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Constraints on the rare tau decays from $\mu \to e\gamma$ in the supersymmetric see-saw model

Alejandro Ibarra[1] and **Cristoforo Simonetto**[2] *

[1] DESY, Theory Group, Notkestrasse 85, 22603 Hamburg, Germany

[2] Physik Department T30, Technische Universität München,
James-Franck-Strasse, 85748 Garching, Germany.

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Abstract

It is now a firmly established fact that all family lepton numbers are violated in Nature. In this paper we discuss the implications of this observation for future searches for rare tau decays in the supersymmetric see-saw model. Using the two loop renormalization group evolution of the soft terms and the Yukawa couplings we show that there exists a lower bound on the rate of the rare process $\mu \to e\gamma$ of the form $BR(\mu \to e\gamma) \gtrsim C \times BR(\tau \to \mu\gamma)BR(\tau \to e\gamma)$, where C is a constant that depends on supersymmetric parameters. Our only assumption is the absence of cancellations among the high-energy see-saw parameters. We also discuss the implications of this bound for future searches for rare tau decays. In particular, for large regions of the mSUGRA parameter space, we show that present B-factories could discover either $\tau \to \mu\gamma$ or $\tau \to e\gamma$, but not both.

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*E-mail addresses: alejandro.ibarra@desy.de, cristoforo.simonetto@ph.tum.de

1 Introduction

The renormalizable part of the Standard Model Lagrangian is invariant under four global $U(1)$ symmetries, namely baryon number, B, and the three family lepton numbers, L_e, L_μ and L_τ. This invariance has been for many years considered accidental and expected to be broken by additional terms in the Lagrangian, possibly non-renormalizable. Whereas experiments searching for proton decay have not provided yet any evidence for baryon number violation, it is nowadays a firmly established experimental fact that the three family lepton numbers are violated in the neutrino sector. Namely, the disappearance of electron neutrinos on their way from the Sun indicated by the Homestake chlorine detector [1], SAGE [2], GALLEX/GNO [3,4], Kamiokande [5], Super-Kamiokande [6] and Borexino [7], and unequivocally confirmed by SNO [8], proves the violation of L_e. This is further supported by the disappearance of electron antineutrinos observed by the reactor experiment KamLAND [9]. On the other hand, the atmospheric neutrino anomaly discovered by Kamiokande and IMB [10], and explained by Super-Kamiokande [11], Soudan2 [12] and MACRO [13] as an oscillation of muon neutrinos into a different neutrino species, proves the violation of L_μ. The disappearance of muon neutrinos reported by the long baseline accelerator experiments K2K [14] and MINOS [15] supports this conclusion. Finally, the observation of tau neutrino appearance in the atmospheric neutrino flux by Super-Kamiokande [16] indicates the violation of L_τ.

The most economical way to accommodate the family lepton number violation in the Standard Model is by adding to the leptonic Lagrangian a dimension five operator [17]

$$\mathcal{L}_{\rm lep} = -e^c_{Ri}\mathbf{Y}_{eij}L_jH^* - \frac{\alpha_{ij}}{\Lambda}(L_iH)(L_jH) + \text{h.c.}\,, \tag{1}$$

where L_i and e^c_R are the left-handed and right-handed leptons respectively, $i,j=1,2,3$, H is the Higgs doublet and Λ a mass parameter. In fact, this operator not only violates the three family lepton numbers but also the total lepton number. After the electroweak symmetry breaking, a Majorana mass term is generated for the left-handed neutrinos, $\mathcal{M}_{ij} = \frac{\alpha_{ij}}{\Lambda}\langle H^0\rangle^2$, being the smallness of neutrino masses attributed to small values of the couplings α_{ij} and/or to a large value of Λ.

In this minimal framework, lepton flavour violation is generated by the same operator that generates neutrino masses. Therefore, the rate of any lepton flavour violating pro-

	present bound	projected bound
$BR(\mu \to e\gamma)$	1.2×10^{-11} [19]	10^{-13} [20]
$BR(\tau \to e\gamma)$	1.1×10^{-7} [21]	10^{-9} [22]
$BR(\tau \to \mu\gamma)$	6.8×10^{-8} [23]	10^{-9} [22]

Table 1: Present and projected bounds on the rare lepton decays.

cess will be proportional to the neutrino masses. Indeed, a detailed calculation shows that [18]:

$$BR(l_j \to l_i\gamma) = \frac{3\alpha}{32\pi}\left|\frac{\Delta m^2_{\rm sol}}{M_W^2}U^*_{i2}U_{j2} + \frac{\Delta m^2_{\rm atm}}{M_W^2}U^*_{i3}U_{j3}\right|^2 BR(l_j \to l_i\nu_j\bar{\nu}_i)\ , \qquad (2)$$

which gives $BR(\tau \to \mu\gamma) \sim 10^{-54}$, $BR(\mu \to e\gamma) \sim 10^{-57}$, $BR(\tau \to e\gamma) \sim 10^{-57}$, far below the sensitivity of present and projected experiments (see Table 1)[1].

Nevertheless, the Lagrangian Eq. (1) describes just an effective theory and new degrees of freedom are expected to arise above the scale Λ. The interactions of the new degrees of freedom with the lepton fields are likely to contain additional sources of flavour violation that can enhance the branching ratios of the rare decays by many orders of magnitude, bringing them to the reach of future experiments. For this reason, rare lepton decays are considered very powerful probes for physics beyond the Standard Model.

The supersymmetric (SUSY) see-saw mechanism is probably the best motivated high energy theory to generate small neutrino masses [25] . In this framework the particle content of the Minimal Supersymmetric Standard Model (MSSM) is extended with three right-handed neutrino superfields, ν_{Ri}, $i = 1, 2, 3$, singlets under the Standard Model gauge group. Imposing R-parity conservation, the leptonic superpotential reads:

$$W_{\rm lep} = e^c_{Ri}\mathbf{Y}_{eij}L_jH_d + \nu^c_{Ri}\mathbf{Y}_{\nu ij}L_jH_u - \frac{1}{2}\nu^c_{Ri}\mathbf{M}_{ij}\nu^c_{Rj}\ , \qquad (3)$$

where H_u and H_d are the hypercharge $+1/2$ and $-1/2$ Higgs doublets, respectively, $\mathbf{Y}_e$ and $\mathbf{Y}_\nu$ are the matrices of charged lepton and neutrino Yukawa couplings, respectively, and $\mathbf{M}$ is a 3×3 Majorana mass matrix. It is natural to assume that the overall scale

[1] In the Table we restrict ourselves to bounds that were published by the time of writing this paper. We note however that the Belle Collaboration has reported the bounds $BR(\tau \to \mu\gamma) < 4.5 \times 10^{-8}$ and $BR(\tau \to e\gamma) < 1.2 \times 10^{-7}$ in the yet unpublished preprint [24].

of $\mathbf{M}$, which we will denote by $M_{\rm maj}$, is much larger than the electroweak scale or any soft mass. If this is the case, at energies below $M_{\rm maj}$ the theory is well described by the following effective superpotential:

$$W_{\rm lep}^{\rm eff} = e^c_{Ri}\mathbf{Y}_{eij}L_jH_d + \frac{1}{2}\left(\mathbf{Y}_\nu^T\mathbf{M}^{-1}\mathbf{Y}_\nu\right)_{ij}(L_iH_u)(L_jH_u)\ , \tag{4}$$

that generates the fermionic Lagrangian Eq. (1). In the phenomenological studies it is convenient to work in the leptonic basis where the charged lepton Yukawa coupling is diagonal, $\mathbf{Y}_e = {\rm diag}(y_e, y_\mu, y_\tau)$. Then, in this basis, the neutrino mass matrix is given by

$$\mathcal{M} = \left(\mathbf{Y}_\nu^T\mathbf{M}^{-1}\mathbf{Y}_\nu\right)\langle H_u^0\rangle^2\ , \tag{5}$$

whose eigenvalues are naturally very small due to the suppression by the large right-handed neutrino mass scale.

From the point of view of generating small neutrino masses, the non-supersymmetric version of the see-saw mechanism is equally natural. Nonetheless, in the non-supersymmetric see-saw model the presence of very heavy new degrees of freedom interacting with the Higgs doublet introduces a serious naturalness problem. The Higgs mass acquires quadratically-divergent radiative corrections which would drive the Higgs mass to values of the order of the Majorana mass scale [26]. However, in the supersymmetric version this divergence is automatically canceled by the presence of right-handed sneutrinos with a mass essentially identical to the mass of their corresponding right-handed neutrinos. Therefore, the supersymmetric see-saw mechanism can accommodate simultaneously tiny neutrino masses and a relatively light electroweak scale without serious fine-tunings.

In addition to the flavour violation stemming from the right-handed neutrino Yukawa couplings, the supersymmetric see-saw model contains additional sources of lepton flavour violation in the soft SUSY breaking Lagrangian [27]:

$$\begin{aligned} -\mathcal{L}_{\rm soft} \;=\; & (\mathbf{m}_L^2)_{ij}\widetilde{L}_i^*\widetilde{L}_j \;+(\mathbf{m}_e^2)_{ij}\widetilde{e}^*_{Ri}\widetilde{e}_{Rj} \;+(\mathbf{m}_\nu^2)_{ij}\widetilde{\nu}^*_{Ri}\widetilde{\nu}_{Rj} \;+ \\ & \left(\mathbf{A}_{eij}\widetilde{e}^*_{Ri}H_d\widetilde{L}_j + \mathbf{A}_{\nu ij}\widetilde{\nu}^*_{Ri}H_u\widetilde{L}_j + {\rm h.c.}\right) + {\rm etc}\ . \end{aligned} \tag{6}$$

where $\widetilde{L}_i$, $\widetilde{e}_{Ri}$ and $\widetilde{\nu}_{Ri}$ are the supersymmetric partners of the left-handed lepton doublets, right-handed charged leptons and right-handed neutrinos, respectively, $\mathbf{m}_L^2$, $\mathbf{m}_e^2$ and $\mathbf{m}_\nu^2$ are their corresponding soft mass matrices squared, and $\mathbf{A}_e$ and $\mathbf{A}_\nu$ are the charged lepton and neutrino soft trilinear terms.

The flavour violation in the slepton sector contributes through one loop diagrams to different flavour violating processes such as rare muon and tau decays, $K_L^0 \to e^\pm \mu^\mp$ or $\mu - e$ conversion in nuclei. The strong bounds on these processes restrict very severely the structure of the soft mass matrices at low energies [28], suggesting an approximately flavour universal structure: $(\mathbf{m}_L^2)_{ij} \simeq m_L^2 \delta_{ij}$, $(\mathbf{m}_e^2)_{ij} \simeq m_e^2 \delta_{ij}$, $(\mathbf{A}_e)_{ij} \simeq A_e \mathbf{Y}_{eij}$. The most plausible explanation for this structure is to assume that supersymmetry is broken in a hidden sector and the breaking is transmitted to the visible sector by a flavour blind mediation mechanism. If this is the case, the soft terms would be strictly flavour universal at some high energy scale:

$$(\mathbf{m}_L^2)_{ij} = m_L^2 \delta_{ij}, \quad (\mathbf{m}_e^2)_{ij} = m_e^2 \delta_{ij}, \quad (\mathbf{m}_\nu^2)_{ij} = m_\nu^2 \delta_{ij},$$
$$(\mathbf{A}_e)_{ij} = A_e \ \mathbf{Y}_{eij}, \quad (\mathbf{A}_\nu)_{ij} = A_\nu \ \mathbf{Y}_{\nu ij} \ . \tag{7}$$

Interestingly, if this high energy scale is larger than the right-handed neutrino masses, the flavour violation in the neutrino Yukawa couplings will propagate through radiative effects to the soft terms [29]. As a consequence, even under the most conservative assumption for the soft terms from the point of view of lepton flavour violation, in the supersymmetric see-saw model some amount of flavour violation is always expected at low energies.

The discovery of small neutrino masses as a hint of the see-saw mechanism and the continuous improvement in sensitivity of the experiments searching for rare lepton decays have stimulated in recent years the interest in the radiative generation of lepton flavour violation. Different groups have estimated the predictions for various lepton flavour violating processes in some specific high-energy frameworks based on Grand Unification [30], flavour models [31], texture zeros [32] or string theory [33], but also pursuing a more phenomenological approach, where the predictions are somehow connected to low energy observables [34–38]. A general expectation of all these analyses is that the observation of rare lepton decays could be possible at the next round of experiments. Nevertheless, it should be stressed that there are many optimistic assumptions underlying this conclusion, and the non-observation of rare decays in future experiments would by no means exclude the supersymmetric see-saw model.

In this paper we will adopt a completely phenomenological approach and we will derive the relation among the branching ratios $BR(\mu \to e\gamma) \gtrsim C \times BR(\tau \to \mu\gamma) BR(\tau \to e\gamma)$ that holds independently of the high-energy see-saw parameters, with the only

assumption of the absence of cancellations. In this relation, however, some model dependence will remain associated to our ignorance on the supersymmetric parameters, which is encoded in the constant C. To derive our main result we will present in Section 2 the scenario that saturates the bound and yields the minimal rate for $\mu \to e\gamma$. In Section 3 we will compute the off-diagonal terms of the slepton mass matrices in this extremal scenario taking into account the two loop renormalization group equations and we will derive relations among them. Using these relations, we will derive in Section 4 the lower bound on $BR(\mu \to e\gamma)$ in terms of the branching ratios of the rare tau decays. We will also discuss the implications of this bound for present and future searches for rare tau decays for different supersymmetric benchmark points. Finally, in Section 5 we will present our conclusions. We will also present an Appendix containing the complete two loop renormalization group equations for the soft SUSY breaking terms, including the effects of the neutrino Yukawa couplings, which to the best of our knowledge have not been given explicitely in the literature.

2 Scenarios with minimal rates for $\mu \to e\gamma$

In the MSSM extended with right-handed neutrinos there are sources of lepton flavour violation both in the soft SUSY breaking Lagrangian and in the SUSY conserving Lagrangian. The latter can be completely encoded in the neutrino Yukawa couplings by means of a basis transformation. Neutrino oscillations require the presence of lepton flavour violation in the neutrino Yukawa couplings, but not in the soft SUSY breaking Lagrangian. Therefore, the minimal rate for $\mu \to e\gamma$ will clearly occur in scenarios where there is no lepton flavour violation in the soft SUSY breaking Lagrangian. There are in fact some well motivated supersymmetric scenarios where the soft breaking parameters are flavour universal at some high energy scale. However, if this scale is larger than the mass of the right-handed neutrinos[2], the flavour violation in the supersymmetric part of the Lagrangian will propagate to the soft SUSY breaking terms through quantum effects.

For generic neutrino Yukawa couplings, the off-diagonal elements of the soft SUSY

[2]This is the case for example in scenarios with minimal supergravity, dilaton-dominated SUSY breaking, or gauge mediation where the masses of the messenger particles are larger than the masses of the right-handed neutrinos.

breaking terms read at low energies, in the leading-log approximation,

$$\begin{aligned}
\left(\mathbf{m}_L^2\right)_{ij} &\simeq -\frac{1}{8\pi^2}(m_L^2 + m_\nu^2 + m_{H_u}^2 + |A_\nu|^2)(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) \ , \\
\left(\mathbf{m}_e^2\right)_{ij} &\simeq 0 \ , \\
(\mathbf{A}_e)_{ij} &\simeq \frac{-1}{8\pi^2}(2A_\nu + A_e)\mathbf{Y}_e(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) \ ,
\end{aligned} \tag{8}$$

where $i \neq j$ and $M_{\rm X}$ is some high energy scale that we identify with the Grand Unification scale, $M_{\rm X} = 2 \times 10^{16}$ GeV. The size of the off-diagonal elements depends very strongly on the flavour structure of the neutrino Yukawa couplings, which in the see-saw model is not directly connected to the flavour structure of the low energy neutrino mass matrix. In fact, there is an infinite set of neutrino Yukawa couplings compatible with a given set of low energy data [36]. Among all those Yukawa couplings, the minimal rate for $\mu \to e\gamma$ will clearly occur in the scenario where

$$\left(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu\right)_{21} (M_{\rm maj}) = y_{12}^* y_{11} + y_{22}^* y_{21} + y_{32}^* y_{31} = 0 \, . \tag{9}$$

Assuming that there are no cancellations among the different terms, this condition is satisfied in the following eight situations:

$$\begin{aligned}
&(a) \quad y_{11} = 0, y_{21} = 0, y_{31} = 0 \ , && (e) \quad y_{12} = 0, y_{21} = 0, y_{31} = 0 \ , \\
&(b) \quad y_{11} = 0, y_{21} = 0, y_{32} = 0 \ , && (f) \quad y_{12} = 0, y_{21} = 0, y_{32} = 0 \ , \\
&(c) \quad y_{11} = 0, y_{22} = 0, y_{31} = 0 \ , && (g) \quad y_{12} = 0, y_{22} = 0, y_{31} = 0 \ , \\
&(d) \quad y_{11} = 0, y_{22} = 0, y_{32} = 0 \ , && (h) \quad y_{12} = 0, y_{22} = 0, y_{32} = 0 \ ,
\end{aligned} \tag{10}$$

where the Yukawa matrix elements are evaluated at $M_{\rm maj}$. The cases (a) and (h) preserve electron and muon lepton numbers respectively. Since in this basis the neutrino Yukawa coupling is the only source of lepton flavour violation, the complete Lagrangian will preserve at least one family lepton number. This invariance is inherited by the effective theory, in conflict with the flavour conversions observed in neutrino oscillations. On the other hand, the remaining six possibilities $(b) - (g)$ violate all family lepton numbers and lead to a neutrino mass matrix that schematically reads

$$\mathcal{M} \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix} \tag{11}$$

and which is allowed by present experiments (this mass matrix leads to the prediction $\sin\theta_{13} \simeq \frac{1}{2}\sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}}\,\sin 2\theta_{\rm sol} \simeq 0.08$ for the case with normal neutrino mass hierarchy and $\sin\theta_{13} \simeq \frac{1}{4}\sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}}\,\sin 2\theta_{\rm sol} \simeq 0.04$ for the case with inverted neutrino mass hierarchy). Therefore, from Eq. (10) it is straightforward to check that the experimentally allowed neutrino Yukawa textures which lead to a vanishing rate for $\mu \to e\gamma$ necessarily lead to $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{31}(M_{\rm maj}) \neq 0$ and $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{32}(M_{\rm maj}) \neq 0$, unless unnatural cancellations in Eq. (9) are taking place.

The presence of exact zeros in the Yukawa matrices as in Eq. (10) can be justified by symmetries. However, symmetries are not expected to hold at the decoupling scale but at the Grand Unification scale, $M_{\rm X}$. Again, and barring cancellations, the minimal rate for $\mu \to e\gamma$ will occur when $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}(M_{\rm X}) = 0$, that as was argued above necessarily implies $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{31}(M_{\rm X}) \neq 0$ and $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{32}(M_{\rm X}) \neq 0$.

The key point of this paper is that even when the soft terms are flavour universal at $M_{\rm X}$ and $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}(M_{\rm X}) = 0$, the flavour violation in the $\tau - \mu$ and the $\tau - e$ sectors, will propagate through radiative corrections to the $\mu - e$ sector, inducing small though non-vanishing values for $(\mathbf{m}^2_{\tilde{L}})_{21}$ and $(\mathbf{A}_e)_{12,21}$ at low energies. We will show in this paper that in this very special case the rate of $\mu \to e\gamma$ is related to the rates of the rare tau decays through

$$BR(\mu \to e\gamma) \simeq C \times BR(\tau \to \mu\gamma) BR(\tau \to e\gamma)\ , \tag{12}$$

where the proportionality constant, C, will be determined in the next sections.

This value is attained by the experimentally allowed neutrino Yukawa textures satisfying $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}(M_{\rm X}) = 0$, which is the scenario producing the minimal rate for $\mu \to e\gamma$. Therefore, and barring cancellations, for any other Yukawa coupling compatible with the low energy neutrino parameters the following bound will hold:

$$\boxed{BR(\mu \to e\gamma) \gtrsim C \times BR(\tau \to \mu\gamma) BR(\tau \to e\gamma)}\ . \tag{13}$$

This inequality is the main result of this paper. As we will see later, this bound has important implications for the searches for rare tau decays in present and future experiments.

To finish this section we would like to comment on the possibility sometimes discussed in the literature of establishing model independent correlations between the branching ratios of the rare decays of the form $BR(\tau \to \mu\gamma)/BR(\mu \to e\gamma) \simeq$ constant.

We would like to clarify this point stressing that in a general see-saw model with three right-handed neutrinos such correlations *cannot* be established on model independent grounds.

It can be proved that there is a one to one mapping between the high energy see-saw parameters, $\mathbf{Y}_\nu$ and $\mathbf{M}$, and the matrices $\mathcal{M}$ and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ [39]. The former is the neutrino mass matrix and the latter, the matrix that participates in the radiative corrections to the soft SUSY breaking terms, which is in principle measurable at low energies once the boundary conditions for the soft terms have been specified. In particular, the off-diagonal elements of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ are directly related to the branching ratios of the rare decays. Being this mapping bijective, one could use as parameters of the see-saw model either the familiar set $\{\mathbf{Y}_\nu, \mathbf{M}\}$ (the top-down parametrization) or the less familiar one $\{\mathcal{M}, \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu\}$ (the bottom-up parametrization). Clearly, in the bottom-up parametrization the branching ratios for the rare decays are inputs, and as such are completely uncorrelated. For the same reason, it is not possible to establish on model independent grounds correlations between the branching ratios of the rare decays and neutrino parameters, such as $\sin\theta_{13}$, the neutrino mass spectrum or the CP violating phases. This result also holds for the most restricted case with just two right-handed neutrinos [40].

A more explicit proof of this result can be derived as follows using the familiar top-down parametrization. Let us first write the effective neutrino mass matrix in terms of the high-energy parameters in the basis where the charged lepton Yukawa coupling and the right-handed mass matrix are simultaneously diagonal:

$$\mathcal{M}_{ij} = \left(\frac{y_{1i} y_{1j}}{M_1} + \frac{y_{2i} y_{2j}}{M_2} + \frac{y_{3i} y_{3j}}{M_3} \right) \langle H_u^0 \rangle^2 \ . \tag{14}$$

One could conceive a scenario where $\frac{y_{3i} y_{3j}}{M_3} \ll \frac{y_{1i} y_{1j}}{M_1}, \frac{y_{2i} y_{2j}}{M_2}$ for all $i, j = 1, 2, 3$. This could occur for instance when the mass of the heaviest right handed neutrino mass is much larger than the masses of the other two right-handed neutrinos. If this is the case, the third row of the neutrino Yukawa matrix is completely unconstrained from neutrino observations. In addition, it could occur that y_{3i} are much larger than y_{1i}, y_{2i}, implying that $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \simeq y_{3i}^* y_{3j}$. Therefore, the branching ratios for the rare decays would be essentially determined by the parameters y_{3i}, that are completely unconstrained from low energy data. As a result, the ratio $BR(\tau \to \mu\gamma)/BR(\mu \to e\gamma)$ can range from zero (when $y_{33} = 0$, $y_{31}, y_{32} \neq 0$) to infinity (when $y_{31} = 0$, $y_{32}, y_{33} \neq 0$), being both

situations compatible with neutrino observations. Additionally, since the parameters that determine the branching ratios, y_{3i}, are unconstrained from low energy data, no model independent correlation can be established between rare decays and neutrino parameters.

In order to establish correlations between two branching ratios, or correlations between one branching ratio and low energy neutrino parameters, it is *necessary* to make assumptions about the high-energy see-saw parameters, for instance motivated by Grand Unified theories or by flavour models. In this paper we make what we consider the most minimal assumption about the high-energy see-saw parameters, namely that in constructing the derived quantities $\mathcal{M}$ and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ no cancellations occur. Then, the no-go theorem presented in [39,40] can be circumvented, opening the possibility of deriving relations among the branching ratios. Although the relation that results, $BR(\mu \to e\gamma) \gtrsim C \times BR(\tau \to \mu\gamma) BR(\tau \to e\gamma)$, is not model independent in the strict sense, it is valid in a very large class of models, characterized by the absence of unnatural cancellations. In addition, this bound will proof to have remarkably strong implications given the generality of our assumptions, as we will see in Section 4.

3 Radiative generation of lepton flavour violation in the soft mass matrices

The Renormalization Group Equations (RGEs) for the relevant soft terms read

$$\frac{d\mathbf{m}_L^2}{dt} = \frac{1}{16\pi^2}\beta^{(1)}_{\mathbf{m}_L^2} + \frac{1}{(16\pi^2)^2}\beta^{(2)}_{\mathbf{m}_L^2} + \dots , \tag{15}$$

$$\frac{d\mathbf{m}_e^2}{dt} = \frac{1}{16\pi^2}\beta^{(1)}_{\mathbf{m}_e^2} + \frac{1}{(16\pi^2)^2}\beta^{(2)}_{\mathbf{m}_e^2} + \dots , \tag{16}$$

$$\frac{d\mathbf{A}_e}{dt} = \frac{1}{16\pi^2}\beta^{(1)}_{\mathbf{A}_e} + \frac{1}{(16\pi^2)^2}\beta^{(2)}_{\mathbf{A}_e} + \dots , \tag{17}$$

where $\beta^{(1)}$ and $\beta^{(2)}$ indicate respectively the one and two loop β-functions. The complete expression for the β-functions of the soft terms in the supersymmetric see-saw model can be found in the Appendix.

The solution to the RGEs can be well approximated by the trapezium rule, which

keeping just the one and two loop contributions reads:

$$\begin{aligned}
\mathbf{m}_L^2(M_{\rm maj}) &\simeq \mathbf{m}_L^2(M_{\rm X}) - \frac{1}{16\pi^2}\frac{1}{2}\left[\beta^{(1)}_{\mathbf{m}_L^2}(M_{\rm X}) + \beta^{(1)}_{\mathbf{m}_L^2}(M_{\rm maj})\right]\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) \\
&\quad -\frac{1}{(16\pi^2)^2}\frac{1}{2}\left[\beta^{(2)}_{\mathbf{m}_L^2}(M_{\rm X}) + \beta^{(2)}_{\mathbf{m}_L^2}(M_{\rm maj})\right]\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) \ , \qquad (18) \\
\mathbf{m}_e^2(M_{\rm maj}) &\simeq \mathbf{m}_e^2(M_{\rm X}) - \frac{1}{16\pi^2}\frac{1}{2}\left[\beta^{(1)}_{\mathbf{m}_e^2}(M_{\rm X}) + \beta^{(1)}_{\mathbf{m}_e^2}(M_{\rm maj})\right]\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) \\
&\quad -\frac{1}{(16\pi^2)^2}\frac{1}{2}\left[\beta^{(2)}_{\mathbf{m}_e^2}(M_{\rm X}) + \beta^{(2)}_{\mathbf{m}_e^2}(M_{\rm maj})\right]\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) \ , \qquad (19) \\
\mathbf{A}_e(M_{\rm maj}) &\simeq \mathbf{A}_e(M_{\rm X}) - \frac{1}{16\pi^2}\frac{1}{2}\left[\beta^{(1)}_{\mathbf{A}_e}(M_{\rm X}) + \beta^{(1)}_{\mathbf{A}_e}(M_{\rm maj})\right]\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) \\
&\quad -\frac{1}{(16\pi^2)^2}\frac{1}{2}\left[\beta^{(2)}_{\mathbf{A}_e}(M_{\rm X}) + \beta^{(2)}_{\mathbf{A}_e}(M_{\rm maj})\right]\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) \ . \qquad (20)
\end{aligned}$$

For the Yukawa couplings leading to a minimal rate for $\mu \to e\gamma$, Eq. (10), the 31 and 32 entries in the left-handed slepton mass matrix are generated at order $\mathcal{O}(Y_\nu^2)$. Therefore, noting that $\beta^{(1)}_{\mathbf{m}_L^2}(M_{\rm maj}) = \beta^{(1)}_{\mathbf{m}_L^2}(M_{\rm X}) + \mathcal{O}(Y_\nu^4)$ and that the two loop β-functions are $\mathcal{O}(Y_\nu^4)$, it follows from Eq. (18) that

$$\mathbf{m}_L^2(M_{\rm maj})_{31,32} = -\frac{1}{16\pi^2}(\beta^{(1)}_{\mathbf{m}_L^2})_{31,32}(M_{\rm X})\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) + \mathcal{O}(Y_\nu^4) \ . \qquad (21)$$

Using Eq. (75) in the Appendix for the one loop β-function we find

$$\mathbf{m}_L^2(M_{\rm maj})_{31,32} = -\frac{1}{8\pi^2}(m_L^2 + m_\nu^2 + m_{H_u}^2 + |A_\nu|^2)(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{31,32}\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) + \mathcal{O}(Y_\nu^4) \ , \qquad (22)$$

where all the soft terms in the right-hand side of this equation and $(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{31,32}$ are understood to be evaluated at the scale M_X.

On the other hand, since $(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{21}(M_{\rm X}) = 0$ the previous equation indicates that $\mathbf{m}_L^2(M_{\rm maj})_{21}$ vanishes at order $\mathcal{O}(Y_\nu^2)$ and is only generated at higher order in perturbation theory. Keeping only terms of $\mathcal{O}(Y_\nu^4)$, it follows that $(\beta^{(2)}_{\mathbf{m}_L^2})_{21}(M_{\rm maj}) = (\beta^{(2)}_{\mathbf{m}_L^2})_{21}(M_{\rm X}) + \mathcal{O}(Y_\nu^6)$ and $(\beta^{(1)}_{\mathbf{m}_L^2})_{21}(M_{\rm X}) = 0$. Therefore,

$$(\mathbf{m}_L^2)_{21}(M_{\rm maj}) = -\frac{1}{16\pi^2}\frac{1}{2}(\beta^{(1)}_{\mathbf{m}_L^2})_{21}(M_{\rm maj})\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) - \frac{1}{(16\pi^2)^2}(\beta^{(2)}_{\mathbf{m}_L^2})_{21}(M_{\rm X})\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) + \mathcal{O}(Y_\nu^6) \ . \qquad (23)$$

The one loop β-function at the Majorana mass scale is proportional to $(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{21}(M_{\rm maj})$, that following Eq. (56) in the Appendix is generated radiatively by the effect of $(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{32}$ and $(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{31}$. The result is:

$$(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{21}(M_{\rm maj}) = -\frac{3}{8\pi^2}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^*_{32}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{31}\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) + \mathcal{O}(Y_\nu^6) \ . \qquad (24)$$

Using Eqs. (23,24) and Eqs. (75,76) in the Appendix, it is straightforward to compute $(\mathbf{m}_L^2)_{21}(M_{\rm maj})$. The result is:

$$\begin{aligned}(\mathbf{m}_L^2)_{21}(M_{\rm maj}) &= \frac{1}{(16\pi^2)^2}(m_L^2+m_{H_u}^2+m_\nu^2+2|A_\nu|^2)\\ &\times\left[12\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)^2+8\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)\right](\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^*_{32}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{31}+\mathcal{O}(Y_\nu^6)\ . \quad (25)\end{aligned}$$

Recall from the previous section that when $(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{21}(M_{\rm X})=0$, the observed pattern of neutrino mixing angles requires $(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{31}(M_{\rm X})\neq 0$ and $(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{32}(M_{\rm X})\neq 0$. Then, $(\mathbf{m}_L^2)_{21}(M_{\rm maj})$ will always be generated *at least* at order $\mathcal{O}(Y_\nu^4)$: it will be generated at order $\mathcal{O}(Y_\nu^2)$ when $(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{21}(M_{\rm X})$ is different from zero and at order $\mathcal{O}(Y_\nu^4)$ when it is equal to zero. Therefore, and barring cancellations, the observed violation of all family lepton numbers in neutrino oscillations *necessarily* implies in the supersymmetric see-saw model a contribution to the process $\mu\to e\gamma$ from the off-diagonal entries of the soft terms.

Following the same steps , one can calculate the off-diagonal elements of the trilinear soft terms. For the elements generated at $\mathcal{O}(Y_\nu^2)$ we obtain

$$(\mathbf{A}_e)_{31,32} = -\frac{1}{16\pi^2}\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)(2A_\nu+A_e)y_\tau(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{31,32}+\mathcal{O}(Y_\nu^4)\ , \quad (26)$$

$$(\mathbf{A}_e)_{13,23} = -\frac{1}{16\pi^2}\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)(2A_\nu+A_e)y_{e,\mu}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^*_{31,32}+\mathcal{O}(Y_\nu^4)\ , \quad (27)$$

and for the elements generated at $\mathcal{O}(Y_\nu^4)$,

$$\begin{aligned}(\mathbf{A}_e)_{21} &= \left(\frac{1}{16\pi^2}\right)^2\left[(14A_\nu+\frac{7}{2}A_e)\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)^2+(8A_\nu+2A_e)\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)\right]\\ &\times y_\mu(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^*_{32}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{31}+\mathcal{O}(Y_\nu^6)\ , \quad (28)\\ (\mathbf{A}_e)_{12} &= \left(\frac{1}{16\pi^2}\right)^2\left[(14A_\nu+\frac{7}{2}A_e)\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)^2+(8A_\nu+2A_e)\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)\right]\\ &\times y_e(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^*_{31}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{32}+\mathcal{O}(Y_\nu^6)\ . \quad (29)\end{aligned}$$

Finally, there are additional sources of lepton flavour violation stemming from the radiatively generated off-diagonal entries in the charged lepton Yukawa coupling. Analogously to the previous discussion, $(\mathbf{Y}_e)(M_{\rm maj})_{12}$ and $(\mathbf{Y}_e)(M_{\rm maj})_{21}$ get values of $\mathcal{O}(Y_\nu^4)$, while the remaining off-diagonal terms get values of $\mathcal{O}(Y_\nu^2)$. Rotating the leptonic fields to bring them to the basis where the charged lepton Yukawa coupling is diagonal will modify the values of the soft terms calculated above. Defining

$\mathbf{Y}_e = V_R \mathbf{Y}_e^{\rm diag} V_L^\dagger$, where $\mathbf{Y}_e^{\rm diag}$ is a diagonal real matrix and $V_{L,R}$ are unitary matrices, the basis transformation $L \to V_L L$, $e_R \to V_R e_R$ yields

$$\begin{aligned}
\mathbf{Y}_e &\to \mathbf{Y}_e^{\rm diag} \,, \\
\mathbf{m}_e^2 &\to V_R^\dagger \mathbf{m}_e^2 V_R \,, \\
\mathbf{m}_L^2 &\to V_L^\dagger \mathbf{m}_L^2 V_L \,, \\
\mathbf{A}_e &\to V_R^\dagger \mathbf{A}_e V_L \,.
\end{aligned} \tag{30}$$

The explicit expression for V_L is

$$V_L \simeq \begin{pmatrix} 1 & V_{L,12} & V_{L,13} \\ V_{L,21} & 1 & V_{L,23} \\ -V_{L,13}^* & -V_{L,23}^* & 1 \end{pmatrix} \,, \tag{31}$$

where

$$\begin{aligned}
V_{L,12} &= \left(\frac{1}{16\pi^2}\right)^2 \left[\frac{3}{2}\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)^2 + 2\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)\right] (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^*_{31} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{32} + \mathcal{O}(Y_\nu^6) \,, \\
V_{L,21} &= -\left(\frac{1}{16\pi^2}\right)^2 \left[\frac{5}{2}\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)^2 + 2\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)\right] (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^*_{32} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{31} + \mathcal{O}(Y_\nu^6) \,, \\
V_{L,13,23} &= -\frac{1}{16\pi^2}\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^*_{31,32} + \mathcal{O}(Y_\nu^4) \,.
\end{aligned} \tag{32}$$

Notice that $V_{L,21} \neq -V_{L,12}^*$, as required by unitarity. On the other hand, the expression for V_R is

$$V_R \simeq \begin{pmatrix} 1 & V_{R,12} & V_{R,13} \\ V_{R,21} & 1 & V_{R,23} \\ -V_{R,13}^* & -V_{R,23}^* & 1 \end{pmatrix} \,, \tag{33}$$

where

$$\begin{aligned}
V_{R,12} &= \left(\frac{1}{8\pi^2}\right)^2 \frac{y_e}{y_\mu} \left[\left(1 - \frac{y_\mu^2}{y_\tau^2}\right)\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)^2 + \log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)\right] (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^*_{31} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{32} + \mathcal{O}(Y_\nu^6) \,, \\
V_{R,21} &= -\left(\frac{1}{8\pi^2}\right)^2 \frac{y_e}{y_\mu} \left[\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)^2 + \log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)\right] (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^*_{32} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{31} + \mathcal{O}(Y_\nu^6) \,, \\
V_{R,13} &= -\frac{1}{8\pi^2}\frac{y_e}{y_\tau}\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^*_{31} + \mathcal{O}(Y_\nu^4) \,, \\
V_{R,23} &= -\frac{1}{8\pi^2}\frac{y_\mu}{y_\tau}\log\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^*_{32} + \mathcal{O}(Y_\nu^4) \,.
\end{aligned} \tag{34}$$

With these expressions for V_L and V_R it is straightforward to compute, using Eq. (30), the off-diagonal elements of the leptonic soft mass terms in the basis where

the charged lepton Yukawa coupling is diagonal. In this "mass basis" the off-diagonal elements of the right-handed slepton mass matrix read:

$$(\mathbf{m}_e^2)^{\rm mb}_{31,32} = \mathcal{O}(Y_\nu^4) \ , \qquad (\mathbf{m}_e^2)^{\rm mb}_{21} = \mathcal{O}(Y_\nu^6) \ , \tag{35}$$

that no longer vanish, but still give negligible contributions to the rare decays compared to the other sources of flavour violation. On the other hand, for the left-handed slepton mass matrix they approximately read

$$\begin{aligned}
(\mathbf{m}_L^2)^{\rm mb}_{31,32}(M_{\rm maj}) &\simeq -\frac{1}{8\pi^2}(m_L^2 + m_\nu^2 + m_{H_u}^2 + |A_\nu|^2)(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{31,32} \log\!\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) \ , \\
(\mathbf{m}_L^2)^{\rm mb}_{21}(M_{\rm maj}) &\simeq \left(\frac{1}{16\pi^2}\right)^2 \left[8(m_L^2 + m_{H_u}^2 + m_\nu^2 + 2|A_\nu|^2) \log\!\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) + \right. \\
&\left. 8(m_L^2 + m_{H_u}^2 + m_\nu^2 + \frac{5}{2}|A_\nu|^2) \log\!\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)^2 \right] (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^*_{32} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{31} \ .
\end{aligned} \tag{36}$$

Note that the diagonalization of the charged-lepton Yukawa coupling does not alter $\mathbf{m}_L^2(M_{\rm maj})_{31,32}$ but only $(\mathbf{m}_L^2)(M_{\rm maj})_{21}$. Finally, the off-diagonal elements of the soft trilinear term read:

$$\begin{aligned}
(\mathbf{A}_e)^{\rm mb}_{31,32} &\simeq -\frac{1}{16\pi^2} \log\!\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) 2A_\nu y_\tau (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{31,32} \ , \\
(\mathbf{A}_e)^{\rm mb}_{13,23} &\simeq -\frac{1}{16\pi^2} \log\!\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) 2A_\nu h_{e,\mu} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^*_{31,32} \ , \\
(\mathbf{A}_e)^{\rm mb}_{21} &\simeq \left(\frac{1}{16\pi^2}\right)^2 \left[8 \log\!\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) + 8 \log\!\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)^2 \right] A_\nu y_\mu (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^*_{32} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{31} \ , \\
(\mathbf{A}_e)^{\rm mb}_{12} &\simeq \left(\frac{1}{16\pi^2}\right)^2 \left[8 \log\!\left(\frac{M_{\rm X}}{M_{\rm maj}}\right) + 8 \log\!\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)^2 \right] A_\nu y_e (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^*_{31} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{32} \ .
\end{aligned} \tag{37}$$

In this case, *all* the off-diagonal elements get modified at the lowest order by the basis transformation.

It is apparent from Eqs. (36,37) that in the scenario with flavour universality of the soft terms at $M_{\rm X}$ and $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}(M_{\rm X}) = 0$ there exists a very precise correlation between the 21 and the 31 and 32 entries of the soft terms. As was argued in the previous section, this scenario yields the minimal amount of flavour violation in the $\mu - e$ sector. Therefore, any other neutrino Yukawa coupling compatible with the experimental data will induce larger $(\mathbf{m}_L^2)^{\rm mb}_{21}$ and $(\mathbf{A}_e)^{\rm mb}_{12,21}$ entries at low energies. For example, under the assumption of complete universality of the soft mass terms at

$M_{\rm X}$, $m_L^2(M_{\rm X}) = m_{H_u}^2(M_{\rm X}) = m_\nu^2(M_{\rm X}) \equiv m_0^2$, the following lower bounds hold at low energies for the 21 entries of the soft terms:

$$\begin{aligned}
\left|\frac{(\mathbf{m}_L^2)_{21}^{\rm mb}}{(\mathbf{m}_L^2)_{31}^{\rm mb}(\mathbf{m}_L^2)_{32}^{\rm mb}}\right| &\gtrsim \frac{2(3m_0^2+2|A_\nu|^2)\log\!\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)+2(3m_0^2+\frac{5}{2}|A_\nu|^2)\log\!\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)^2}{\left[(3m_0^2+|A_\nu|^2)\log\!\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)\right]^2}\,, \\
\left|\frac{(\mathbf{A}_e)_{21}^{\rm mb}}{(\mathbf{A}_e)_{23}^{\rm mb}(\mathbf{A}_e)_{31}^{\rm mb}}\right| &\simeq \left|\frac{(\mathbf{A}_e)_{12}^{\rm mb}}{(\mathbf{A}_e)_{13}^{\rm mb}(\mathbf{A}_e)_{32}^{\rm mb}}\right| \gtrsim \frac{2}{y_\tau|A_\nu|}\left(1+\frac{1}{\log\!\left(\frac{M_{\rm X}}{M_{\rm maj}}\right)}\right)\,. \qquad (38)
\end{aligned}$$

These bounds will eventually translate into a bound on $BR(\mu\to e\gamma)$ involving the branching ratios of the rare tau decays.

4 Lower bound on $\mu \to e\gamma$ from rare tau decays

After computing the radiatively generated off-diagonal elements of the soft SUSY breaking terms, it is straightforward to calculate the branching ratios for the rare lepton decays. In order to understand qualitatively the results we will use in this section approximate formulas for the branching ratios, although in our numerical analysis we used the general expressions existing in the literature [34] and we solved numerically the renormalization group equations including the two-loop β-functions.

A very useful tool to treat analytically the complicated exact expressions for the branching ratios is the mass insertion approximation, where the small off-diagonal elements of the soft terms are treated as insertions in the sfermion propagators in the loops [28,35]. This rationale can also be applied to the gaugino-higgsino sector, yielding at the end of the day relatively compact expressions for the branching ratios.

It was argued in [35] that for the rare tau decays, the dominant contributions correspond to the mass insertion diagrams enhanced by $\tan\beta$ factors. Namely,

$$\begin{aligned}
BR(\tau\to e\gamma) &\simeq \frac{\alpha^3}{G_F^2}\frac{|(\mathbf{m}_L^2)_{31}|^2}{m_S^8}\tan^2\beta BR(\tau\to e\nu_\tau\bar\nu_e) \\
BR(\tau\to \mu\gamma) &\simeq \frac{\alpha^3}{G_F^2}\frac{|(\mathbf{m}_L^2)_{32}|^2}{m_S^8}\tan^2\beta BR(\tau\to \mu\nu_\tau\bar\nu_\mu) \qquad (39)
\end{aligned}$$

where $BR(\tau\to\mu\nu_\tau\bar\nu_\mu)\simeq 0.17$, $BR(\tau\to e\nu_\tau\bar\nu_e)\simeq 0.18$ and m_S is a mass scale of the order of typical SUSY masses. In the case of the Constrained MSSM it is best approximated by $m_S^8 \simeq 0.5 m_0^2 M_{1/2}^2(m_0^2+0.6M_{1/2}^2)^2$ [38], where m_0 is the universal scalar mass and $M_{1/2}$ is the universal gaugino mass at $M_{\rm X}$.

On the other hand, being $|(\mathbf{m}_L^2)_{21}|$ and $|(\mathbf{A}_e)_{21}|$ generated at order $\mathcal{O}(Y_\nu^4)$, we keep for consistency the contributions to $BR(\mu \to e\gamma)$ induced not only by a single mass insertion (where $(\mathbf{m}_L^2)_{21}$ or $(\mathbf{A}_e)_{21}$ is inserted) but also by a double mass insertion (where $(\mathbf{m}_L^2)_{32}$ or $(\mathbf{A}_e)_{32}$, and $(\mathbf{m}_L^2)_{31}$ or $(\mathbf{A}_e)_{13}$ are inserted). The result reads:

$$BR(\mu \to e\gamma) \simeq \frac{\alpha^3}{G_F^2} \left| \frac{(\mathbf{m}_L^2)_{21}}{m_S^4} - \frac{(\mathbf{m}_L^2)^*_{32}(\mathbf{m}_L^2)_{31}}{m_{S'}^6} \right|^2 \tan^2\beta \ , \tag{40}$$

where $m_{S'}$ is another mass scale of the order of m_S, although in general different (note that the single and the double mass insertion contributions have opposite signs). Inserting the bound Eq. (38) into Eq. (40), we obtain the following bound:

$$BR(\mu \to e\gamma) \gtrsim \frac{\alpha^3}{G_F^2} \frac{|(\mathbf{m}_L^2)_{32}|^2 |(\mathbf{m}_L^2)_{31}|^2}{m_{S''}^{12}} \tan^2\beta \ , \tag{41}$$

where $m_{S''}$ is another mass scale, again of the same order of m_S, $m_{S'}$. Using the expressions for the rare tau decays in terms of $|(\mathbf{m}_L^2)_{31}|^2$, $|(\mathbf{m}_L^2)_{32}|^2$, Eq. (39), this bound can be casted as:

$$BR(\mu \to e\gamma) \gtrsim \frac{G_F^2}{\alpha^3 \tan^2\beta} \frac{m_S^{16}}{m_{S''}^{12}} \frac{BR(\tau \to \mu\gamma)}{BR(\tau \to \mu\nu_\tau\bar{\nu}_\mu)} \frac{BR(\tau \to e\gamma)}{BR(\tau \to e\nu_\tau\bar{\nu}_e)} \ . \tag{42}$$

This equation is a more explicit expression of Eq. (13) and constitutes the main result of this paper[3].

The numerical values of m_S and $m_{S''}$ depend crucially on the supersymmetric scenario. Before presenting the exact numerical results for some common supersymmetric scenarios, let us obtain first a rough estimate of the bound Eq. (42). To this end, we will make the approximation $m_{S''} \sim m_S$. Then, the previous bound reads:

$$BR(\mu \to e\gamma) \gtrsim 10^{-9} \left(\frac{m_S}{200\ \text{GeV}}\right)^4 \left(\frac{\tan\beta}{10}\right)^{-2} \left(\frac{BR(\tau \to \mu\gamma)}{6.8 \times 10^{-8}}\right) \left(\frac{BR(\tau \to e\gamma)}{1.1 \times 10^{-7}}\right) \ . \tag{43}$$

Therefore, if the rates for the rare tau decays were just below the present experimental bounds, the see-saw scenario with universal soft terms at $M_{\rm X}$ would predict a branching ratio for $\mu \to e\gamma$ typically larger than 10^{-9}, in conflict with the present bound $BR(\mu \to e\gamma) \leq 1.2 \times 10^{-11}$. Furthermore, if the observation of *both* rare tau decays is at the reach of present B-factories, $BR(\tau \to l_i\gamma)$ have to be larger than $\sim 10^{-8}$, which would

[3] One loop QED corrections to the electric and magnetic dipole operators reduce the theoretical prediction for $BR(l_j \to l_i\gamma)$ by a factor $\left(1 - \frac{8\alpha}{\pi} \log \frac{m_S}{m_j}\right)$ [41]. This correction makes the bound Eq. (42) a 2-6% stronger for $m_S = 100 - 1000$ GeV.

imply $BR(\mu \to e\gamma) \gtrsim 2 \times 10^{-11}$, barely compatible with the present upper bound from MEGA. One can make this argument more quantitative turning Eq. (43) to obtain an upper bound on $BR(\tau \to e\gamma)$, or alternatively on $BR(\tau \to \mu\gamma)$, from the stringent experimental constraint on $BR(\mu \to e\gamma)$:

$$BR(\tau \to e\gamma) \lesssim 10^{-9} \left(\frac{m_S}{200\,\mathrm{GeV}}\right)^{-4} \left(\frac{\tan\beta}{10}\right)^{2} \left(\frac{BR(\mu \to e\gamma)}{1.2 \times 10^{-11}}\right) \left(\frac{BR(\tau \to \mu\gamma)}{6.8 \times 10^{-8}}\right)^{-1},$$
$$BR(\tau \to \mu\gamma) \lesssim 7 \times 10^{-10} \left(\frac{m_S}{200\,\mathrm{GeV}}\right)^{-4} \left(\frac{\tan\beta}{10}\right)^{2} \left(\frac{BR(\mu \to e\gamma)}{1.2 \times 10^{-11}}\right) \left(\frac{BR(\tau \to e\gamma)}{1.1 \times 10^{-7}}\right)^{-1}. \quad (44)$$

Then, if $BR(\tau \to \mu\gamma) \gtrsim 10^{-8}$, thus making the observation of $\tau \to \mu\gamma$ accessible to present B-factories, the previous equation would imply that $BR(\tau \to e\gamma) \lesssim 8 \times 10^{-9}$, which would make the observation of $\tau \to e\gamma$ difficult (analogous conclusions can be drawn for $BR(\tau \to \mu\gamma)$). Therefore, whereas the observation of one rare tau decay at present B-factories is indeed possible, in the see-saw scenario the observation of *both* rare tau decays is unlikely.

This qualitative discussion shows that in the supersymmetric see-saw model the so far negative searches for $\mu \to e\gamma$ have crucial implications for the future searches for $\tau \to \mu\gamma$ and $\tau \to e\gamma$. Clearly, these implications will become stronger as the MEG experiment at PSI improves the bound on $\mu \to e\gamma$. However, one should bear in mind that this is just a qualitative discussion and that the actual impact of the bound Eq. (42) on future searches for rare decays depends on the particular point of the SUSY parameter space, through the values of m_S and $m_{S''}$. We will see, however, that these strong conclusions hold for a wide choice of supersymmetric parameters.

We have investigated in detail the 'Snowmass Points and Slopes' (SPS) [42], which constitute a set of benchmark points in the supersymmetric parameter space aiming to describe typical points, but also extreme although well motivated possibilities. We have analyzed the six mSUGRA benchmark points, that are defined at $M_{\mathrm{X}} = 2 \times 10^{16}$ GeV by five parameters: the universal scalar mass (m_0), gaugino mass ($M_{1/2}$) and trilinear term (A_0), $\tan\beta$ and the sign of μ. For each SPS point, the values of these parameters are given in Table 2.

The SPS1a and SPS1b points are "typical" mSUGRA points with intermediate and relatively high values of $\tan\beta$ respectively. Assuming that the neutralino constitutes the dominant component of dark matter of the Universe, these two points lie in the "bulk" region of the allowed mSUGRA parameter space. On the other hand, the

	m_0 (GeV)	$M_{1/2}$ (GeV)	A_0 (GeV)	$\tan\beta$	sign μ
SPS1a	100	250	-100	10	+
SPS1b	200	400	0	30	+
SPS2	1450	300	0	10	+
SPS3	90	400	0	10	+
SPS4	400	300	0	50	+
SPS5	150	300	-1000	5	+

Table 2: Parameters at $M_{\rm X}$ for the six mSUGRA SPS benchmark points.

SPS2 point is characterized by heavy squarks and sleptons and fairly light neutralinos, charginos and gluinos[4]. The SPS3 point has a small stau-neutralino mass difference and lies in the stau coannihilation region. The SPS4 point has a large $\tan\beta$ and lies in the "funnel" region. Lastly, the SPS5 point is characterized by a relatively light stop. We have also analyzed for completeness the mSUGRA-like scenario SPS6, characterized by having non-universal gaugino masses, and defined at the GUT scale by $m_0 = 150$ GeV, $M_3 = M_2 = 300$ GeV, $M_1 = 480$ GeV, $A_0 = 0$, $\tan\beta = 10$ and positive μ.

In Fig. 1 we show the allowed values for $BR(\tau \to e\gamma)$ and $BR(\tau \to \mu\gamma)$ in the supersymmetric see-saw model for the mSUGRA benchmark points SPS1-5. The area above (to the right of) the dashed line at $BR(\tau \to \mu\gamma) = 6.8 \times 10^{-8}$ ($BR(\tau \to e\gamma) = 1.1 \times 10^{-7}$) is excluded by the present experimental bounds on the rare tau decays. On the other hand, the area above the diagonal line labeled $BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$ is excluded from the present experimental bound on $\mu \to e\gamma$, as a consequence of Eq. (42). We find remarkable that for *all* the mSUGRA SPS points the bound Eq. (42) excludes values for the branching ratios of the rare tau decays that are otherwise allowed by direct searches. Furthermore, if the MEG experiment reaches the sensitivity $BR(\mu \to e\gamma) \sim 10^{-13}$ without observing a positive signal, the region of the parameter space excluded by Eq. (42) would enlarge considerably. Finally, in the plots we assumed an intermediate decoupling scale, $M_{\rm maj} = 5 \times 10^{13}$ GeV. Had we used a larger value for $M_{\rm maj}$, the excluded region would also enlarge, as is apparent from Eq. (38)[5].

[4]The low energy predictions for this benchmark point are extremely sensitive to the value of the top quark mass [43]. In particular, assuming $M_t = 175$ GeV this point would lie in the "focus point" region of the allowed mSUGRA parameter space. However, the most recent measurement of the top quark mass at CDF II, $M_t = 170.8 \pm 2.2(\text{stat.}) \pm 1.4(\text{syst.})$ GeV [44], pushes the SPS2 benchmark point out of the "focus point" region.

[5]Recall that to bring the decay rates to the reach of present and future experiments the neutrino

The bound Eq. (42) also has implications for future searches for rare tau decays. In Fig. 1 we show with a dash-dotted line the projected sensitivity of present B-factories to rare tau decays ($BR(\tau \to \mu\gamma), BR(\tau \to e\gamma) \gtrsim 10^{-8}$). Therefore, the area shaded in green is the region of this parameter space accessible to present B-factories, that we call for definiteness the "observable window of present B-factories". Using Eq. (42) we find that large regions of the observable window of present B-factories are excluded from the present bound $BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$ (light green shaded area). In particular, for the benchmark points SPS1a, SPS1b and SPS3 the region where *both* $\tau \to \mu\gamma$ and $\tau \to e\gamma$ could be discovered at present B-factories is excluded. Nevertheless, the discovery of one of them, either $\tau \to \mu\gamma$ or $\tau \to e\gamma$, is still possible. Let us remind that SPS1a and SPS1b correspond to "typical" mSUGRA points, and thus this conclusion holds for a large region of the parameter space. On the other hand, for SPS2 and SPS5 the discovery of both rare decays is still possible, although only if their branching ratios are close to the experimental sensitivity of present B-factories[6]. Finally, for SPS4 the present bound $BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$ has only little impact for present B-factories. An improvement of the bound on $BR(\mu \to e\gamma)$ by two orders of magnitude, as planned by the MEG experiment at PSI, would exclude the possibility of observing both rare tau decays at present B-factories in most mSUGRA parameter space. Hence, would present B-factories refute this expectation, the supersymmetric see-saw model with mSUGRA would have to be abandoned, unless a certain amount of fine tuning is accepted. The same conclusions hold for the mSUGRA-like scenario SPS6, which has non-universal gaugino masses, as can be realized from Fig. 2.

We also show for completeness the observable window of the projected super$B$-factories, shown as a yellow shaded area. It is defined by the region between the dashed lines indicating the present bounds on the rare tau decays and the dotted lines showing the projected bounds ($BR(\tau \to \mu\gamma), BR(\tau \to \mu\gamma) \gtrsim 10^{-9}$). The present bound on $\mu \to e\gamma$ practically does not exclude any region of the observable window of the projected super$B$-factories, except for the benchmark point SPS3. On the other hand, if the MEG experiment reaches the projected sensitivity $BR(\mu \to e\gamma) \sim 10^{-13}$

Yukawa couplings have to be sizable. This typically requires large values of $M_{\rm maj}$ in order to produce small neutrino masses.

[6] One should bear in mind, however, that the SPS2 point is extremely sensitive to the input value of the top quark mass. Therefore, the conclusions for this benchmark point should be taken with a pinch of salt.

without observing a positive signal, again a large portion of the observable window of the projected super$B$-factories would be excluded by Eq. (42). In particular, whereas the observation of either $\tau \to \mu\gamma$ or $\tau \to e\gamma$ will indeed be possible at the projected super$B$-factories, the observation of both would only be possible for the benchmark point SPS4 and marginally for SPS2 and SPS5. For the rest of the benchmark points analyzed in this paper, SPS1a, SPS1b, SPS3 and SPS6, the observation of both rare decays will not be possible in the supersymmetric see-saw model, unless a certain amount of fine tuning is accepted.

5 Conclusions

A series of neutrino experiments have demonstrated that *all* family lepton numbers are violated in Nature. Therefore, there is no symmetry reason forbidding the rare lepton decays $\mu \to e\gamma$, $\tau \to \mu\gamma$ or $\tau \to e\gamma$. In this paper we have discussed the implications of this observation for the supersymmetric see-saw model.

We have shown that even in the very special situation where the flavour violation vanishes at high-energies in one of the sectors, for instance in the $\mu-e$ sector, the flavour violation in the other two sectors will induce through two loop radiative corrections a small though non-vanishing flavour violation at low energies in the $\mu-e$ sector, that will be correlated to the flavour violation in the $\tau-\mu$ and $\tau-e$ sectors. Barring cancellations, this scenario will produce the minimal rate for the rare decay $\mu \to e\gamma$. Therefore, in any other scenario the lower bound $BR(\mu \to e\gamma) \gtrsim C \times BR(\tau \to \mu\gamma)BR(\tau \to e\gamma)$ will hold, where C is a constant that depends on supersymmetric parameters.

We have analyzed the implications of this bound on the possible values of the rates of the rare tau decays for the supersymmetric benchmark points SPS1-6. We have found that values for $BR(\tau \to \mu\gamma)$ and $BR(\tau \to e\gamma)$ that are allowed by present experiments searching for rare tau decays are forbidden by our bound $BR(\mu \to e\gamma) \gtrsim C \times BR(\tau \to \mu\gamma)BR(\tau \to e\gamma)$ and the present constraint on $BR(\mu \to e\gamma)$ from MEGA. In particular, we have found that, for large regions of the Constrained MSSM parameter space, present B-factories could discover either $\tau \to \mu\gamma$ or $\tau \to e\gamma$, but not both. We have also discussed the implications of the non-observation of the process $\mu \to e\gamma$ by the MEG experiment at PSI for the future searches for rare tau decays at present B-factories and at projected super$B$-factories. This analysis could also be extended to

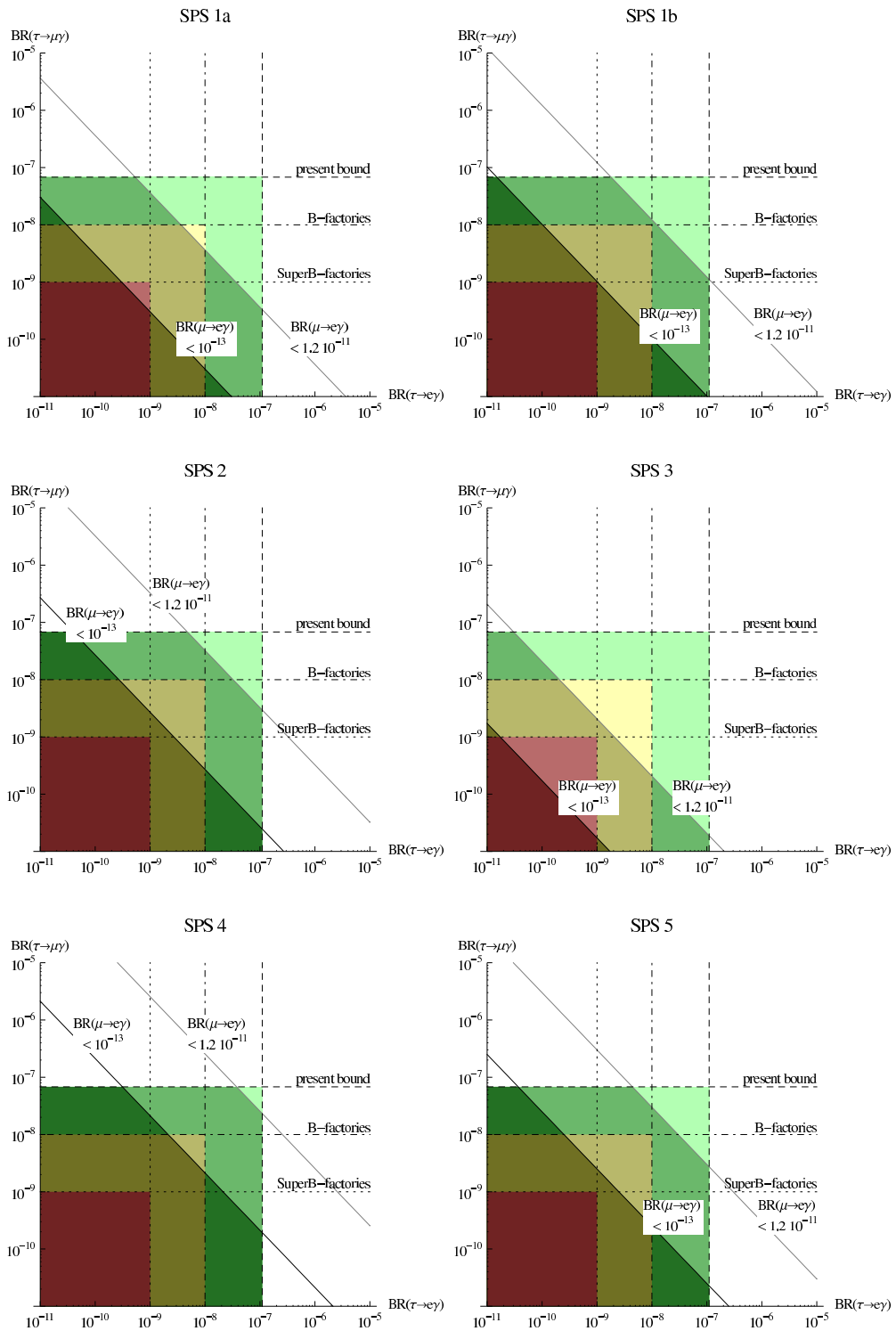

Figure 1: Allowed values for the branching ratios of the rare tau decays $\tau \to e\gamma$ and $\tau \to \mu\gamma$ from present experiments and from the bound $BR(\mu \to e\gamma) \gtrsim C \times BR(\tau \to \mu\gamma) BR(\tau \to e\gamma)$ for the mSUGRA scenarios SPS1-5. The area in green indicates the observable window of present B-factories, and in yellow the observable window of future super$B$-factories. Excluded regions are shown with light shading, whereas allowed regions are shown with dark shading. In the plots it is assumed $M_{\rm maj} = 5 \times 10^{13}$ GeV.

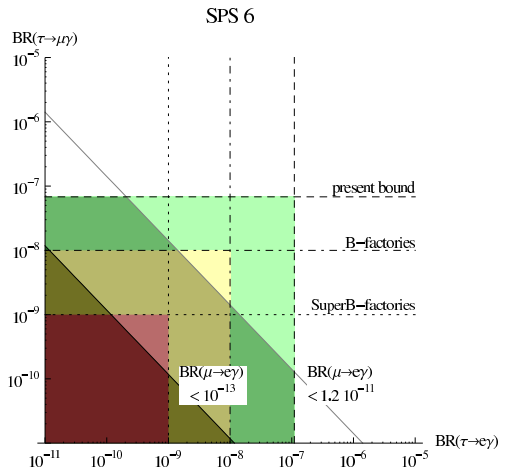

Figure 2: As in Fig. 1, but for the mSUGRA-like scenario SPS6.

more general classes of models. Work along these lines is already in progress [45].

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Appendix

In this appendix we report the full set of one- and two-loop Renormalization Group Equations (RGEs) for the parameters of the Minimal Supersymmetric Standard Model extended with three right-handed neutrino superfields. Partial results for the Yukawa couplings, the gauge couplings and the neutrino mass matrix can be found in [46]. The RGEs for the soft SUSY breaking terms and the μ term have been derived particularizing the general formulas in [47] to the supersymmetric see-saw model[7].

The RGE for any supersymmetric or soft-breaking parameter can be schematically written as

$$\frac{d}{dt}X = \frac{1}{16\pi^2}\beta_X^{(1)} + \frac{1}{(16\pi^2)^2}\beta_X^{(2)} \ . \tag{45}$$

[7] All the RGEs are written in the $\overline{DR}'$ scheme.

The one- and two-loop β functions for the gauge couplings are given by

$$\beta_{g_a}^{(1)} = g_a^3 B_a^{(1)} \,, \tag{46}$$

$$\beta_{g_a}^{(2)} = g_a^3 \left[\sum_{b=1}^{3} B_{ab}^{(2)} g_b^2 - \sum_{x=u,d,e,\nu} C_a^x \, \mathrm{Tr}(\mathbf{Y}_x^\dagger \mathbf{Y}_x) \right] \,, \tag{47}$$

where g_1, g_2 and g_3 are the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gauge couplings, respectively, and

$$B_a^{(1)} = (33/5, 1, -3) \,, \tag{48}$$

$$B_{ab}^{(2)} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix} \,,$$

$$C_a^{u,d,e,\nu} = \begin{pmatrix} 26/5 & 14/5 & 18/5 & 6/5 \\ 6 & 6 & 2 & 2 \\ 4 & 4 & 0 & 0 \end{pmatrix} \,.$$

(In these expressions, g_1 has been normalized as in $SU(5)$.)

On the other hand, the complete superpotential of the MSSM extended with right-handed neutrinos reads, imposing R-parity conservation,

$$W = d_{Ri}^c \mathbf{Y}_{dij} Q_j H_d + u_{Ri}^c \mathbf{Y}_{uij} Q_j H_u + e_{Ri}^c \mathbf{Y}_{eij} L_j H_d + \nu_{Ri}^c \mathbf{Y}_{\nu ij} L_j H_u + \mu H_u H_d - \frac{1}{2} \nu_{Ri}^c \mathbf{M}_{ij} \nu_{Rj}^c \,. \tag{49}$$

The one- and two-loop β functions for these SUSY conserving parameters are:

$$\beta_{\mathbf{Y}_d}^{(1)} = \mathbf{Y}_d \left\{ \mathrm{Tr}(3\mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{Y}_e \mathbf{Y}_e^\dagger) + 3\mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_u^\dagger \mathbf{Y}_u - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{15} g_1^2 \right\} \,, \tag{50}$$

$$\begin{aligned} \beta_{\mathbf{Y}_d}^{(2)} = \mathbf{Y}_d \Big\{ & - \mathrm{Tr}(9\mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger + 3\mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger + 3\mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_e^\dagger + \mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_\nu^\dagger) \\ & - \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathrm{Tr}(3\mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) - 3\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathrm{Tr}(3\mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{Y}_e \mathbf{Y}_e^\dagger) \\ & - 4\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d - 2\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u - 2\mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \\ & + \left[16 g_3^2 - \frac{2}{5} g_1^2 \right] \mathrm{Tr}(\mathbf{Y}_d \mathbf{Y}_d^\dagger) + \frac{6}{5} g_1^2 \mathrm{Tr}(\mathbf{Y}_e \mathbf{Y}_e^\dagger) + \frac{4}{5} g_1^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u + \left[6 g_2^2 + \frac{4}{5} g_1^2 \right] \mathbf{Y}_d^\dagger \mathbf{Y}_d \\ & - \frac{16}{9} g_3^4 + 8 g_3^2 g_2^2 + \frac{8}{9} g_3^2 g_1^2 + \frac{15}{2} g_2^4 + g_2^2 g_1^2 + \frac{287}{90} g_1^4 \Big\} \,, \end{aligned} \tag{51}$$

$$\beta_{\mathbf{Y}_u}^{(1)} = \mathbf{Y}_u \left\{ \mathrm{Tr}(3\mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) + 3\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2 \right\} \,, \tag{52}$$

$$\beta^{(2)}_{\mathbf{Y}_u} = \mathbf{Y}_u\Big\{ -\mathrm{Tr}(9\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + 3\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + \mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger) \quad (53)$$
$$- \mathbf{Y}_d^\dagger\mathbf{Y}_d\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - 3\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)$$
$$- 4\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u - 2\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d - 2\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u$$
$$+ \Big[16g_3^2 + \frac{4}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + \Big[6g_2^2 + \frac{2}{5}g_1^2\Big]\mathbf{Y}_u^\dagger\mathbf{Y}_u + \frac{2}{5}g_1^2\mathbf{Y}_d^\dagger\mathbf{Y}_d$$
$$- \frac{16}{9}g_3^4 + 8g_3^2g_2^2 + \frac{136}{45}g_3^2g_1^2 + \frac{15}{2}g_2^4 + g_2^2g_1^2 + \frac{2743}{450}g_1^4\Big\}\,,$$

$$\beta^{(1)}_{\mathbf{Y}_e} = \mathbf{Y}_e\Big\{\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) + 3\mathbf{Y}_e^\dagger\mathbf{Y}_e + \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - 3g_2^2 - \frac{9}{5}g_1^2\Big\}\,, \quad (54)$$

$$\beta^{(2)}_{\mathbf{Y}_e} = \mathbf{Y}_e\Big\{ -\mathrm{Tr}(9\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + 3\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + 3\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger + \mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger) \quad (55)$$
$$- \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger) - 3\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger)$$
$$- 4\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e - 2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - 2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e$$
$$+ \Big[16g_3^2 - \frac{2}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) + 6g_2^2\mathbf{Y}_e^\dagger\mathbf{Y}_e$$
$$+ \frac{15}{2}g_2^4 + \frac{9}{5}g_2^2g_1^2 + \frac{27}{2}g_1^4\Big\}\,,$$

$$\beta^{(1)}_{\mathbf{Y}_\nu} = \mathbf{Y}_\nu\Big\{\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger) + 3\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + \mathbf{Y}_e^\dagger\mathbf{Y}_e - 3g_2^2 - \frac{3}{5}g_1^2\Big\}\,, \quad (56)$$

$$\beta^{(2)}_{\mathbf{Y}_\nu} = \mathbf{Y}_\nu\Big\{ -\mathrm{Tr}(9\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + 3\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + \mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger) \quad (57)$$
$$- \mathbf{Y}_e^\dagger\mathbf{Y}_e\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - 3\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)$$
$$- 4\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - 2\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e - 2\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$$
$$+ \Big[16g_3^2 + \frac{4}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + \Big[6g_2^2 + \frac{6}{5}g_1^2\Big]\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + \frac{6}{5}g_1^2\mathbf{Y}_e^\dagger\mathbf{Y}_e$$
$$+ \frac{15}{2}g_2^4 + \frac{9}{5}g_2^2g_1^2 + \frac{207}{50}g_1^4\Big\}\,,$$

$$\beta^{(1)}_{\mu} = \mu\Big\{\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger) - 3g_2^2 - \frac{3}{5}g_1^2\Big\}\,, \quad (58)$$

$$\beta^{(2)}_{\mu} = \mu\Big\{ -\mathrm{Tr}(\,9\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + 9\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + 6\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger \quad (59)$$
$$+ 3\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger + 3\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + 2\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger)$$
$$+ \Big[16g_3^2 + \frac{4}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + \Big[16g_3^2 - \frac{2}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger)$$
$$+ \frac{15}{2}g_2^4 + \frac{9}{5}g_1^2g_2^2 + \frac{207}{50}g_1^4\Big\}\,,$$

$$\beta^{(1)}_{\mathbf{M}} = \mathbf{M}\Big[2\mathbf{Y}^*_\nu\mathbf{Y}^T_\nu\Big] + \Big[2\mathbf{Y}_\nu\mathbf{Y}^\dagger_\nu\Big]\mathbf{M}\,, \tag{60}$$

$$\begin{aligned}\beta^{(2)}_{\mathbf{M}} = \mathbf{M}\Big[& -2\mathbf{Y}^*_\nu\mathbf{Y}^T_e\mathbf{Y}^*_e\mathbf{Y}^T_\nu - 2\mathbf{Y}^*_\nu\mathbf{Y}^T_\nu\mathbf{Y}^*_\nu\mathbf{Y}^T_\nu - 2\mathbf{Y}^*_\nu\mathbf{Y}^T_\nu\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}^\dagger_u + \mathbf{Y}_\nu\mathbf{Y}^\dagger_\nu) \\ & + \frac{6}{5}g_1^2\mathbf{Y}^*_\nu\mathbf{Y}^T_\nu + 6g_2^2\mathbf{Y}^*_\nu\mathbf{Y}^T_\nu\Big] + \\ & \Big[-2\mathbf{Y}_\nu\mathbf{Y}^\dagger_e\mathbf{Y}_e\mathbf{Y}^\dagger_\nu - 2\mathbf{Y}_\nu\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu\mathbf{Y}^\dagger_\nu - 2\mathbf{Y}_\nu\mathbf{Y}^\dagger_\nu\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}^\dagger_u + \mathbf{Y}_\nu\mathbf{Y}^\dagger_\nu) \\ & + \frac{6}{5}g_1^2\mathbf{Y}_\nu\mathbf{Y}^\dagger_\nu + 6g_2^2\mathbf{Y}_\nu\mathbf{Y}^\dagger_\nu\Big]\mathbf{M}\,.\end{aligned} \tag{61}$$

The soft SUSY breaking Lagrangian of the MSSM extended with right-handed neutrinos reads:

$$\begin{aligned}-\mathcal{L}_{\rm soft} = \; & m^2_{H_u}H^*_uH_u + m^2_{H_d}H^*_dH_d + (\mathbf{m}^2_Q)_{ij}\widetilde{Q}^*_i\widetilde{Q}_j + (\mathbf{m}^2_d)_{ij}\widetilde{d}^*_{Ri}\widetilde{d}_{Rj} + (\mathbf{m}^2_u)_{ij}\widetilde{u}^*_{Ri}\widetilde{u}_{Rj} + \\ & (\mathbf{m}^2_L)_{ij}\widetilde{L}^*_i\widetilde{L}_j \; + (\mathbf{m}^2_e)_{ij}\widetilde{e}^*_{Ri}\widetilde{e}_{Rj} \; + (\mathbf{m}^2_\nu)_{ij}\widetilde{\nu}^*_{Ri}\widetilde{\nu}_{Rj} \; + \Big(M_1\widetilde{B}\widetilde{B} + M_2\widetilde{W}\widetilde{W} + M_3\widetilde{g}\widetilde{g} \\ & \mathbf{A}_{dij}\widetilde{d}^*_{Ri}H_d\widetilde{Q}_j + \mathbf{A}_{uij}\widetilde{u}^*_{Ri}H_u\widetilde{Q}_j + \mathbf{A}_{eij}\widetilde{e}^*_{Ri}H_d\widetilde{L}_j + \mathbf{A}_{\nu ij}\widetilde{\nu}^*_{Ri}H_u\widetilde{L}_j + \\ & BH_uH_d - \frac{1}{2}\widetilde{\nu}_{Ri}\mathbf{B}_{\mathbf{M}ij}\widetilde{\nu}_{Rj} + \text{h.c.}\Big)\,.\end{aligned} \tag{62}$$

The β-functions of the soft gaugino masses are given by

$$\beta^{(1)}_{M_a} = 2g_a^2B^{(1)}_aM_a\,, \tag{63}$$

$$\beta^{(2)}_{M_a} = 2g_a^2\left[\sum_{b=1}^{3}B^{(2)}_{ab}g_b^2(M_a+M_b) + \sum_{x=u,d,e,\nu}C^x_a\left(\mathrm{Tr}[\mathbf{Y}^\dagger_x\mathbf{A}_x] - M_a\mathrm{Tr}[\mathbf{Y}^\dagger_x\mathbf{Y}_x]\right)\right]\,, \tag{64}$$

where $B^{(1)}_a$, $B^{(2)}_{ab}$ and $C^{u,d,e,\nu}_a$ were defined in Eq. (48). On the other hand, the β-functions of the soft scalar masses read:

$$\begin{aligned}\beta^{(1)}_{m^2_{H_u}} = \mathrm{Tr}[\,& 6(m^2_{H_u}+\mathbf{m}^2_Q)\mathbf{Y}^\dagger_u\mathbf{Y}_u + 6\mathbf{Y}^\dagger_u\mathbf{m}^2_u\mathbf{Y}_u + 6\mathbf{A}^\dagger_u\mathbf{A}_u + 2(m^2_{H_u}+\mathbf{m}^2_L)\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu \\ & + 2\mathbf{Y}^\dagger_\nu\mathbf{m}^2_\nu\mathbf{Y}_\nu + 2\mathbf{A}^\dagger_\nu\mathbf{A}_\nu] - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 + \frac{3}{5}g_1^2\mathcal{S}\,,\end{aligned} \tag{65}$$

$$
\begin{aligned}
\beta^{(2)}_{m^2_{H_u}} \;=\; -2\mathrm{Tr}\Big[& +18(m^2_{H_u}+\mathbf{m}^2_Q)\mathbf{Y}^\dagger_u\mathbf{Y}_u\mathbf{Y}^\dagger_u\mathbf{Y}_u + 18\mathbf{Y}^\dagger_u\mathbf{m}^2_u\mathbf{Y}_u\mathbf{Y}^\dagger_u\mathbf{Y}_u \\
& +3(m^2_{H_u}+m^2_{H_d}+\mathbf{m}^2_Q)\mathbf{Y}^\dagger_u\mathbf{Y}_u\mathbf{Y}^\dagger_d\mathbf{Y}_d + 3\mathbf{Y}^\dagger_u\mathbf{m}^2_u\mathbf{Y}_u\mathbf{Y}^\dagger_d\mathbf{Y}_d \\
& +3\mathbf{Y}^\dagger_u\mathbf{Y}_u\mathbf{m}^2_Q\mathbf{Y}^\dagger_d\mathbf{Y}_d + 3\mathbf{Y}^\dagger_u\mathbf{Y}_u\mathbf{Y}^\dagger_d\mathbf{m}^2_d\mathbf{Y}_d \\
& +6(m^2_{H_u}+\mathbf{m}^2_L)\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu + 6\mathbf{Y}^\dagger_\nu\mathbf{m}^2_\nu\mathbf{Y}_\nu\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu \\
& +(m^2_{H_u}+m^2_{H_d}+\mathbf{m}^2_L)\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu\mathbf{Y}^\dagger_e\mathbf{Y}_e + \mathbf{Y}^\dagger_\nu\mathbf{m}^2_\nu\mathbf{Y}_\nu\mathbf{Y}^\dagger_e\mathbf{Y}_e \\
& +\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu\mathbf{m}^2_L\mathbf{Y}^\dagger_e\mathbf{Y}_e + \mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu\mathbf{Y}^\dagger_e\mathbf{m}^2_e\mathbf{Y}_e + 18\mathbf{A}^\dagger_u\mathbf{A}_u\mathbf{Y}^\dagger_u\mathbf{Y}_u \\
& +18\mathbf{A}^\dagger_u\mathbf{Y}_u\mathbf{Y}^\dagger_u\mathbf{A}_u + 3\mathbf{A}^\dagger_d\mathbf{A}_d\mathbf{Y}^\dagger_u\mathbf{Y}_u + 3\mathbf{Y}^\dagger_d\mathbf{Y}_d\mathbf{A}^\dagger_u\mathbf{A}_u \\
& +3\mathbf{A}^\dagger_d\mathbf{Y}_d\mathbf{Y}^\dagger_u\mathbf{A}_u + 3\mathbf{Y}^\dagger_d\mathbf{A}_d\mathbf{A}^\dagger_u\mathbf{Y}_u + 6\mathbf{A}^\dagger_\nu\mathbf{A}_\nu\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu + 6\mathbf{A}^\dagger_\nu\mathbf{Y}_\nu\mathbf{Y}^\dagger_\nu\mathbf{A}_\nu \\
& +\mathbf{A}^\dagger_e\mathbf{A}_e\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu + \mathbf{Y}^\dagger_e\mathbf{Y}_e\mathbf{A}^\dagger_\nu\mathbf{A}_\nu + \mathbf{A}^\dagger_e\mathbf{Y}_e\mathbf{Y}^\dagger_\nu\mathbf{A}_\nu + \mathbf{Y}^\dagger_e\mathbf{A}_e\mathbf{A}^\dagger_\nu\mathbf{Y}_\nu\Big] \\
& +\Big[32g_3^2+\frac{8}{5}g_1^2\Big]\mathrm{Tr}[(m^2_{H_u}+\mathbf{m}^2_Q)\mathbf{Y}^\dagger_u\mathbf{Y}_u + \mathbf{Y}^\dagger_u\mathbf{m}^2_u\mathbf{Y}_u + \mathbf{A}^\dagger_u\mathbf{A}_u] \\
& +32g_3^2\Big\{2|M_3|^2\mathrm{Tr}[\mathbf{Y}^\dagger_u\mathbf{Y}_u] - M_3^*\mathrm{Tr}[\mathbf{Y}^\dagger_u\mathbf{A}_u] - M_3\mathrm{Tr}[\mathbf{A}^\dagger_u\mathbf{Y}_u]\Big\} \\
& +\frac{8}{5}g_1^2\Big\{2|M_1|^2\mathrm{Tr}[\mathbf{Y}^\dagger_u\mathbf{Y}_u] - M_1^*\mathrm{Tr}[\mathbf{Y}^\dagger_u\mathbf{A}_u] - M_1\mathrm{Tr}[\mathbf{A}^\dagger_u\mathbf{Y}_u]\Big\} + \frac{6}{5}g_1^2\mathcal{S}' \\
& +33g_2^4|M_2|^2 + \frac{18}{5}g_2^2g_1^2(|M_2|^2+|M_1|^2+\Re[M_1M_2^*]) + \frac{621}{25}g_1^4|M_1|^2 \\
& +3g_2^2\sigma_2 + \frac{3}{5}g_1^2\sigma_1 \;,
\end{aligned}
\tag{66}
$$

$$
\begin{aligned}
\beta^{(1)}_{m^2_{H_d}} = \mathrm{Tr}\Big[& 6(m^2_{H_d}+\mathbf{m}^2_Q)\mathbf{Y}^\dagger_d\mathbf{Y}_d + 6\mathbf{Y}^\dagger_d\mathbf{m}^2_d\mathbf{Y}_d + 2(m^2_{H_d}+\mathbf{m}^2_L)\mathbf{Y}^\dagger_e\mathbf{Y}_e + 2\mathbf{Y}^\dagger_e\mathbf{m}^2_e\mathbf{Y}_e \\
& +6\mathbf{A}^\dagger_d\mathbf{A}_d + 2\mathbf{A}^\dagger_e\mathbf{A}_e\Big] - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 - \frac{3}{5}g_1^2\mathcal{S} \;,
\end{aligned}
\tag{67}
$$

$$
\begin{aligned}
\beta^{(2)}_{m^2_{H_d}} \;=\; -2\mathrm{Tr}\Big[& +18(m^2_{H_d}+\mathbf{m}^2_Q)\mathbf{Y}^\dagger_d\mathbf{Y}_d\mathbf{Y}^\dagger_d\mathbf{Y}_d + 18\mathbf{Y}^\dagger_d\mathbf{m}^2_d\mathbf{Y}_d\mathbf{Y}^\dagger_d\mathbf{Y}_d \\
& +3(m^2_{H_u}+m^2_{H_d}+\mathbf{m}^2_Q)\mathbf{Y}^\dagger_u\mathbf{Y}_u\mathbf{Y}^\dagger_d\mathbf{Y}_d + 3\mathbf{Y}^\dagger_u\mathbf{m}^2_u\mathbf{Y}_u\mathbf{Y}^\dagger_d\mathbf{Y}_d \\
& +3\mathbf{Y}^\dagger_u\mathbf{Y}_u\mathbf{m}^2_Q\mathbf{Y}^\dagger_d\mathbf{Y}_d + 3\mathbf{Y}^\dagger_u\mathbf{Y}_u\mathbf{Y}^\dagger_d\mathbf{m}^2_d\mathbf{Y}_d \\
& +6(m^2_{H_d}+\mathbf{m}^2_L)\mathbf{Y}^\dagger_e\mathbf{Y}_e\mathbf{Y}^\dagger_e\mathbf{Y}_e + 6\mathbf{Y}^\dagger_e\mathbf{m}^2_e\mathbf{Y}_e\mathbf{Y}^\dagger_e\mathbf{Y}_e \\
& +(m^2_{H_u}+m^2_{H_d}+\mathbf{m}^2_L)\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu\mathbf{Y}^\dagger_e\mathbf{Y}_e + \mathbf{Y}^\dagger_\nu\mathbf{m}^2_\nu\mathbf{Y}_\nu\mathbf{Y}^\dagger_e\mathbf{Y}_e \\
& +\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu\mathbf{m}^2_L\mathbf{Y}^\dagger_e\mathbf{Y}_e + \mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu\mathbf{Y}^\dagger_e\mathbf{m}^2_e\mathbf{Y}_e + 18\mathbf{A}^\dagger_d\mathbf{A}_d\mathbf{Y}^\dagger_d\mathbf{Y}_d \\
& +18\mathbf{A}^\dagger_d\mathbf{Y}_d\mathbf{Y}^\dagger_d\mathbf{A}_d + 3\mathbf{A}^\dagger_u\mathbf{A}_u\mathbf{Y}^\dagger_d\mathbf{Y}_d + 3\mathbf{Y}^\dagger_u\mathbf{Y}_u\mathbf{A}^\dagger_d\mathbf{A}_d \\
& +3\mathbf{A}^\dagger_u\mathbf{Y}_u\mathbf{Y}^\dagger_d\mathbf{A}_d + 3\mathbf{Y}^\dagger_u\mathbf{A}_u\mathbf{A}^\dagger_d\mathbf{Y}_d + 6\mathbf{A}^\dagger_e\mathbf{A}_e\mathbf{Y}^\dagger_e\mathbf{Y}_e + 6\mathbf{A}^\dagger_e\mathbf{Y}_e\mathbf{Y}^\dagger_e\mathbf{A}_e \\
& +\mathbf{A}^\dagger_\nu\mathbf{A}_\nu\mathbf{Y}^\dagger_e\mathbf{Y}_e + \mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu\mathbf{A}^\dagger_e\mathbf{A}_e + \mathbf{A}^\dagger_\nu\mathbf{Y}_\nu\mathbf{Y}^\dagger_e\mathbf{A}_e + \mathbf{Y}^\dagger_\nu\mathbf{A}_\nu\mathbf{A}^\dagger_e\mathbf{Y}_e\Big]
\end{aligned}
\tag{68}
$$

$$
\begin{aligned}
&+\Big[32g_3^2-\frac{4}{5}g_1^2\Big]\mathrm{Tr}[(m_{H_d}^2+\mathbf{m}_Q^2)\mathbf{Y}_d^\dagger\mathbf{Y}_d+\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d+\mathbf{A}_d^\dagger\mathbf{A}_d]\\
&+32g_3^2\Big\{2|M_3|^2\mathrm{Tr}[\mathbf{Y}_d^\dagger\mathbf{Y}_d]-M_3^*\mathrm{Tr}[\mathbf{Y}_d^\dagger\mathbf{A}_d]-M_3\mathrm{Tr}[\mathbf{A}_d^\dagger\mathbf{Y}_d]\Big\}\\
&-\frac{4}{5}g_1^2\Big\{2|M_1|^2\mathrm{Tr}[\mathbf{Y}_d^\dagger\mathbf{Y}_d]-M_1^*\mathrm{Tr}[\mathbf{Y}_d^\dagger\mathbf{A}_d]-M_1\mathrm{Tr}[\mathbf{A}_d^\dagger\mathbf{Y}_d]\Big\}\\
&+\frac{12}{5}g_1^2\Big\{\mathrm{Tr}[(m_{H_d}^2+\mathbf{m}_L^2)\mathbf{Y}_e^\dagger\mathbf{Y}_e+\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e+\mathbf{A}_e^\dagger\mathbf{A}_e]\\
&\qquad\qquad+2|M_1|^2\mathrm{Tr}[\mathbf{Y}_e^\dagger\mathbf{Y}_e]-M_1\mathrm{Tr}[\mathbf{A}_e^\dagger\mathbf{Y}_e]-M_1^*\mathrm{Tr}[\mathbf{Y}_e^\dagger\mathbf{A}_e]\Big\}\\
&-\frac{6}{5}g_1^2\mathcal{S}'+33g_2^4|M_2|^2+\frac{18}{5}g_2^2g_1^2(|M_2|^2+|M_1|^2+\Re[M_1M_2^*])\\
&+\frac{621}{25}g_1^4|M_1|^2+3g_2^2\sigma_2+\frac{3}{5}g_1^2\sigma_1\ ,
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(1)}_{\mathbf{m}_Q^2}=&(\mathbf{m}_Q^2+2m_{H_u}^2)\mathbf{Y}_u^\dagger\mathbf{Y}_u+(\mathbf{m}_Q^2+2m_{H_d}^2)\mathbf{Y}_d^\dagger\mathbf{Y}_d+2\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u+2\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d\\
&+[\mathbf{Y}_u^\dagger\mathbf{Y}_u+\mathbf{Y}_d^\dagger\mathbf{Y}_d]\mathbf{m}_Q^2+2\mathbf{A}_u^\dagger\mathbf{A}_u+2\mathbf{A}_d^\dagger\mathbf{A}_d\\
&-\frac{32}{3}g_3^2|M_3|^2-6g_2^2|M_2|^2-\frac{2}{15}g_1^2|M_1|^2+\frac{1}{5}g_1^2\mathcal{S}\ ,
\end{aligned}
\tag{69}
$$

$$
\begin{aligned}
\beta^{(2)}_{\mathbf{m}_Q^2}=&-(2\mathbf{m}_Q^2+8m_{H_u}^2)\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u-4\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u-4\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_u^\dagger\mathbf{Y}_u\\
&-4\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u-2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{m}_Q^2-(2\mathbf{m}_Q^2+8m_{H_d}^2)\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\\
&-4\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d-4\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_d^\dagger\mathbf{Y}_d-4\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d-2\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{m}_Q^2\\
&-\Big[(\mathbf{m}_Q^2+4m_{H_u}^2)\mathbf{Y}_u^\dagger\mathbf{Y}_u+2\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u+\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{m}_Q^2\Big]\mathrm{Tr}(3\mathbf{Y}_u^\dagger\mathbf{Y}_u+\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)\\
&-\Big[(\mathbf{m}_Q^2+4m_{H_d}^2)\mathbf{Y}_d^\dagger\mathbf{Y}_d+2\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d+\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{m}_Q^2\Big]\mathrm{Tr}(3\mathbf{Y}_d^\dagger\mathbf{Y}_d+\mathbf{Y}_e^\dagger\mathbf{Y}_e)\\
&-\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathrm{Tr}(6\mathbf{m}_Q^2\mathbf{Y}_u^\dagger\mathbf{Y}_u+6\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u+2\mathbf{m}_L^2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu+2\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2\mathbf{Y}_\nu)\\
&-\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathrm{Tr}(6\mathbf{m}_Q^2\mathbf{Y}_d^\dagger\mathbf{Y}_d+6\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d+2\mathbf{m}_L^2\mathbf{Y}_e^\dagger\mathbf{Y}_e+2\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e)\\
&-4\Big\{\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{A}_u^\dagger\mathbf{A}_u+\mathbf{A}_u^\dagger\mathbf{A}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u+\mathbf{Y}_u^\dagger\mathbf{A}_u\mathbf{A}_u^\dagger\mathbf{Y}_u+\mathbf{A}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{A}_u\Big\}\\
&-4\Big\{\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{A}_d^\dagger\mathbf{A}_d+\mathbf{A}_d^\dagger\mathbf{A}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d+\mathbf{Y}_d^\dagger\mathbf{A}_d\mathbf{A}_d^\dagger\mathbf{Y}_d+\mathbf{A}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{A}_d\Big\}\\
&-\mathbf{A}_u^\dagger\mathbf{A}_u\mathrm{Tr}[6\mathbf{Y}_u^\dagger\mathbf{Y}_u+2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu]-\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathrm{Tr}[6\mathbf{A}_u^\dagger\mathbf{A}_u+2\mathbf{A}_\nu^\dagger\mathbf{A}_\nu]\\
&-\mathbf{A}_u^\dagger\mathbf{Y}_u\mathrm{Tr}[6\mathbf{Y}_u^\dagger\mathbf{A}_u+2\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu]-\mathbf{Y}_u^\dagger\mathbf{A}_u\mathrm{Tr}[6\mathbf{A}_u^\dagger\mathbf{Y}_u+2\mathbf{A}_\nu^\dagger\mathbf{Y}_\nu]\\
&-\mathbf{A}_d^\dagger\mathbf{A}_d\mathrm{Tr}[6\mathbf{Y}_d^\dagger\mathbf{Y}_d+2\mathbf{Y}_e^\dagger\mathbf{Y}_e]-\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathrm{Tr}[6\mathbf{A}_d^\dagger\mathbf{A}_d+2\mathbf{A}_e^\dagger\mathbf{A}_e]\\
&-\mathbf{A}_d^\dagger\mathbf{Y}_d\mathrm{Tr}[6\mathbf{Y}_d^\dagger\mathbf{A}_d+2\mathbf{Y}_e^\dagger\mathbf{A}_e]-\mathbf{Y}_d^\dagger\mathbf{A}_d\mathrm{Tr}[6\mathbf{A}_d^\dagger\mathbf{Y}_d+2\mathbf{A}_e^\dagger\mathbf{Y}_e]\\
&+\frac{2}{5}g_1^2\Big\{(2\mathbf{m}_Q^2+4m_{H_u}^2)\mathbf{Y}_u^\dagger\mathbf{Y}_u+4\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u+2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{m}_Q^2+4\mathbf{A}_u^\dagger\mathbf{A}_u-4M_1\mathbf{A}_u^\dagger\mathbf{Y}_u\\
&\qquad-4M_1^*\mathbf{Y}_u^\dagger\mathbf{A}_u+8|M_1|^2\mathbf{Y}_u^\dagger\mathbf{Y}_u+(\mathbf{m}_Q^2+2m_{H_d}^2)\mathbf{Y}_d^\dagger\mathbf{Y}_d+2\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d\\
&\qquad+\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{m}_Q^2+2\mathbf{A}_d^\dagger\mathbf{A}_d-2M_1\mathbf{A}_d^\dagger\mathbf{Y}_d-2M_1^*\mathbf{Y}_d^\dagger\mathbf{A}_d+4|M_1|^2\mathbf{Y}_d^\dagger\mathbf{Y}_d\Big\}
\end{aligned}
\tag{70}
$$

$$
\begin{aligned}
&+\frac{2}{5}g_1^2\mathcal{S}' - \frac{128}{3}g_3^4|M_3|^2 + 32g_3^2g_2^2(|M_3|^2+|M_2|^2+\Re[M_2M_3^*])\\
&+\frac{32}{45}g_3^2g_1^2(|M_3|^2+|M_1|^2+\Re[M_1M_3^*]) + 33g_2^4|M_2|^2\\
&+\frac{2}{5}g_2^2g_1^2(|M_2|^2+|M_1|^2+\Re[M_1M_2^*]) + \frac{199}{75}g_1^4|M_1|^2\\
&+\frac{16}{3}g_3^2\sigma_3 + 3g_2^2\sigma_2 + \frac{1}{15}g_1^2\sigma_1 \ ,
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(1)}_{\mathbf{m}_d^2} =& (2\mathbf{m}_d^2 + 4m_{H_d}^2)\mathbf{Y}_d\mathbf{Y}_d^\dagger + 4\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_d^\dagger + 2\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{m}_d^2 + 4\mathbf{A}_d\mathbf{A}_d^\dagger \\
&- \frac{32}{3}g_3^2|M_3|^2 - \frac{8}{15}g_1^2|M_1|^2 + \frac{2}{5}g_1^2\mathcal{S} \ ,
\end{aligned} \tag{71}
$$

$$
\begin{aligned}
\beta^{(2)}_{\mathbf{m}_d^2} =& -(2\mathbf{m}_d^2 + 8m_{H_d}^2)\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger - 4\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger - 4\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d\mathbf{Y}_d^\dagger \\
&- 4\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_d^\dagger - 2\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{m}_d^2 - (2\mathbf{m}_d^2 + 4m_{H_u}^2 + 4m_{H_d}^2)\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger \\
&- 4\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger - 4\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u\mathbf{Y}_d^\dagger - 4\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_d^\dagger - 2\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{m}_d^2 \\
&- \Big[(\mathbf{m}_d^2 + 4m_{H_d}^2)\mathbf{Y}_d\mathbf{Y}_d^\dagger + 2\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_d^\dagger + \mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{m}_d^2\Big]\mathrm{Tr}(6\mathbf{Y}_d^\dagger\mathbf{Y}_d + 2\mathbf{Y}_e^\dagger\mathbf{Y}_e) \\
&- 4\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathrm{Tr}(3\mathbf{m}_Q^2\mathbf{Y}_d^\dagger\mathbf{Y}_d + 3\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d + \mathbf{m}_L^2\mathbf{Y}_e^\dagger\mathbf{Y}_e + \mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e) \\
&- 4\Big\{\mathbf{A}_d\mathbf{A}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{A}_d\mathbf{A}_d^\dagger + \mathbf{A}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{A}_d^\dagger + \mathbf{Y}_d\mathbf{A}_d^\dagger\mathbf{A}_d\mathbf{Y}_d^\dagger\Big\} \\
&- 4\Big\{\mathbf{A}_d\mathbf{A}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger + \mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{A}_u\mathbf{A}_d^\dagger + \mathbf{A}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{A}_d^\dagger + \mathbf{Y}_d\mathbf{A}_u^\dagger\mathbf{A}_u\mathbf{Y}_d^\dagger\Big\} \\
&- 4\mathbf{A}_d\mathbf{A}_d^\dagger\mathrm{Tr}(3\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_e^\dagger\mathbf{Y}_e) - 4\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathrm{Tr}(3\mathbf{A}_d^\dagger\mathbf{A}_d + \mathbf{A}_e^\dagger\mathbf{A}_e) \\
&- 4\mathbf{A}_d\mathbf{Y}_d^\dagger\mathrm{Tr}(3\mathbf{A}_d^\dagger\mathbf{Y}_d + \mathbf{A}_e^\dagger\mathbf{Y}_e) - 4\mathbf{Y}_d\mathbf{A}_d^\dagger\mathrm{Tr}(3\mathbf{Y}_d^\dagger\mathbf{A}_d + \mathbf{Y}_e^\dagger\mathbf{A}_e) \\
&+ \Big[6g_2^2 + \frac{2}{5}g_1^2\Big]\Big\{(\mathbf{m}_d^2 + 2m_{H_d}^2)\mathbf{Y}_d\mathbf{Y}_d^\dagger + 2\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_d^\dagger + \mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{m}_d^2 + 2\mathbf{A}_d\mathbf{A}_d^\dagger\Big\} \\
&+ 12g_2^2\Big\{2|M_2|^2\mathbf{Y}_d\mathbf{Y}_d^\dagger - M_2^*\mathbf{A}_d\mathbf{Y}_d^\dagger - M_2\mathbf{Y}_d\mathbf{A}_d^\dagger\Big\} \\
&+ \frac{4}{5}g_1^2\Big\{2|M_1|^2\mathbf{Y}_d\mathbf{Y}_d^\dagger - M_1^*\mathbf{A}_d\mathbf{Y}_d^\dagger - M_1\mathbf{Y}_d\mathbf{A}_d^\dagger\Big\} + \frac{4}{5}g_1^2\mathcal{S}' \\
&- \frac{128}{3}g_3^4|M_3|^2 + \frac{128}{45}g_3^2g_1^2(|M_3|^2+|M_1|^2+\Re[M_1M_3^*]) + \frac{808}{75}g_1^4|M_1|^2 \\
&+ \frac{16}{3}g_3^2\sigma_3 + \frac{4}{15}g_1^2\sigma_1 \ ,
\end{aligned} \tag{72}
$$

$$
\begin{aligned}
\beta^{(1)}_{\mathbf{m}_u^2} =& (2\mathbf{m}_u^2 + 4m_{H_u}^2)\mathbf{Y}_u\mathbf{Y}_u^\dagger + 4\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_u^\dagger + 2\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{m}_u^2 + 4\mathbf{A}_u\mathbf{A}_u^\dagger \\
&- \frac{32}{3}g_3^2|M_3|^2 - \frac{32}{15}g_1^2|M_1|^2 - \frac{4}{5}g_1^2\mathcal{S} \ ,
\end{aligned} \tag{73}
$$

$$
\begin{aligned}
\beta^{(2)}_{\mathbf{m}_u^2} =& -(2\mathbf{m}_u^2 + 8m_{H_u}^2)\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger - 4\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger - 4\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u\mathbf{Y}_u^\dagger \\
&- 4\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_u^\dagger - 2\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{m}_u^2 - (2\mathbf{m}_u^2 + 4m_{H_u}^2 + 4m_{H_d}^2)\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger \\
&- 4\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger - 4\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d\mathbf{Y}_u^\dagger - 4\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{m}_Q^2\mathbf{Y}_u^\dagger - 2\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{m}_u^2
\end{aligned} \tag{74}
$$

$$
\begin{aligned}
&- \Big[(\mathbf{m}_u^2 + 4m_{H_u}^2)\mathbf{Y}_u\mathbf{Y}_u^\dagger + 2\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_u^\dagger + \mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{m}_u^2\Big]\mathrm{Tr}[6\mathbf{Y}_u^\dagger\mathbf{Y}_u + 2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu] \\
&- 4\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathrm{Tr}[3\mathbf{m}_Q^2\mathbf{Y}_u^\dagger\mathbf{Y}_u + 3\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u + \mathbf{m}_L^2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + \mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2\mathbf{Y}_\nu] \\
&- 4\left\{\mathbf{A}_u\mathbf{A}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{A}_u\mathbf{A}_u^\dagger + \mathbf{A}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{A}_u^\dagger + \mathbf{Y}_u\mathbf{A}_u^\dagger\mathbf{A}_u\mathbf{Y}_u^\dagger\right\} \\
&- 4\left\{\mathbf{A}_u\mathbf{A}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + \mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{A}_d\mathbf{A}_u^\dagger + \mathbf{A}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{A}_u^\dagger + \mathbf{Y}_u\mathbf{A}_d^\dagger\mathbf{A}_d\mathbf{Y}_u^\dagger\right\} \\
&- 4\mathbf{A}_u\mathbf{A}_u^\dagger\mathrm{Tr}(3\mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu) - 4\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathrm{Tr}(3\mathbf{A}_u^\dagger\mathbf{A}_u + \mathbf{A}_\nu^\dagger\mathbf{A}_\nu) \\
&- 4\mathbf{A}_u\mathbf{Y}_u^\dagger\mathrm{Tr}(3\mathbf{A}_u^\dagger\mathbf{Y}_u + \mathbf{A}_\nu^\dagger\mathbf{Y}_\nu) - 4\mathbf{Y}_u\mathbf{A}_u^\dagger\mathrm{Tr}(3\mathbf{Y}_u^\dagger\mathbf{A}_u + \mathbf{Y}_\nu^\dagger\mathbf{A}_\nu) \\
&+ \Big[6g_2^2 - \frac{2}{5}g_1^2\Big]\Big\{(\mathbf{m}_u^2 + 2m_{H_u}^2)\mathbf{Y}_u\mathbf{Y}_u^\dagger + 2\mathbf{Y}_u\mathbf{m}_Q^2\mathbf{Y}_u^\dagger + \mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{m}_u^2 + 2\mathbf{A}_u\mathbf{A}_u^\dagger\Big\} \\
&+ 12g_2^2\Big\{2|M_2|^2\mathbf{Y}_u\mathbf{Y}_u^\dagger - M_2^*\mathbf{A}_u\mathbf{Y}_u^\dagger - M_2\mathbf{Y}_u\mathbf{A}_u^\dagger\Big\} \\
&- \frac{4}{5}g_1^2\Big\{2|M_1|^2\mathbf{Y}_u\mathbf{Y}_u^\dagger - M_1^*\mathbf{A}_u\mathbf{Y}_u^\dagger - M_1\mathbf{Y}_u\mathbf{A}_u^\dagger\Big\} - \frac{8}{5}g_1^2\mathcal{S}' \\
&- \frac{128}{3}g_3^4|M_3|^2 + \frac{512}{45}g_3^2g_1^2(|M_3|^2 + |M_1|^2 + \Re[M_1M_3^*]) + \frac{3424}{75}g_1^4|M_1|^2 \\
&+ \frac{16}{3}g_3^2\sigma_3 + \frac{16}{15}g_1^2\sigma_1 \ ,
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(1)}_{\mathbf{m}_L^2} =\ & + (\mathbf{m}_L^2 + 2m_{H_u}^2)\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + (\mathbf{m}_L^2 + 2m_{H_d}^2)\mathbf{Y}_e^\dagger\mathbf{Y}_e + 2\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2\mathbf{Y}_\nu + 2\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e \\
&+ [\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + \mathbf{Y}_e^\dagger\mathbf{Y}_e]\mathbf{m}_L^2 + 2\mathbf{A}_\nu^\dagger\mathbf{A}_\nu + 2\mathbf{A}_e^\dagger\mathbf{A}_e \\
&- 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 - \frac{3}{5}g_1^2\mathcal{S} \ ,
\end{aligned} \tag{75}
$$

$$
\begin{aligned}
\beta^{(2)}_{\mathbf{m}_L^2} =\ & - (2\mathbf{m}_L^2 + 8m_{H_u}^2)\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - 4\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - 4\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{m}_L^2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu \\
&- 4\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2\mathbf{Y}_\nu - 2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{m}_L^2 - (2\mathbf{m}_L^2 + 8m_{H_d}^2)\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e \\
&- 4\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e - 4\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{m}_L^2\mathbf{Y}_e^\dagger\mathbf{Y}_e - 4\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e - 2\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{m}_L^2 \\
&- \Big[(\mathbf{m}_L^2 + 4m_{H_u}^2)\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + 2\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2\mathbf{Y}_\nu + \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{m}_L^2\Big]\mathrm{Tr}(3\mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu) \\
&- \Big[(\mathbf{m}_L^2 + 4m_{H_d}^2)\mathbf{Y}_e^\dagger\mathbf{Y}_e + 2\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e + \mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{m}_L^2\Big]\mathrm{Tr}(3\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_e^\dagger\mathbf{Y}_e) \\
&- \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathrm{Tr}[6\mathbf{m}_Q^2\mathbf{Y}_u^\dagger\mathbf{Y}_u + 6\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u + 2\mathbf{m}_L^2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + 2\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2\mathbf{Y}_\nu] \\
&- \mathbf{Y}_e^\dagger\mathbf{Y}_e\mathrm{Tr}[6\mathbf{m}_Q^2\mathbf{Y}_d^\dagger\mathbf{Y}_d + 6\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d + 2\mathbf{m}_L^2\mathbf{Y}_e^\dagger\mathbf{Y}_e + 2\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e] \\
&- 4\Big\{\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{A}_\nu^\dagger\mathbf{A}_\nu + \mathbf{A}_\nu^\dagger\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + \mathbf{Y}_\nu^\dagger\mathbf{A}_\nu\mathbf{A}_\nu^\dagger\mathbf{Y}_\nu + \mathbf{A}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu\Big\} \\
&- 4\Big\{\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{A}_e^\dagger\mathbf{A}_e + \mathbf{A}_e^\dagger\mathbf{A}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e + \mathbf{Y}_e^\dagger\mathbf{A}_e\mathbf{A}_e^\dagger\mathbf{Y}_e + \mathbf{A}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{A}_e\Big\} \\
&- \mathbf{A}_\nu^\dagger\mathbf{A}_\nu\mathrm{Tr}[6\mathbf{Y}_u^\dagger\mathbf{Y}_u + 2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu] - \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathrm{Tr}[6\mathbf{A}_u^\dagger\mathbf{A}_u + 2\mathbf{A}_\nu^\dagger\mathbf{A}_\nu] \\
&- \mathbf{A}_\nu^\dagger\mathbf{Y}_\nu\mathrm{Tr}[6\mathbf{Y}_u^\dagger\mathbf{A}_u + 2\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu] - \mathbf{Y}_\nu^\dagger\mathbf{A}_\nu\mathrm{Tr}[6\mathbf{A}_u^\dagger\mathbf{Y}_u + 2\mathbf{A}_\nu^\dagger\mathbf{Y}_\nu] \\
&- \mathbf{A}_e^\dagger\mathbf{A}_e\mathrm{Tr}[6\mathbf{Y}_d^\dagger\mathbf{Y}_d + 2\mathbf{Y}_e^\dagger\mathbf{Y}_e] - \mathbf{Y}_e^\dagger\mathbf{Y}_e\mathrm{Tr}[6\mathbf{A}_d^\dagger\mathbf{A}_d + 2\mathbf{A}_e^\dagger\mathbf{A}_e] \\
&- \mathbf{A}_e^\dagger\mathbf{Y}_e\mathrm{Tr}[6\mathbf{Y}_d^\dagger\mathbf{A}_d + 2\mathbf{Y}_e^\dagger\mathbf{A}_e] - \mathbf{Y}_e^\dagger\mathbf{A}_e\mathrm{Tr}[6\mathbf{A}_d^\dagger\mathbf{Y}_d + 2\mathbf{A}_e^\dagger\mathbf{Y}_e]
\end{aligned} \tag{76}
$$

$$
\begin{aligned}
&+\frac{6}{5}g_1^2\Big\{\,(\mathbf{m}_L^2+2m_{H_d}^2)\mathbf{Y}_e^\dagger\mathbf{Y}_e+2\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e+\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{m}_L^2+2\mathbf{A}_e^\dagger\mathbf{A}_e\\
&\qquad\qquad-2M_1\mathbf{A}_e^\dagger\mathbf{Y}_e-2M_1^*\mathbf{Y}_e^\dagger\mathbf{A}_e+4|M_1|^2\mathbf{Y}_e^\dagger\mathbf{Y}_e\Big\}\\
&-\frac{6}{5}g_1^2\mathcal{S}'+33g_2^4|M_2|^2+\frac{18}{5}g_2^2g_1^2(|M_2|^2+|M_1|^2+\Re[M_1M_2^*])\\
&+\frac{621}{25}g_1^4|M_1|^2+3g_2^2\sigma_2+\frac{3}{5}g_1^2\sigma_1\ ,
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(1)}_{\mathbf{m}_e^2}=&\,(2\mathbf{m}_e^2+4m_{H_d}^2)\mathbf{Y}_e\mathbf{Y}_e^\dagger+4\mathbf{Y}_e\mathbf{m}_L^2\mathbf{Y}_e^\dagger+2\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{m}_e^2+4\mathbf{A}_e\mathbf{A}_e^\dagger \qquad (77)\\
&-\frac{24}{5}g_1^2|M_1|^2+\frac{6}{5}g_1^2\mathcal{S}\ ,\\
\beta^{(2)}_{\mathbf{m}_e^2}=&-(2\mathbf{m}_e^2+8m_{H_d}^2)\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger-4\mathbf{Y}_e\mathbf{m}_L^2\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger-4\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e\mathbf{Y}_e^\dagger \qquad (78)\\
&-4\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{m}_L^2\mathbf{Y}_e^\dagger-2\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{m}_e^2-(2\mathbf{m}_e^2+4m_{H_u}^2+4m_{H_d}^2)\mathbf{Y}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\\
&-4\mathbf{Y}_e\mathbf{m}_L^2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger-4\mathbf{Y}_e\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2\mathbf{Y}_\nu\mathbf{Y}_e^\dagger-4\mathbf{Y}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{m}_L^2\mathbf{Y}_e^\dagger-2\mathbf{Y}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{m}_e^2\\
&-\Big[(\mathbf{m}_e^2+4m_{H_d}^2)\mathbf{Y}_e\mathbf{Y}_e^\dagger+2\mathbf{Y}_e\mathbf{m}_L^2\mathbf{Y}_e^\dagger+\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{m}_e^2\Big]\mathrm{Tr}(6\mathbf{Y}_d^\dagger\mathbf{Y}_d+2\mathbf{Y}_e^\dagger\mathbf{Y}_e)\\
&-4\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathrm{Tr}(3\mathbf{m}_Q^2\mathbf{Y}_d^\dagger\mathbf{Y}_d+3\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d+\mathbf{m}_L^2\mathbf{Y}_e^\dagger\mathbf{Y}_e+\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e)\\
&-4\Big\{\mathbf{A}_e\mathbf{A}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger+\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{A}_e\mathbf{A}_e^\dagger+\mathbf{A}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{A}_e^\dagger+\mathbf{Y}_e\mathbf{A}_e^\dagger\mathbf{A}_e\mathbf{Y}_e^\dagger\Big\}\\
&-4\Big\{\mathbf{A}_e\mathbf{A}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger+\mathbf{Y}_e\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu\mathbf{A}_e^\dagger+\mathbf{A}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{A}_e^\dagger+\mathbf{Y}_e\mathbf{A}_\nu^\dagger\mathbf{A}_\nu\mathbf{Y}_e^\dagger\Big\}\\
&-4\mathbf{A}_e\mathbf{A}_e^\dagger\mathrm{Tr}(3\mathbf{Y}_d^\dagger\mathbf{Y}_d+\mathbf{Y}_e^\dagger\mathbf{Y}_e)-4\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathrm{Tr}(3\mathbf{A}_d^\dagger\mathbf{A}_d+\mathbf{A}_e^\dagger\mathbf{A}_e)\\
&-4\mathbf{A}_e\mathbf{Y}_e^\dagger\mathrm{Tr}(3\mathbf{A}_d^\dagger\mathbf{Y}_d+\mathbf{A}_e^\dagger\mathbf{Y}_e)-4\mathbf{Y}_e\mathbf{A}_e^\dagger\mathrm{Tr}(3\mathbf{Y}_d^\dagger\mathbf{A}_d+\mathbf{Y}_e^\dagger\mathbf{A}_e)\\
&+\Big[6g_2^2-\frac{6}{5}g_1^2\Big]\Big\{(\mathbf{m}_e^2+2m_{H_d}^2)\mathbf{Y}_e\mathbf{Y}_e^\dagger+2\mathbf{Y}_e\mathbf{m}_L^2\mathbf{Y}_e^\dagger+\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{m}_e^2+2\mathbf{A}_e\mathbf{A}_e^\dagger\Big\}\\
&+12g_2^2\Big\{2|M_2|^2\mathbf{Y}_e\mathbf{Y}_e^\dagger-M_2^*\mathbf{A}_e\mathbf{Y}_e^\dagger-M_2\mathbf{Y}_e\mathbf{A}_e^\dagger\Big\}\\
&-\frac{12}{5}g_1^2\Big\{2|M_1|^2\mathbf{Y}_e\mathbf{Y}_e^\dagger-M_1^*\mathbf{A}_e\mathbf{Y}_e^\dagger-M_1\mathbf{Y}_e\mathbf{A}_e^\dagger\Big\}\\
&+\frac{12}{5}g_1^2\mathcal{S}'+\frac{2808}{25}g_1^4|M_1|^2+\frac{12}{5}g_1^2\sigma_1\ ,
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(1)}_{\mathbf{m}_\nu^2}=&\,(2\mathbf{m}_\nu^2+4m_{H_u}^2)\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger+4\mathbf{Y}_\nu\mathbf{m}_L^2\mathbf{Y}_\nu^\dagger+2\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2+4\mathbf{A}_\nu\mathbf{A}_\nu^\dagger\ , \qquad (79)\\
\beta^{(2)}_{\mathbf{m}_\nu^2}=&-(2\mathbf{m}_\nu^2+8m_{H_u}^2)\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger-4\mathbf{Y}_\nu\mathbf{m}_L^2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger-4\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger \qquad (80)\\
&-4\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{m}_L^2\mathbf{Y}_\nu^\dagger-2\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2-(2\mathbf{m}_\nu^2+4m_{H_u}^2+4m_{H_d}^2)\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger\\
&-4\mathbf{Y}_\nu\mathbf{m}_L^2\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger-4\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e\mathbf{Y}_\nu^\dagger-4\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{m}_L^2\mathbf{Y}_\nu^\dagger-2\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2\\
&-\Big[(\mathbf{m}_\nu^2+4m_{H_u}^2)\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger+2\mathbf{Y}_\nu\mathbf{m}_L^2\mathbf{Y}_\nu^\dagger+\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2\Big]\mathrm{Tr}[6\mathbf{Y}_u^\dagger\mathbf{Y}_u+2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu]\\
&-4\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathrm{Tr}[3\mathbf{m}_Q^2\mathbf{Y}_u^\dagger\mathbf{Y}_u+3\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u+\mathbf{m}_L^2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu+\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2\mathbf{Y}_\nu]
\end{aligned}
$$

$$
\begin{aligned}
&- 4\left\{\mathbf{A}_\nu\mathbf{A}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu\mathbf{A}_\nu^\dagger + \mathbf{A}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{A}_\nu^\dagger + \mathbf{Y}_\nu\mathbf{A}_\nu^\dagger\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger\right\}\\
&- 4\left\{\mathbf{A}_\nu\mathbf{A}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger + \mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{A}_e\mathbf{A}_\nu^\dagger + \mathbf{A}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{A}_\nu^\dagger + \mathbf{Y}_\nu\mathbf{A}_e^\dagger\mathbf{A}_e\mathbf{Y}_\nu^\dagger\right\}\\
&- 4\mathbf{A}_\nu\mathbf{A}_\nu^\dagger\mathrm{Tr}(3\mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu) - 4\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathrm{Tr}(3\mathbf{A}_u^\dagger\mathbf{A}_u + \mathbf{A}_\nu^\dagger\mathbf{A}_\nu)\\
&- 4\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger\mathrm{Tr}(3\mathbf{A}_u^\dagger\mathbf{Y}_u + \mathbf{A}_\nu^\dagger\mathbf{Y}_\nu) - 4\mathbf{Y}_\nu\mathbf{A}_\nu^\dagger\mathrm{Tr}(3\mathbf{Y}_u^\dagger\mathbf{A}_u + \mathbf{Y}_\nu^\dagger\mathbf{A}_\nu)\\
&+ \left[6g_2^2 + \frac{6}{5}g_1^2\right]\left\{(\mathbf{m}_\nu^2 + 2m_{H_u}^2)\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + 2\mathbf{Y}_\nu\mathbf{m}_L^2\mathbf{Y}_\nu^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{m}_\nu^2 + 2\mathbf{A}_\nu\mathbf{A}_\nu^\dagger\right\}\\
&+ 12g_2^2\left\{2|M_2|^2\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger - M_2^*\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger - M_2\mathbf{Y}_\nu\mathbf{A}_\nu^\dagger\right\}\\
&+ \frac{12}{5}g_1^2\left\{2|M_1|^2\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger - M_1^*\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger - M_1\mathbf{Y}_\nu\mathbf{A}_\nu^\dagger\right\}\,,
\end{aligned}
$$

where we have defined

$$
\begin{aligned}
\mathcal{S} &= m_{H_u}^2 - m_{H_d}^2 + \mathrm{Tr}[\mathbf{m}_Q^2 - \mathbf{m}_L^2 - 2\mathbf{m}_u^2 + \mathbf{m}_d^2 + \mathbf{m}_e^2]\,, \qquad (81)\\
\mathcal{S}' &= \mathrm{Tr}\Big[- (3m_{H_u}^2 + \mathbf{m}_Q^2)\mathbf{Y}_u^\dagger\mathbf{Y}_u + 4\mathbf{Y}_u^\dagger\mathbf{m}_u^2\mathbf{Y}_u + (3m_{H_d}^2 - \mathbf{m}_Q^2)\mathbf{Y}_d^\dagger\mathbf{Y}_d - 2\mathbf{Y}_d^\dagger\mathbf{m}_d^2\mathbf{Y}_d\\
&\qquad + (m_{H_d}^2 + \mathbf{m}_L^2)\mathbf{Y}_e^\dagger\mathbf{Y}_e - 2\mathbf{Y}_e^\dagger\mathbf{m}_e^2\mathbf{Y}_e + (-m_{H_u}^2 + \mathbf{m}_L^2)\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\Big]\\
&\quad + \left[\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right]\left\{m_{H_u}^2 - m_{H_d}^2 - \mathrm{Tr}(\mathbf{m}_L^2)\right\} + \left[\frac{8}{3}g_3^2 + \frac{3}{2}g_2^2 + \frac{1}{30}g_1^2\right]\mathrm{Tr}(\mathbf{m}_Q^2)\\
&\quad - \left[\frac{16}{3}g_3^2 + \frac{16}{15}g_1^2\right]\mathrm{Tr}(\mathbf{m}_u^2) + \left[\frac{8}{3}g_3^2 + \frac{2}{15}g_1^2\right]\mathrm{Tr}(\mathbf{m}_d^2) + \frac{6}{5}g_1^2\mathrm{Tr}(\mathbf{m}_e^2)\,,\\
\sigma_1 &= \frac{1}{5}g_1^2\left\{3(m_{H_u}^2 + m_{H_d}^2) + \mathrm{Tr}[\mathbf{m}_Q^2 + 3\mathbf{m}_L^2 + 8\mathbf{m}_u^2 + 2\mathbf{m}_d^2 + 6\mathbf{m}_e^2]\right\}\,,\\
\sigma_2 &= g_2^2\left\{m_{H_u}^2 + m_{H_d}^2 + \mathrm{Tr}[3\mathbf{m}_Q^2 + \mathbf{m}_L^2]\right\}\,,\\
\sigma_3 &= g_3^2\mathrm{Tr}[2\mathbf{m}_Q^2 + \mathbf{m}_u^2 + \mathbf{m}_d^2]\,.
\end{aligned}
$$

Finally, the β functions for the trilinear and bilinear soft terms are:

$$
\begin{aligned}
\beta_{\mathbf{A}_d}^{(1)} &= \mathbf{A}_d\Big\{\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) + 5\mathbf{Y}_d^\dagger\mathbf{Y}_d + \mathbf{Y}_u^\dagger\mathbf{Y}_u - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\Big\} + \qquad (82)\\
&\quad \mathbf{Y}_d\Big\{\mathrm{Tr}(6\mathbf{A}_d\mathbf{Y}_d^\dagger + 2\mathbf{A}_e\mathbf{Y}_e^\dagger) + 4\mathbf{Y}_d^\dagger\mathbf{A}_d + 2\mathbf{Y}_u^\dagger\mathbf{A}_u\\
&\qquad + \frac{32}{3}g_3^2M_3 + 6g_2^2M_2 + \frac{14}{15}g_1^2M_1\Big\}\,,\\
\beta_{\mathbf{A}_d}^{(2)} &= \mathbf{A}_d\Big\{ - \mathrm{Tr}(9\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + 3\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + 3\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger + \mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger) \qquad (83)\\
&\qquad - \mathbf{Y}_u^\dagger\mathbf{Y}_u\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger) - 5\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger)\\
&\qquad - 6\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d - 2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u - 4\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\\
&\qquad + \left[16g_3^2 - \frac{2}{5}g_1^2\right]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) + \frac{4}{5}g_1^2\mathbf{Y}_u^\dagger\mathbf{Y}_u + \left[12g_2^2 + \frac{6}{5}g_1^2\right]\mathbf{Y}_d^\dagger\mathbf{Y}_d\\
&\qquad - \frac{16}{9}g_3^4 + 8g_3^2g_2^2 + \frac{8}{9}g_3^2g_1^2 + \frac{15}{2}g_2^4 + g_2^2g_1^2 + \frac{287}{90}g_1^4\Big\} +
\end{aligned}
$$

$$
\begin{aligned}
&\mathbf{Y}_d\Big\{ -2\mathrm{Tr}(\,18\mathbf{A}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + 3\mathbf{A}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + 3\mathbf{A}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger \\
&\qquad + 6\mathbf{A}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger + \mathbf{A}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger + \mathbf{A}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger) \\
&\quad - \mathbf{Y}_u^\dagger\mathbf{Y}_u\mathrm{Tr}(6\mathbf{A}_u\mathbf{Y}_u^\dagger + 2\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger) - 6\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathrm{Tr}(3\mathbf{A}_d\mathbf{Y}_d^\dagger + \mathbf{A}_e\mathbf{Y}_e^\dagger) \\
&\quad - \mathbf{Y}_u^\dagger\mathbf{A}_u\mathrm{Tr}(6\mathbf{Y}_u\mathbf{Y}_u^\dagger + 2\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger) - 4\mathbf{Y}_d^\dagger\mathbf{A}_d\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) \\
&\quad - 6\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{A}_d - 8\mathbf{Y}_d^\dagger\mathbf{A}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d - 4\mathbf{Y}_u^\dagger\mathbf{A}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u \\
&\quad - 4\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{A}_u - 4\mathbf{Y}_u^\dagger\mathbf{A}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d - 2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{A}_d \\
&\quad + \Big[32g_3^2 - \frac{4}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{A}_d\mathbf{Y}_d^\dagger) + \frac{12}{5}g_1^2\mathrm{Tr}(\mathbf{A}_e\mathbf{Y}_e^\dagger) + \frac{8}{5}g_1^2\mathbf{Y}_u^\dagger\mathbf{A}_u \\
&\quad + \Big[6g_2^2 + \frac{6}{5}g_1^2\Big]\mathbf{Y}_d^\dagger\mathbf{A}_d - \Big[32g_3^2M_3 - \frac{4}{5}g_1^2M_1\Big]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) - \frac{12}{5}g_1^2M_1\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) \\
&\quad - \Big[12g_2^2M_2 + \frac{8}{5}g_1^2M_1\Big]\mathbf{Y}_d^\dagger\mathbf{Y}_d - \frac{8}{5}g_1^2M_1\mathbf{Y}_u^\dagger\mathbf{Y}_u \\
&\quad + \frac{64}{9}g_3^4M_3 - 16g_3^2g_2^2(M_3+M_2) - \frac{16}{9}g_3^2g_1^2(M_3+M_1) \\
&\quad - 30g_2^4M_2 - 2g_2^2g_1^2(M_2+M_1) - \frac{574}{45}g_1^4M_1\Big\}\,,
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(1)}_{\mathbf{A}_u} = \mathbf{A}_u&\Big\{\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger) + 5\mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathbf{Y}_d^\dagger\mathbf{Y}_d - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\Big\} + \qquad (84)\\
&\mathbf{Y}_u\Big\{\mathrm{Tr}(6\mathbf{A}_u\mathbf{Y}_u^\dagger + 2\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger) + 4\mathbf{Y}_u^\dagger\mathbf{A}_u + 2\mathbf{Y}_d^\dagger\mathbf{A}_d \\
&\quad + \frac{32}{3}g_3^2M_3 + 6g_2^2M_2 + \frac{26}{15}g_1^2M_1\Big\}\,,
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(2)}_{\mathbf{A}_u} = \mathbf{A}_u&\Big\{ -\mathrm{Tr}(9\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + 3\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + \mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger) \quad (85)\\
&\quad - \mathbf{Y}_d^\dagger\mathbf{Y}_d\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - 5\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger) \\
&\quad - 6\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u - 2\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d - 4\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u \\
&\quad + \Big[16g_3^2 + \frac{4}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + 12g_2^2\mathbf{Y}_u^\dagger\mathbf{Y}_u + \frac{2}{5}g_1^2\mathbf{Y}_d^\dagger\mathbf{Y}_d \\
&\quad - \frac{16}{9}g_3^4 + 8g_3^2g_2^2 + \frac{136}{45}g_3^2g_1^2 + \frac{15}{2}g_2^4 + g_2^2g_1^2 + \frac{2743}{450}g_1^4\Big\} + \\
&\mathbf{Y}_u\Big\{ -2\mathrm{Tr}(\,18\mathbf{A}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{A}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + 3\mathbf{A}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger \\
&\qquad + 6\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + \mathbf{A}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger + \mathbf{A}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger) \\
&\quad - 6\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathrm{Tr}(3\mathbf{A}_u\mathbf{Y}_u^\dagger + \mathbf{A}_\nu\mathbf{Y}_\nu^\dagger) - \mathbf{Y}_d^\dagger\mathbf{Y}_d\mathrm{Tr}(6\mathbf{A}_d\mathbf{Y}_d^\dagger + 2\mathbf{A}_e\mathbf{Y}_e^\dagger) \\
&\quad - 4\mathbf{Y}_u^\dagger\mathbf{A}_u\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger) - \mathbf{Y}_d^\dagger\mathbf{A}_d\mathrm{Tr}(6\mathbf{Y}_d\mathbf{Y}_d^\dagger + 2\mathbf{Y}_e\mathbf{Y}_e^\dagger) \\
&\quad - 6\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{A}_u - 8\mathbf{Y}_u^\dagger\mathbf{A}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u - 4\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{A}_d \\
&\quad - 4\mathbf{Y}_d^\dagger\mathbf{A}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d - 2\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{A}_u - 4\mathbf{Y}_d^\dagger\mathbf{A}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u
\end{aligned}
$$

$$
\begin{aligned}
&+\Big[32g_3^2+\frac{8}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{A}_u\mathbf{Y}_u^\dagger)+\Big[6g_2^2+\frac{6}{5}g_1^2\Big]\mathbf{Y}_u^\dagger\mathbf{A}_u+\frac{4}{5}g_1^2\mathbf{Y}_d^\dagger\mathbf{A}_d\\
&-\Big[32g_3^2M_3+\frac{8}{5}g_1^2M_1\Big]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger)-\Big[12g_2^2M_2+\frac{4}{5}g_1^2M_1\Big]\mathbf{Y}_u^\dagger\mathbf{Y}_u\\
&-\frac{4}{5}g_1^2M_1\mathbf{Y}_d^\dagger\mathbf{Y}_d+\frac{64}{9}g_3^4M_3-16g_3^2g_2^2(M_3+M_2)-\frac{272}{45}g_3^2g_1^2(M_3+M_1)\\
&-30g_2^4M_2-2g_2^2g_1^2(M_2+M_1)-\frac{5486}{225}g_1^4M_1\Big\}\,,
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(1)}_{\mathbf{A}_e}=\mathbf{A}_e\Big\{&\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger+\mathbf{Y}_e\mathbf{Y}_e^\dagger)+5\mathbf{Y}_e^\dagger\mathbf{Y}_e+\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu-3g_2^2-\frac{9}{5}g_1^2\Big\}+\\
\mathbf{Y}_e\Big\{&\mathrm{Tr}(6\mathbf{A}_d\mathbf{Y}_d^\dagger+2\mathbf{A}_e\mathbf{Y}_e^\dagger)+4\mathbf{Y}_e^\dagger\mathbf{A}_e+2\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu+6g_2^2M_2+\frac{18}{5}g_1^2M_1\Big\}\,,
\end{aligned}
\tag{86}
$$

$$
\begin{aligned}
\beta^{(2)}_{\mathbf{A}_e}=\mathbf{A}_e\Big\{&-\mathrm{Tr}(9\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger+3\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger+3\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger+\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger)\\
&-\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger+\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)-5\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger+\mathbf{Y}_e\mathbf{Y}_e^\dagger)\\
&-6\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e-2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu-4\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\\
&+\Big[16g_3^2-\frac{2}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger)+\frac{6}{5}g_1^2\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger)+\Big[12g_2^2-\frac{6}{5}g_1^2\Big]\mathbf{Y}_e^\dagger\mathbf{Y}_e\\
&+\frac{15}{2}g_2^4+\frac{9}{5}g_2^2g_1^2+\frac{27}{2}g_1^4\Big\}+\\
\mathbf{Y}_e\Big\{&-2\mathrm{Tr}(\,18\mathbf{A}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger+3\mathbf{A}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger+3\mathbf{A}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\\
&\qquad+6\mathbf{A}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger+\mathbf{A}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger+\mathbf{A}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger)\\
&-\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathrm{Tr}(6\mathbf{A}_u\mathbf{Y}_u^\dagger+2\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger)-6\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathrm{Tr}(3\mathbf{A}_d\mathbf{Y}_d^\dagger+\mathbf{A}_e\mathbf{Y}_e^\dagger)\\
&-\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu\mathrm{Tr}(6\mathbf{Y}_u\mathbf{Y}_u^\dagger+2\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)-4\mathbf{Y}_d^\dagger\mathbf{A}_d\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger+\mathbf{Y}_e\mathbf{Y}_e^\dagger)\\
&-6\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{A}_e-8\mathbf{Y}_e^\dagger\mathbf{A}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e-4\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\\
&-4\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu-4\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e-2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{A}_e\\
&+\Big[32g_3^2-\frac{4}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{A}_d\mathbf{Y}_d^\dagger)+\frac{12}{5}g_1^2\mathrm{Tr}(\mathbf{A}_e\mathbf{Y}_e^\dagger)+\Big[6g_2^2+\frac{6}{5}g_1^2\Big]\mathbf{Y}_e^\dagger\mathbf{A}_e\\
&-\Big[32g_3^2M_3-\frac{4}{5}g_1^2M_1\Big]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger)-\frac{12}{5}g_1^2M_1\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger)-12g_2^2M_2\mathbf{Y}_e^\dagger\mathbf{Y}_e\\
&-30g_2^4M_2-\frac{18}{5}g_2^2g_1^2(M_1+M_2)-54g_1^4M_1\Big\}\,,
\end{aligned}
\tag{87}
$$

$$
\begin{aligned}
\beta^{(1)}_{\mathbf{A}_\nu}=\mathbf{A}_\nu\Big\{&\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger+\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)+5\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu+\mathbf{Y}_e^\dagger\mathbf{Y}_e-3g_2^2-\frac{3}{5}g_1^2\Big\}+\\
\mathbf{Y}_\nu\Big\{&\mathrm{Tr}(6\mathbf{A}_u\mathbf{Y}_u^\dagger+2\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger)+4\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu+2\mathbf{Y}_e^\dagger\mathbf{A}_e+6g_2^2M_2+\frac{6}{5}g_1^2M_1\Big\}\,,
\end{aligned}
\tag{88}
$$

$$
\begin{aligned}
\beta^{(2)}_{\mathbf{A}_\nu} = \mathbf{A}_\nu \Big\{ & -\mathrm{Tr}(9\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + 3\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + \mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger) \\
& - \mathbf{Y}_e^\dagger\mathbf{Y}_e\mathrm{Tr}(3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - 5\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger) \\
& - 6\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - 2\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e - 4\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu \\
& + \Big[16g_3^2 + \frac{4}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + \Big[12g_2^2 + \frac{12}{5}g_1^2\Big]\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + \frac{6}{5}g_1^2\mathbf{Y}_e^\dagger\mathbf{Y}_e \\
& + \frac{15}{2}g_2^4 + \frac{9}{5}g_2^2g_1^2 + \frac{207}{50}g_1^4\Big\} + \\
\mathbf{Y}_\nu \Big\{ & -2\mathrm{Tr}(\,18\mathbf{A}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{A}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + 3\mathbf{A}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger \\
& \qquad + 6\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + \mathbf{A}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger + \mathbf{A}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger) \\
& - 6\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathrm{Tr}(3\mathbf{A}_u\mathbf{Y}_u^\dagger + \mathbf{A}_\nu\mathbf{Y}_\nu^\dagger) - \mathbf{Y}_e^\dagger\mathbf{Y}_e\mathrm{Tr}(6\mathbf{A}_d\mathbf{Y}_d^\dagger + 2\mathbf{A}_e\mathbf{Y}_e^\dagger) \\
& - 4\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger) - \mathbf{Y}_e^\dagger\mathbf{A}_e\mathrm{Tr}(6\mathbf{Y}_d\mathbf{Y}_d^\dagger + 2\mathbf{Y}_e\mathbf{Y}_e^\dagger) \\
& - 6\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu - 8\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - 4\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{A}_e - 4\mathbf{Y}_e^\dagger\mathbf{A}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e \\
& - 2\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu - 4\mathbf{Y}_e^\dagger\mathbf{A}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + \Big[32g_3^2 + \frac{8}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{A}_u\mathbf{Y}_u^\dagger) \\
& + \Big[6g_2^2 + \frac{6}{5}g_1^2\Big]\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu + \frac{12}{5}g_1^2\mathbf{Y}_e^\dagger\mathbf{A}_e - \Big[32g_3^2M_3 + \frac{8}{5}g_1^2M_1\Big]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) \\
& - \Big[12g_2^2M_2 + \frac{12}{5}g_1^2M_1\Big]\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - \frac{12}{5}g_1^2M_1\mathbf{Y}_e^\dagger\mathbf{Y}_e \\
& - 30g_2^4M_2 - \frac{18}{5}g_2^2g_1^2(M_2+M_1) - \frac{414}{25}g_1^4M_1\Big\}\,,
\end{aligned}
\tag{89}
$$

$$
\beta^{(1)}_B = B\Big\{\mathrm{Tr}(3\mathbf{Y}_u\mathbf{Y}_u^\dagger + 3\mathbf{Y}_d\mathbf{Y}_d^\dagger + \mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + \mathbf{Y}_e\mathbf{Y}_e^\dagger) - 3g_2^2 - \frac{3}{5}g_1^2\Big\} + \mu\Big\{\mathrm{Tr}(6\mathbf{A}_u\mathbf{Y}_u^\dagger + 6\mathbf{A}_d\mathbf{Y}_d^\dagger + 2\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger + 2\mathbf{A}_e\mathbf{Y}_e^\dagger) + 6g_2^2M_2 + \frac{6}{5}g_1^2M_1\Big\}\,,
\tag{90}
$$

$$
\begin{aligned}
\beta^{(2)}_B = B\Big\{ & -\mathrm{Tr}(\,9\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + 9\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + 6\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger \\
& \qquad + 3\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + 3\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger + 2\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger) \\
& + \Big[16g_3^2 + \frac{4}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) + \Big[16g_3^2 - \frac{2}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger) \\
& + \frac{15}{2}g_2^4 + \frac{9}{5}g_1^2g_2^2 + \frac{207}{50}g_1^4\Big\} + \\
\mu\Big\{ & -4\mathrm{Tr}(\,9\mathbf{A}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_u^\dagger + 9\mathbf{A}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_d^\dagger + 3\mathbf{A}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger + 3\mathbf{A}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger \\
& \qquad + 3\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger + 3\mathbf{A}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_e^\dagger + \mathbf{A}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{Y}_\nu^\dagger + \mathbf{A}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_e^\dagger) \\
& + \Big[32g_3^2 + \frac{8}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{A}_u\mathbf{Y}_u^\dagger) + \Big[32g_3^2 - \frac{4}{5}g_1^2\Big]\mathrm{Tr}(\mathbf{A}_d\mathbf{Y}_d^\dagger) + \frac{12}{5}g_1^2\mathrm{Tr}(\mathbf{A}_e\mathbf{Y}_e^\dagger) \\
& - \Big[32g_3^2M_3 + \frac{8}{5}g_1^2M_1\Big]\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger) - \Big[32g_3^2M_3 - \frac{4}{5}g_1^2M_1\Big]\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger)
\end{aligned}
\tag{91}
$$

$$- \frac{12}{5} g_1^2 M_1 \mathrm{Tr}(\mathbf{Y}_e \mathbf{Y}_e^\dagger) - 30 g_2^4 M_2 - \frac{18}{5} g_1^2 g_2^2 (M_1 + M_2) - \frac{414}{25} g_1^4 M_1 \Bigg\} \,,$$

$$\beta^{(1)}_{\mathbf{B_M}} = \mathbf{B_M} \Big[2\mathbf{Y}_\nu^* \mathbf{Y}_\nu^T \Big] + \mathbf{M} \Big[4\mathbf{Y}_\nu^* \mathbf{A}_\nu^T \Big] + \Big[2\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \Big] \mathbf{B_M} + \Big[4\mathbf{A}_\nu \mathbf{Y}_\nu^\dagger \Big] \mathbf{M} \,, \tag{92}$$

$$\begin{aligned}
\beta^{(2)}_{\mathbf{B_M}} = \mathbf{B_M} \Big[& - 2\mathbf{Y}_\nu^* \mathbf{Y}_e^T \mathbf{Y}_e^* \mathbf{Y}_\nu^T - 2\mathbf{Y}_\nu^* \mathbf{Y}_\nu^T \mathbf{Y}_\nu^* \mathbf{Y}_\nu^T - 2\mathbf{Y}_\nu^* \mathbf{Y}_\nu^T \mathrm{Tr}(3\mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) \\
& + \frac{6}{5} g_1^2 \mathbf{Y}_\nu^* \mathbf{Y}_\nu^T + 6 g_2^2 \mathbf{Y}_\nu^* \mathbf{Y}_\nu^T \Big] + \\
\mathbf{M} \Big[& - 4\mathbf{Y}_\nu^* \mathbf{A}_e^T \mathbf{Y}_e^* \mathbf{Y}_\nu^T - 4\mathbf{Y}_\nu^* \mathbf{A}_\nu^T \mathbf{Y}_\nu^* \mathbf{Y}_\nu^T - 4\mathbf{Y}_\nu^* \mathbf{Y}_\nu^T \mathrm{Tr}(3\mathbf{A}_u \mathbf{Y}_u^\dagger + \mathbf{A}_\nu \mathbf{Y}_\nu^\dagger) \\
& - 4\mathbf{Y}_\nu^* \mathbf{Y}_e^T \mathbf{Y}_e^* \mathbf{A}_\nu^T - 4\mathbf{Y}_\nu^* \mathbf{Y}_\nu^T \mathbf{Y}_\nu^* \mathbf{A}_\nu^T - 4\mathbf{Y}_\nu^* \mathbf{A}_\nu^T \mathrm{Tr}(3\mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) \\
& + \frac{12}{5} g_1^2 \mathbf{Y}_\nu^* \mathbf{A}_\nu^T + 12 g_2^2 \mathbf{Y}_\nu^* \mathbf{A}_\nu^T - \frac{12}{5} g_1^2 M_1 \mathbf{Y}_\nu^* \mathbf{Y}_\nu^T - 12 g_2^2 M_2 \mathbf{Y}_\nu^* \mathbf{Y}_\nu^T \Big] + \\
\Big[& - 2\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_\nu^\dagger - 2\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger - 2\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathrm{Tr}(3\mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) \\
& + \frac{6}{5} g_1^2 \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger + 6 g_2^2 \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \Big] \mathbf{B_M} + \\
\Big[& - 4\mathbf{Y}_\nu \mathbf{Y}_e^\dagger \mathbf{A}_e \mathbf{Y}_\nu^\dagger - 4\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{A}_\nu \mathbf{Y}_\nu^\dagger - 4\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathrm{Tr}(3\mathbf{A}_u \mathbf{Y}_u^\dagger + \mathbf{A}_\nu \mathbf{Y}_\nu^\dagger) \\
& - 4\mathbf{A}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_\nu^\dagger - 4\mathbf{A}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger - 4\mathbf{A}_\nu \mathbf{Y}_\nu^\dagger \mathrm{Tr}(3\mathbf{Y}_u \mathbf{Y}_u^\dagger + \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger) \\
& + \frac{12}{5} g_1^2 \mathbf{A}_\nu \mathbf{Y}_\nu^\dagger + 12 g_2^2 \mathbf{A}_\nu \mathbf{Y}_\nu^\dagger - \frac{12}{5} g_1^2 M_1 \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger - 12 g_2^2 M_2 \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \Big] \mathbf{M} \,.
\end{aligned} \tag{93}$$

References

[1] R. J. Davis, D. S. Harmer and K. C. Hoffman, Phys. Rev. Lett. **20** (1968) 1205.

[2] J. N. Abdurashitov *et al.* [SAGE Collaboration], J. Exp. Theor. Phys. **95** (2002) 181 [Zh. Eksp. Teor. Fiz. **122** (2002) 211].

[3] W. Hampel *et al.* [GALLEX Collaboration], Phys. Lett. B **447** (1999) 127.

[4] M. Altmann *et al.* [GNO Collaboration], Phys. Lett. B **616** (2005) 174.

[5] Y. Fukuda *et al.* [Kamiokande Collaboration], Phys. Rev. Lett. **77** (1996) 1683.

[6] M. B. Smy *et al.* [Super-Kamiokande Collaboration], Phys. Rev. D **69** (2004) 011104.

[7] Borexino Collaboration, Phys. Lett. B **658**, 101 (2008).

[8] Q. R. Ahmad *et al.* [SNO Collaboration], Phys. Rev. Lett. **87** (2001) 071301; Phys. Rev. Lett. **89**, 011301 (2002); Phys. Rev. Lett. **89** (2002) 011302.

[9] K. Eguchi *et al.* [KamLAND Collaboration], Phys. Rev. Lett. **90** (2003) 021802.

[10] R. Becker-Szendy *et al.*, Phys. Rev. D **46** (1992) 3720.

[11] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **81** (1998) 1562.

[12] M. C. Sanchez *et al.* [Soudan 2 Collaboration], Phys. Rev. D **68** (2003) 113004.

[13] M. Ambrosio *et al.* [MACRO Collaboration], Phys. Lett. B **517** (2001) 59.

[14] E. Aliu *et al.* [K2K Collaboration], Phys. Rev. Lett. **94** (2005) 081802.

[15] D. G. Michael *et al.* [MINOS Collaboration], Phys. Rev. Lett. **97** (2006) 191801.

[16] K. Abe *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **97** (2006) 171801.

[17] S. Weinberg, Phys. Rev. Lett. **43** (1979) 1566.

[18] S. M. Bilenkii, S. T. Petcov and B. Pontecorvo, Phys. Lett. B **67** (1977) 309; T. P. Cheng and L. Li, Phys. Rev. Lett. **45** (1980) 1908; W. J. Marciano and A. I. Sanda, Phys. Lett. B **67** (1977) 303; B. W. Lee, S. Pakvasa, R. E. Shrock and H. Sugawara, Phys. Rev. Lett. **38** (1977) 937 [Erratum-ibid. **38** (1977) 937].

[19] M. L. Brooks *et al.* [MEGA Collaboration], Phys. Rev. Lett. **83** (1999) 1521; M. Ahmed *et al.* [MEGA Collaboration], Phys. Rev. D **65** (2002) 112002.

[20] T. Mori *et al.* "Search for $\mu \to e\gamma$ Down to 10^{-14} Branching Ratio". Research Proposal to Paul Scherrer Institut. See also http://meg.web.psi.ch/

[21] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **96** (2006) 041801.

[22] A. G. Akeroyd *et al.* [SuperKEKB Physics Working Group], arXiv:hep-ex/0406071.

[23] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **95** (2005) 041802.

[24] K. Hayasaka *et al.* [Belle Collaboration], arXiv:0705.0650 [hep-ex].

[25] P. Minkowski, Phys. Lett. B **67** (1977) 421. M. Gell-Mann, P. Ramond and R. Slansky, *Proceedings of the Supergravity Stony Brook Workshop*, New York 1979, eds. P. Van Nieuwenhuizen and D. Freedman; T. Yanagida, *Proceedinds of the Workshop on Unified Theories and Baryon Number in the Universe*, Tsukuba, Japan 1979, eds. A. Sawada and A. Sugamoto; R. N. Mohapatra, G. Senjanovic, *Phys.Rev.Lett.* **44** (1980)912, *ibid. Phys.Rev.* **D23** (1981) 165; S. L. Glashow, *The Future Of Elementary Particle Physics*, *In *Cargese 1979, Proceedings, Quarks and Leptons*, 687-713 and Harvard Univ.Cambridge - HUTP-79-A059 (79,REC.DEC.) 40p.* J. Schechter and J. W. F. Valle, Phys. Rev. D **22** (1980) 2227.

[26] J. A. Casas, V. Di Clemente, A. Ibarra and M. Quiros, Phys. Rev. D **62** (2000) 053005; J. A. Casas, J. R. Espinosa and I. Hidalgo, JHEP **0411** (2004) 057.

[27] S. Dimopoulos and H. Georgi, Nucl. Phys. B **193** (1981) 150; J. R. Ellis and D. V. Nanopoulos, Phys. Lett. B **110** (1982) 44; R. Barbieri and R. Gatto, Phys. Lett. B **110** (1982) 211; M. J. Duncan, Nucl. Phys. B **221** (1983) 285; J. F. Donoghue, H. P. Nilles and D. Wyler, Phys. Lett. B **128**, 55 (1983); A. Bouquet, J. Kaplan and C. A. Savoy, Phys. Lett. B **148** (1984) 69; L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B **267** (1986) 415.

[28] F. Gabbiani and A. Masiero, Nucl. Phys. B **322** (1989) 235; J. S. Hagelin, S. Kelley and T. Tanaka, Nucl. Phys. B **415** (1994) 293; D. Choudhury, F. Eberlein, A. Konig, J. Louis and S. Pokorski, Phys. Lett. B **342** (1995) 180; B. de Carlos, J. A. Casas and J. M. Moreno, Phys. Rev. D **53**, 6398 (1996); F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B **477** (1996) 321; I. Masina and C. A. Savoy, Nucl. Phys. B **661**, 365 (2003); P. Paradisi, JHEP **0510** (2005) 006.

[29] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986).

[30] J. Hisano, D. Nomura and T. Yanagida, Phys. Lett. B **437** (1998) 351; A. Masiero, S. K. Vempati and O. Vives, Nucl. Phys. B **649** (2003) 189 L. Calibbi, A. Faccia, A. Masiero and S. K. Vempati, Phys. Rev. D **74** (2006) 116002; C. H. Albright and M. C. Chen, arXiv:0802.4228 [hep-ph].

[31] M. E. Gomez, G. K. Leontaris, S. Lola and J. D. Vergados, Phys. Rev. D **59**, 116009 (1999); W. Buchmuller, D. Delepine and L. T. Handoko, Nucl. Phys. B **576** (2000) 445; W. Buchmuller, D. Delepine and F. Vissani, Phys. Lett. B **459** (1999) 171; J. Sato and K. Tobe, Phys. Rev. D **63** (2001) 116010.

[32] A. Kageyama, S. Kaneko, N. Shimoyama and M. Tanimoto, Phys. Rev. D **65** (2002) 096010; M. Raidal and A. Strumia, Phys. Lett. B **553** (2003) 72; A. Ibarra and G. G. Ross, Phys. Lett. B **591** (2004) 285.

[33] S. F. King and M. Oliveira, Phys. Rev. D **60** (1999) 035003; J. R. Ellis and O. Lebedev, Phys. Lett. B **653** (2007) 411.

[34] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D **53** (1996) 2442. J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, Phys. Lett. B **357** (1995) 579.

[35] J. Hisano and D. Nomura, Phys. Rev. D **59** (1999) 116005.

[36] J. A. Casas and A. Ibarra, Nucl. Phys. B **618** (2001) 171.

[37] S. Lavignac, I. Masina and C. A. Savoy, Phys. Lett. B **520**, 269 (2001); T. Blazek and S. F. King, Nucl. Phys. B **662** (2003) 359; S. Pascoli, S. T. Petcov and C. E. Yaguna, Phys. Lett. B **564** (2003) 241; S. T. Petcov, T. Shindou and Y. Takanishi, Nucl. Phys. B **738** (2006) 219; S. T. Petcov, W. Rodejohann, T. Shindou and Y. Takanishi, Nucl. Phys. B **739** (2006) 208; E. Arganda and M. J. Herrero, Phys. Rev. D **73** (2006) 055003; G. C. Branco, A. J. Buras, S. Jager, S. Uhlig and A. Weiler, JHEP **0709** (2007) 004.

[38] S. T. Petcov, S. Profumo, Y. Takanishi and C. E. Yaguna, Nucl. Phys. B **676** (2004) 453.

[39] S. Davidson and A. Ibarra, JHEP **0109** (2001) 013. See also J. R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Rev. D **66** (2002) 115013.

[40] A. Ibarra, JHEP **0601** (2006) 064.

[41] A. Czarnecki and E. Jankowski, Phys. Rev. D **65** (2002) 113004.

[42] B. C. Allanach *et al.*, in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. N. Graf, *In the Proceedings of APS / DPF / DPB Summer Study on the Future of Particle Physics (Snowmass 2001), Snowmass, Colorado, 30 Jun - 21 Jul 2001, pp P125* [arXiv:hep-ph/0202233].

[43] B. C. Allanach, S. Kraml and W. Porod, JHEP **0303** (2003) 016.

[44] A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **99** (2007) 182002.

[45] A. Ibarra, T. Shindou, C. Simonetto, to appear.

[46] N. Haba, N. Okamura and M. Sugiura, Prog. Theor. Phys. **103** (2000) 367; J. A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, Phys. Rev. D **63**, 097302 (2001), Nucl. Phys. B **569** (2000) 82; S. Antusch and M. Ratz, JHEP **0207** (2002) 059.

[47] S. P. Martin and M. T. Vaughn, Phys. Rev. D **50** (1994) 2282.