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Racetrack Inflation and Cosmic Strings

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Abstract. We consider the coupling of racetrack inflation to matter fields as realised in the D3/D7 brane system. In particular, we investigate the possibility of cosmic string formation in this system. We find that string formation before or at the onset of racetrack inflation is possible, but they are then inflated away. Furthermore, string formation at the end of inflation is prevented by the presence of the moduli sector. As a consequence, no strings survive racetrack inflation.

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1. Introduction

Racetrack inflation is a promising inflationary model, which can be realised within string theory [1, 2]. In this scenario, inflation is driven by Kähler moduli fields. The original string theory scenario is based on the KKLT mechanism for moduli fixing, extending it to include a racetrack-type superpotential, i.e. the superpotential contains more than one exponential of the Kähler modulus field [1]. In a later development, based on an explicit compactification of type IIB string theory, racetrack inflation with two complex Kähler moduli has been considered [2]. In these papers, the KKLT set-up has been used, in which a flux potential stabilises the dilaton and the complex structure moduli. A non-trivial potential for the Kähler moduli is generated by non-perturbative effects (for example via gaugino condensation). This alone results in an Anti-de Sitter (AdS) vacuum, which is then uplifted by the presence of anti-branes, which break supersymmetry explicitly. In [3] a racetrack model was constructed, in which the uplifting is obtained by D-terms, as suggested in [4, 5].

In all cases the possible presence of additional fields during racetrack inflation has been ignored (apart from [3], where additional meson fields on D7 branes have been included). In supergravity, interactions between the Kähler modulus field and other fields are inevitable and one may wonder whether the dynamics of the modulus field are slightly modified. With the meson field considered in [3], the features of racetrack inflation were not significantly altered. Additionally, topological defects, such as cosmic strings, may form during or after inflation. In this paper, we extend the racetrack inflation scenario and study the impact of such additional fields on inflation. The model we consider is inspired by the D3/D7 system in type IIB string theory compactified on $K3 \times T^2/Z_2$ [6]. On top of a Kähler modulus field T, the system contains a neutral field ϕ, describing the interbrane distance and two charged fields $\phi^{\pm}$, describing open strings stretching between the D3 and the D7 brane. This construction is the stringy analogue of D-term hybrid inflation. The main difference we introduce here is that inflation is due to the racetrack sector and is not driven by the interbrane potential. Cosmic strings are formed when the charged fields condense. We investigate whether the formation of these strings is still possible in this scenario.

The condensation scale depends on a Fayet-Iliopoulos (FI) term which originates from a non-trivial flux on D7 branes. In the low energy supergravity description, such FI terms are either constant when a $U(1)_R$ gauged R-symmetry is present or field dependent when the $U(1)$ gauge symmetry is pseudo-anomalous. Here the structure of the superpotential, after the dilaton and the complex structure moduli have been stabilised by fluxes, prevents the existence of the R-symmetry. Pseudo-anomalous symmetries with a field dependent FI term exist and can be used to uplift the potential [3]. D-term cosmic strings necessitate the introduction of either two FI terms, one for uplifting and one for condensation‡ or a single FI term together with an uplifting anti-brane. In fact, we will see that even when no FI term is introduced the condensation of the charged fields can be triggered by the F-term potential. This is a purely supergravity effect; the U(1) symmetry breaking is induced by the moduli sector. If this occurs the F-term strings that form during the phase transition have a tension that depends on the vacuum expectation values of the moduli fields. In all cases, we find that either strings are inflated away when created before or during inflation, or they are dynamically prevented from forming in the first place.

The paper is organised as follows. In Section 2 we describe a simple toy model in which cosmic strings form, and whose string tension is field dependent. In Section 3 we review the setup of racetrack inflation. In Section 4 we couple the racetrack model to matter fields and study the dynamics of the fields during inflation. We discuss the conditions under which cosmic strings form. We discuss and summarise our findings in Section 5.

‡ With only one FI term, the uplifting term is removed by the charged field condensation.

2. Field dependent F-term strings

In this section, we describe a simple situation where field dependent cosmic strings may form. By "field dependent" we mean that the string tension depends on the vacuum expectation values (VEVs) of other fields in the theory. Cosmic strings form during a phase transition if the vacuum manifold has non-contractible loops, as is the case in a U(1) symmetry breaking.

A U(1)-breaking phase transition will occur in a supersymmetric theory with the superpotential and D-term potentials

$$W = \lambda\Phi(\Phi^+\Phi^- - x^2)\,, \qquad V_D = \frac{g^2}{2}\left(|\Phi^+|^2 - |\Phi^-|^2 - \xi\right)^2\,, \tag{1}$$

where λ is a coupling constant, g the U(1) gauge coupling, x a mass parameter, and ξ a Fayet-Iliopolous term. The fields $\Phi^\pm$ are oppositely charged, while the third field Φ is neutral. If the value of Φ is below some critical value, the charged fields will have a tachyonic instability and will condense. For a F-term driven model with no FI term, $\Phi^+\Phi^- = x^2$ in the true vacuum. For D-term driven models $x = 0$, and the global minimum has $\Phi^- = 0$ and $\Phi^+ = \sqrt{\xi}$. In both cases the U(1) symmetry breaking produces cosmic strings. We will refer to these two types of strings as F-term and D-term strings§.

We note that apart from symmetry breaking, the above theory can also give rise to hybrid inflation. In this paper we will be mainly be interested in models in which inflation is produced by a different sector of the full theory. However some features of supersymmetric hybrid inflation will be relevant to our analysis, so we will briefly review it here. In hybrid inflation models Φ is the inflaton. For $\langle\Phi\rangle$ larger than the critical value, the potential has a valley of local minima where inflation takes place. The U(1) gauge symmetry is unbroken and the "waterfall" fields $\Phi^\pm$ are zero. At tree level the inflationary direction is flat, but it is lifted by loop corrections which induce the slow rolling of the inflaton. When Φ falls below the critical value, the phase transition described above ends inflation, and cosmic strings form.

Finding a non-vanishing constant x-term in string theory has proved to be difficult. Instead an FI term, and thus D-term symmetry breaking, is easily realised by a D3/D7 system. This, and the corresponding hybrid inflation model, is described in [6]. When moduli fields are introduced, the presence of a constant FI term is forbidden‖. A moduli dependent FI term is required instead, leading to the formation of field dependent D-term strings. However, as was shown in [7], in this set-up it is hard to combine inflation and moduli stabilisation in a working model. In fact, the problems of combining inflation with moduli stabilisation occur in a wide range of models [8], and are not restricted to D-term hybrid inflation.

§ Not to be confused with the stringy usage of F- and D-strings, which refer to fundamental and Dirichlet strings respectively.

‖ In supergravity a constant FI term can only be introduced if the theory is invariant under a gauged R-symmetry. However the constant W_0 in the KKLT moduli superpotential breaks this R-symmetry.

In the following we describe a set-up in which an effective x-term is induced by the presence of the moduli sector. This overcomes the difficulty of finding a constant x-parameter. Just as in hybrid inflation models we assume a shift-symmetric Kähler potential for the gauge singlet [9]. In our set-up inflation is not driven by the gauge singlet but by the moduli fields, implemented in the form of racetrack inflation. In the remainder of this section we outline the basic idea.

Our set-up is loosely based on the D3/D7-matter system in type IIB string theory compactified on $K3 \times T^2/Z_2$ [6]. Although we will not derive an exact correspondence, we will say a bit more on this later on. For now, just consider a supergravity theory with the following super- and Kähler potentials

$$W = \lambda \Phi \Phi^+ \Phi^- , \qquad K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |\Phi_+|^2 + |\Phi_-|^2 . \tag{2}$$

Note that there is no constant x-term in the superpotential. We embed this model in a supersymmetry breaking ($m_{3/2} \neq 0$) and moduli stabilised background. The moduli scalar potential comprises F- and uplifting terms such that $V_{\rm mod} \sim H_*^2$ during inflation and $V_{\rm mod} \sim 0$ in the Minkowski vacuum after inflation (the above relations are up to corrections coming from the matter sector). Here we have defined H_* as the Hubble rate during inflation. We assume that the matter sector parameterised by Φ and $\Phi^\pm$ is a small perturbation to the moduli dynamics, i.e. $\Phi,\ \Phi^\pm \ll 1$. In this limit, for $\Phi = 0$, the potential reads

$$\begin{aligned} V = {} & \frac{g^2}{2}\left(|\Phi^+|^2 - |\Phi^-|^2\right)^2 + V_{\rm mod}^F + V_{\rm up} + {\rm e}^{K_{\rm mod}} \lambda^2 |\Phi^+|^2 |\Phi^-|^2 \\ & + \left(\frac{V_{\rm mod}^F}{2} + m_{3/2}^2\right)\left(|\Phi^+|^2 + |\Phi^-|^2\right)^2 + \left(V_{\rm mod}^F + m_{3/2}^2\right)\left(|\Phi^+|^2 + |\Phi^-|^2\right) \end{aligned} \tag{3}$$

with $K_{\rm mod}$ the Kähler potential of the moduli sector, and $m_{3/2}$ is the gravitino mass (defined in the absence of the matter fields). The moduli stabilisation potential is $V_{\rm mod} = V_{\rm mod}^F + V_{\rm up}$, where $V_{\rm mod}^F$ and $V_{\rm up}$ are respectively the F- and uplifting terms. Note that $V_{\rm mod}^F < 0$. For $2m_{3/2}^2 > -V_{\rm mod}^F > m_{3/2}^2$, the final three terms of the above potential resemble the usual symmetry breaking terms that are generated by a non-zero x or FI term. Since these terms arise in the F-term potential, it appears that an effective field-dependent x-term is generated.

The dynamics of the model are reminiscent of what happens in hybrid inflation (remember that in our set-up, the moduli sector is responsible for inflation, and not the matter fields). The $\phi = {\rm Re}(\Phi)$ direction is flat at tree level, and only lifted by loop corrections. If the loop corrections are small, the ϕ-field is frozen during inflation. The masses of the waterfall fields are ϕ-dependent. If initially $\phi > \phi_c$ with ϕ_c some critical value, the charged fields are minimised at $\Phi^\pm = 0$ and the U(1) symmetry is unbroken. Some time after inflation, when $H \sim \partial_\phi^2 V_{\rm loop}$, the ϕ field starts rolling down its potential dropping below the critical value. In the usual hybrid inflation this triggers the U(1) breaking phase transition. In our set-up the situation is more complicated, and as we will see whether the waterfall fields actually condense at low energies depends on the specifics of the moduli sector.

The complete model will be studied in section 4, where both the inflaton/moduli and the matter sectors are treated dynamically. As discussed above, this full treatment is needed to determine whether cosmic strings can form.

3. Racetrack Inflation

In this section we will briefly review racetrack inflation in a supergravity setting. The original racetrack model [1] is formulated in a flux compactification of type IIB string on a Calabi-Yau space. In the low energy effective action, there is only the volume modulus field T, with a no-scale Kähler potential

$$K^{\rm RT} = -3\log(T+\bar{T})\,. \tag{4}$$

The superpotential is of the modified racetrack form

$$W^{\rm RT} = W_0 + A{\rm e}^{-aT} + B{\rm e}^{-bT}\,. \tag{5}$$

The constant W_0 arises from integrating out the stabilised dilaton and the complex structure moduli. The non-perturbative terms come from gaugino condensation on D7 branes or from instanton effects; in both cases the parameters a, b depend on the specifics of the gauge group. In addition there is an uplifting term

$$V_{\rm up} = \frac{E}{(T+\bar{T})^n} \tag{6}$$

originating from an anti-D3 in the bulk ($n=3$) or in the throat ($n=2$). The constant E is tuned to get a Minkowski vacuum in the minimum after inflation.

The full potential is a series of cosines in Y, where we defined $T = X + {\rm i}Y$. Inflation takes place near a saddle point, which is unstable in the Y-direction but stable in the X-direction (in order that X does not run off to infinity during inflation). The overall scale of the potential is set by the WMAP normalisation and we find that the Hubble parameter is $H_* \sim m_{3/2} \sim 10^{-8}$ with $m_{3/2} = {\rm e}^{K/2}|W|$ the vacuum gravitino mass. This fixes the constant term of the superpotential $W_0 \sim 10^{-4} - 10^{-5}$. The spectral index $n_s \leq 0.95$, in good agreement with the latest WMAP data [10].

In the above set-up the uplifting term breaks SUSY explicitly. This can be cured by using instead an uplifting D-term [3, 4, 5]; in this case additional meson fields have to be introduced to make the potential gauge invariant. In the improved racetrack model [2], a set-up is discussed with two Kähler moduli. This all suggests that racetrack is very robust, and does not depend on the details of the uplifting or the superpotential (though at least two exponents are needed). This is in line with the observation made in [11] that saddle points are ubiquitous in the string landscape.

4. Cosmic Strings and Racetrack Inflation

In this section we discuss the dynamics of racetrack inflation in the presence of a matter sector. Our set-up are inspired by the D3/D7 system described in [6], in the context of

type IIB theory compactified on $K3 \times T^2/Z_2$. Consider a D7 and D3 brane which are located far from the gaugino condensation brane. Then the light D3/D7 matter fields are uncharged under the moduli sector symmetries. In the same vein we neglect the backreaction of the D3 on the geometry, which makes the A and B in the superpotential on the gaugino D7 brane dependent on the D3 position [12]. We thus assume that the racetrack fields couple only gravitationally to the matter fields.

The fields in the matter D3/D7 sector are a neutral field Φ representing the interbrane distance, and two oppositely charged fields $\Phi^{\pm}$ corresponding to strings stretching between the D3 and D7 branes. Approximate translational invariance of the brane system, which is a consequence of the background isometries, translates into a shift symmetry for the Φ field in the Kähler. For simplicity we assume canonical kinetic terms for the charged fields¶; we expect that more complicated Kähler potentials give qualitatively similar results.

For the moduli sector we use the no-scale modulus with a modified racetrack potential discussed in the previous section. Note that in the explicit example based on $K3 \times T^2/Z_2$, there are many more Kähler moduli [13]. In particular, in addition to the $K3$ volume modulus (that we are calling T), there is the volume modulus of the torus. We assume that all these additional moduli are stabilised by instanton effects [14]. If the torus is stabilised with the same radius as the $K3$ manifold the effective Kähler potential for T is of the no-scale form [9]. In any case, as we remarked in the previous section, racetrack inflation is very robust. In that spirit we can use the racetrack model (4, 5, 6) as a useful toy model for a possibly more complex set-up.

The model we study is then

$$K = K^{\rm RT} + K^{\rm m} = -3\log(T+\bar{T}) - \frac{1}{2}(\Phi-\bar{\Phi})^2 + |\Phi^+|^2 + |\Phi^-|^2\,, \tag{7}$$

$$W = W^{\rm RT} + W^{\rm m} = W_0 + A{\rm e}^{-aT} + B{\rm e}^{-bT} + \lambda\Phi\Phi^+\Phi^-\,. \tag{8}$$

The scalar potential reads

$$V = \frac{{\rm e}^{K^{\rm m}}}{(T+\bar{T})^3}\left(\frac{(T+\bar{T})^2}{3}|F_T|^2 + \sum_i |F_{\Phi_i}|^2 - 3|W|^2\right) + V_{\rm up} + \frac{g^2}{2}\left(|\Phi_+|^2 - |\Phi_-|^2\right)^2 \tag{9}$$

where

$$F_T = F_T^{\rm RT} - \frac{3}{T+\bar{T}}\lambda\Phi\Phi_+\Phi_-\,, \tag{10}$$

and

$$F_{\Phi^\pm} = \lambda\Phi^{\mp}\Phi + \bar{\Phi}^{\pm}(W^{\rm RT} + \lambda\Phi\Phi^+\Phi^-)\,, \tag{11}$$

$$F_\Phi \;\; = \lambda\Phi^+\Phi^- - (\Phi-\bar{\Phi})(W^{\rm RT} + \lambda\Phi\Phi^+\Phi^-)\,. \tag{12}$$

The matter fields $\Phi^{\pm}$ are oppositely charged under a U(1) symmetry. The corresponding D-term enforces $|\Phi_+| = |\Phi_-|$ in the minimum. By an overall phase rotation, W_0 can

¶ In a set-up with the D3 at a fixed point, or in the limit that the stack of D3s is heavy, we can treat the D7 as a probe brane. Then the matter fields have unit modular weight, and in the small field limit can be expanded to get minimal kinetic terms at lowest order.

be made real and positive. $\mathrm{Im}(T)$ adjusts to minimise $W^{\rm RT}$. Then there is an overall phase between $W^{\rm RT}$ and the matter potential $W^{\rm m}$. The charged fields can be rotated to be real, and the phase of λ can be absorbed in Φ. We can take any residual phase dependence to reside in Φ. We thus define the real fields

$$T = X + \mathrm{i}Y\,, \qquad \Phi = \phi + \mathrm{i}\alpha\,, \qquad \Phi^- = \Phi^+ = \phi^+\,. \tag{13}$$

The U(1) symmetry is unbroken for a zero VEV of the charged fields $\phi^+ = 0$. Cosmic strings form in a U(1) breaking phase transition. To see whether such a phase transition can take place during or after inflation, we consider the stability of the potential for $\phi^+ = 0$. We will assume that moduli stabilisation is not disrupted by the presence of the matter fields, and that the moduli are fixed at some value T_0. We will check this assertion numerically.

The potential is indeed extremised for $\phi^+ = 0$, i.e. $\partial_{\phi^+} V|_{\phi^+=0} = 0$. Whether this is a stable minimum, a saddle or a maximum depends on the mass matrix. The mass matrix is block diagonal in the charged fields and the neutral Φ field. Let us start with the latter first. As a consequence of the shift symmetry the ϕ direction is flat for $\phi^+ = 0$. The potential is extremised with respect to α, i.e. $\partial_\alpha V|_{\phi^+=0} = 0$, for $\alpha^2 = 0$ or $\alpha^2 = \alpha_0^2 \equiv -1/2 - V_{\rm RT}/(4m^2)$. The mass of α at these extrema is

$$m_\alpha^2 = \begin{cases} 2m^2(2-y) & \alpha = 0 \\ -4m^2 \mathrm{e}^{2\alpha_0^2}(2-y) & \alpha = \alpha_0 \end{cases} \tag{14}$$

where we defined

$$y = -V_{\rm RT}/m\,. \tag{15}$$

Here $V_{\rm RT}$ and $m = \mathrm{e}^{K^{\rm RT}/2}|W^{\rm RT}|$ are the racetrack potential and gravitino mass as defined in the absence of matter fields. In particular $m \approx m_{3/2} \equiv \mathrm{e}^{K/2}|W|$ up to small Φ_i dependent corrections, and with abuse of language we will sometimes refer to it as the gravitino mass. With D-term or D-brane uplifting, $y \approx 3$ after inflation and $\alpha = \alpha_0$ is the minimum. However, during inflation $y \approx 1$ and $\alpha = 0$ is the minimum. This implies that the phase field obtains a VEV during inflation. As we will see, since the α-field starts rolling only near the very end of inflation, it affects the inflationary results only mildly (keep also in mind that (14) is only valid for unbroken U(1) with $\phi^+ = 0$).

Consider now the bosonic mass matrix of the charged fields along the ϕ-flat direction. The diagonal entries of the mass matrix are

$$m^2_{\Phi^+\bar\Phi^+} = m^2_{\Phi^-\bar\Phi^-} = \mathrm{e}^{2\alpha^2}\left(V_{\rm RT} + (1+4\alpha^2)m^2 + \tilde\lambda^2|\Phi|^2\right), \tag{16}$$

and the off-diagonal term reads

$$m^2_{\Phi^+\Phi^-} = \frac{\tilde\lambda \mathrm{e}^{2\alpha^2}}{(T+\bar T)^{3/2}}\left(2\mathrm{i}\alpha\bar W^{\rm RT} - (T+\bar T)\partial_{\bar T}\bar W^{\rm RT}\Phi + \bar W^{\rm RT}(2+4\alpha^2)\Phi\right). \tag{17}$$

Here we defined the rescaled coupling via

$$\tilde\lambda = \frac{\lambda}{(T+\bar T)^{3/2}}\,. \tag{18}$$

The bosonic mass eigenstates are $m_\pm^2 = m^2_{\Phi^+\bar\Phi^+} \pm |m^2_{\Phi^+\Phi^-}|$. The fermion mass eigenstates are two Weyl fermions with masses $m_\psi^2 = \mathrm{e}^{2\alpha^2}\tilde\lambda^2|\Phi|^2$.

What does this imply for inflation? Consider first the situation at the beginning of inflation, near the racetrack saddle point. As discussed above $\alpha = 0$, and the mass matrix elements simplify to

$$m^2_{\Phi^+\bar\Phi^+} \approx (1-y)m^2 + \tilde\lambda^2\phi^2\,, \qquad m^2_{\Phi^+\Phi^-} \sim \tilde\lambda\phi m\,. \tag{19}$$

The $\phi_\pm = 0$ extremum is only stable for large ϕ

$$\phi > \phi_c \approx \frac{m}{\tilde\lambda}(1+\sqrt{4y-3}) \tag{20}$$

assuring both bosonic mass eigenstates to be positive definite. In terms of the original parameters in the potential $m/\tilde\lambda \sim W_0/\lambda$. In racetrack inflation $m \sim 10^{-8}$ which gives $\phi_c \ll 1$. If follows that the resulting behaviour will be very different depending on whether ϕ is smaller or larger than the critical value at the onset of inflation. The flat direction is lifted by loop corrections. If these corrections give a large mass to the ϕ field, $m_\phi > H_*$, the field will settle in the minimum, whereas in the opposite limit a large VEV may be expected.

The loop corrections originate from the splitting between the boson and fermion masses as a consequence of SUSY breaking. Notice that $\mathrm{Str}M^2$ is ϕ-independent (but does not vanish in supergravity) so the ϕ potential arises from the log correction

$$V = \frac{1}{32\pi^2}\Bigg\{(m^2_{\Phi^+\bar\Phi^+} - |m^2_{\Phi^+\Phi^-}|)^2 \ln\frac{(m^2_{\Phi^+\bar\Phi^+} - |m^2_{\Phi^+\Phi^-}|)}{\Lambda^2} + (m^2_{\Phi^+\bar\Phi^+} + |m^2_{\Phi^+\Phi^-}|)^2 \ln\frac{(m^2_{\Phi^+\bar\Phi^+} + |m^2_{\Phi^+\Phi^-}|)}{\Lambda^2} - 2m_\psi^4 \ln\frac{m_\psi^2}{\Lambda^2}\Bigg\}\,. \tag{21}$$

If ϕ (or α) is very large the mass splitting, and thus the potential, vanishes as in supersymmetry. Hence a far away D7 is weakly attracted by the D3. We can estimate the potential in the limit that $m^2 \ll \tilde\lambda^2(\phi^2+\alpha^2)$:

$$V \sim \frac{1}{32\pi^2}\Big\{\tilde\lambda^2(\phi^2+\alpha^2)m^2 + \mathrm{O}(m^4)\Big\}\,. \tag{22}$$

Then $m_\phi^2/H_*^2 \sim \tilde\lambda^2/(32\pi^2)$ is indeed small unless $\tilde\lambda$ approaches non-perturbative values. We therefore conclude that ϕ is light during inflation, and remains frozen. We generically expect $\phi > \phi_c$ at the onset of inflation. But since smaller VEVs are not excluded, we discuss both cases in turn.

4.1. Unbroken U(1)

Consider first the case that $\phi > \phi_c$, and the U(1) preserving phase $\phi^+ = 0$ is a minimum of the potential at the saddle. As discussed above the ϕ field is frozen during inflation. The evolution of the fields X, Y, α is shown in Figure 1. At the end of racetrack inflation, the phase field α becomes tachyonic and starts rolling towards the minimum $\alpha = \alpha_0$ starting from $\alpha = 0$ during racetrack inflation, as follows from (14). The potential near $\alpha = 0$ can be approximated as

$$V(\alpha) = V_0(1+\eta_\alpha\alpha^2)\,. \tag{23}$$

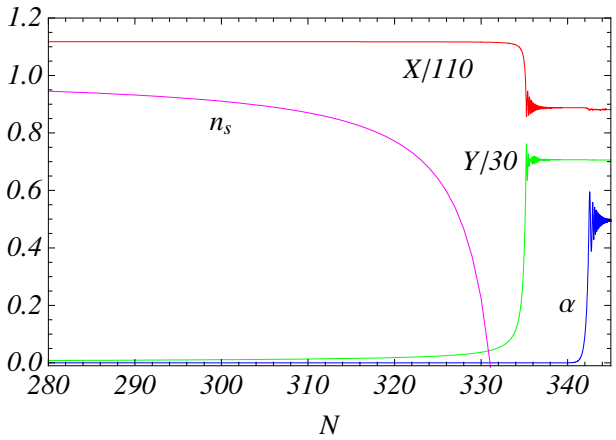

Figure 1. Evolution of the (rescaled) fields X, Y and α as function of the number of e-folds N since the beginning of inflation, for the case $\phi > \phi_c$ and the U(1) symmetry is preserved. The parameters are $A = 1/50$, $B = -7/200$, $a = \pi/50$, $b = \pi/45$, $W_0 = -3.5868 \times 10^{-5}$, $\lambda = 1$.

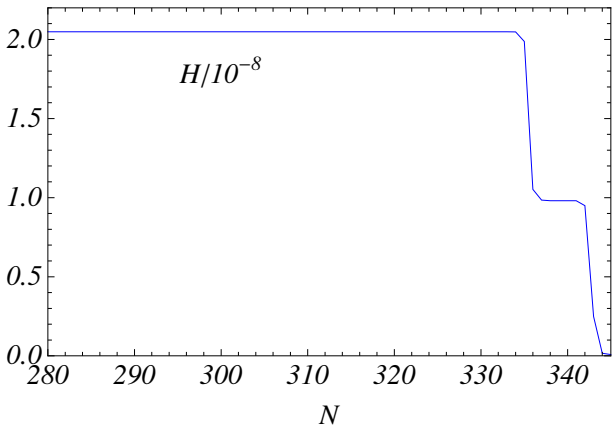

Figure 2. Evolution of the Hubble rate as a function of the number of e-folds N. The parameters are the same as in Figure 1.

where it should be noted that the canonically normalised field is $\alpha/\sqrt{2}$, and that $\eta_\alpha < 0$. This gives rise to a period of fast-roll α-driven inflation, which ends when α departs from the origin too much and feels the minimum of the potential. Figure 2 shows the Hubble constant during inflation, which is nearly constant both during racetrack and the following period of fast-roll inflation. Since $|\eta_\alpha| = \mathrm{O}(10)$ ($|\eta_\alpha| \approx 7$ for the parameters used in Figure 1) is rather large, this extra bout of inflation is short. The number of extra e-folds can be approximated by [15]

$$N \approx \frac{1}{F} \log\left(\frac{\alpha_0}{\alpha_*}\right), \qquad F = \sqrt{\frac{9}{4} + 3\eta_\alpha} - \frac{3}{2} \tag{24}$$

with $\alpha_0 \sim 0.5$ the value at the minimum, and α_* the initial value. During the period of racetrack inflation $\eta_\alpha \approx 1$ and the α-field is rapidly damped to zero. Thus we expect α_* to be small, set by the scale of quantum fluctuations $\alpha_* \sim H_* \sim 10^{-8}$. Plugging in the numbers gives $N_{\text{fast roll}} \sim 5$, in good agreement with the numerical results shown in

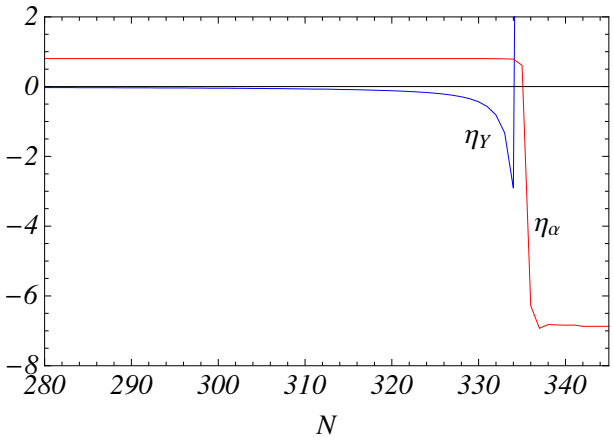

Figure 3. Plotted is η_Y and η_α parameter during inflation. Same parameters as in Figure 1.

Figures 1 – 3. This extra burst of fast-roll inflation lowers the spectral index, as now observable scales leave the horizon $N_* - 5$ e-folds before the end of the first period of racetrack inflation (instead of the usual $N_* \approx 55$). But the change is minimal, lowering n_s by less than 0.01.

In the minimum after inflation $\alpha^2 = \alpha_0^2 \approx 1/4$. The situation is now very different from the one at the saddle point. Indeed, the diagonal and off-diagonal terms of the bosonic mass matrix are now

$$m^2_{\Phi_+\bar{\Phi}_+} \approx \left(-m^2 + \tilde{\lambda}^2(\alpha^2 + \phi^2)\right), \qquad m^2_{\Phi+\Phi_-} \sim \tilde{\lambda}(\phi + \alpha)m \tag{25}$$

where in the off-diagonal term we neglected order one coefficients. It is clear that for $\phi < \alpha$ the VEV of the flat direction field ϕ will only affect the mass eigenstates at subleading order. The result is that the charged fields are non-tachyonic for all values of ϕ. There is no equivalent of ϕ_c as found at the saddle point. The minimum is U(1) preserving and stable.

To summarise, the U(1) matter symmetry is preserved to low energies if at the onset of inflation $\phi > \phi_c$. Racetrack inflation proceeds as before, but is followed by a short period of α-driven fast-roll inflation. This lowers the spectral index a little bit. No cosmic strings are formed at any time.

4.2. Broken U(1)

If the initial field value is small $\phi < \phi_c$ the charged fields condense towards a $U(1)$ breaking minimum with Φ and $\phi^+ = \phi_-$ non-zero. From the mass matrix (16, 17) it follows that the mass eigenstates approach $(1 - y)m$ in this limit. Since y is slightly bigger than one at the saddle (extracted from the numerics), this gives negative mass eigenstates, and $\eta_{\phi^+} \sim (1 - y) = -\mathrm{O}(0.1)$. Inflation still proceeds, but the U(1) is broken both during and after inflation. Any strings formed in the U(1) breaking phase transition are inflated away. Figures 4 and 5 show the evolution of the various fields during inflation.

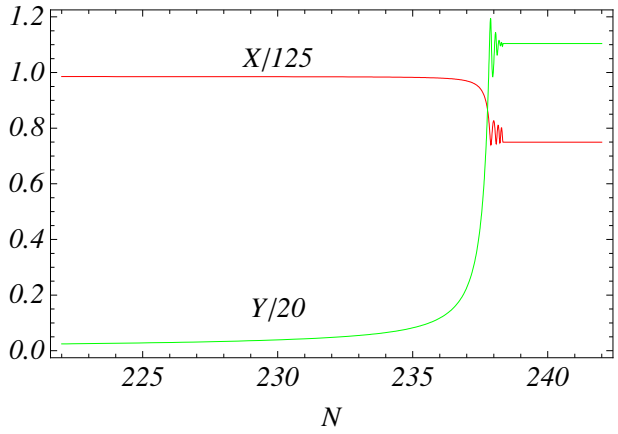

Figure 4. The evolution of the (rescaled) moduli fields X and Y during inflation for the case $\phi < \phi_c$. The parameters values are $A = 1/50$, $B = -7/200$, $a = \pi/50$, $b = \pi/45$, $W_0 = -1/25000 \times 10^{-5}$, $.\lambda = 1$.

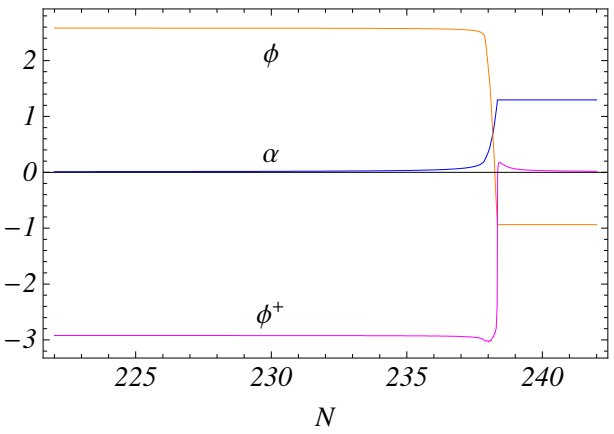

Figure 5. The evolution of the charged field ϕ^+ and α during inflation when $\phi < \phi_c$. The parameters values are the same as in Figure 4

The minimum after inflation can be found in the small field limit, by solving $F_{\Phi_i} = 0$. The result is that all matter fields $\phi, \alpha, \phi^\pm$ obtain a VEV of order $m/\tilde{\lambda}$. The system thus ends up in a different minimum depending on whether ϕ larger or smaller than ϕ_c initially. The U(1) preserving minimum with $\phi^\pm = 0$ is lower than the U(1) broken minimum discussed in this subsection, by an amount $\delta V \sim m^2$. The two minima are separated by a barrier.

4.3. F-term lifting

Up to now we have discussed a set-up where lifting was done by either D-terms or anti-D-branes, and $y > 1$ during and after inflation (see (15)). Depending on the initial conditions, we found that either the U(1) symmetry is never broken, or else breaking occurs before the onset of inflation and strings are inflated away. Since the y value determines the α-mass, and thus whether there is a critical value ϕ_c or not, it may be

interesting to consider F-term uplifting as well as it gives $y = 0$ in the vacuum (see e.g. [16] and references therein for F-term uplifting). Although no explicit model of racetrack inflation with F-term lifting exists, considering its robustness we expect it to be possible.

With F-term lifting $\alpha = 0$ is the minimum during and after inflation, and the mass matrix for the charge fields is well approximated by (19) throughout. One may be inclined to think that as ϕ drops below its critical value a U(1) breaking phase transition takes place. But this is not the case. As y drops below $3/4$, which is the case in the post-inflationary minimum where $y = 0$, the charged fields become non-tachyonic for all ϕ-values. This is reflected in (20) by the fact that ϕ_c becomes imaginary, i.e. non-existent.

The situation is thus similar to that for the D-term or D-brane lifting case. If $\phi > \phi_c$ initially the U(1) symmetry is preserved throughout, and no strings form. But since $\alpha = 0$ both during and after inflation, there is no period of α-driven fast-roll inflation. In the opposite limit $\phi < \phi_c$, the matter U(1) is broken throughout, and strings are inflated away.

5. Discussion

In this paper we discussed racetrack inflation coupled gravitationally to a matter sector with a U(1) symmetry. We found that cosmic strings, if formed at all, do not survive to low energies. But how generic are these results? After all, we assumed a matter sector with a shift symmetry for the neutral field, and we set possible FI terms to zero. Let us comment on possible modifications to this scenario.

First of all, we could have introduced a FI term in the D-term for the matter fields, i.e. we could have made the U(1) pseudo-anomalous. This can be implemented by making the modulus field transform under the U(1) (the U(1) transformation acts as a shift symmetry on T) [17]. To preserve gauge invariance of the moduli superpotential, additional charged meson fields are needed [5]. The upshot of all of this is that if the modulus and the meson fields are stabilised at some non-zero value, this generates an effective FI term for the matter fields. If the same D-term is used for uplifting, the model does not work as the charged fields will cancel the D-term and therefore remove the uplifting. On the other hand additional lifting terms could be introduced. For the sake of argument, let us introduce an anti-brane for uplifting, which does not depend on the matter fields. The D-term enforces Φ^- zero throughout. With a shift symmetry for the Φ field the F-term potential in the U(1) phase with $\phi^+ = 0$ is the same as the one we have investigated. The α minimum is also the same . The mass eigenvalues for the charged fields are now $m_{\Phi^+\Phi^+} \pm \sqrt{|m_{\Phi^+\Phi^-}|^2 + g^2\xi}$. The gauge coupling g and FI term ξ depend on the moduli fields. The F-term contribution to the mass terms is the same as for $\xi = 0$, and still given by (16,17). As a result, the whole discussion of the previous section follows through, although with slightly altered parameters. Therefore depending on initial values, the U(1) stays either broken or preserved, both during and

after inflation.

The shift symmetry is an essential part of the set-up we have discussed. Without it there is no flat direction. Performing the same analysis again with a canonical Kähler potential leads to the result that the U(1) symmetry is preserved to low energies for $\lambda \gtrsim W_0 \sim 10^{-5}$. The matter fields are minimised at $\phi^{\pm} = 0$ and $\phi \approx \sqrt{2}$. Racetrack inflation proceeds as before, with the same predictions for the power spectrum (of course the parameter values needed to tune $\eta \ll 1$ and to get a Minkowski minimum are slightly altered due to the presence of the matter fields).

It is less clear what happens in the small coupling limit $\lambda < W_0$, or equivalently $\tilde{\lambda} < m$. The situation is similar for all set-ups, with or without a shift symmetry or FI term. We find that the U(1) symmetry is broken. For the set-up discussed in section 4, this can be seen from (20), as for such small couplings the critical value ϕ_c is pushed above unity. The VEVs for the matter fields are large. In the small field limit we found $\phi^{\pm}, \phi, \alpha \sim W_0/\lambda$. Extrapolating, we expect field values of order one. As a result racetrack inflation is greatly affected. Numerically, we did not find a working model with Y the only unstable direction at the saddle point. This does not mean there is not such a saddle as the potential is complex with many extrema. It does show that if racetrack inflation works the parameters and field values are rather different from the case without the matter fields. This is rather surprising as in the $\lambda \to 0$ limit one expects the effects of the matter fields to be small. Although in this limit W no longer depend on Φ_i, but the Kähler potential and thus the scalar potential does. All in all, this case leads to large modifications of racetrack inflation, possibly even destroying it.

To conclude, in this paper we studied racetrack inflation augmented by a matter sector. The coupling between the moduli fields and the matter fields is only gravitational. The matter sector is inspired by a D3/D7 system, and consists of a neutral field Φ and two fields $\Phi^{\pm}$ with opposite charges under a U(1) symmetry; the moduli fields are neutral under this U(1). Due to a shift symmetry the $\phi = \mathrm{Re}(\Phi)$ direction is flat. The matter superpotential is $W^{\mathrm{m}} = \lambda \Phi \Phi^{+} \Phi^{-}$. We investigated whether the matter sector can affect inflation, and vice versa, and whether the moduli sector can induce interesting effects in the matter sector such as cosmic string formation.

The resulting scalar potential has both a U(1) preserving and breaking minimum, the symmetry breaking induced by supergravity effects due to the presence of the moduli sector. One may be led to think this can give rise to cosmic string formation. We investigated this possibility in this paper. The inflationary dynamics depend on initial conditions. In particular if ϕ is larger than some critical value at the onset of inflation, then the U(1) symmetry is preserved both during and after inflation. If lifting is done by D-terms or D-branes (and not by F-terms) racetrack inflation proceeds as before, but is followed by a short period of α-driven fast-roll inflation. This lowers the spectral index by a small amount, less than 0.01. No cosmic strings are formed at any time. In the opposite limit that ϕ is small initially, the U(1) symmetry is broken at all times. Any strings formed at early times are inflated away.

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