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Quark mixing renormalization effects in the determination of the CKM parameters $|V_{ij}|$

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Abstract

We briefly review existing proposals for the renormalization of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and study the numerical effects of several of them on the W-boson hadronic partial decay widths. We then use these results to evaluate the relative shifts on the CKM parameters $|V_{ij}|^2$ induced by the quark mixing renormalization effects, as well as their scheme dependence. We also discuss the implications of this analysis for the most precise unitarity test of the CKM matrix.

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I. INTRODUCTION

The renormalizability of the Standard Model (SM) without quark-flavor mixing was proved in the early seventies [1]. Since the elements of the quark mixing matrices appear as basic parameters in the bare Lagrangian, they are subject to renormalization, too. This is a problem of old vintage [2], the solution of which was first realized for the Cabibbo angle in the SM with two fermion generations in a pioneering paper by Marciano and Sirlin [3] in 1975. The extension to the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix of the three-generation SM was addressed fifteen years later [4]. In the subsequent years, interest on the subject increased significantly, and new renormalization prescriptions were proposed [5, 6, 7, 8, 9, 10, 11, 12, 13].

The on-shell (OS) prescription of Ref. [4] is compact and plausible, but the proposed expression for the CKM matrix counterterm, δV, is given in terms of wave-function renormalization constants and is thus gauge dependent, as was noticed later [5, 6, 14]. This prescription was employed to study the electroweak effects on the B^0–$\overline{B}^0$ mixing in Ref. [15], where also the scheme and scale dependences were estimated. In Ref. [7], the gauge-dependence problem of Ref. [4] was remedied by adopting the pinch technique. In Ref. [5], an alternative OS-like prescription was proposed that avoids this problem at one loop. The characteristic feature of this prescription is that the quark self-energies that enter the definition of δV are not evaluated on their respective mass shells, but at the common subtraction point $q^2 = 0$. To work exclusively in terms of OS renormalization constants, the authors of Ref. [6] proposed to renormalize the CKM matrix with respect to a reference theory in which no quark mixing occurs. However, this prescription does not comply with the unitarity constraint for the renormalized CKM matrix, as was shown in Ref. [8], where this drawback was successfully eliminated. The renormalization prescriptions of Refs. [9, 11] are similar in spirit to the two-step procedure of Ref. [8] and reach beyond the one-loop level. The prescription of Ref. [10] is based on an *ad hoc* separation of the one-loop quark self-energies into ultraviolet(UV)-divergent, gauge-independent parts to be absorbed into the CKM matrix counterterm and UV-finite, gauge-dependent parts to be combined with the vertex corrections. A genuine OS renormalization condition for the CKM matrix, which satisfies the criteria of UV finiteness, gauge independence, and unitarity has been found recently [12]. It is based on a novel procedure to separate the external-leg mixing corrections into gauge-

independent self-mass and gauge-dependent wave-function renormalization contributions. Very recently, a variant of the prescription of Ref. [12] was proposed that is flavor democratic and formulated in terms of the invariant self-energy functions appearing in the quark mixing amplitudes [13]. The prescriptions of Refs. [8, 12, 13] have the important property that they are based on explicit OS renormalization conditions.

The plan of this paper is the following. In Sec. II, we study numerically the effects of CKM matrix renormalization on the hadronic partial decay widths of the W boson at one loop, on the basis of the CKM matrix elements V_{ij} obtained in the global analysis [16]. For definiteness, we focus on the prescriptions of Refs. [4, 5, 8, 12, 13], which we also compare to the modified minimal-subtraction ($\overline{\rm MS}$) scheme. We believe that these prescriptions are representative, since the others are either based on ideas similar to those in Refs. [4, 5, 8] and/or do not comply with all the properties which the renormalized CKM quark mixing matrix should have, namely UV finiteness, gauge independence, and unitarity. Although the renormalization proposal of Ref. [4] does not fulfill the second criterion, we include it in our analysis, as implemented in 't Hooft-Feynman gauge, because it is the first attempt to renormalize the three-generation CKM matrix. In Sec. III, we use the results of Sec. II to evaluate the relative shifts in the $|V_{ij}|^2$ parameters induced by the incorporation of the quark mixing renormalization effects. This section contains also a discussion of the scheme dependence of these shifts and their implications for the most precise unitarity test of the CKM matrix, involving its first row. Section IV summarizes our conclusions.

II. EVALUATION OF THE W-BOSON HADRONIC WIDTHS

We consider the two-particle decay of the W^+-boson to generic quarks,

$$W^+(k) \to u_i(p_1)\overline{d}_j(p_2). \tag{1}$$

The partial decay width in the Born approximation is given by

$$\Gamma_0^{W u_i \overline{d}_j} = \frac{N_c \alpha \left|V_{ij}\right|^2}{24 s_w^2 m_W^3} \kappa(m_W^2, m_{u,i}^2, m_{d,j}^2) \left[2m_W^2 - m_{u,i}^2 - m_{d,j}^2 - \frac{(m_{u,i}^2 - m_{d,j}^2)^2}{m_W^2}\right], \tag{2}$$

where $N_c = 3$, $\alpha = e^2/(4\pi)$ is the fine-structure constant, and

$$\kappa(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 2(xy + yz + zx)} \tag{3}$$

is Källén's function.

The one-loop-corrected partial decay width is calculated by including the renormalization constants for the parameters e, s_w, and V_{ij}, those for the W^+, u_i, and $\overline{d}_j$ fields, and the proper vertex corrections. The results can be expressed in the form:

$$\Gamma_1^{W u_i \overline{d}_j} = \Gamma_0^{W u_i \overline{d}_j} (1 + \delta^{\rm ew} + \delta^{\rm QCD}), \tag{4}$$

where $\delta^{\rm ew}$ and $\delta^{\rm QCD}$ are the electroweak and QCD corrections, respectively. Analytical expressions for $\delta^{\rm ew}$ and $\delta^{\rm QCD}$ in the R_ξ gauges may be found, for example, in Ref. [14].

Note that $\delta^{\rm ew}$ and $\delta^{\rm QCD}$ also receive contributions from the bremsstrahlung of a single real photon and gluon, respectively. Going beyond one loop, it is important to redefine the W^+-boson partial decay widths so that they remain infrared-safe observables. An obvious way of doing this is to generalize Eq. (4) to any order beyond one loop by including all final-state configurations of the type $u_i \overline{d}_j + X$, where X comprises all possible sets of additional particles, possibly including further u_i or $\overline{d}_j$ quarks. This represents a fully inclusive quantity, which is manifestly free of infrared (soft and collinear) singularities by the Kinoshita-Lee-Nauenberg theorem. This definition also avoids the use of jet algorithms and fragmentation functions altogether, which could dilute the sensitivity to the CKM matrix elements.

We now proceed with our numerical analysis of Eq. (4). We perform all the calculations with the aid of the `LOOPTOOLS` [17] package embedded into the `MATHEMATICA` environment. As a check, we reproduce the numerical results of Ref. [14] when adopting the definition of δV_{ij} and the values of the input parameters employed in that paper.

In our analysis, we use the following input parameters [16]:

$\alpha = 1/137.035999679$, $G_F = 1.16637 \times 10^{-5}$ GeV^{-2}, $\alpha_s^{(5)}(m_Z) = 0.1176$,
$m_W = 80.398$ GeV, $m_Z = 91.1876$ GeV,
$m_e = 0.510998910$ MeV, $m_\mu = 105.658367$ MeV, $m_\tau = 1776.84$ MeV,
$m_u = 2.4$ MeV, $m_d = 4.8$ MeV, $m_s = 100$ MeV,
$m_c = 1.25$ GeV, $m_b = 4.25$ GeV, $m_t = 172.4$ GeV.

The standard parameterization of the CKM matrix, in terms of the three mixing angles

Partial width	Ref. [4]	Ref. [5]	Ref. [8]	Ref. [12]	Ref. [13]	$\overline{\rm MS}$ scheme	$\delta V_{ij} = 0$
$\Gamma(W^+ \to u\overline{d})$	0.6697016	0.6697016	0.6697016	0.6697016	0.6697016	0.6696999	0.6697012
$\Gamma(W^+ \to u\overline{s}) \times 10$	0.3594604	0.3594604	0.3594604	0.3594604	0.3594604	0.3594804	0.3590518
$\Gamma(W^+ \to u\overline{b}) \times 10^5$	0.9345792	0.9309188	0.9345444	0.9345781	0.9345797	0.9040684	0.9065685
$\Gamma(W^+ \to c\overline{d}) \times 10$	0.3589746	0.3589746	0.3589746	0.3589746	0.3589746	0.3589738	0.3556135
$\Gamma(W^+ \to c\overline{s})$	0.6684818	0.6684818	0.6684819	0.6684818	0.6684818	0.6684267	0.6614634
$\Gamma(W^+ \to c\overline{b}) \times 10^2$	0.1211309	0.1211315	0.1211263	0.1211318	0.1211315	0.1266316	0.1196919
$\Gamma(W \to \text{hadrons})$	1.4112476	1.4112476	1.4112476	1.4112476	1.4112476	1.4112474	1.4038372

TABLE I: Partial widths (in GeV) of the hadronic W-boson decay channels evaluated at one loop using the quark mixing renormalization prescriptions of Refs. [4, 5, 8, 12, 13] and the $\overline{\rm MS}$ scheme. The entries of the last column are obtained by neglecting quark mixing renormalization.

θ_{ij} and the CP-violating phase δ, reads [16]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{5}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. The choice

$$s_{12} = \lambda, \qquad s_{23} = A\lambda^2, \qquad s_{13}e^{i\delta} = \frac{A\lambda^3(\overline{\rho} + i\overline{\eta})\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\overline{\rho} + i\overline{\eta})]} \tag{6}$$

ensures that the CKM matrix written in terms of λ, A, $\overline{\rho}$, and $\overline{\eta}$ is unitary to all orders in λ. In our analysis, we evaluate the CKM matrix elements from Eqs. (5) and (6) using the values $\lambda = 0.2257$, $A = 0.814$, $\overline{\rho} = 0.135$, and $\overline{\eta} = 0.349$ [16].

III. RESULTS

In Table I, the one-loop-corrected partial widths of the various hadronic W-boson decay channels are presented for the selected definitions of the CKM counterterm matrix δV_{ij} [4, 5, 8, 12, 13], assuming $m_H = 120$ GeV. The first and second columns in Table I (not counting the one labeled *Partial width*) describe the partial widths of the W boson when adopting the CKM matrix renormalization conditions proposed in Refs. [4, 5], respectively.

This has been already done in the literature, for example in Ref. [14]. We emphasize that we find full agreement, provided we adopt the same values for the input parameters. Note that the prescription of Ref. [4] leads to a gauge-dependent result, so that the gauge choice must be specified. We perform the calculation in 't Hooft-Feynman gauge.

New results are those from the third, fourth, and fifth columns, which refer to the three genuine OS renormalization proposals of Refs. [8, 12, 13], respectively. The prescription of Ref. [8] entails the minor complication that one needs to consider a reference theory with zero quark mixing. It is important to note that the proposals of Refs. [8, 12, 13] have the important property that they lead to renormalized amplitudes that are non-singular in the limit in which any two fermions become mass degenerate and are thus suitable for the generalization to theories where maximal mixing could appear. A generalization of Ref. [12] to lepton mixing in Majorana-neutrino theories has recently been carried out in Ref. [18]. For reference, we have included in the sixth column of Table I the results based on the $\overline{\rm MS}$ scheme with 't Hooft mass scale $\mu = m_W$. Finally, in order to assess the significance of quark mixing renormalization, we have included in the last column the results of calculations where $V_{ij} = \delta_{ij}$ is substituted in loops inserted in the external quark legs, so that the criteria of UV finiteness, gauge independence, and unitarity may be satisfied with the trivial choice $\delta V_{ij} = 0$. This corresponds to the conventional calculations in which mixing effects in the external quark legs are neglected, on the grounds that their UV divergences are canceled by the counterterms and their finite contributions are very small. The numbers in Table I are not meant to give the W-boson decay widths with the stated accuracy, since they are based on a one-loop calculation. However, it is necessary to exhibit 7 digits in the one-loop results in order to illustrate their differences.

It is important to note that these corrections also affect the theoretical calculations of the accurate observables underpinning the determination of the CKM elements V_{ij}. When inserted into those calculations, they lead to modified parameters $|V'_{ij}|^2$ that cancel, at the one-loop electroweak level, the very small scheme dependence portrayed in Table I. In order to show this cancellation, we call δ^α_{ij} the one-loop correction in renormalization scheme α, and δ^0_{ij} the one corresponding to the last column in Table I. Taking into account that in the conventional determination of the V_{ij} parameters quark mixing effects in the external legs

Δ_{ij}^{α}	Ref. [4]	Ref. [5]	Ref. [8]	Ref. [12]	Ref. [13]	$\overline{\rm MS}$ scheme	$\vert V_{ij}\vert^2$ [16]
ud	-5.29×10^{-5}	-5.29×10^{-5}	-5.26×10^{-5}	-5.29×10^{-5}	-5.29×10^{-5}	2.00×10^{-4}	0.94905
us	-0.114	-0.114	-0.114	-0.114	-0.114	-0.119	5.0940×10^{-2}
ub	-3.00	-2.62	-2.99	-3.00	-3.00	0.277	1.2888×10^{-5}
cd	-0.936	-0.936	-0.936	-0.936	-0.936	-0.936	5.0895×10^{-2}
cs	-1.05	-1.05	-1.05	-1.05	-1.05	-1.04	0.94739
cb	-1.19	-1.19	-1.18	-1.19	-1.19	-5.48	1.7223×10^{-3}

TABLE II: Relative shifts Δ_{ij}^{α} (in %) in the central values of $|V_{ij}|^2$ [16] induced by quark mixing renormalization effects according to the prescriptions α of Refs. [4, 5, 8, 12, 13] and the $\overline{\rm MS}$ scheme.

are generally neglected, as is also the case in δ_{ij}^0, we readily find the relation:

$$|V_{ij}^{\prime\alpha}|^2(1+\delta_{ij}^{\alpha}) = |V_{ij}|^2(1+\delta_{ij}^0), \tag{7}$$

where α labels the renormalization scheme employed. In turn, this implies

$$\frac{|V_{ij}^{\prime\alpha}|^2}{|V_{ij}|^2} = R_{ij}^{\alpha}, \tag{8}$$

where R_{ij}^{α} are the ratios of the entries in the last column in Table I and those in the α column. In order to incorporate the modified CKM parameters in the calculation of the partial widths, we multiply the entries in the first six columns of that Table by $|V_{ij}^{\prime\alpha}|^2/|V_{ij}|^2$ and, using Eq. (8), we see that they become equal to those in the last column, independently of the chosen renormalization scheme α. In summary, when the calculations of the W-boson decay widths incorporate the modified CKM parameters $|V_{ij}^{\prime\alpha}|^2$, the very small scheme dependence portrayed in Table I cancels.

On the other hand, Eq. (8) permits us to evaluate the relative shifts,

$$\Delta_{ij}^{\alpha} = \frac{|V_{ij}^{\prime\alpha}|^2 - |V_{ij}|^2}{|V_{ij}|^2} = R_{ij}^{\alpha} - 1, \tag{9}$$

in the $|V_{ij}|^2$ parameters induced by the quark mixing renormalization effects, an issue of considerable interest given the fundamental importance of the CKM parameters. The results are portrayed in Table II. (In order to compute some of the entries in Table II, we have used more precise values than those displayed in Table I.)

From Table II we see that the scheme dependence of Δ_{ij}^{α} among the five prescriptions [4, 5, 8, 12, 13] is extremely small, of $\mathcal{O}(10^{-2}\%)$ or less, except in the single case of $|V'_{ub}|^2$ in scheme [5], where it reaches 0.38%. The differences in the Δ_{ij}^{α} between those five schemes and the $\overline{\rm MS}$ evaluations are also very small, of $\mathcal{O}(10^{-2}\%)$ or less, except in $|V'_{ub}|^2$ and $|V'_{cb}|^2$, where they reach 3.3% and 4.3%, respectively.

A matter of considerable interest is the magnitude of Δ_{ij}^{α}. With only two exceptions in the $\overline{\rm MS}$ scheme, a general feature is that the incorporation of the quark mixing renormalization effects decreases the values of the $|V_{ij}|^2$ parameters. In particular, using the results in the first five columns of Table II, we see that $|V_{ud}|^2$ is not modified to a high degree of accuracy, $|V_{us}|^2$ is decreased by 0.11%, $|V_{ub}|^2$ by 3.0%, $|V_{cd}|^2$ by 0.94%, $|V_{cs}|^2$ by 1.1%, and $|V_{cb}|^2$ by 1.2%.

We now consider the effect of these shifts on the most precise unitarity test of the CKM matrix, involving the elements in its first row. The latest update [19] employs $|V_{ud}| = 0.97425(23)$ and $|V_{us}| = 0.2252(9)$, values that differ slightly from those reported in Ref. [16]. They lead to

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999(6), \tag{10}$$

in excellent agreement with unitarity.

Including the quark mixing renormalization effects discussed in this paper, we have $|V'_{ud}| = 0.97425(23)$, since $|V_{ud}|$ is not altered, $|V'_{us}| = 0.2251(9)$, and $|V'_{ub}| = 0.00354(16)$, leading to

$$|V'_{ud}|^2 + |V'_{us}|^2 + |V'_{ub}|^2 = 0.9998(6). \tag{11}$$

We note that the shifts in the $|V_{ij}|$ parameters and the unitarity test are considerable smaller than the current errors in their evaluation. On the other hand, Eq. (11) remains an impressive test of the SM at the level of its quantum corrections! In fact, it is worth remembering that the electroweak corrections in this test amount to roughly 4% [20]. Thus, if they were neglected, the unitarity test of the CKM matrix would fail by about 66 standard deviations!

IV. CONCLUSIONS

In summary, we have reviewed a number of schemes for the renormalization of the CKM matrix and studied the numerical effects of several of them on the W-boson hadronic partial decay widths, using the V_{ij} values obtained in the global analysis. We have then employed

these results to infer the relative shifts in the $|V_{ij}|^2$ parameters due to the quark mixing renormalization corrections. Finally, we have discussed the effect of these shifts on the most precise unitarity test of the CKM matrix.

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