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A new treatment of mixed virtual and real IR-singularities

Janusz Gluza
Department of Field Theory and Particle Physics, Institute of Physics, University of Silesia, Uniwersytecka 4, PL-40-007 Katowice, Poland
E-mail: gluza@us.edu.pl

Tord Riemann*
Deutsches Elektronen-Synchrotron, DESY, Platanenallee 6, 15738 Zeuthen, Germany
E-mail: Tord.Riemann@desy.de

We discuss the determination of the infrared singularities of massive one-loop 5-point functions with Mellin-Barnes (MB) representations. Massless internal lines may lead to poles in the ε expansion of the Feynman diagram, while unresolved massless final state particles give endpoint singularities of the phase space integrals. MB integrals are an elegant tool for their common treatment. An evaluation by taking residues leads to inverse binomial sums.

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*Speaker.

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1. Introduction

We study the infrared (IR) singularities of some massive one-loop n-point Feynman integrals,

$$I = \frac{e^{\varepsilon\gamma_E}}{i\pi^{d/2}} \int \frac{d^dk \;\; T(k)}{(q_1^2-m_1^2)^{\nu_1}\ldots(q_i^2-m_i^2)^{\nu_j}\ldots(q_n^2-m_n^2)^{\nu_n}}, \tag{1.1}$$

by representing them with standard Feynman parameter integrals with characteristic F and U forms:

$$I = \frac{e^{\varepsilon\gamma_E}(-1)^{N_\nu}\Gamma\left(N_\nu-\frac{d}{2}\right)}{\prod_{i=1}^n\Gamma(\nu_i)} \int_0^1 \prod_{j=1}^n dx_j\, x_j^{\nu_j-1}\delta(1-\sum_{i=1}^n x_i) \frac{U(x)^{N_\nu-d}}{F(x)^{N_\nu-d/2}}\, P(T), \tag{1.2}$$

with $N_\nu = \sum_{i=1}^n \nu_i$. For one loop integrals, the $U = \sum x_i$ may be set to one. The F-form is bilinear in the x_i and may be represented in turn by a multiple Mellin-Barnes (MB) integral, using the representation:

$$\frac{1}{(A(x)+B(x))^\nu} = \frac{1}{2\pi i} \int_{-i\infty+R}^{i\infty+R} dz\, A(x)^z\, B(x)^{-\nu-z}\, \frac{\Gamma(-z)\Gamma(\nu+z)}{\Gamma(\nu)}, \tag{1.3}$$

where the integration contour separates the poles of the Γ-functions.

Afterwards, the Feynman parameters may be integrated out and one has to solve the resulting MB integral. This is in general quite non-trivial. However, there is an interesting kind of problems where a systematic approach might be developed, namely the evaluation of the IR divergent parts of the Feynman integrals. They are at the begin of the ε-expansion ($\varepsilon = (4-d)/2$) of the Feynman integral and so of smaller dimensionality in the variables z. In fact, usually one subtracts them from the rest of the integral and treats them separately.

The MB representation allows to do this in a special way which might be of some practical usefulness. We will discuss here only scalar one-loop functions, $T(k) = 1$, but tensors don't show additional problems. For basic definitions and formulae we refer to [1−3] and references cited therein. We use here and in the following the Mathematica packages AMBRE [1] and MB [4] for the derivations of the MB representations and for the ε-expansions. In section 2 we apply the MB-approach to the massive Bhabha vertex and box functions and extract their ε-poles. Section 3 contains the treatment of both the virtual ε-poles and the endpoint singularities from an unresolved, massless particles in a pentagon diagram of massive Bhabha scattering. The method may be generalized to more complex cases, including higher loop orders, but explicit evaluations become then more and more complicated.

2. Simple ε-poles: Massive QED vertex and box

We will set $m = 1$, and s, t are the usual Mandelstam variables. The QED vertex function has the F-form:

$$F(s) = [X[2]+X[3]]^2 + [-s]\, X[2]X[3], \tag{2.1}$$

leading, without a continuation in ε, to a one-dimensional MB representation and a series over residues [2]:

$$V(s) = \frac{1}{2s\varepsilon}\frac{e^{\varepsilon\gamma_E}}{2\pi i}\int\limits_{-i\infty-1/2}^{-i\infty-1/2} dz\ (-s)^{-z}\frac{\Gamma^2(-z)\Gamma(-z+\varepsilon)\Gamma(1+z)}{\Gamma(-2z)}$$
$$= -\frac{e^{\varepsilon\gamma_E}}{2\varepsilon}\sum_{n=0}^{\infty}\frac{s^n}{\binom{2n}{n}(2n+1)}\frac{\Gamma(n+1+\varepsilon)}{\Gamma(n+1)}. \tag{2.2}$$

The complete series may be summed directly with Mathematica[1], and the vertex becomes:

$$V(s) = -\frac{e^{\varepsilon\gamma_E}}{2\varepsilon}\Gamma(1+\varepsilon)\ {}_2F_1\left[1,1+\varepsilon;3/2;s/4\right]. \tag{2.3}$$

Alternatively, one may derive the ε-expansion by exploiting the well-known relation with harmonic numbers $S_k(n) = \sum_{i=1}^{n} 1/i^k$:

$$\frac{\Gamma(n+a\varepsilon)}{\Gamma(n)} = \Gamma(1+a\varepsilon)\exp\left[-\sum_{k=1}^{\infty}\frac{(-a\varepsilon)^k}{k}S_k(n-1)\right]. \tag{2.4}$$

The product $\exp(\varepsilon\gamma_E)\Gamma(1+\varepsilon) = 1+\frac{1}{2}\zeta[2]\varepsilon^2+O(\varepsilon^3)$ yields expressions with zeta numbers $\zeta[n]$, and, taking all terms together, one gets a collection of inverse binomial sums[2]; the first of them is the IR divergent part:

$$V(s) = \frac{V_{-1}(s)}{\varepsilon}+V_0(s)+\cdots \tag{2.5}$$
$$V_{-1}(s) = \frac{1}{2}\sum_{n=0}^{\infty}\frac{s^n}{\binom{2n}{n}(2n+1)} = \frac{1}{2}\frac{4\arcsin(\sqrt{s}/2)}{\sqrt{4-s}\sqrt{s}}. \tag{2.6}$$

This procedure applies similarly to the Bhabha box diagram [7]. We take for definiteness the s channel scalar loop integral. The F-form is (again with $m=1$):

$$F(s,t) = [X[2]+X[4]]^2+[-s]\ X[1]X[3]+[-t]\ X[2]X[4], \tag{2.7}$$

and an MB representation is, after continuation to small ε, a sum of two terms:

$$B(s,t) = \frac{(-s)^{-\varepsilon}}{2st}\frac{\Gamma[1+\varepsilon]\Gamma[-\varepsilon]^2}{\Gamma[-2\varepsilon]}\frac{e^{\varepsilon\gamma_E}}{2\pi i}\int\limits_{-i\infty-7/16}^{+i\infty-7/16} dz_1(-t)^{-z_1}\frac{\Gamma^3[-z_1]\Gamma[1+z_1]}{\Gamma[-2z_1]} \tag{2.8}$$
$$+\frac{1}{t^2}\frac{1}{\Gamma[-2\varepsilon]}\frac{e^{\varepsilon\gamma_E}}{(2\pi i)^2}\int\limits_{-i\infty-3/4}^{+i\infty-3/4} dz_1\left(\frac{s}{t}\right)^{z_1}\Gamma[-z_1]\Gamma[-2(1+\varepsilon+z_1)]\Gamma[1+z_1]^2$$
$$\times\int\limits_{-i\infty-7/16}^{+i\infty-7/16} dz_2(-t)^{-\varepsilon-z_2}\Gamma[-z_2]\frac{\Gamma[-1-\varepsilon-z_1-z_2]^2}{\Gamma[-2(1+\varepsilon+z_1+z_2)]}\Gamma[2+\varepsilon+z_1+z_2].$$

[1]The expression for $V(s)$ was also derived in [5]; see additionally [6].

[2]For the first four terms of the ε-expansion in terms of inverse binomial sums or of polylogarithmic functions, see [2].

Due to the pre-factors, $\Gamma[-\varepsilon]^2/\Gamma[-2\varepsilon] = -2/\varepsilon + 2\zeta[2]\varepsilon + O(\varepsilon^2)$ and $1/\Gamma[-2\varepsilon] = -2\varepsilon + 4\gamma_E\varepsilon^2 + O(\varepsilon^3)$, only the first integral contributes to the first two terms of the ε-expansion,

$$B(s,t) \;=\; \left[\frac{1}{\varepsilon} - \ln(-s)\right] \frac{V_{-1}(t)}{(-s)} + O(\varepsilon), \tag{2.9}$$

where V_{-1} is from (2.6), and the IR divergency is:

$$B_{-1}(s,t) \;=\; \frac{V_{-1}(t)}{(-s)}. \tag{2.10}$$

We reproduce here the well-known fact that IR-divergences of vertices and boxes are algebraically related, see e.g. [3].

3. Mixed virtual and real IR-singularities: massive Bhabha pentagon

Things become more interesting for pentagon diagrams (se also [3, 8, 9]). We again use Bhabha scattering as an example. A compact F-form is:

$$F(s,t,t',V_2,V_4) \;=\; (x_2+x_4+x_5)^2 + [-s]x_1x_3 + [-V_4]x_3x_5 + [-t]x_2x_4 + [-t']x_2x_5 + [-V_2]x_1x_4. \tag{3.1}$$

It exhibits a set of five invariants (out of a set of 10 scalar products at choice) describing the kinematics of a $2 \to 3$ process (here assuming a final state emission of an unresolved photon from an s channel box diagram). The s,t,t' are the usual Mandelstam variables for the fermions in $e^+e^- \to e^+e^-\gamma$, and:

$$V_i = 2p_{f_i}p_\gamma, \quad i = 1,\ldots 4. \tag{3.2}$$

The V_i are proportional to the energy of the potentially unresolved massless particle.

From a subsequent phase space integration, we have to expect endpoint singularities arising from terms proportional to $1/V_2 \sim 1/E_\gamma$ and $1/V_4 \sim 1/E_\gamma$, so we have to control, for a complete treatment of the IR-problem, not only the ε-expansion, but also the first terms of the V_2, V_4 expansions for small V_2, V_4.

In fact, the F-form (3.1), written here in its shortest form, depends on those two of the four V_i which are related to the phase space of the given topology.

The MB-representation is a useful tool for that problem. For the IR limit, we may approximate $t' = t$, and the scalar pentagon may be written as:

$$I_5 \;=\; \frac{-e^{\varepsilon\gamma_E}}{(2\pi i)^4} \prod_{i=1}^{4} \int_{-i\infty+u_i}^{+i\infty+u_i} dz_i (-s)^{z_2} (-t)^{z_4} (-V_2)^{z_3} (-V_4)^{-3-\varepsilon-z_1-z_2-z_3-z_4} \frac{\prod_{j=1}^{12}\Gamma_j}{\Gamma_0\Gamma_{13}\Gamma_{14}}, \tag{3.3}$$

with $u_i = (-5/8, -7/8, \ldots)$ and with a normalization $\Gamma_0 = \Gamma[-1-2\varepsilon]$, and the other Γ-functions are:

$$\Gamma_1 \;=\; \Gamma[-z_1],\;\; \Gamma_2 = \Gamma[-z_2],\;\; \Gamma_3 = \Gamma[-z_3],\;\; \Gamma_4 = \Gamma[1+z_3],$$

$$\Gamma_5 = \Gamma[1+z_2+z_3],\ \Gamma_6 = \Gamma[-z_4],\ \Gamma_7 = \Gamma[1+z_4],\ \Gamma_8 = \Gamma[-1-\varepsilon-z_1-z_2],$$
$$\Gamma_9 = \Gamma[-2-\varepsilon-z_1-z_2-z_3-z_4],\ \Gamma_{10} = \Gamma[-2-\varepsilon-z_1-z_3-z_4],$$
$$\Gamma_{11} = \Gamma[-\varepsilon+z_1-z_2+z_4],\ \Gamma_{12} = \Gamma[3+\varepsilon+z_1+z_2+z_3+z_4], \tag{3.4}$$

and, in the denominator:

$$\Gamma_{13} = \Gamma[-1-\varepsilon-z_1-z_2-z_4],\ \Gamma_{14} = \Gamma[-\varepsilon-z_1-z_2+z_4]. \tag{3.5}$$

Leaving out here the details of derivation (see [3] for that), we just mention that we have to consider, after continuation in ε, eleven MB-integrals, being at most 4-dimensional (for $t' = t$). The resulting IR-sensible part is:

$$I_5^{IR} = I_5^{IR}(V_2) + I_5^{IR}(V_4), \tag{3.6}$$
$$I_5^{IR}(V_i) = \frac{I^s_{-1}(V_i)}{\varepsilon} + I^s_0(V_i). \tag{3.7}$$

The ε-pole is again proportional to that of the vertex:

$$\frac{I^s_{-1}(V_i)}{\varepsilon} = \frac{1}{2sV_i\varepsilon}\sum_{n=0}^{\infty}\frac{(t)^n}{\binom{2n}{n}(2n+1)} = \frac{V_{-1}(t)}{sV_i\varepsilon}, \tag{3.8}$$

and:

$$I^s_0(V_i) = \frac{1}{2sV_i}\sum_{n=0}^{\infty}\frac{(t)^n}{\binom{2n}{n}(2n+1)}\left[-2\ln(-V_i) - 3S_1(n) + 2S_1(2n+1)\right], \tag{3.9}$$

where we have to understand $\ln(-V_i) = \ln(V_i/s) + \ln[-(s+i\delta)/m^2]$. The series for $I^s_0(V_i)$ may be summed up in terms of polylogarithmic functions with the aid of Table 1 of Appendix D of [10], see also [3].

Equations (3.6) – (3.9) are the main physical result of the study. One may express the complete IR-divergent part of an amplitude with 5-point functions in terms of those expressions, subtract it from the complete, divergent amplitude and get a matrix element, which is integrable in four dimensions.

In the rest of this short write-up we would like to demonstrate why we have here besides the harmonic numbers $S_1(n)$ also those of the kind $S_1(2n+1)$. As mentioned, the scalar 5-point function may be written as a sum of eleven MB-integrals after ε-continuation, before ε-expansion. Two types of them contribute to the IR-part (there are four such integrals [in an ad-hoc notations J_3, J_4, J_7, J_9], but with a symmetry $V_2 \leftrightarrow V_4$). The first one is:

$$J_7 = -\frac{(V_2/s)^{2\varepsilon}}{sV_4}\Gamma[-2\varepsilon]\Gamma[1+2\varepsilon]\frac{e^{\varepsilon\gamma_E}}{2\pi i}\int\limits_{-i\infty-5/8}^{+i\infty-5/8} dz(-t)^{-1-z}\frac{\Gamma[\varepsilon-z]\Gamma[2\varepsilon-z]\Gamma[-z]\Gamma[1+z]}{\Gamma[2\varepsilon-2z]}$$
$$= -\frac{(V_2/s)^{2\varepsilon}}{sV_4}\ \frac{e^{\varepsilon\gamma_E}\varepsilon\sqrt{\pi}}{2^{2\varepsilon}}\ \frac{\Gamma[-2\varepsilon]\Gamma[2\varepsilon]\Gamma[1+2\varepsilon]}{\Gamma[3/2+\varepsilon]}\ {}_2F_1[1,1+2\varepsilon,3/2+\varepsilon,t/4]$$

$$= -\frac{(V_2/s)^{2\varepsilon}}{sV_4} e^{\varepsilon\gamma_E}\ \Gamma[-2\varepsilon]\Gamma[1+2\varepsilon]\ \sum_{n=1}^{\infty} t^{n-1} \frac{\Gamma[\varepsilon+n]\Gamma[2\varepsilon+n]}{\Gamma[2\varepsilon+2n]}. \tag{3.10}$$

The J_7 is proportional to $1/V_4$. We have a second integral of the same type, being proportional to $1/V_2$:

$$J_3 = J_7(V_4 \leftrightarrow V_2). \tag{3.11}$$

The other type of integrals J_4, J_9, with:

$$J_4 = J_9(V_4 \leftrightarrow V_2), \tag{3.12}$$

are two-fold MB-integrals:

$$\begin{aligned} J_9 = & \frac{\Gamma[-2\varepsilon]}{\Gamma[-1-2\varepsilon]} \frac{e^{\varepsilon\gamma_E}}{(2\pi i)^2} \int\limits_{-i\infty-5/8}^{+i\infty-5/8} dz_1 \int\limits_{-i\infty-7/8}^{+i\infty-7/8} dz_2 (-s)^{z_2} (-t)^{\varepsilon-z_1+z_2} (-V_2)^{-1-2\varepsilon-z_2} (-V_4)^{-2-z_2} \\ & \times \Gamma[-z_1]\Gamma_A[-1-z_2]\Gamma[-2\varepsilon-z_2]\Gamma[-1-\varepsilon-z_1-z_2]\Gamma_C[-\varepsilon+z_1-z_2]\Gamma[-z_2] \\ & \times \frac{\Gamma[2+z_2]\ \ \Gamma_B[1+2\varepsilon+z_2]\ \ \Gamma[1+\varepsilon-z_1+z_2]}{\Gamma[-2z_1]\ \ \Gamma[-1-2\varepsilon-2z_2]}. \end{aligned} \tag{3.13}$$

The integral looks like being, in the limit $V_4 \to 0$, too singular. This limit is an endpoint of the phase space integration. Let us close the contour to the left. We *shift* now the integration contour in z_2 to the left, raising in this way the (real part of the) power of $(-V_4)$ to a value which makes the photon phase space integral explicitely integrable at $V_4 \to 0$ in $d = 3-2\varepsilon$ space dimensions. If singularities of the integrand (from Γ-functions) at some values $z_2 = z_R$ are crossed one has to add the corresponding residues $J_9^R(z_R)$, so getting one-dimensional MB-integrals to be considered:

$$J_9 = 2\pi i \sum_{z_R} J_9^R(z_R) + J_9^{shift}. \tag{3.14}$$

The resulting integral J_9^{shift} differs from J_9 only by the shifted integration path, but will now not contribute to the IR-singular part and will not be considered here any more. We see that only the residues of crossed singular points in z_2 contain the IR-relevant endpoint singularities in V_2, V_4. Here, two of them (at $z_2 = -1$ [argument of Γ_A in (3.13)] and at $z_2 = -1-2\varepsilon$ [argument of $\Gamma_B$in (3.13)]) contribute due to a shift from $\Re z_2 = -7/8$ to $\Re z_2 = -7/8-1 = -15/8$. The first of them is:

$$\begin{aligned} J_{9A} = & -\frac{(-V_2)^{-2\varepsilon}}{sV_4} \frac{\Gamma[-2\varepsilon]\Gamma_B[2\varepsilon]}{\Gamma[-1-2\varepsilon]} \frac{e^{\varepsilon\gamma_E}}{2\pi i} \\ & \times \int\limits_{-i\infty-5/8}^{+i\infty-5/8} dz_1 (-t)^{-1-z_1} \frac{\Gamma[-2\varepsilon-z_1]\Gamma[-\varepsilon-z_1]\Gamma[-z_1]\Gamma_C[1+z_1])}{\Gamma[-1-2\varepsilon]\Gamma[-2\varepsilon-2z_1)]} \\ = & \frac{(-V_2)^{-2\varepsilon}2^{2\varepsilon}}{sV_4} e^{\varepsilon\gamma_E} \frac{\varepsilon\sqrt{\pi}\Gamma[-2\varepsilon]^2\Gamma_B[2\varepsilon]}{\Gamma[-1-2\varepsilon]\Gamma[3/2-\varepsilon]}\ \ {}_2F_1[1, 1-2\varepsilon, 3/2-\varepsilon, t/4]. \end{aligned} \tag{3.15}$$

We performed here an irrelevant shift $z_1 \to z_1+\varepsilon$ in order to make the argument of Γ_C independent of ε. This will be here the only Γ-function producing residues in z_1 when closing again the contour

to the left. The explicit ε-expansion of the hypergeometric function in (3.15) may be obtained with the Mathematica package HypExp2 [11, 5]. Alternatively, we may perform the sum of residues arising from $\Gamma_C(1+z_1)$ directly:

$$J_{9A} = -\frac{(-V_2)^{-2\varepsilon}}{sV_4} e^{\varepsilon\gamma_E} \frac{\Gamma[-2\varepsilon]\Gamma[2\varepsilon]}{\Gamma[-1-2\varepsilon]} \sum_{n=1}^{\infty} t^{n-1} \frac{\Gamma[-2\varepsilon+n]\Gamma[-\varepsilon+n]}{\Gamma[-2\varepsilon+2n]}, \tag{3.16}$$

and apply then (2.4) to it. It is here where we may see why the harmonic numbers $S_1(2n+1)$ appear, which were not contributing to the vertex or the box: There was no Γ-function with an ε-shifted, doubled argument in the denominator of the final sums, while here this appears. In the general massive case, this will usually happen.

The second residue crossed by the contour shifting in the z_2-plane gives the third kind of contribution to be added; after a shift $z_1 \to z_1 - \varepsilon$:

$$\begin{aligned}
J_{9B} &= \frac{(V_4/s)^{2\varepsilon}}{sV_4} \Gamma[1-2\varepsilon]\Gamma[-2\varepsilon]\Gamma_A[2\varepsilon] \frac{e^{\varepsilon\gamma_E}}{2\pi i} \int\limits_{-i\infty-5/8}^{+i\infty-5/8} dz_1 (-t)^{-1-z_1} \\
&\quad \times \frac{\Gamma[\varepsilon-z_1]\Gamma[2\varepsilon-z_1]\Gamma[-z_1]\Gamma_C[1+z_1]}{\Gamma[-1-2\varepsilon]\Gamma[2\varepsilon-2z_1]} \\
&= \frac{(V_4/s)^{2\varepsilon}}{sV_4} e^{\varepsilon\gamma_E} \varepsilon\sqrt{\pi} \frac{\Gamma[1-2\varepsilon]\Gamma[-2\varepsilon]\Gamma[2\varepsilon]^2}{2^{2\varepsilon}\Gamma[-1-2\varepsilon]\Gamma[3/2+\varepsilon]} \;\; {}_2F_1[1,1+2\varepsilon,3/2+\varepsilon,t/4] \\
&= \frac{(V_4/s)^{2\varepsilon}}{sV_4} e^{\varepsilon\gamma_E} \frac{\Gamma[1-2\varepsilon]\Gamma[-2\varepsilon]\Gamma[2\varepsilon]}{\Gamma[-1-2\varepsilon]} \sum_{n=1}^{\infty} t^{n-1} \frac{\Gamma[\varepsilon+n]\Gamma[2\varepsilon+n]}{\Gamma[2\varepsilon+2n]}
\end{aligned} \tag{3.17}$$

Again, this may be expanded into an ε-series over inverse binomial sums by use of (2.4).

Collecting everything together, we rediscover (3.6) (plus additional terms of no relevance for the IR-treatment):

$$J_3 + J_{4A} + J_{4B} + J_7 + J_{9A} + J_{9B} = \frac{1}{\varepsilon}\left[I^s_{-1}(V_2) + I^s_{-1}(V_4)\right] + I^s_0(V_2) + I^s_0(V_4) + \cdots \tag{3.18}$$

4. Conclusions

We gave a pedagogical introduction to the treatment of mixed real and virtual IR-singularities. This kind of problems arises in NNLO problems, where one has to treat the unresolved massless particle phase space for loop integrals. The presented method was exemplified for a scalar integral, but it may be easily applied to general tensor functions. For the QED pentagon, this is discussed in [3], which may be considered as an introduction to this presentation. A *derivation* of MB-representations for higher n point functions or for multi-loop integrals is more or less straightforward, although an *analytical evaluation* will become more and more troublesome. It is an interesting open question how useful the MB techniques might appear for realistic, so far unsolved applications.

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