulate the moments of the corresponding higher twist correlation functions, in the future.

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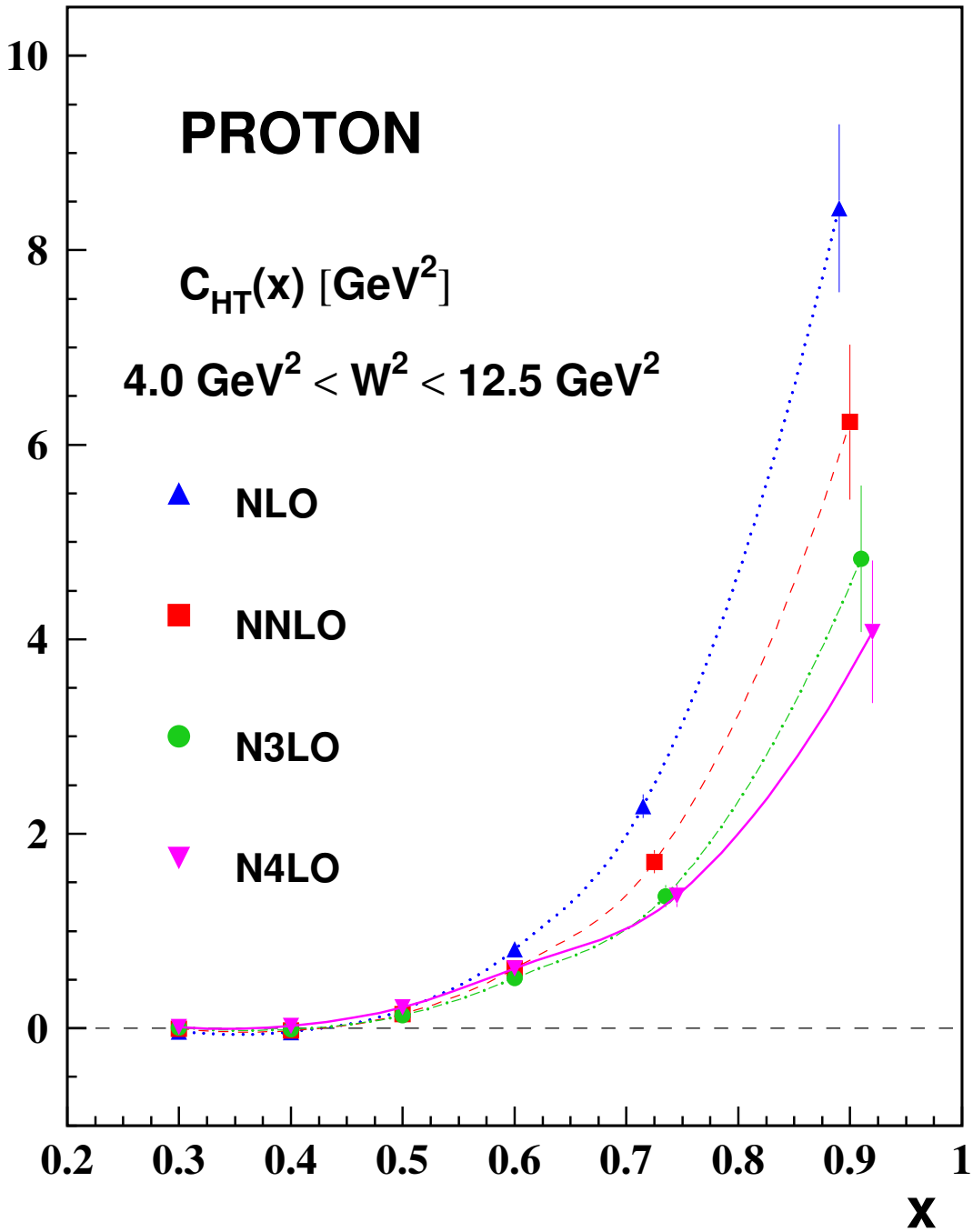

Figure 1: Comparison of the higher twist coefficient $C_{\rm HT}(x)$ in the large x region for the proton data as function of x in a NLO (dotted line), NNLO (dashes line), N^{3}LO analysis (dash-dotted line) and adding the large x terms in $O(\alpha_s^4)$ for the non-singlet QCD Wilson coefficient (full line). Some bin centers are slightly shifted for better visibility.

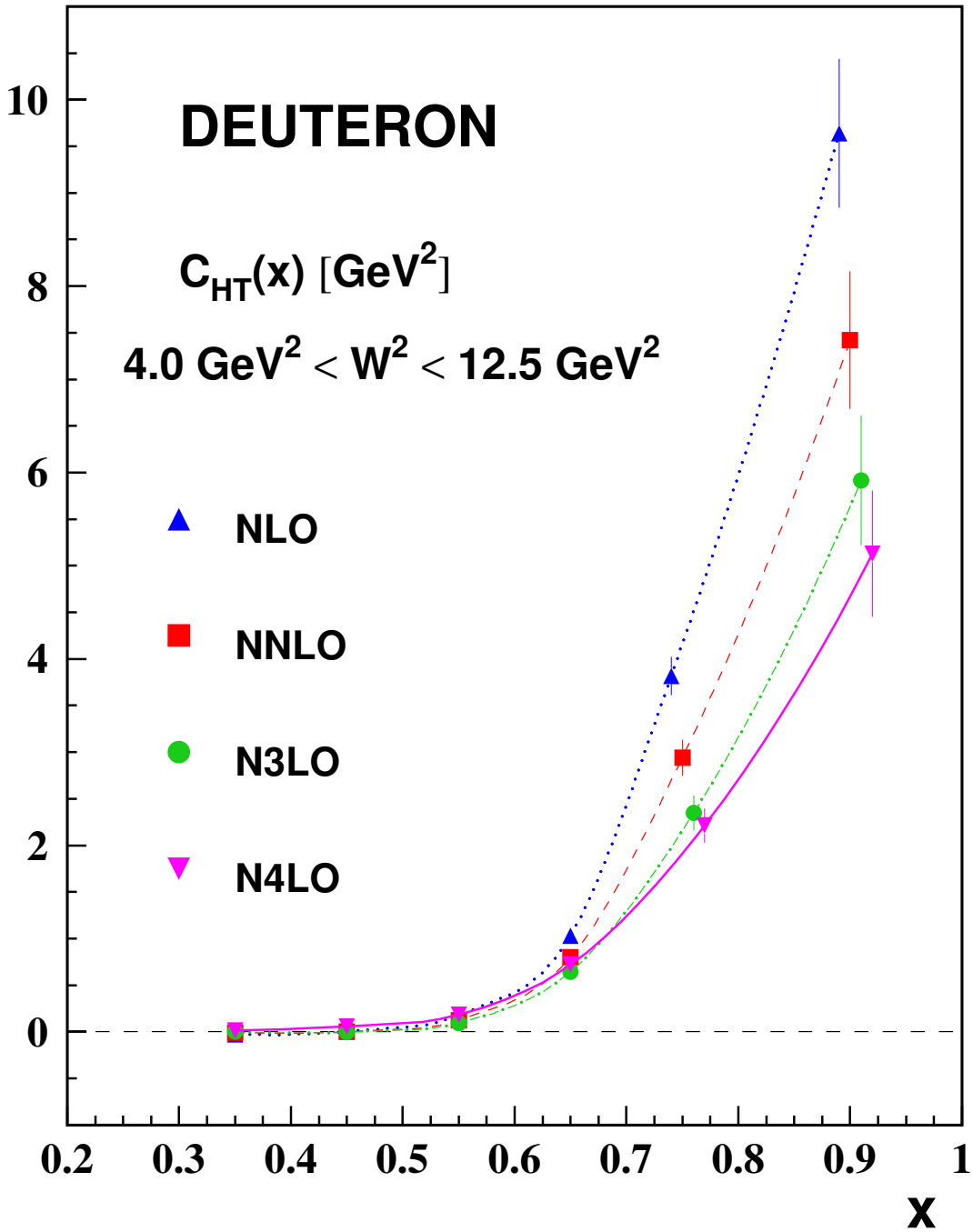

Figure 2: The coefficient $C_{\rm HT}(x)$ for the deuteron data. The curves have the same meaning as in Figure 1.