DESY 08-002
January 2008

ISSN 0418-9833

Heavy-quark contributions to the ratio F_L/F_2 at low x

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arXiv:0801.1502v1 [hep-ph] 9 Jan 2008

Abstract

We study the heavy-quark contribution to the proton structure functions $F_2^i(x,Q^2)$ and $F_L^i(x,Q^2)$, with $i=c,b$, for small values of Bjorken's x variable at next-to-lading order and provide compact formulas for their ratios $R_i=F_L^i/F_2^i$ that are useful to extract $F_2^i(x,Q^2)$ from measurements of the doubly differential cross section of inclusive deep-inelastic scattering at DESY HERA. Our approach naturally explains why R_i is approximately independent of x and the details of the parton distributions in the small-x regime.

PACS: 12.38.-t, 12.38.Bx, 13.66.Bc, 13.85.Lg

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1 Introduction

The totally inclusive cross section of deep-inelastic lepton-proton scattering (DIS) depends on the square s of the centre-of-mass energy, Bjorken's variable $x = Q^2/(2pq)$, and the inelasticity variable $y = Q^2/(xs)$, where p and q are the four-momenta of the proton and the virtual photon, respectively, and $Q^2 = -q^2 > 0$. The doubly differential cross section is parameterized in terms of the structure function F_2 and the longitudinal structure function F_L, as

$$\frac{d^2\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{xQ^4}\{[1+(1-y)^2]F_2(x,Q^2) - y^2F_L(x,Q^2)\}, \tag{1}$$

where α is Sommerfeld's fine-structure constant. At small values of x, F_L becomes non-negligible and its contribution should be properly taken into account when the F_2 is extracted from the measured cross section. The same is true also for the contributions F_2^i and F_L^i of F_2 and F_L due to the heavy quarks $i = c, b$.

Recently, the H1 [1, 2, 3] and ZEUS [4, 5, 6] Collaborations at HERA presented new data on F_2^c and F_2^b. At small x values, of order 10^{-4}, F_2^c was found to be around 25% of F_2, which is considerably larger than what was observed by the European Muon Collaboration (EMC) at CERN [7] at larger x values, where it was only around 1% of F_2. Extensive theoretical analyses in recent years have generally served to establish that the F_2^c data can be described through the perturbative generation of charm within QCD (see, for example, the review in Ref. [8] and references cited therein).

In the framework of Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) dynamics [9], there are two basic methods to study heavy-flavour physics. One of them [10] is based on the massless evolution of parton distributions and the other one on the photon-gluon fusion (PGF) process [12]. There are also some interpolating schemes (see Ref. [13] and references cited therein). The present HERA data on F_2^c [1, 2, 3, 4, 5, 6] are in good agreement with the modern theoretical predictions.

In earlier HERA analyses [1, 4], F_L^c and F_L^b were taken to be zero for simplicity. Four years ago, the situation changed: in the ZEUS paper [5], the F_L^c contribution at next-to-leading order (NLO) was subtracted from the data; in Refs. [2, 3], the H1 Collaboration introduced the reduced cross sections

$$\tilde{\sigma}^{i\bar{i}} = \frac{xQ^4}{2\pi\alpha^2[1+(1-y)^2]}\,\frac{d^2\sigma^{i\bar{i}}}{dx\,dy} = F_2^i(x,Q^2) - \frac{y^2}{1+(1-y)^2}F_L^i(x,Q^2) \tag{2}$$

for $i = c, b$ and thus extracted F_2^i at NLO by fitting their data. Very recently, a similar analysis, but for the doubly differential cross section $d^2\sigma^{i\bar{i}}/(dx\,dy)$ itself, has been performed by the ZEUS Collaboration [6].

In this letter, we present a compact formula for the ratio $R_i = F_L^i/F_2^i$, which greatly simplifies the extraction of F_2^i from measurements of $d^2\sigma^{i\bar{i}}/(dx\,dy)$.

2 Master formula

We now derive our master formula for $R_i(x,Q^2)$ appropriate for small values of x, which has the advantage of being independent of the parton distribution functions (PDFs) $f_a(x,Q^2)$, with parton label $a = g,q,\overline{q}$, where q generically denotes the light-quark flavours. In the small-x range, where only the gluon and quark-singlet contributions matter, while the non-singlet contributions are negligibly small, we have[1]

$$F_k^i(x,Q^2) = \sum_{a=g,q,\overline{q}} \sum_{l=+,-} C_{k,a}^l(x,Q^2) \otimes x f_a^l(x,Q^2), \tag{3}$$

where $l = \pm$ labels the usual $+$ and $-$ linear combinations of the gluon and quark-singlet contributions, $C_{k,a}^l(x,Q^2)$ are the DIS coefficient functions, which can be calculated perturbatively in the parton model of QCD, μ is the renormalization scale appearing in the strong-coupling constant $\alpha_s(\mu)$, and the symbol $\otimes$ denotes convolution according to the usual prescription, $f(x) \otimes g(x) = \int_x^1 (dy/y) f(y) g(x/y)$. Massive kinematics requires that $C_{k,a}^l = 0$ for $x > b_i = 1/(1+4a_i)$, where $a_i = m_i^2/Q^2$. We take m_i to be the solution of $\overline{m}_i(m_i) = m_i$, where $\overline{m}_i(\mu)$ is defined in the modified minimal-subtraction ($\overline{\rm MS}$) scheme.

Exploiting the small-x asymptotic behaviour of $f_a^l(x,Q^2)$ [12],

$$f_a^l(x,Q^2) \stackrel{x\to 0}{\to} \frac{1}{x^{1+\delta_l}}, \tag{4}$$

Eq. (3) can be rewritten as

$$F_k^i(x,Q^2) \approx \sum_{a=g,q,\overline{q}} \sum_{l=+,-} M_{k,a}^l(1+\delta_l,Q^2) x f_a^l(x,Q^2), \tag{5}$$

where

$$M_{k,a}^l(n,Q^2) = \int_0^{b_i} dx\, x^{n-2} C_{k,a}^l(x,Q^2) \tag{6}$$

is the Mellin transform, which is to be analytically continued from integer values n to real values $1+\delta_l$.

As demonstrated in Ref. [14], HERA data support the modified Bessel-like behavior of PDFs at low x values predicted in the framework of the so-called generalized double-asymptotic scaling regime. In this approach, one has $M_{k,a}^+(1,Q^2) = M_{k,a}^-(1,Q^2)$ if $M_{k,a}^l(n,Q^2)$ are devoid of singularities in the limit $\delta_l \to 0$, as in our case. Defining $M_{k,a}(1,Q^2) = M_{k,a}^\pm(1,Q^2)$ and using $f_a(x,Q^2) = \sum_{l=\pm} f_a^l(x,Q^2)$, Eq. (5) may be simplified to become

$$F_k^i(x,Q^2) \approx \sum_{a=g,q,\overline{q}} M_{k,a}(1,Q^2) x f_a(x,Q^2). \tag{7}$$

A further simplification is obtained by neglecting the contributions due to incoming light quarks and antiquarks in Eq. (7), which is justified because they vanish at LO and are

[1] Here and in the following, we suppress the variables μ and m_i in the argument lists of the structure and coefficient functions for the ease of notation.

numerically suppressed at NLO for small values of x. One is thus left with the contribution due to PGF [12],

$$F_k^i(x,Q^2) \approx M_{k,g}(1,Q^2) x f_g(x,Q^2). \tag{8}$$

In fact, the non-perturbative input $f_g(x,Q^2)$ does cancels in the ratio

$$R_i(x,Q^2) \approx \frac{M_{L,g}(1,Q^2)}{M_{2,g}(1,Q^2)}, \tag{9}$$

which is very useful for practical applications. Through NLO, $M_{k,g}(1,Q^2)$ exhibits the structure

$$M_{k,g}(1,Q^2) = e_i^2 a(\mu) \left\{ M_{k,g}^{(0)}(1,a_i) + a(\mu) \left[M_{k,g}^{(1)}(1,a_i) + M_{k,g}^{(2)}(1,a_i) \times \ln\frac{\mu^2}{m_i^2} \right] \right\} + \mathcal{O}(a^3), \tag{10}$$

where e_i is the fractional electric charge of heavy quark i and $a(\mu) = \alpha_s(\mu)/(4\pi)$ is the couplant. Inserting Eq. (10) into Eq. (9), we arrive at our master formula

$$R_i(x,Q^2) \approx \frac{M_{L,g}^{(0)}(1,a_i) + a(\mu)\left[M_{L,g}^{(1)}(1,a_i) + M_{L,g}^{(2)}(1,a_i)\ln(\mu^2/m_i^2)\right]}{M_{2,g}^{(0)}(1,a_i) + a(\mu)\left[M_{2,g}^{(1)}(1,a_i) + M_{2,g}^{(2)}(1,a_i)\ln(\mu^2/m_i^2)\right]} + \mathcal{O}(a^2). \tag{11}$$

We observe that the right-hand side of Eq. (11) is independent of x, a remarkable feature that is automatically exposed by our procedure. In the next two sections, we present compact analytic results for the LO ($j = 0$) and NLO ($j = 1, 2$) coefficients $M_{k,g}^{(j)}(1,a_i)$, respectively.

3 LO results

The LO coefficient functions of PGF can be obtained from the QED case [15] by adjusting coupling constants and colour factors, and they read [16, 17]

$$\begin{aligned} C_{2,g}^{(0)}(x,a) &= -2x\{[1-4x(2-a)(1-x)]\beta - [1-2x(1-2a) \\ &\quad + 2x^2(1-6a-4a^2)]L(\beta)\}, \\ C_{L,g}^{(0)}(x,a) &= 8x^2[(1-x)\beta - 2axL(\beta)], \end{aligned} \tag{12}$$

where

$$\beta = \sqrt{1-\frac{4ax}{1-x}}, \qquad L(\beta) = \ln\frac{1+\beta}{1-\beta}. \tag{13}$$

Using the auxiliary formulas

$$\int_0^b x^m \beta = \begin{cases} 1-2aJ(a), & \text{if } m=0 \\ \frac{b}{2}[1-2a-4a(1+3a)J(a)], & \text{if } m=1 \\ \frac{b^2}{3}[(1+3a)(1+10a)-6a(1+6a+10a^2)J(a)], & \text{if } m=2 \end{cases}, \tag{14}$$

$$\int_0^b x^m L(\beta) = \begin{cases} J(a), & \text{if } m=0 \\ -\frac{b}{2}[1-(1+2a)J(a)], & \text{if } m=1 \\ -\frac{b^2}{3}[3(1+2a)-2(1+4a+6a^2)J(a)], & \text{if } m=2 \end{cases}, \tag{15}$$

where

$$J(a) = -\sqrt{b}\ln t, \qquad t = \frac{1-\sqrt{b}}{1+\sqrt{b}}, \tag{16}$$

we perform the Mellin transformation in Eq. (6) to find

$$\begin{aligned} M_{2,g}^{(0)}(1,a) &= \frac{2}{3}[1+2(1-a)J(a)], \\ M_{L,g}^{(0)}(1,a) &= \frac{4}{3}b[1+6a-4a(1+3a)J(a)]. \end{aligned} \tag{17}$$

At LO, the small-x approximation formula thus reads

$$R_i \approx 2b_i \frac{1+6a_i-4a_i(1+3a_i)J(a_i)}{1+2(1-a_i)J(a_i)}. \tag{18}$$

4 NLO results

The NLO coefficient functions of PGF are rather lengthy and not published in print; they are only available as computer codes [18]. For the purpose of this letter, it is sufficient to work in the high-energy regime, defined by $a_i \ll 1$, where they assume the compact form [19]

$$C_{k,g}^{(j)}(x,a) = \beta R_{k,g}^{(j)}(1,a), \tag{19}$$

with

$$\begin{aligned} R_{2,g}^{(1)}(1,a) &= \frac{8}{9}C_A[5+(13-10a)J(a)+6(1-a)I(a)], \\ R_{L,g}^{(1)}(1,a) &= -\frac{16}{9}C_A b\{1-12a-[3+4a(1-6a)]J(a)+12a(1+3a)I(a)\}, \\ R_{k,g}^{(2)}(1,a) &= -4C_A M_{k,g}^{(0)}(1,a), \end{aligned} \tag{20}$$

where $C_A = N$ for the colour gauge group SU(N), $J(a)$ is defined by Eq. (16), and

$$I(a) = -\sqrt{b}\left[\zeta(2) + \frac{1}{2}\ln^2 t - \ln(ab)\ln t + 2\,\mathrm{Li}_2(-t)\right]. \tag{21}$$

Here, $\zeta(2) = \pi^2/6$ and $\mathrm{Li}_2(x) = -\int_0^1 (dy/y)\ln(1-xy)$ is the dilogarithmic function. Using Eq. (14) for $m=0$, we find the Mellin transform (6) of Eq. (19) to be

$$M_{k,g}^{(j)}(1,a) = [1-2aJ(a)]R_{k,g}^{(j)}(1,a). \tag{22}$$

Table 1: Values of $F_2^c(x,Q^2)$ extracted from the H1 measurements of $\tilde{\sigma}^{c\overline{c}}$ at low [3] and high [2] values of Q^2 (in GeV^2) at various values of x (in units of 10^{-3}) using our approach at NLO for $\mu^2 = \xi Q^2$ with $\xi = 1, 100$. The LO results agree with the NLO results for $\xi = 1$ within the accuracy of this table. For comparison, also the results determined in Refs. [2, 3] are quoted.

Q^2	x	H1	$\mu^2 = Q^2$	$\mu^2 = 100\,Q^2$
12	0.197	0.435 ± 0.078	0.433	0.432
12	0.800	0.186 ± 0.024	0.185	0.185
25	0.500	0.331 ± 0.043	0.329	0.329
25	2.000	0.212 ± 0.021	0.212	0.212
60	2.000	0.369 ± 0.040	0.368	0.368
60	5.000	0.201 ± 0.024	0.200	0.200
200	0.500	0.202 ± 0.046	0.201	0.201
200	1.300	0.131 ± 0.032	0.130	0.130
650	1.300	0.213 ± 0.057	0.212	0.213
650	3.200	0.092 ± 0.028	0.091	0.091

5 Results

We are now in a position to explore the phenomenological implications of our results. As for our input parameters, we choose $m_c = 1.25$ GeV and $m_b = 4.2$ GeV. While the LO result for R_i in Eq. (18) is independent of the unphysical mass scale μ, the NLO formula (11) does depend on it, due to an incomplete compensation of the μ dependence of $a(\mu)$ by the terms proportional to $\ln(\mu^2/Q^2)$, the residual μ dependence being formally beyond NLO. In order to estimate the theoretical uncertainty resulting from this, we put $\mu^2 = \xi Q^2$ and vary ξ. Besides our default choice $\xi = 1$, we also consider the extreme choice $\xi = 100$, which is motivated by the observation that NLO corrections are usually large and negative at small x values [20]. A large ξ value is also advocated in Ref. [21], where the choice $\xi = 1/x^a$, with $0.5 < a < 1$, is proposed.

We now extract $F_2^i(x,Q^2)$ $(i = c, b)$ from the H1 measurements of the reduced cross sections in Eq. (2) at low ($12 < Q^2 < 60$ GeV^2) [3] and high ($Q^2 > 150$ GeV^2) [2] values of Q^2 using the LO and NLO results for R_i derived in Sections 3 and 4, respectively. Our NLO results for $\mu^2 = \xi Q^2$ with $\xi = 1, 100$ are presented for $i = c, b$ in Tables 1 and 2, respectively, where they are compared with the values determined by H1. We refrain from showing our results for other popular choices, such as $\mu^2 = 4m_i^2, Q^2 + 4m_i^2$ because they are very similar. We observe that the theoretical uncertainty related to the freedom in the choice of μ is negligibly small and find good agreement with the results obtained by the H1 Collaboration using a more accurate, but rather cumbersome procedure [2, 3]. The experimental data from the ZEUS Collaboration [6] do not allow for such an analysis because they do not come in the form of Eq. (2).

In order to assess the significance of and the theoretical uncertainty in the NLO corrections to R_i, we show in Fig. 1 the Q^2 dependences of R_c, R_b, and R_t evaluated at LO

Table 2: Values of $F_2^b(x,Q^2)$ extracted from the H1 measurements of $\tilde{\sigma}^{b\overline{b}}$ at low [3] and high [2] values of Q^2 (in GeV^2) at various values of x (in units of 10^{-3}) using our approach at NLO for $\mu^2 = \xi Q^2$ with $\xi = 1, 100$. The LO results agree with the NLO results for $\xi = 1$ within the accuracy of this table. For comparison, also the results determined in Refs. [2, 3] are quoted.

Q^2	x	H1	$\mu^2 = Q^2$	$\mu^2 = 100\,Q^2$
12	0.197	0.0045 ± 0.0027	0.0047	0.0046
12	0.800	0.0048 ± 0.0022	0.0048	0.0048
25	0.500	0.0123 ± 0.0038	0.0124	0.0124
25	2.000	0.0061 ± 0.0024	0.0061	0.0061
60	2.000	0.0190 ± 0.0055	0.0190	0.0190
60	5.000	0.0130 ± 0.0047	0.0130	0.0130
200	0.500	0.0413 ± 0.0128	0.0400	0.0400
200	1.300	0.0214 ± 0.0079	0.0212	0.0212
650	1.300	0.0243 ± 0.0124	0.0238	0.0238
650	3.200	0.0125 ± 0.0055	0.0125	0.0125

from Eq. (18) and at NLO from Eq. (11) with $\mu^2 = 4m_i^2, Q^2 + 4m_i^2$. We observe from Fig. 1 that the NLO predictions are rather stable under scale variations and practically coincide with the LO ones in the lower Q^2 regime. On the other hand, for $Q^2 \gg 4m_i^2$, the NLO predictions overshoot the LO ones and exhibit an appreciable scale dependence. We encounter the notion that the fixed-flavour-number scheme used here for convenience is bound to break down in the large-Q^2 regime due to unresummed large logarithms of the form $\ln(Q^2/m_i^2)$. In our case, such logarithms do appear linearly at LO and quadratically at NLO. In the standard massless factorization, such terms are responsible for the Q^2 evolution of the PDFs and do not contribute to the coefficient functions. In fact, in the variable-flavour-number scheme, they are $\overline{\rm MS}$-subtracted from the coefficient functions and absorbed into the Q^2 evolution of the PDFs. Thereafter, the asymptotic large-Q^2 dependences of R_i at NLO should be proportional to $\alpha_s(Q^2)$ and thus decreasing. This is familiar from the Callan-Gross ratio $R = F_L/(F_2 - F_L)$, as may be seen from its (x,Q^2) parameterizations in Ref. [22]. Fortunately, this large-Q^2 problem does not affect our results in Tables 1 and 2 because the bulk of the H1 data is located in the range of moderate Q^2 values. Furthermore, R_i enters Eq. (2) with the suppression factor $y^2/[1 + (1-y)^2]$.

The ratio R_c was previously studied in the framework of the k_t-factorization approach [17] and found to weakly depend on the choice of unintegrated gluon PDF and to be approximately x independent in the small-x regime (see Fig. 8 in Ref. [17]). Both features are inherent in our approach, as may be seen at one glance from Eq. (11). The prediction for R_c from Ref. [17], which is included in Fig. 1 for comparison, agrees well with our results in the lower Q^2 range, but it continues to rise with Q^2, while our results reach maxima, beyond which they fall. In fact, the k_t-factorization approach is likely to overestimate R_c for $Q^2 \gg 4m_i^2$, due to the unresummed large logarithms of the form $\ln(Q^2/m_i^2)$ discussed above.

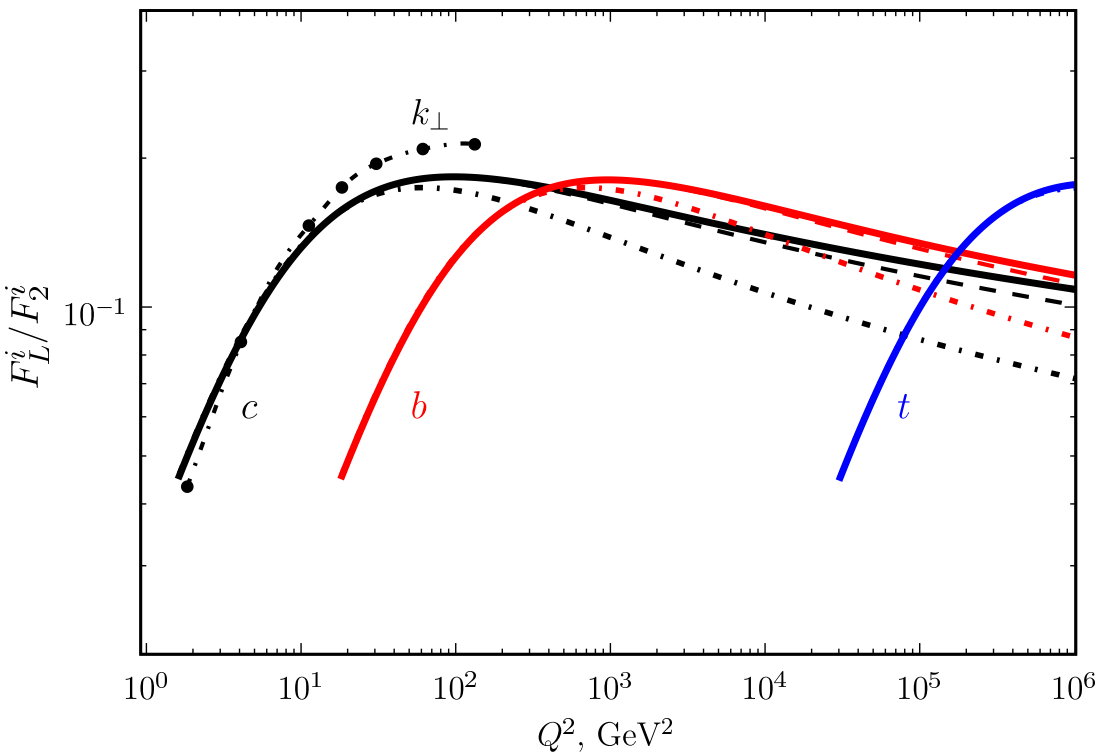

Figure 1: R_c, R_b, and R_t evaluated as functions of Q^2 at LO from Eq. (18) (dot-dashed lines) and at NLO from Eq. (11) with $\mu^2 = 4m_i^2$ (dashed lines) and $\mu^2 = Q^2 + 4m_i^2$ (solid lines). For comparison, the prediction for R_c in the k_t-factorization approach (dot-dot-dashed line) [17] is also shown.

6 Conclusions

In this letter, we derived a compact formula for the ratio $R_i = F_L^i/F_2^i$ of the heavy-flavour contributions to the proton structure functions F_2 and F_L valid through NLO at small values of Bjorken's x variable. We demonstrated the usefulness of this formula by extracting F_2^c and F_2^b from the doubly differential cross section of DIS recently measured by the H1 Collaboration [2, 3] at HERA. Our results agree with those extracted in Refs. [2, 3] well within errors. In the Q^2 range probed by the H1 data, our NLO predictions agree very well with the LO ones and are rather stable under scale variations. Since we worked in the fixed-flavour-number scheme, our results are bound to break down for $Q^2 \gg 4m_i^2$, which manifests itself by appreciable QCD correction factors and scale dependences. As is well known, this problem is conveniently solved by adopting the variable-flavour-number scheme, which we leave for future work. Our approach also simply explains the feeble dependence of R_i on x and the details of the PDFs in the small-x regime.

Acknowledgements

We are grateful to Sergei Chekanov, Vladimir Chekelian, Achim Geiser, Leonid Gladilin, and Zakaria Merebashvili for useful discussions. A.Yu.I. is grateful to the Scuola Internazionale Superiore di Studi Avanzati (SISSA), where most of his work has been done. A.V.K. was supported in part by the Alexander von Humboldt Foundation and the Heisenberg-Landau Programme. This work was supported in part by BMBF Grant No. 05 HT4GUA/4, HGF Grant No. NG–VH–008, DFG Grant No. KN 365/7–1, and RFBR Grant No. 07-02-01046-a.

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