DESY 08-086
LTH 794

arXiv:0807.0321v1 [hep-ph] 2 Jul 2008

DESY 08-086, LTH 794

NNLO Time-like Splitting Functions in QCD

S. Moch[1] and A. Vogt[2]

1- Deutsches Elektronensynchrotron DESY
Platanenallee 6, D–15738 Zeuthen - Germany

2- Department of Mathematical Sciences, University of Liverpool
Liverpool L69 3BX - United Kingdom

We review the status of the calculation of the time-like splitting functions for the evolution of fragmentation functions to the next-to-next-to-leading order in perturbative QCD. By employing relations between space-like and time-like deep-inelastic processes, all quark-quark and the gluon-gluon time-like splitting functions have been obtained to three loops. The corresponding quantities for the quark-gluon and gluon-quark splitting at this order are presently still unknown except for their second Mellin moments.

1 Introduction

The transition from coloured quarks and gluons to colourless hadrons – the so-called fragmentation or hadronization process – is a Quantum Chromodynamics (QCD) phenomenon with many important theoretical and phenomenological implications for the physics at high-energy colliders (see, e.g., Ref. [2] for a recent summary).

For a given fractional momentum x carried by the outgoing hadron h the fragmentation functions (FFs) $D_f^h(x,Q^2)$ of the final-state partons f obey evolution equations given by

$$\frac{d}{d\ln Q^2}\,D_i^h(x,Q^2) \;=\; \int_x^1 \frac{dz}{z}\,P_{ji}^T\left(z,\alpha_s(Q^2)\right)\,D_j^h\Big(\frac{x}{z},Q^2\Big) \tag{1}$$

where the summation over $j = q,\,\bar{q},\,g$ is understood. Q^2 is a time-like hard scale such as the squared four-momentum of the gauge boson in the reaction $e^+e^- \to \gamma\,,\,Z \to h + X$.

The FFs are a basic ingredient at colliders, e.g., in the cross sections for particle production at high transverse momenta. However, being non-perturbative quantities, the D_f^h have to be determined from global analyses (fits) of various experimental hadron and photon production data, if possible including flavour identification. The available high-precision results for the FFs constrain and, eventually, improve the perturbative calculation of hadron and photon production in a way similar to global analyses of parton distribution functions (PDFs) for the initial protons. With a new kinematical range accessible in proton collisions at LHC energies and at much increased luminosities, new opportunities for precision determinations of FFs will arise. At the same time, improved evolution equations for the FFs beyond the next-to-leading order (NLO) of perturbative QCD will become important.

2 Time-like splitting functions at three loops

The scale dependence in Eq. (1) is controlled by the time-like splitting functions P_{ji}^T which admit an expansion in powers of the strong coupling α_s,

$$P_{ji}^T\left(x,\alpha_s(Q^2)\right) \;=\; a_s\,P_{ji}^{(0)\,T}(x) \;+\; a_s^2\,P_{ji}^{(1)\,T}(x) \;+\; a_s^3 P_{ji}^{(2)\,T}(x) \;+\; \ldots\;, \tag{2}$$

where we normalize the expansion parameter as $a_{\rm s} \equiv \alpha_{\rm s}(Q^2)/(4\pi)$. The three-loop contributions $P_{ji}^{(2)\,T}$ are needed to complete predictions to next-to-next-to-leading order (NNLO) accuracy in perturbative QCD.

The decomposition into flavour-singlet and non-singlet parts for the time-like evolution equations (1) reads

$$\frac{d}{d\ln Q^2}\, D_{\rm ns}^{\xi} \;=\; P_{{\rm ns},\,\xi}^{T} \otimes D_{\rm ns}^{\xi}\,, \tag{3}$$

$$\frac{d}{d\ln Q^2} \left(\begin{array}{c} D_S \\ D_g \end{array} \right) \;=\; \left(\begin{array}{cc} P_{\rm qq}^T & P_{\rm gq}^T \\ P_{\rm qg}^T & P_{\rm gg}^T \end{array} \right) \otimes \left(\begin{array}{c} D_S \\ D_g \end{array} \right) \quad \text{with} \quad D_S \;\equiv\; \sum_{r=1}^{n_f} (D_{q_r} + D_{\bar{q}_r})\,, \tag{4}$$

where D_{ns}^{ξ} with $\xi = \pm, {\rm v}$ collectively denotes the three types of non-singlet distributions $D_{ik}^{\pm} = D_{q_i} \pm D_{\bar{q}_i} - (D_{q_k} \pm D_{\bar{q}_k})$ and $D^{\rm v} = \sum_{r=1}^{n_f} (D_{q_r} - D_{\bar{q}_r})$. Here $\otimes$ abbreviates the Mellin convolution written out in Eq. (1), while n_f stands for the number of effectively massless quark flavors.

Motivated by the strong similarities between the evolution equations of the time-like FFs in Eq. (1) and their space-like counter-parts for the PDFs, we have calculated in two publications [3, 4] several of the splitting functions P_{ji}^T entering the perturbative expansion in Eq. (2) to NNLO. The leading-order (LO) terms in Eq. (2) are identical to the space-like case of the initial-state PDFs, a fact often referred to as the Gribov-Lipatov relation [5]. Also the next-to-leading order contributions $P_{ji}^{(1)\,T}(x)$ have been known for long [6–8] and shown to be related to their space-like counter-parts by a suitable analytic continuation [6–9].

Following these ideas, we have derived all three-loop non-singlet functions corresponding to D_{ns}^{ξ} and the diagonal singlet quantities $P_{\rm qq}^{(2),T}$ and $P_{\rm gg}^{(2),T}$ from the space-like NNLO results computed in Refs. [10,11]. To that end, we have applied two independent methods. One approach relied on mass factorization, knowledge of the infrared singularities and an analytic continuation in the momentum fraction from x to $1/x$. The other approach implemented the concept of universal (kinematics independent) splitting functions of Refs. [12, 13].

The first method started from the unrenormalized partonic structure functions in deep-inelastic scattering (DIS) [10,11] in dimensional regularization. After iteratively subtracting virtual contributions due to the quark or gluon form factors [14, 15] (which are well known also in the time-like region) we applied an analytic continuation to the x-dependent real-emission functions $\mathcal{R}_n$ from x to $1/x$ at each order in $a_{\rm s}$ similar to Refs. [6–9]. During this step we have also taken into account the (complex) continuation of the DIS scale q^2 and the additional D-dimensional prefactor x^{D-3} originating from the phase space of the detected parton in the time-like case (see, e.g., Refs. [16, 17]). The subtle point in all this is the treatment of logarithmic singularities for $x \to 1$ starting with

$$\ln(1-x) \;\to\; \ln(1-x) - \ln x + i\,\pi\ . \tag{5}$$

These steps lead to the (unrenormalized) one-parton inclusive fragmentation function in time-like kinematics, which can be re-assembled order by order in $a_{\rm s}$, keeping the real parts of the continued functions $\mathcal{R}_n$ only. Finally the time-like splitting functions (and coefficient functions) can be extracted iteratively from the mass factorization relations. Up to two loops all these the steps have been checked by a direct calculation for one-parton inclusive electron-positron annihilation [17].

At three loops we have several consistency conditions, for instance the cancellation of all higher poles in dimensional regularization. Most importantly, sum rules exist for the first moment of $P_{\rm ns,-}^{(2),T}$ (number of fermions) and the second moment of $P_{\rm gg}^{(2),T}$ at $n_f = 0$ (momentum in pure gluodynamics). These indicate different coefficients for the terms $C_F^3\, p_{\rm qq}(x)\, \pi^2\, \ln^2 x$ of $P_{\rm qq}^{(2),T}$ and $C_A^3\, p_{\rm gg}(x)\, \pi^2\, \ln^2 x$ of $P_{\rm gg}^{(2),T}$, where $p_{ij}(x)$ are proportional to the LO splitting functions. Such differences are not unexpected in view of the imperfect real-virtual separation at three loops in the approach described above.

As an independent confirmation of our analytic continuation approach including the sum-rule fix we thus needed a second, completely independent method. For this purpose we have adopted the relations between DIS and fragmentation processes of Ref. [12] which exploit ideas of universal, i.e., kinematics independent parton-parton splittings based on a physical picture of strongly ordered life-times of successive parton fluctuations. Introducing universal (non-singlet) splitting functions $P_{\rm ns}^{\rm univ}$, postulated to be identical for the time-like and space-like cases, Eq. (1) can be represented in the following manner [13]

$$\frac{d}{d\ln Q^2}\, D_{\rm ns}^{\sigma}(x,Q^2) \;=\; \int_x^1 \frac{dz}{z}\, P_{\rm ns}^{\rm univ}\left(z,\alpha_{\rm s}(Q^2)\right)\; D_{\rm ns}^{\sigma}\Big(\frac{x}{z},z^{\sigma}Q^2\Big)\;, \tag{6}$$

where the notation covers both the (time-like q, $\sigma = 1$) FFs and the (space-like q, $Q^2 \equiv -q^2$, $\sigma = -1$) PDFs. An additional shift in the argument of $\alpha_{\rm s}$ in Eq. (6) also included in [13] is irrelevant for our discussion.

The $\alpha_{\rm s}$-expansion of Eq. (6) correctly accounts for the NLO difference $\delta P_{\rm ns}^{(1)}(x) = P_{\rm ns}^{(1),\,\sigma=+1}(x) - P_{\rm ns}^{(1),\,\sigma=-1}(x)$ between the space- and time-like splitting functions [6–8]. At NNLO it leads to the remarkably compact expression

$$\delta P_{{\rm ns},\,\xi}^{(2)}(x) \;=\; 2\left\{\left[\ln x\cdot \widetilde{P}_{{\rm ns},\,\xi}^{(1)}\right]\otimes P_{\rm ns}^{(0)} + \left[\ln x\cdot P_{\rm ns}^{(0)}\right]\otimes \widetilde{P}_{{\rm ns},\,\xi}^{(1)}\right\} \tag{7}$$

where $\xi = \pm$, v, the Mellin convolution is again denoted by $\otimes$, and

$$2\,\widetilde{P}_{{\rm ns},\,\xi}^{(n)}(x) \;=\; P_{{\rm ns},\,\xi}^{(n),\,\sigma=+1}(x) + P_{{\rm ns},\,\xi}^{(n),\,\sigma=-1}(x)\;. \tag{8}$$

The evaluation of Eq. (7) exactly coincides with the result of the above analytic continuation including the correct coefficient for the term ${C_F}^3\, p_{\rm qq}(x)\, \pi^2\, \ln^2 x$ of $P_{{\rm ns},\,\xi}^{(2),T}$, as the required vanishing first moment is manifest in Eq. (7). This result provides both the desired confirmation of the analytic continuation and strong evidence in support of the ansatz (6).

The physical picture leading to Eq. (6) assuming strong ordering in the life-times of successive parton fluctuations can be applied in complete analogy to the case of gluodynamics. Hence a relation similar to Eq. (7) can be derived for the space- and time-like difference of the gluon-gluon splitting function $P_{\rm gg}$. In particular, this reasoning again solves the above mentioned problem with the term $C_A^3\, p_{\rm gg}(x)\, \pi^2\, \ln^2 x$ of $P_{\rm gg}^{(2),T}$, automatically yielding the correct coefficient in agreement with the momentum sum rule for $n_f = 0$. Moreover both methods, the ansatz (6) and the analytic continuation, agree for all remaining terms proportional to C_A^3, $C_A^2\, n_f$ and $C_A n_f^2$ (i.e., those not involving gluon emission from quarks).

The analysis of the large- and small-x limits of the time-like splitting functions displays a number of interesting features. The large-x behavior of the diagonal quantities $P_{\rm qq}^{(2)T}$ and $P_{\rm gg}^{(2)T}$ is identical to that of their space-like counter-parts up to the sign of the sub-leading

$\ln(1-x)$ contribution as predicted in Ref. [13]. In contrast, the small-x behaviour in the time-like region is markedly different as, e.g., $xP_{\rm gg}^T$ receives double-logarithmic contributions (up to $a_{\rm s}^n \ln^{2n} x$) with large coefficients. Despite large cancellations between leading and subleading terms, both singlet quantities $xP_{\rm ps}^{(2)T}$ and $xP_{\rm gg}^{(2)T}$ show a huge enhancement already at values of $x \gtrsim 10^{-3}$, see the plots in Ref. [4]. Although the FFs for final state hadrons generally do sample regions of larger x than the corresponding PDFs for initial protons, it will be very interesting to investigate the range of perturbative stability in x once the NNLO time-like splitting functions are known completely.

3 Conclusions

We have briefly summarized the current knowledge on the time-like splitting functions in massless perturbative QCD. To NLO, the complete set of functions for the evolution kernel in Eq. (1) has been known for a long time [6–8]. At NNLO accuracy, progress has been made by relating space-like and time-like deep-inelastic processes. Presently only the off-diagonal quantities $P_{\rm gq}^{(2),T}$ and $P_{\rm qg}^{(2),T}$ are still not completely known as a 'naive' analytic continuation fails, unlike at two loops, for the π^2 contributions. Access to these functions hence requires an extension of the methods employed so far, either by extending the concept of universal splitting functions [13] beyond non-singlet quantities or by refining the technique based on mass factorization and analytic continuation [3, 4]. In particular the latter approach also offers other applications, e.g., to Higgs decay mediated through top-quarks as in Ref. [4] where highly non-trivial three-loop results of Ref. [18] were successfully confirmed in this manner. Thus any progress on the method of analytic continuation beyond the state of the art presented here may have far reaching applications.

References

[1] Slides:
`http://indico.cern.ch/contributionDisplay.py?contribId=56&sessionId=27&confId=24657`

[2] S. Albino et al., ECT* workshop, Trento, Feb. 2008, arXiv:0804.2021 [hep-ph]

[3] A. Mitov, S. Moch and A. Vogt, Phys. Lett. B638 (2006) 61, hep-ph/0604053

[4] S. Moch and A. Vogt, Phys. Lett. B659 (2008) 290, arXiv:0709.3899 [hep-ph]

[5] V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438

[6] G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B175 (1980) 27

[7] W. Furmanski and R. Petronzio, Phys. Lett. 97B (1980) 437

[8] E.G. Floratos, C. Kounnas and R. Lacaze, Nucl. Phys. B192 (1981) 417

[9] M. Stratmann and W. Vogelsang, Nucl. Phys. B496 (1997) 41, hep-ph/9612250

[10] S. Moch, J.A.M. Vermaseren and A. Vogt, Nucl. Phys. B688 (2004) 101, hep-ph/0403192

[11] A. Vogt, S. Moch and J.A.M. Vermaseren, Nucl. Phys. B691 (2004) 129, hep-ph/0404111

[12] Y.L. Dokshitzer, V.A. Khoze and S.I. Troian, Phys. Rev. D53 (1996) 89, hep-ph/9506425

[13] Y.L. Dokshitzer, G. Marchesini and G.P. Salam, Phys. Lett. B634 (2006) 504, hep-ph/0511302

[14] S. Moch, J.A.M. Vermaseren and A. Vogt, JHEP 08 (2005) 049, hep-ph/0507039

[15] S. Moch, J.A.M. Vermaseren and A. Vogt, Phys. Lett. B625 (2005) 245, hep-ph/0508055

[16] P.J. Rijken and W.L. van Neerven, Nucl. Phys. B487 (1997) 233, hep-ph/9609377

[17] A. Mitov and S. Moch, Nucl. Phys. B751 (2006) 18, hep-ph/0604160

[18] P.A. Baikov and K.G. Chetyrkin, Phys. Rev. Lett. 97 (2006) 061803, hep-ph/0604194.