arXiv:0806.4613v1 [hep-ph] 27 Jun 2008

DESY 08–082; SFB-/CPP-08-38

First $O(\alpha_s^3)$ heavy flavor contributions to deeply inelastic scattering *

I. Bierenbaum[a], J. Blümlein[a] and S. Klein[a]

[a]Deutsches Elektronen-Synchrotron, DESY, Platanenallee 6, D-15738 Zeuthen, Germany

In the asymptotic limit $Q^2 \gg m^2$, the heavy flavor Wilson coefficients for deep–inelastic scattering factorize into the massless Wilson coefficients and the universal heavy flavor operator matrix elements resulting from light–cone expansion. In this way, one can calculate all but the power corrections in $(m^2/Q^2)^k, k > 0$. The heavy flavor operator matrix elements are known to NLO. We present the last 2–loop result missing in the unpolarized case for the renormalization at 3–loops and first 3–loop results for terms proportional to the color factor T_F^2 in Mellin–space. In this calculation, the corresponding parts of the NNLO anomalous dimensions [1,2] are obtained as well.

1. Introduction

The unpolarized deep–inelastic double differential scattering cross-section can be expressed in terms of the structure functions $F_2(x,Q^2)$ and $F_L(x,Q^2)$ in case of single photon exchange. In the small x region, both structure functions contain large $c\overline{c}$–contributions of the order of 20-40 %. Precision extractions of parton distribution functions and the measurement of $\Lambda_{\rm QCD}$ therefore require to extend the description of these contributions to the $O(\alpha_s^3)$ terms as reached in the massless case. The complete NLO corrections were calculated semi–analytically in x–space, [3], for which a fast implementation in Mellin N–space was given in [4]. For $Q^2 \gtrsim 10\, m_c^2$, one observes that $F_2^{c\bar{c}}(x,Q^2)$ is very well described by its asymptotic expression in the limit $Q^2 \gg m^2$, [5]. In this kinematic range, one can calculate the heavy flavor Wilson coefficients, the perturbative part of the structure functions $F_2^{c\bar{c}}(x,Q^2)$ and $F_L^{c\bar{c}}(x,Q^2)$, analytically, which has been done for $F_2^{c\bar{c}}(x,Q^2)$ to 2–loop order in [5,6] and for $F_L^{c\bar{c}}(x,Q^2)$ to 3–loop order in [7]. First steps towards an asymptotic 3–loop calculation for $F_2^{c\bar{c}}(x,Q^2)$ have been made by the present authors by calculating the O(ε) terms of the 2–loop heavy operator matrix elements (OMEs), [8,9], contributing to the 3–loop heavy flavor Wilson coefficients via renormalization. In the present paper we also report on new results contributing to the 3–loop operator matrix elements and the progress towards a full 3–loop calculation of moments of the heavy flavor Wilson coefficients. As part of the calculation, the T_F^2–terms of the even moments $N = 2...10$ of the NNLO non-singlet (NS) and pure-singlet (PS) anomalous dimensions given in [1,2], cf. also [10] in the NS-case, are confirmed in an independent calculation.

2. Heavy flavor Wilson coefficients in the asymptotic limit

As outlined in Ref. [5], the heavy flavor Wilson coefficients[2] in the limit $Q^2 \gg m^2$ are obtained as a convolution of the light flavor Wilson coefficients with the corresponding massive operator matrix elements of the flavor decomposed quarkonic and gluonic operators between massless parton states. Here we consider the level of twist–2 operators. The light Wilson coefficients are known up to three loops [11] and carry all the process dependence, whereas the OMEs are universal, process–independent objects. Their logarithmic contributions in m^2/μ^2, as well as all pole terms in $1/\varepsilon$, are completely determined by renormalization. The latter provide checks on the calculation. Here, the single pole terms contain the respective contributions of the 3-loop anoma-

*Presented at Loops and Legs in Quantum Field Theory, Sondershausen, Germany, 2008. This paper was supported in part by SFB-TR-9: Computergestütze Theoretische Teilchenphysik, and Studienstiftung des Deutschen Volkes.

[2]We consider extrinsic charm production only.

lous dimensions.

3. Renormalization

For the calculation of the bare heavy flavor OMEs we use dimensional regularization in $D = 4 + \varepsilon$ dimensions and work in the $\overline{\rm MS}$–scheme, if not stated otherwise, using Feynman–gauge. The renormalization is performed in four steps. For mass renormalization we use the on–shell scheme [12], whereas charge renormalization is done using the $\overline{\rm MS}$ scheme. For the latter we make the requirement that the heavy quark loop contributions to the gluon self–energy, $\Pi(p^2, m^2)$, are renormalized in such a way that $\Pi(0, m^2) = 0$, cf. [5,6,8]. The origin of the remaining divergences is twofold. The UV singularities are renormalized via the operator Z–factors, whereas the collinear singularities are removed via mass factorization through the transition functions Γ. We denote the completely unrenormalized OMEs by a double–hat, $\hat{\hat{A}}$, and those for which mass and coupling renormalization have already been performed by a single hat. Operator renormalization and mass factorization then proceeds via

$$\mathbf{A} \quad = \quad \mathbf{Z}^{-1} \hat{\mathbf{A}} \mathbf{\Gamma}^{-1} \; . \tag{1}$$

Note that in the singlet case, Eq. (1) should be read as a matrix equation, contrary to the NS–case. The Z–factors are related to the anomalous dimensions of the twist–2 operators via

$$\gamma = \mu \partial \ln \mathbf{Z}(\mu) / \partial \mu \; , \tag{2}$$

which allows to express the Z–factors in terms of the anomalous dimensions up to an arbitrary order in the strong coupling constant $a_s := \alpha_s/(4\pi)$ (cf. [8] up to $O(a_s^3)$). If all quarks were massless, the transition functions Γ would be given by $\mathbf{\Gamma} = \mathbf{Z}^{-1}$, but since we are dealing with diagrams containing at least one heavy quark, this equation has to be modified in such a way that mass factorization is applied to those parts of the diagrams containing massless lines only. Finally, the PS and NS terms we are calculating are related by

$$Z_{qq}^{\rm PS} + Z_{qq}^{\rm NS} = Z_{qq} \; . \tag{3}$$

From Eqs. (1,2) one thus can infer that for operator renormalization and mass factorization at $O(a_s^3)$, the anomalous dimensions up to NNLO, [1,2], together with the 1–loop heavy OMEs up to $O(\varepsilon^2)$ and the 2–loop heavy OMEs up to $O(\varepsilon)$ are needed. The last two quantities enter since they multiply $Z-$ and Γ–factors containing poles in ε. This has been worked out in some detail in Ref. [8], where we presented the $O(\varepsilon)$ terms $\overline{a}_{Qg}^{(2)}$, $\overline{a}_{qq,Q}^{(2),{\rm NS}}$ and $\overline{a}_{Qq}^{(2),{\rm PS}}$ in the unpolarized case. The term $\overline{a}_{gg,Q}^{(2)}$ was given in [9]. The missing 2–loop $O(\varepsilon)$ term corresponds to the heavy OME $A_{gq,Q}^{(2)}$ and was calculated for the first time in Ref. [13]. It contributes through operator mixing to the T_F^2–term of $A_{Qq}^{(3),{\rm PS}}$, which we consider in this paper.

4. $A_{gq,Q}^{(2)}$

The term $A_{gq,Q}$ emerges for the first time at $O(a_s^2)$. By applying Eq. (1), one obtains at $O(a_s^2)$ the renormalized OME

$$A_{gq,Q}^{(2)} = \hat{A}_{gq}^{(2)} + Z_{gq}^{-1,(2)} + \left(Z_{gg}^{-1,(1)} + \hat{A}_{gg,Q}^{(1)} \right) \Gamma_{gq}^{-1,(1)} .$$

Here, the term $\hat{A}_{gg,Q}^{(1)}$, cf. [13], enters through mixing. Note that since we consider only terms involving at least one heavy quark, we adopt the following definition for the anomalous dimensions

$$\hat{\gamma} \equiv \gamma(n_f + 1) - \gamma(n_f) \tag{4}$$

in order to obtain the correct color projection. Now we can predict the structure of the unrenormalized result to be

$$\hat{A}_{gq,Q}^{(2)} \quad = \quad \left(\frac{m^2}{\mu^2}\right)^{\varepsilon} \left[\frac{2\beta_{0,Q}}{\varepsilon^2} \gamma_{gq}^{(0)} + \frac{\hat{\gamma}_{gq}^{(1)}}{2\varepsilon} + a_{gq,Q}^{(2)} + \varepsilon \overline{a}_{gq,Q}^{(2)} \right] , \tag{5}$$

where the LO and NLO anomalous dimensions are given by

$$\gamma_{gq}^{(0)} \quad = \quad -4 C_F \frac{N^2 + N + 2}{(N-1)N(N+1)} \; , \tag{6}$$

$$\hat{\gamma}_{gq}^{(1)} \quad = \quad C_F T_F \left(-\frac{32}{3} \frac{N^2 + N + 2}{(N-1)N(N+1)} S_1 \right.$$

$$+\frac{32}{9}\frac{8N^3+13N^2+27N+16}{(N-1)N(N+1)^2}\Biggr)\;, \quad (7)$$

and $\beta_{0,Q} = -4/3T_F$. The calculation in Mellin–space in terms of Feynman–parameters is straightforward, cf. [6], and a representation in Euler–Γ functions can be obtained even to all orders in ε We reproduce the pole terms of Eq. (5) and obtain for the constant term in ε

$$\begin{aligned} a_{gq,Q}^{(2)} &= T_F C_F \Biggl\{ \frac{4}{3}\frac{N^2+N+2}{(N-1)N(N+1)}\Bigl(S_2+S_1^2 \\ &\quad +2\zeta_2\Bigr) - \frac{8}{9}\frac{8N^3+13N^2+27N+16}{(N-1)N(N+1)^2}S_1 \\ &\quad +\frac{8}{27}\frac{P_1}{(N-1)N(N+1)^3}\Biggr\}\;, \quad (8) \\ P_1 &= 43N^4+105N^3+224N^2+230N+86\;. \end{aligned}$$

The last term has first been calculated in [13], with which we agree. The $O(\varepsilon)$ term is new and we obtain

$$\begin{aligned} \overline{a}_{gq,Q}^{(2)} &= T_F C_F \Biggl\{ \frac{2}{9}\frac{N^2+N+2}{(N-1)N(N+1)}\Bigl(-2S_3 \\ &\quad -3S_2S_1 - S_1^3 + 4\zeta_3 - 6\zeta_2 S_1\Bigr) \\ &\quad +\frac{2}{9}\frac{8N^3+13N^2+27N+16}{(N-1)N(N+1)^2}\Bigl(2\zeta_2 \\ &\quad +S_2+S_1^2\Bigr) - \frac{4P_1S_1}{27(N-1)N(N+1)^3} \\ &\quad +\frac{4P_2}{81(N-1)N(N+1)^4}\Biggr\}\;, \quad (9) \\ P_2 &= 248N^5+863N^4+1927N^3+2582N^2 \\ &\quad +1820N+496\;. \end{aligned}$$

Eqs. (7–9) are given in terms of harmonic sums, [14,15], the argument of which we have set equal to N. The representation in Mellin–space allowed us to use various analytic and algebraic relations between harmonic sums, [16,17], to obtain a more compact result. Together with the result of Eq. (9), all 2–loop $O(\varepsilon)$ terms of the heavy OMEs in the unpolarized case are known by now. As a last remark, note that we consider charm quark contributions here, while for heavier quarks decoupling [18] has to be applied.

5. Fixed values of N at three loops

The diagrams we need to calculate are of the 3–loop self energy type with the external particle being massless and on–shell and contain two inner scales. One is set by the mass of the heavy quark, the other by the Mellin–variable N of the operator insertions emerging in the light–cone expansion. At this order, new operator vertices appear with three and four gluonic lines, for which the Feynman–rules had not yet been derived. The necessary diagrams are generated using QGRAF [19]. The number of diagrams contributing to $A_{Qg}^{(3)}$, e.g., is 1478 diagrams with one and 489 diagrams with two quark loops.

The steps for the calculation of these diagrams are the following: The diagrams are genuinely given as tensor integrals due to the operators contracted with the light–cone vector Δ, $\Delta^2 = 0$. The idea is, to first undo this contraction and to develop a projector, which, applied to the tensor integrals, provides the results for the diagrams for the specific (even) Mellin N under consideration. So far, we implemented the projector for the first 5 contributing Mellin moments, $N = 2,...,10$, where the color factors are calculated using [20]. A generalization to higher moments is straightforward, however, one quickly runs into computing time problems. The diagrams are then translated into a form, which is suitable for the program MATAD [21], which does the expansion in ε for the corresponding massive three–loop tadpole–type diagrams. We have implemented these steps into a FORM–program, cf. [22], and tested it against various two–loop results, including the result for $\hat{A}_{gq,Q}^{(2)}$, Eq. (5), and found agreement.

6. First results

The first 3–loop objects being investigated are the terms $\propto T_F^2$ of the OMEs $A_{qq,Q}^{\mathsf{NS}}$ and A_{Qq}^{PS}. Note that in the NS–case, the $\pm$ and v terms do not differ. All diagrams contain two inner quark loops, where the quark to which the operator insertion couples is heavy and the other one may be heavy or light. The latter two cases can be distinguished by a factor n_f, denoting the number of light flavors, in the result.

Table 1
Mellin moments 2 to 10 for the T_F^2 terms of the $O(\varepsilon^0)$ terms of the unrenormalized OME $\hat{\hat{A}}_{Qq}^{(3),\mathsf{PS}}$ as obtained from MATAD.

N	$a_{Qq}^{(3),\mathsf{PS}}\vert_{T_F^2}$
2	$-\frac{36880}{2187} - \frac{736}{81}\zeta_2 - \frac{4096}{81}\zeta_3 + n_f\left(-\frac{76408}{2187} - \frac{112}{81}\zeta_2 + \frac{896}{81}\zeta_3\right)$
4	$-\frac{2879939}{5467500} - \frac{1118}{2025}\zeta_2 - \frac{15488}{2025}\zeta_3 + n_f\left(-\frac{474827503}{109350000} - \frac{851}{20250}\zeta_2 + \frac{3388}{2025}\zeta_3\right)$
6	$\frac{146092097}{1093955625} - \frac{7592}{99225}\zeta_2 - \frac{61952}{19845}\zeta_3 + n_f\left(-\frac{82616977}{45378900} - \frac{16778}{694575}\zeta_2 + \frac{1936}{2835}\zeta_3\right)$
8	$\frac{48402207241}{272211166080} + \frac{1229}{142884}\zeta_2 - \frac{43808}{25515}\zeta_3 + n_f\left(-\frac{16194572439593}{15122842560000} - \frac{343781}{14288400}\zeta_2 + \frac{1369}{3645}\zeta_3\right)$
10	$\frac{430570223624411}{2780024890190625} + \frac{319072}{11026125}\zeta_2 - \frac{802816}{735075}\zeta_3$ $+n_f\left(-\frac{454721266324013}{624087220246875} - \frac{547424}{24257475}\zeta_2 + \frac{175616}{735075}\zeta_3\right)$

Table 2
Mellin moments 2 to 10 of $\gamma_{\mathsf{PS}}^{(2)}\vert_{T_F^2}$ as obtained from MATAD and Eq. (11).

N	2	4	6	8	10
$\gamma_{\mathsf{PS}}^{(2)}\vert_{T_F^2}$	$-\frac{5024}{243}$	$-\frac{618673}{151875}$	$-\frac{126223052}{72930375}$	$-\frac{13131081443}{13502538000}$	$-\frac{265847305072}{420260754375}$

6.1. $A_{Qq}^{(3),\mathsf{PS}}$

From Eq. (1), we obtain the pole structure of the completely unrenormalized PS OME. Considering only the T_F^2 terms, one finds

$$\begin{aligned}\hat{\hat{A}}_{Qq}^{(3),\mathsf{PS}}\Big|_{T_F^2} &= \left(\frac{m^2}{\mu^2}\right)^{3\varepsilon/2}\Bigg\{2\frac{n_f+4}{3\varepsilon^3}\beta_{0,Q}\hat{\gamma}_{qg}^{(0)}\gamma_{gq}^{(0)}\\ &+\frac{1}{\varepsilon^2}\left(\frac{2-n_f}{6}\hat{\gamma}_{qg}^{(0)}\hat{\gamma}_{gq}^{(1)} - (n_f+1)\frac{4}{3}\beta_{0,Q}\hat{\gamma}_{\mathsf{PS}}^{(1)}\right)\\ &+\frac{1}{\varepsilon}\left(\frac{n_f+1}{3}\hat{\bar{\gamma}}_{\mathsf{PS}}^{(2)} - 4(n_f+1)\beta_{0,Q}a_{Qq}^{(2),\mathsf{PS}}\right.\\ &\left.-n_f\frac{\zeta_2\beta_{0,Q}}{4}\hat{\gamma}_{qg}^{(0)}\gamma_{gq}^{(0)} + \hat{\gamma}_{qg}^{(0)}a_{gq,Q}^{(2)}\right) + a_{Qq}^{(3),\mathsf{PS}}\Big|_{T_F^2}\Bigg\},\end{aligned} \tag{10}$$

where we have written the n_f dependence explicitly and with $\hat{\bar{\gamma}}_{\mathsf{PS}}^{(2)}$ being the term $\propto n_f^2$ of the NNLO anomalous dimension $\gamma_{\mathsf{PS}}^{(2)}$. It is not possible to factor out n_f+1, not even in the triple pole term. This is due to the interplay of the prescription for coupling constant renormalization we have adopted, cf. [5,8], and the fact that the transition functions Γ apply to sub graphs containing massless lines only. We have calculated the above term using MATAD for $N = 2, ..., 10$ and all pole terms agree with Eq. (11). In Table 1, we show the constant terms in ε we have obtained.

Using Eqs. (8,11), one can obtain moments for the 3–loop anomalous dimension $\gamma_{\mathsf{PS}}^{(2)}\vert_{T_F^2}$, which we show in Table 2. These latter results agree with the results from Refs. [2]. Here one has to make the replacement $n_f \to n_f(2T_F)$, with $T_F = 1/2$, and multiply with 2, to account for the

Table 3
Mellin moments 2 to 10 for the T_F^2 terms of the $O(\varepsilon^0)$ terms of the unrenormalized OME $\hat{A}_{qq,Q}^{(3),\mathsf{NS}}$ as obtained from MATAD.

N	$a_{qq,Q}^{(3),\mathsf{NS}}\vert_{T_F^2}$
2	$-\frac{28736}{2187} - \frac{512}{81}\zeta_2 - \frac{2048}{81}\zeta_3 + n_f\left(-\frac{100096}{2187} - \frac{256}{81}\zeta_2 + \frac{896}{81}\zeta_3\right)$
4	$-\frac{151928299}{5467500} - \frac{26542}{2025}\zeta_2 - \frac{20096}{405}\zeta_3 + n_f\left(-\frac{1006358899}{10935000} - \frac{13271}{2025}\zeta_2 + \frac{8792}{405}\zeta_3\right)$
6	$-\frac{26884517771}{729303750} - \frac{1712476}{99225}\zeta_2 - \frac{181504}{2835}\zeta_3 + n_f\left(-\frac{524427335513}{4375822500} - \frac{856238}{99225}\zeta_2 + \frac{11344}{405}\zeta_3\right)$
8	$-\frac{740566685766263}{17013197880000} - \frac{36241943}{1786050}\zeta_2 - \frac{632512}{8505}\zeta_3 + n_f\left(-\frac{4763338626853463}{34026395760000} - \frac{36241943}{3572100}\zeta_2 + \frac{39532}{1215}\zeta_3\right)$
10	$-\frac{6080478350275977191}{124545115080540000} - \frac{2451995507}{108056025}\zeta_2 - \frac{1543040}{18711}\zeta_3$ $+n_f\left(-\frac{38817494524177585991}{249090230161080000} - \frac{2451995507}{216112050}\zeta_2 + \frac{96440}{2673}\zeta_3\right)$

Table 4
Mellin moments 2 to 10 of $\gamma_{\mathsf{NS}}^{(2)}\vert_{T_F^2}$ as obtained from MATAD and Eq. (1).

N	2	4	6	8	10
$\gamma_{\mathsf{NS}}^{(2)}\vert_{T_F^2}$	$-\frac{1792}{243}$	$-\frac{384277}{30375}$	$-\frac{160695142}{10418625}$	$-\frac{38920977797}{2250423000}$	$-\frac{27995901056887}{1497656506500}$

different convention for the Z–factors we adopted, see Eq. (2). Note that in Tables (1–4), there is an overall factor $C_F T_F^2$, which we do not show explicitly.
As an example consider the renormalized result for the second moment. Applying Eq. (1), we obtain

$$A_{Qq}^{(3),\mathsf{PS}}\Big|_{N=2,T_F^2} = C_F T_F^2 \left\{ -\frac{128}{81}\ln^3\left(\frac{m^2}{\mu^2}\right) - \frac{32}{27}\ln^2\left(\frac{m^2}{\mu^2}\right) - \frac{5344}{243}\ln\left(\frac{m^2}{\mu^2}\right) + \frac{53144}{2187} - \frac{3584}{81}\zeta_3 + n_f\left(-\frac{128}{81}\ln^3\left(\frac{m^2}{\mu^2}\right) + \frac{32}{27}\ln^2\left(\frac{m^2}{\mu^2}\right) - \frac{5104}{243}\ln\left(\frac{m^2}{\mu^2}\right) - \frac{34312}{2187} + \frac{1024}{81}\zeta_3 \right) \right\}. \quad (11)$$

As in Eq. (11), we observe for all moments in the NS and PS case that the terms $\propto \zeta_2$ disappear after renormalization, since the corresponding terms in the light flavor Wilson coefficients do not contain even ζ-values. This provides us with a further check on our calculation, since it is a general observation made in many $D = 4$ calculations.

6.2. $A_{qq,Q}^{(3),\mathsf{NS}}$

For the T_F^2–terms of the heavy OME $A_{qq,Q}^{(3),\mathsf{NS}}$, a formula similar to Eq. (11) can be derived, which we do not show here. Using again MATAD, we have calculated the first 5 non-vanishing moments of the completely unrenormalized expression. We list the constant terms in ε, $a_{qq,Q}^{(3),\mathsf{NS}}$, of our results in Table 3. The pole terms we obtain agree with what one expects from Eq. (1) and after renor-

malization, we again observe that there are no ζ_2's left anymore. The values for the moments of the terms $\propto T_F^2$ in $\gamma_{\sf NS}^{(2)}$ we thus obtain are shown in Table 4. These values agree with those in Refs. [1,10].

7. Conclusions and Outlook

We calculated the last missing $O(\varepsilon)$ contribution to the unpolarized heavy OMEs for general Mellin variable N at $O(a_s^2)$, needed for the renormalization at $O(a_s^3)$. Furthermore, we installed a program chain to calculate the corresponding 3–loop diagrams to $O(a_s^3)$ using `MATAD`. This chain is now existing and we expect first complete results in the near future. As a first step, we presented moments of the terms $\propto T_F^2$ of the heavy OMEs $\hat{A}_{qq,Q}^{(3),\sf NS}$ and $\hat{A}_{Qq}^{(3),\sf PS}$, for which we found agreement with the general pole structure expected from renormalization. This provides us with a good check on the method we apply for our calculation. For the calculation of high moments we will apply `TFORM`, [23], in the future.

Acknowledgments. We would like to thank M. Steinhauser and J. Vermaseren for useful discussions and M. Steinhauser for a `FORM 3.0` compatible form of the code `MATAD`.

REFERENCES

1. S. A. Larin, T. van Ritbergen and J. A. M. Vermaseren, Nucl. Phys. **B427** (1994) 41; S. Moch, J. A. M. Vermaseren and A. Vogt, Nucl. Phys. **B688** (2004) 101.
2. S. A. Larin, P. Nogueira, T. van Ritbergen and J. A. M. Vermaseren, Nucl. Phys. **B492** (1997) 338; A. Vogt, S. Moch and J. A. M. Vermaseren, Nucl. Phys. **B691** (2004) 129;
3. E. Laenen, S. Riemersma, J. Smith and W. L. van Neerven, Nucl. Phys. **B392** (1993) 162; S. Riemersma, J. Smith and W. L. van Neerven, Phys. Lett. **B347** (1995) 143.
4. S. I. Alekhin and J. Blümlein, Phys. Lett. **B594** (2004) 299.
5. M. Buza, Y. Matiounine, J. Smith, R. Migneron and W. L. van Neerven, Nucl. Phys. **B472** (1996) 611.
6. I. Bierenbaum, J. Blümlein and S. Klein, Nucl. Phys. B **780** (2007) 40; Phys. Lett. **B648** (2007) 195.
7. J. Blümlein, A. De Freitas, W. L. van Neerven and S. Klein, Nucl. Phys. **B755** (2006) 272.
8. I. Bierenbaum, J. Blümlein, S. Klein and C. Schneider, arXiv:0803.0273 [hep-ph], Nucl. Phys. **B** in print; arXiv:0707.4659 [math-ph].
9. I. Bierenbaum, J. Blümlein and S. Klein, arXiv:0806.0451 [hep-ph] and in preparation.
10. J. A. Gracey, Phys. Lett. **B322** (1994) 141.
11. J. A. M. Vermaseren, A. Vogt and S. Moch, Nucl. Phys. **B724** (2005) 3.
12. N. Gray, D. J. Broadhurst, W. Grafe and K. Schilcher, Z. Phys. **C48** (1990) 673; D. J. Broadhurst, N. Gray and K. Schilcher, Z. Phys. **C52** (1991) 111.
13. M. Buza, Y. Matiounine, J. Smith and W. L. van Neerven, Eur. Phys. J. **C1** (1998) 301.
14. J. Blümlein and S. Kurth, Phys. Rev. **D60** (1999) 014018.
15. J. A. M. Vermaseren, Int. J. Mod. Phys. **A14** (1999) 2037.
16. J. Blümlein, Comput. Phys. Commun. **159** (2004) 19.
17. J. Blümlein, DESY 07–042; J. Blümlein and S. Klein, arXiv:0706.2426 [hep-ph].
18. B. A. Ovrut and H. J. Schnitzer, Nucl. Phys. **B179** (1981) 381; **B189** (1981) 509; W. Bernreuther and W. Wetzel, Nucl. Phys. **B197** (1982) 228; E: **B513** (1998) 758; W. Bernreuther, Annals Phys. **151** (1983) 127.
19. P. Nogueira, J. Comput. Phys. **105** (1993) 279.
20. T. van Ritbergen, A. N. Schellekens and J. A. M. Vermaseren, Int. J. Mod. Phys. **A14** (1999) 41.
21. M. Steinhauser, Comput. Phys. Commun. **134** (2001) 335 and `MATAD 3.0`.
22. J. A. M. Vermaseren, New features of FORM, arXiv:math-ph/0010025.
23. M. Tentyukov and J. A. M. Vermaseren, arXiv:hep-ph/0702279.