DESY 08-078

arXiv:0807.4206v1 [hep-ph] 26 Jul 2008

IFJPAN-IV-2008-3
HU-EP-08/22
DESY 08-078
June 2008

The running fine structure constant $\alpha(E)$ via the Adler function

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*Work supported in part by the EU grants MTKD-CT2004-510126 in partnership with the CERN Physics Department and with the TARI Program under contract RII3-CT-2004-506078.
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We present an up-to-date analysis for a precise determination of the effective fine structure constant and discuss the prospects for future improvements. We advocate to use a determination monitored by the Adler function which allows us to exploit perturbative QCD in an optimal well controlled way. Together with a long term program of hadronic cross section measurements at energies up to a few GeV, a determination of $\alpha(M_Z)$ at a precision comparable to the one of the Z mass M_Z should be feasible. Presently $\alpha(E)$ at $E > 1$ GeV is the least precisely known of the fundamental parameters of the SM. Since, in spite of substantial progress due to new BaBar exclusive data, the region 1.4 to 2.4 GeV remains the most problematic one a major step in the reduction of the uncertainties are expected from VEPP-2000 [1] and from a possible "high-energy" option DAFNE-2 at Frascati [2]. The up-to-date evaluation reads $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027515 \pm 0.000149$ or $\alpha^{-1}(M_Z^2) = 128.957 \pm 0.020$.

1. INTRODUCTION

The accuracy of theoretical predictions of precision observables often is limited as soon as low energy hadronic physics comes into play. In fact, one of the main non-perturbative hadronic effect contributing to many electroweak precision observables is the hadronic vacuum polarization which affects the effective fine structure "constant" $\alpha(E)$. For precise SM predictions one thus needs to know the running α very precisely. As $\alpha(E)$ is steeply increasing at low E, substantial corrections show up at low scales already. Furthermore, in the time–like region, non–perturbative resonance effects make $\alpha(E)$ to be a complicated function, as illustrated in Fig. 1.

2. $\alpha(M_Z)$ IN PRECISION PHYSICS

For SM predictions the most precisely known parameters α, G_μ and M_Z are chosen as the basic input parameters. However, for processes beyond the very low energy region not α itself but $\alpha(E)$ plays the role of α. This has dramatic consequences for precision physics: the uncertainties of the hadronic contributions to the effective α represent a major limitation for electroweak precision physics. In fact $\alpha(E)$ above about 1GeV is a factor of 10 less well known than the next worse which is the Z mass M_Z. This is particularly important for a precise investigation of Z and W gauge boson physics, for example. The present accuracies of the main SM parameters read $\delta\alpha/\alpha \sim 3.7 \times 10^{-10}$, $\delta G_\mu/G_\mu \sim 8.6 \times 10^{-6}$, $\delta M_Z/M_Z \sim 2.4 \times 10^{-5}$, but $\delta\alpha(M_Z)/\alpha(M_Z) \sim 1.1 \div 2.6 \times 10^{-4}$ [$\delta\alpha(E)/\alpha(E) \sim 4.5 \times 10^{-5}$ at $E = 1$ GeV space-like]. Thus at present we loose a factor 10^5 in precision in the replacement $\alpha \to \alpha(M_Z)$. For precision physics at the ILC one would require $\alpha(M_Z)$ to be determined as precise as M_Z [3], typically, which would require an improvement by a factor about 10 to obtain

$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 2.5 \times 10^{-5} . \quad (1)$$

At present, an important example is the LEP/SLD measurement of $\sin^2\Theta_{\rm eff} = (1 - g_{Vl}/g_{Al})/4 = 0.23148 \pm 0.00017$ from which the Higgs mass bound depends most sensitively. An

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†Work supported in part by the EU grants MTKD-CT2004-510126 in partnership with the CERN Physics Department and with the TARI Program under contract RII3-CT-2004-506078.

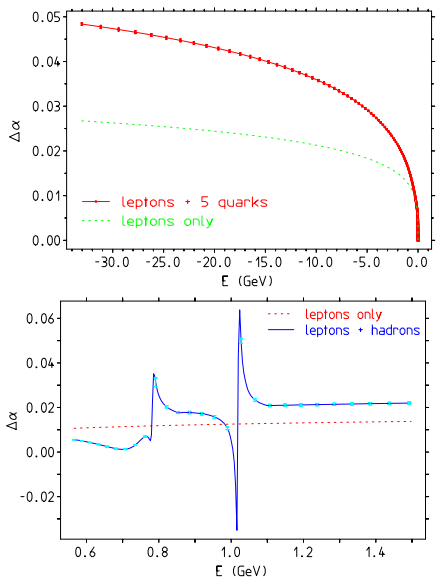

Figure 1. The running of α. The "negative" E axis is assigned to space-like momentum transfer. In the time-like region the resonances lead to pronounced variations of the effective charge.

uncertainty of $\delta\Delta\alpha(M_Z) = 0.00036$ leads to an error $\delta \sin^2\Theta_{\rm eff} = 0.00013$ in the prediction of $\sin^2\Theta_{\rm eff}$, and affects the Higgs mass bound, precision tests and new physics searches. Note that once m_H will have been measured by the LHC $\sin^2\Theta_{\rm eff}$ will be an excellent monitor for new physics! This will be particularly important once the top mass m_t will have been determined with higher accuracy.

One also should keep in mind that for calculations of perturbative QCD contributions precise QCD parameters α_s, m_c, m_b, m_t are mandatory.

3. UPDATED EVALUATION OF $\alpha(M_Z)$

Since my last major update in August 2006 a number of new results mainly from BaBar [4] were published. In fact a series of new channels have been measured in a range which covers the problematic region between 1.4 and 2.4 GeV. This means that we have almost completely new data for the exclusive measurements in this region. In contrast the inclusive measurements date back to the early 1980's. Important new cross–section measurements were also presented by KLOE at this meeting [5] (see also status reports from CMD-2/SND, BaBar, Belle, CLEO and BES at this meeting). The standard evaluation of the non-perturbative hadronic contributions in terms of measured cross-sections $\sigma(e^+e^- \to \text{hadrons})$ is based on the dispersion integral:

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\mathrm{P}\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R_\gamma(s')}{s'(s'-s)}\right), \quad (2)$$

where the e^+e^-–data are encoded in $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$.

The evaluation of the integral at $M_Z = 91.19$ GeV is performed by using $R(s)$ data up to $\sqrt{s} = E_{\rm cut} = 5$ GeV and for the Υ resonances region between 9.6 and 13 GeV. Perturbative QCD is applied from 5.0 to 9.6 GeV and for the high energy tail above 13 GeV. The result is

$$\begin{aligned}\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) &= 0.027594 \pm 0.000219 \\ \alpha^{-1}(M_Z^2) &= 128.946 \pm 0.030\,. \end{aligned} \quad (3)$$

Note that BaBar exclusive radiative return measurements in this evaluation play an essential role up to 2 GeV [lower end of BES inclusive measurement]. In the problematic region from 1.4 to 2 GeV the exclusive measurements actually dominate in comparison to the much older inclusive measurements from Frascati [MEA, $\gamma\gamma$2, M3N, B$\overline{\rm B}$]. More detail are given in Table 1.

4. TESTING NON–PERTURBATIVE HADRONIC EFFECTS VIA THE ADLER FUNCTION

The non-perturbative Adler function related to the photon vacuum polarization can be calculated in terms of experimental e^+e^- annihilation data by the dispersion integral

$$D(Q^2) = Q^2\left(\int_{4m_\pi^2}^{E^2_{\rm cut}} \frac{R^{\rm data}(s)}{(s+Q^2)^2}\,ds + \int_{E^2_{\rm cut}}^{\infty} \frac{R^{\rm pQCD}(s)}{(s+Q^2)^2}\,ds\right). \quad (4)$$

Here $Q^2 = -q^2$ is the squared Euclidean momentum transfer and s the center of mass en-

Table 1
Contributions and uncertainties for $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)^{\rm data} \times 10^4$.

final state	range (GeV)	result	(stat)	(syst)	[tot]	rel	abs
ρ	(0.28, 0.81)	25.95	(0.09)	(0.14)	[0.17]	0.6%	0.6%
ω	(0.42, 0.81)	2.91	(0.03)	(0.08)	[0.09]	3.0%	0.2%
ϕ	(1.00, 1.04)	4.42	(0.06)	(0.10)	[0.12]	2.7%	0.3%
J/ψ		11.14	(0.53)	(0.58)	[0.79]	7.1%	12.9%
Υ		1.18	(0.05)	(0.06)	[0.08]	6.9%	0.1%
had	(0.81, 1.40)	13.21	(0.04)	(0.35)	[0.35]	2.7%	2.6%
had	(1.40, 2.00)	11.34	(0.07)	(1.26)	[1.26]	11.2%	33.3%
had	(2.00, 3.10)	15.73	(0.11)	(0.87)	[0.88]	5.6%	16.2%
had	(3.10, 3.60)	5.26	(0.11)	(0.10)	[0.15]	2.8%	0.5%
had	(3.60, 9.46)	50.58	(0.11)	(0.24)	[0.26]	0.5%	1.5%
had	(9.46,13.00)	18.52	(0.25)	(1.21)	[1.23]	6.7%	31.8%
pQCD	(13.0,∞)	115.71	(0.00)	(0.06)	[0.06]	0.0%	0.1%
data	(0.28,13.00)	160.23	(0.63)	(2.10)	[2.19]	1.4%	0.0%
total		275.94	(0.63)	(2.10)	[2.19]	0.8%	100.0%

ergy squared for hadron production in e^+e^-–annihilation. Formally the Adler function is defined as the derivative of the shift in the fine structure constant

$$\frac{D(Q^2)}{Q^2} = (12\pi^2)\,\frac{d\Pi'_\gamma\,(q^2)}{dq^2} = -\frac{3\pi}{\alpha}\frac{d}{dq^2}\Delta\alpha_{\rm had}(q^2)\;,$$

evaluated in the Euclidean at $Q^2 = -q^2$. $\Pi'_\gamma\,(q^2)$ is the photon vacuum polarization amplitude defined by

$$\begin{aligned}\Pi^\gamma_{\mu\nu}(q) &= i\int d^4x e^{iqx} < 0|T J^\gamma_\mu\,(x)\; J^\gamma_\nu\,(0)\;|0> \\ &= -\left(q^2\,g_{\mu\nu} - q_\mu q_\nu\right)\;\Pi'_\gamma\,(q^2)\;. \qquad (5)\end{aligned}$$

The perturbative result is given in [7]. Crucial for this prediction are known full massive QCD results [10,11,12]. Note that the main Q^2 dependence of $D(Q^2)$ is due to the quark masses m_c and m_b. Without mass effects, up to small effects from the running of α_s, $D(Q^2) = 3\sum_f Q_f^2\,(1 + O(\alpha_s))$ is a constant depending on the number of active flavors. We also include the 4–loop [13,14] and 5–loop [15] contributions in the high energy limit (massless approximation)

$$D(Q^2) \simeq 3\sum_f Q_f^2\;\left(1 + a + d_2 a^2 + d_3 a^3 + d_4 a^4\right)$$

with $a = \alpha_s(Q^2)/\pi$, $d_2 = 1.9857 - 0.1153\,n_f$, $d_3 = 18.2428 - 4.2159\,n_f + 0.0862\,n_f^2 - 1.2395\,(\sum Q_f)^2/(3\sum Q_f^2)$ and $d_4 = -0.010\,n_f^3 + 1.88\,n_f^2 - 34.4\,n_f + 135.8$. The corresponding formula for $R(s)$ only differs at the 4–loop and 5–loop level due to the effect from the analytic continuation from the Euclidean to the Minkowski region which yields $r_3^R = d_3 - \pi^2\beta_0^2\frac{d_1}{3}$ with $\beta_0 = (11 - 2/3\,n_f)/4$, $d_1 = 1$ and $r_4^R = d_4 - \pi^2\beta_0^2\left(d_2 + \frac{5\beta_1}{6\beta_0}\,d_1\right)$ with $\beta_1 = (102 - 38/3\,n_f)/16$. Numerically the 4–loop term proportional to d_3 amounts to -0.0036% at 100 GeV and increases to about 0.32% at 2.5 GeV. The higher order massless results only improve the perturbative high energy tail (see Fig. 2). Towards low Q^2 we also approach the Landau pole of $\alpha_s(Q^2)$, present typically in $\overline{\rm MS}$ type schemes, and pQCD ceases to "converge".

5. $\Delta\alpha^{\rm had}$ VIA THE ADLER FUNCTION

Figure 2 provides convincing evidence that pQCD works well to predict $D(Q^2)$ down to $Q \sim M_0 = 2.5\,{\rm GeV}$. This may be used to calculate

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int dQ'^2\frac{D(Q'^2)}{Q'^2}$$

and we may write

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = \Delta\alpha^{(5)}_{\rm had}(-M_0^2)^{\rm data}$$

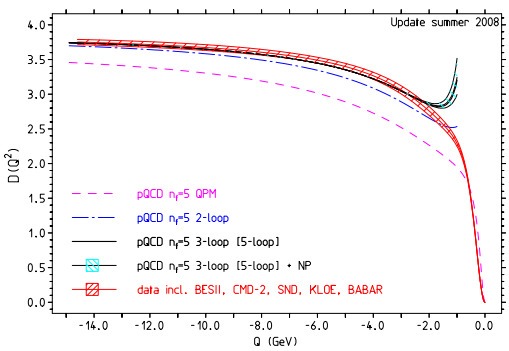

Figure 2. The "experimental" non-perturbative Adler–function versus theory (pQCD + NP). The error includes statistical + systematic here (in contrast to most R-plots showing statistical errors only!). "[5-loop]" indicates that 4- and 5-loop contribution in the massless limit are taken into account. For more details see Ref. [7].

$$+\left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2)-\Delta\alpha^{(5)}_{\rm had}(-M_0^2)\right]^{\rm pQCD}$$
$$+\left[\Delta\alpha^{(5)}_{\rm had}(M_Z^2)-\Delta\alpha^{(5)}_{\rm had}(-M_Z^2)\right]^{\rm pQCD} \quad (6)$$

and obtain, for $M_0 = 2.5\,\mathrm{GeV}$

$$\Delta\alpha^{(5)}_{\rm had}(-M_0^2)^{\rm data} = 0.007354 \pm 0.000107 \quad (7)$$
$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.027477 \pm 0.000149$$
$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027515 \pm 0.000149 \quad (8)$$

where a tiny shift of $+0.000008$ results from the 5-loop contribution. An error ± 0.000103 added in quadrature comes form the perturbative part. For the perturbative calculation of $\left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2)-\Delta\alpha^{(5)}_{\rm had}(-M_0^2)\right]^{\rm pQCD}$ we use the QCD parameters: $\alpha_s(M_Z) = 0.1189(20)$,
$m_c(m_c) = 1.286(13)$ $[M_c = 1.666(17)]$ GeV,
$m_b(m_c) = 4.164(25)$ $[M_b = 4.800(29)]$ GeV,
based on a complete 3–loop massive QCD analysis [16,17] (see contributions by Kühn and Sturm). Note that due to a dramatic improvement in the determination of the quark masses m_c and m_b, for the first time the pQCD error included in (8) is smaller than the one from the data which also has been improved substantially. A very important long term project here is the lattice determinations of the basic QCD parameters [18].

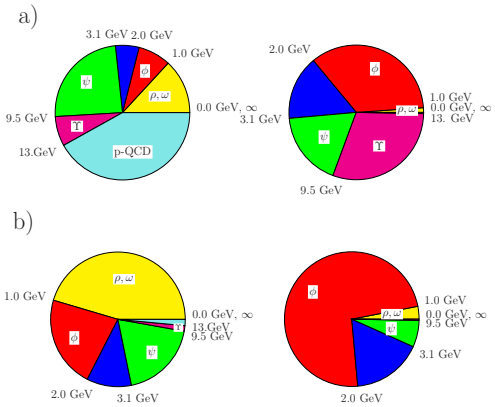

Figure 3. Present distribution of contributions [left] and errors^2 [right] for a) $\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2)$; b) $\Delta\alpha^{(5)}_{\rm had}(-M_0^2)^{\rm data}$ ($M_0 = 2.5$ GeV); both obtained by direct integration of (2).

Mandatory pQCD improvements required are:
• 4–loop massive pQCD calculation of Adler function; required are a number of terms in the low and high momentum series expansions which allow for the appropriate Padé improvements [essentially equivalent to a massive 4–loop calculation of $R(s)$];
• m_c improvement by sum rule and/or lattice QCD evaluations;
• improved α_s in low Q^2 region.

Renormalization schemes which exhibit a Landau pole, like the $\overline{\rm MS}$ scheme, evidently fail in parametrizing the low energy tail of the Adler function. Therefore modeling the Adler function at low Q^2 by testable models may be useful, such as "analytized" α_s [19] and the instanton liquid model [20] or – others.

The contribution and error profiles of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ and $\Delta\alpha^{(5)}_{\rm had}(-M_0^2)$, are shown in Fig. 3. Fig. 4 illustrates where more precise measurements are particularly important. For the Adler function approach in particular low energy machines below 2.5 GeV most successfully can contribute to improve the precise determination of $\alpha(E)$. At the same time machines in this regime substantially contribute to further reduce the error of the leading hadronic contribution to the muon $g-2$.

Table 2
Contributions and uncertainties for $\Delta\alpha^{(5)}_{\rm had}(-M_0^2)^{\rm data} \times 10^4$ ($M_0 = 2.5$ GeV).

final state	range (GeV)	result	(stat)	(syst)	[tot]	rel	abs
ρ	(0.28, 0.81)	24.06	(0.09)	(0.13)	[0.16]	0.6%	2.1%
ω	(0.42, 0.81)	2.65	(0.03)	(0.07)	[0.08]	3.0%	0.5%
ϕ	(1.00, 1.04)	3.79	(0.05)	(0.09)	[0.10]	2.7%	0.9%
J/ψ		3.95	(0.19)	(0.18)	[0.26]	6.6%	5.9%
Υ		0.07	(0.00)	(0.00)	[0.00]	6.7%	0.0%
had	(0.81, 1.40)	11.33	(0.03)	(0.29)	[0.29]	2.6%	7.3%
had	(1.40, 2.00)	7.81	(0.05)	(0.87)	[0.87]	11.2%	65.8%
had	(2.00, 3.10)	7.91	(0.05)	(0.44)	[0.44]	5.6%	16.7%
had	(3.10, 3.60)	1.88	(0.04)	(0.04)	[0.05]	2.8%	0.2%
had	(3.60, 9.46)	8.11	(0.02)	(0.05)	[0.05]	0.6%	0.2%
had	(9.46,13.00)	0.89	(0.01)	(0.06)	[0.06]	6.6%	0.3%
pQCD	(13.0,∞)	1.09	(0.00)	(0.00)	[0.00]	0.1%	0.0%
data	(0.28,13.00)	72.45	(0.23)	(1.05)	[1.08]	1.5%	0.0%
total		73.54	(0.23)	(1.05)	[1.08]	1.5%	100.0%

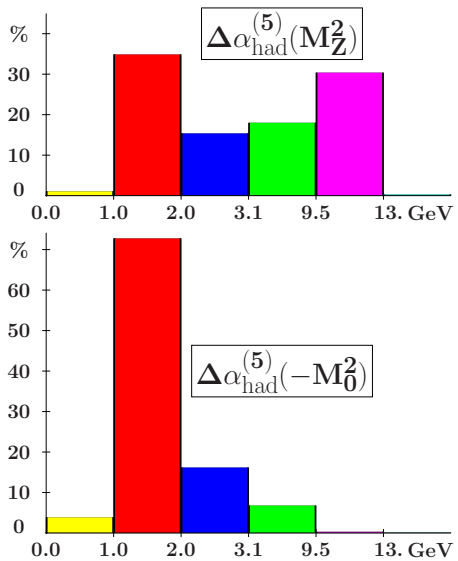

Figure 4. Present error profiles for $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ and $\Delta\alpha^{(5)}_{\rm had}(-M_0^2)$.

6. POSSIBLE IMPROVEMENT BY VEPP-2000 and DAFNE-2

Next generation precision physics experiments, not only the ones possible at an ILC, in many cases require a more precise determination of $\alpha(E)$. A reasonable goal could be an improvement by about a factor 10 in accuracy which would match the precision of the Z mass. The options are

- the standard approach by direct integration of the e^+e^-–data: in this case 58% of the contribution is obtained from the data and 42% from pQCD. My analysis yields $\Delta\alpha^{(5)\,\rm data}_{\rm had} \times 10^4 = 160.12 \pm 2.24$ (1.4%) and thus increasing the overall accuracy to 1% would yield an uncertainty ± 1.63. However, for independent measurements in ranges as used in the Tab. 1 a 1% accuracy for each region and errors including systematic ones added in quadrature would yield ± 0.85. The improvement on the data ([2.24] vs. [0.85]) thus would yield an improvement factor of 2.6. The pQCD part in this case is $\Delta\alpha^{(5)\,\rm pQCD}_{\rm had} \times 10^4 = 115.71 \pm 0.06$ (0.05%) and for the theory part this means that no improvement would be needed.
- With the "Adler function approach" we get 26% of the contribution from data and 74% from pQCD. Here $\Delta\alpha^{(5)\,\rm data}_{\rm had} \times 10^4 = 72.35 \pm 1.10$ (1.5%) and a 1% overall accuracy would mean an uncertainty ± 0.74. Again, a subdivision of ranges as used in Tab. 2 and assuming that a 1% accuracy can be reached for each region and adding up errors in quadrature in this case would lead to a precision of ± 0.40. The improvement

from the data ([1.10] vs. [0.40]) again yields a similar improvement factor of 2.7. If we compare the standard approach of direct integration with the Adler function controlled approach ([2.24] vs. [0.40]) we have an improvement factor 5.6. However, now a much larger fraction $\Delta\alpha_{\rm had}^{(5)\,\rm pQCD} \times 10^4 = 201.15 \pm 1.03$ is coming from pQCD and an improvement of the QCD prediction is mandatory in order to profit in an optimal way from the improvement on the data. A factor 3 to 5 at least should be possible in a long term effort on higher order effects and more importantly on QCD parameters. An accuracy of about ± 0.20 would be a high goal.

Our study shows that the requirement Eq. (1) could be achieved by

• pinning down experimental errors to the 1% level in all non-perturbative regions up to 10 GeV
• safely use pQCD in the Euclidean region monitored by the Adler function
• improve on pQCD and QCD parameters.

In any case as we see from Fig. 4 by far the largest improvement factor will come from precise cross–section measurements in the region from 1.4 to 2.4 GeV. A unique challenge and chance for VEPP-2000 and DAFNE-2.

Acknowledgments: I am very grateful to the organizers of the Frascati Int. Workshop "e^+e^- Collisions from Φ to Ψ" for the kind invitation and for support. It was a very interesting and inspiring event. I also thank S. Jadach and his team from the Niewodniczanski Nuclear Physics Institute at Krakow for the kind hospitality extended to me.

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