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The Gluonic Operator Matrix Elements at $O(\alpha_s^2)$ for DIS Heavy Flavor Production

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Abstract

We calculate the $O(\alpha_s^2)$ gluonic operator matrix elements for the twist–2 operators, which contribute to the heavy flavor Wilson coefficients in unpolarized deeply inelastic scattering in the region $Q^2 \gg m^2$, up to the linear terms in the dimensional parameter ε, ($D = 4+\varepsilon$). These quantities are required for the description of parton distribution functions in the variable flavor number scheme (VFNS). The $O(\alpha_s^2\varepsilon)$ terms contribute at the level of the $O(\alpha_s^3)$ corrections through renormalization. We also comment on additional terms, which have to be considered in the fixed (FFNV) and variable flavor number scheme, adopting the $\overline{\rm MS}$ scheme for the running coupling constant.

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1 Introduction

Both unpolarized and polarized deep-inelastic structure functions receive contributions from light partons and heavy quarks. In the unpolarized case, the charm quark contribution may amount to 25-35% in the small x region, [1]. Since the scaling violations in case of the heavy quark contributions differ significantly from those of the light partons in a rather wide range starting from lower values of Q^2, a detailed description of the heavy quark contributions is required. In the FFNS the corresponding Wilson coefficients were calculated to next-to-leading order (NLO) in a semi-analytic approach in [2]. [2] Consistent QCD analyzes to 3-loop order require the description of both the light and the heavy flavor contributions at this level to allow for an accurate measurement of the QCD scale $\Lambda_{\rm QCD}$ in singlet analyzes [4] [3] and the measurement of the parton distribution functions. The calculation of the 3-loop heavy flavor Wilson coefficients in the whole Q^2 region is currently not within reach. However, as noticed in [6], a very precise description of the heavy flavor Wilson coefficients contributing to the structure function $F_2(x,Q^2)$ is obtained for $Q^2 \gtrsim 10\ m_Q^2$, disregarding the power corrections $\propto (m_Q^2/Q^2)^k, k \geq 1$, which covers the main region for deep–inelastic physics at HERA. In this case, the Wilson coefficients are even obtained in analytic form. The heavy flavor Wilson coefficients factorize into universal massive operator matrix elements (OMEs) $A_{ij}(\mu^2/m_Q^2)$ and the light flavor Wilson coefficients $c_j(Q^2/\mu^2)$ [7] in this limit,

$$H_i(Q^2/\mu^2, m_Q^2/\mu^2, z) = A_{ji}(m_Q^2/\mu^2, z) \otimes c^j(Q^2/\mu^2, z),\ \ i = 2, L\ . \tag{1}$$

Here, μ denotes the factorization scale and z the longitudinal momentum fraction of the parton in the hadron.

In the strict sense, only massless particles can be interpreted as partons in hard scattering processes since the lifetime of these quantum-fluctuations off the hadronic background $\tau_{\rm life} \propto 1/(k_\perp^2 + m_Q^2)$ has to be large against the interaction time $\tau_{\rm int} \propto 1/Q^2$ in the infinite momentum frame, [8]. In the massive case, $\tau_{\rm life}$ is necessarily finite and there exists a larger scale Q_0^2 below which any partonic description fails. From this it follows, that the heavy quark effects are genuinely described by the process dependent Wilson coefficients. Since parton-densities are process independent quantities, only those pieces out of the Wilson coefficients can be used to define them for heavy quarks at all. Clearly this is impossible in the region close to threshold but requires $Q^2/m_Q^2 = r \gg 1$, with $r \gtrsim 10$ in case of $F_2(x,Q^2)$. For $F_L(x,Q^2)$ the corresponding ratio even turns out to be $r \gtrsim 800$, [6,9,10]. Heavy flavor parton distributions can thus be constructed only for scales $\mu^2 \gg m_Q^2$. Their use in observables is restricted to a region, in which the power corrections can be safely neglected. This range may strongly depend on the observable considered as the examples of F_2 and F_L show.

For processes in the high $p_\perp$ region at the LHC, in which the above conditions are fulfilled, one may use heavy flavor parton distributions by proceeding as follows. In the region $Q^2 \gtrsim 10 m_Q^2$ the heavy flavor contributions to the $F_2(x,Q^2)$–world data are very well described by the asymptotic representation in the FFNS. For large scales one can then form a variable flavor representation including one heavy flavor distribution, [11]. This process can be iterated towards the next heavier flavor, provided the universal representation holds and all power corrections can be safely neglected. One has to take special care of the fact, that the matching scale in the coupling

[2] For a fast implementation of these corrections in Mellin space see [3].

[3] For a determination of $\Lambda_{\rm QCD}$ effectively analyzing the scaling violations of the non-singlet world data to $O(a_s^4)$, $(a_s = \alpha_s/(4\pi))$, cf. Ref. [5].

constant, at which the transition $N_f \to N_f+1$ is to be performed, often differs rather significantly from m_Q, cf. [12],

For the procedure outlined above, besides the quarkonic heavy flavor OMEs [6,13], the gluonic matrix elements are required. These have been calculated to $O(a_s^2)$ in Ref. [11]. Here we verify this calculation and extend it to the terms of $O(a_s^2\varepsilon)$, which enter the $O(a_s^3)$ matrix elements through renormalization. The corresponding contributions for the quarkonic matrix elements were calculated in [14]. [4]

The paper is organized as follows. In Section 2 we summarize the relations needed to describe heavy flavor parton densities out of parton densities of only light flavors in terms of massive operator matrix elements. Furthermore, we point out terms to be added to the FFNS description in the $\overline{\rm MS}$ scheme if compared to [2,6], which are of numerical relevance, cf. [16]. We also comment on the question of the effective number of flavors considering the renormalization of the process. In Section 3 the massive gluonic 2–loop operator matrix elements are presented and Section 4 contains the conclusions.

2 Heavy Flavor Parton Densities

In the asymptotic region $Q^2 \gg m_Q^2$ one may define heavy flavor parton densities. This is done under the further assumption that for the other heavy flavors the masses m_{Q_i} form a hierarchy $m_{Q_1}^2 \ll m_{Q_2}^2 \ll$ etc. Allowing for one heavy quark of mass m_Q and N_f light quarks one obtains the following light and heavy-quark parton distribution functions in Mellin space, [11],

$$\begin{aligned}
f_k(N_f+1,\mu^2,N) + f_{\overline{k}}(N_f+1,\mu^2,N) &= A_{qq,Q}^{\rm NS}\left(N_f,\frac{\mu^2}{m_Q^2},N\right)\cdot\left[f_k(N_f,\mu^2,N) + f_{\overline{k}}(N_f,\mu^2,N)\right] \\
&\quad + \tilde{A}_{qq,Q}^{\rm PS}\left(N_f,\frac{\mu^2}{m_Q^2},N\right)\cdot\Sigma(N_f,\mu^2,N) \\
&\quad + \tilde{A}_{qg,Q}^{\rm S}\left(N_f,\frac{\mu^2}{m_Q^2},N\right)\cdot G(N_f,\mu^2,N), \qquad (2) \\
f_Q(N_f+1,\mu^2,N) + f_{\overline{Q}}(N_f+1,\mu^2,N) &= A_{Qq}^{\rm PS}\left(N_f,\frac{\mu^2}{m_Q^2},N\right)\cdot\Sigma(N_f,\mu^2,N) \\
&\quad + A_{Qg}^{\rm S}\left(N_f,\frac{\mu^2}{m_Q^2},N\right)\cdot G(N_f,\mu^2,N)\ . \qquad (3)
\end{aligned}$$

Here $f_k(f_{\bar{k}})$ denote the light quark and anti–quark densities, $f_Q(f_{\bar{Q}})$ the heavy quark densities, and G the gluon density. The flavor singlet, non-singlet and gluon densities for (N_f+1) flavors are given by

$$\begin{aligned}
\Sigma(N_f+1,\mu^2,N) &= \left[A_{qq,Q}^{\rm NS}\left(N_f,\frac{\mu^2}{m_Q^2},N\right) + N_f\tilde{A}_{qq,Q}^{\rm PS}\left(N_f,\frac{\mu^2}{m_Q^2},N\right)\right. \\
&\quad \left. + A_{Qq}^{\rm PS}\left(N_f,\frac{\mu^2}{m_Q^2},N\right)\right]\cdot\Sigma(N_f,\mu^2,N)
\end{aligned}$$

[4] For the first few values of the Mellin moment N the pure-singlet and non-singlet quarkonic OMEs were calculated to $O(a_s^3)$ in Refs. [15].

$$+\left[N_f \tilde{A}^{\rm S}_{qg,Q}\left(N_f,\frac{\mu^2}{m_Q^2},N\right) + A^{\rm S}_{Qg}\left(N_f,\frac{\mu^2}{m_Q^2},N\right)\right]\cdot G(N_f,\mu^2,N) \quad (4)$$

$$\Delta(N_f+1,\mu^2,N) = f_k(N_f+1,\mu^2,N) + f_{\overline{k}}(N_f+1,\mu^2,N) - \frac{1}{N_f+1}\Sigma(N_f+1,\mu^2,N) \quad (5)$$

$$G(N_f+1,\mu^2,N) = A^{\rm S}_{gq,Q}\left(N_f,\frac{\mu^2}{m_Q^2},N\right)\cdot\Sigma(N_f,\mu^2,N)$$
$$+A^{\rm S}_{gg,Q}\left(N_f,\frac{\mu^2}{m_Q^2},N\right)\cdot G(N_f,\mu^2,N)\ . \quad (6)$$

Here,

$$A_{ij}^{\rm NS(PS,S)} = \langle j|O_i^{\rm NS(PS,S)}|j\rangle = \delta_{ij} + \sum_{l=1}^{\infty} a_s^l A_{ij}^{(l),\rm NS(PS,S)} \quad (7)$$

are the operator matrix elements of the local twist–2 non-singlet (NS), pure singlet (PS) and singlet (S) operators $O_j^{\rm NS(PS,S)}$ between on–shell partonic states $|j\rangle,\ j = q, g$ and

$$A_{ij} = N_f \tilde{A}_{ij}\ . \quad (8)$$

Note that in the pure-singlet case the term δ_{ij} in (7) is absent. The normalization of the quarkonic and gluonic operators obtained in the light cone expansion can be chosen arbitrarily. It is, however, convenient to chose the relative factor such, that the non-perturbative nucleon-state expectation values, $\Sigma(N_f,\mu^2,N)$ and $G(N_f,\mu^2,N)$, obey

$$\Sigma(N_f,\mu^2,N=2) + G(N_f,\mu^2,N=2) = 1 \quad (9)$$

due to 4-momentum conservation. As a consequence, the OMEs fulfill the relations

$$A^{\rm NS}_{qq,Q}(N_f,N=2) + N_f\tilde{A}^{\rm PS}_{qq,Q}(N_f,N=2) + \tilde{A}^{\rm PS}_{Qq}(N_f,N=2) + A^{\rm S}_{gq,Q}(N_f,N=2) = 1\ , \quad (10)$$
$$N_f\tilde{A}^{\rm S}_{qg,Q}(N_f,N=2) + \tilde{A}^{\rm S}_{Qg}(N_f,N=2) + A^{\rm S}_{gg,Q}(N_f,N=2) = 1\ . \quad (11)$$

The above scenario can be easily followed up to 2-loop order. Also here diagrams contribute which carry two different heavy quark flavors. At this level, the heavy degree of freedom may be absorbed into the coupling constant and thus being decoupled temporarily. Beginning with 3-loop order the situation becomes more involved since there are graphs in which two different heavy quark flavors occur in nested topologies, i.e. the corresponding diagrams depend on the ratio $\rho = m_c^2/m_b^2$ yielding power corrections in ρ. There is no strong hierarchy between these two masses. The above picture, leading to heavy flavor parton distributions whenever $Q^2 \gg m_Q^2$ will not hold anymore, since one cannot decide immediately in case of the two-flavor graphs, whether they belong to the c– or the b–quark distribution. Hence, the partonic description can only be maintained within a certain approximation by assuming $\rho \ll 1$.

At this point we would like to add some remarks directed to readers who are not acquainted with the details of the calculation of heavy flavor Wilson coefficients. In Refs. [2,6] the calculation to $O(a_s^2)$ was performed in a scheme, in which the heavy quark insertion in the external gluon legs[5] were absorbed into the strong coupling constant through the relation

$$\hat{a}_s = a_s(\mu^2)\left[1 + a_s(\mu^2)\delta a_s(\mu^2,m_Q^2)\right] \quad (12)$$

$$\delta a_s(\mu^2,m_Q^2) = S_\varepsilon\left\{\frac{2\beta_0(N_f)}{\varepsilon} + \frac{2\beta_{0,Q}}{\varepsilon}\left(\frac{m_Q^2}{\mu^2}\right)^{\varepsilon/2}\left(1 + \frac{\zeta_2}{8}\varepsilon^2 + \frac{\zeta_3}{24}\varepsilon^3\right)\right\}\ , \quad (13)$$

[5]In the present case, these are the terms $\propto\ T_F^2$.

for N_f light and one heavy flavor. Here $\hat{a}_s$ denotes the bare coupling constant $\hat{a}_s = \hat{g}_s^2/(16\pi^2)$, $\beta_0(N_f) = (11/3)C_A - (4/3)T_F N_f, \beta_{0,Q} = -(4/3)T_F$, $C_A = 3, T_F = 1/2$ for $SU(3)_c$. The spherical factor $S_\varepsilon = \exp[(\varepsilon/2)(\gamma_E - \ln(4\pi)]$ is set to one in the $\overline{\rm MS}$ scheme. The bare coupling constant is thus given by

$$\hat{a}_s = a_s(\mu^2)\left[1 + a_s(\mu^2)\frac{2\beta_0(N_f+1)}{\varepsilon}\right] - a_s^2(\mu^2)\beta_{0,Q}\ln\left(\frac{\mu^2}{m_Q^2}\right) + O(\varepsilon)\ . \tag{14}$$

If the above scheme is applied, cf. [6], the renormalized OMEs do not contain terms $\propto T_F^2$. However, to express a_s in the $\overline{\rm MS}$ scheme, only the first term in Eq. (14) has to be used, while the second remains as a prefactor of the 1–loop contributions. [6] Hence, as has been lined out in [11] later, the latter term appears e.g. in $A_{Qg}^{(2)}$ in front of $A_{Qg}^{(1)}$ in the case $Q^2 \gg m_Q^2$. This is encountered as well in the present paper for $A_{gg,Q}^{(2)}$. Additionally, one has to do the same for the complete heavy flavor Wilson coefficients, leading to the extra terms

$$a_s^2(\mu^2)\beta_{0,Q}\ln\left(\frac{\mu^2}{m_Q^2}\right) H_{F_i}^{(1)}\left(\frac{m_Q^2}{\mu^2}, z\right) \tag{15}$$

in the scattering cross section in the $\overline{\rm MS}$ scheme. Here,

$$\begin{aligned} H_{F_2}^{(1)}\left(\frac{m_Q^2}{Q^2}, z\right) &= 8T_F\left\{\beta\left[-\frac{1}{2} + 4z(1-z) + \frac{m_Q^2}{Q^2}z(2z-1)\right]\right. \\ &+ \left.\left[-\frac{1}{2} + z - z^2 + 2\frac{m_Q^2}{Q^2}z(3z-1) - 4\frac{m^4}{Q^4}z^2\right]\ln\left(\frac{1-\beta}{1+\beta}\right)\right\}\ , \end{aligned} \tag{16}$$

$$H_{F_L}^{(1)}\left(\frac{m_Q^2}{Q^2}, z\right) = 16T_F\left[z(1-z)\beta - \frac{m_Q^2}{Q^2}z^2\ln\left|\frac{1+\beta}{1-\beta}\right|\right]\ , \tag{17}$$

$$\beta = \sqrt{1 - \frac{4m_Q^2 z}{Q^2(1-z)}}\ , \tag{18}$$

denote the leading order Wilson coefficients for massive quarks with the strong coupling constant taken out. In the same manner the contributions $\propto T_F^2$ in the non-1PI-contribution in A_{Qg}, Ref. [11], have to be removed, to avoid double counting if the asymptotic representation for the heavy flavor Wilson coefficients is referred to. Since the light flavor Wilson coefficients are calculated in the $\overline{\rm MS}$ scheme, the same scheme has to be used for the massive OMEs. It should also be thoroughly used for renormalization, as the case for light flavors, to derive consistent results in QCD analyzes of deep-inelastic scattering data.

In Refs. [2, 6, 11] another contribution, which belongs to the inclusive heavy flavor contributions to the structure functions $F_{2,L}(x, Q^2)$, was not dealt with. To $O(a_s^2)$ these are heavy quark loop insertions on the initial state gluon line for the 1st order light flavor Wilson coefficient $c_{2(L),g}^{(1)}(x, Q^2)$. The corresponding contribution is

$$a_s^2(\mu^2)\beta_{0,Q}\ln\left(\frac{\mu^2}{m_Q^2}\right) c_{F_i,g}^{(1)}(z)\ , \tag{19}$$

[6] The running coupling constant including heavy flavors in the MOM–scheme was presented in [17] to $O(a_s^3)$ recently, showing Applequist–Carrazone [18] decoupling of the heavy quark contributions.

see also [19]. We also note that virtual corrections to $A_{qq,Q}^{(2),\rm NS}$, resp. $H_{2,L}^{(2),\rm NS}$, and $A_{gg,Q}^{(2),\rm S}$ need to be accounted for. In the asymptotic case $Q^2 \gg m_Q^2$ they lead to +-functions, which regularize the soft singularity, cf. [6, 11, 13, 14]. Here we always considered only one heavy quark contribution.

The above expressions are derived for the FFNS. Charge– and mass–renormalization are performed multiplicatively for the observables. As evident from Eq. (13), $a_s(\mu^2)$ has to be calculated for $N_f + 1$ flavors upon passing the $N_f + 1$st flavor threshold. In the FFNS the structure functions contain separate contributions of the strictly light and heavy flavors. The corresponding expressions for the Wilson coefficient contain anomalous dimensions which partly depend on N_f. In the case of the heavy flavor contributions to $O(a_s^2)$, [6, 11, 13, 14], (2–6), no closed light fermion lines contribute, however. The evolution of the three light flavors proceeds with $N_f = 3$. Due to this, there is no arbitrariness in the choice of N_f as sometimes anticipated in the literature.

3 The Gluonic Operator Matrix Elements

The description of heavy quark parton densities, Eqs. (2–6), requires the massive operator matrix elements given by the partonic on-shell expectation values $\langle p|O^K|p\rangle,\ p = q, g$, of the operators, cf. [20],

$$O^{F,0}_{\mu_1,\ldots,\mu_n} = i^{n-1}\mathbf{S}[\overline{\psi}\gamma_{\mu_1}D_{\mu_2}\ldots D_{\mu_n}\psi] - \text{trace terms}\ , \tag{20}$$

$$O^{V}_{\mu_1,\ldots,\mu_n} = 2i^{n-2}\mathbf{S}\mathrm{Sp}[F_{\mu_1\alpha}D_{\mu_2}\ldots D_{\mu_{n-1}}F^{\alpha}_{\mu_n}] - \text{trace terms}\ . \tag{21}$$

Here $D_\mu = \partial_\mu - ig_s t_a A_\mu^a$ denotes the covariant derivative, t_a are the generators of $SU(3)_c$, ψ the quark fields, A_μ^a the gluon fields, $F_{\mu\nu}$ the gluonic field strength tensors, Sp the color trace, and $\mathbf{S}$ the operator which symmetrizes the Lorentz indices. The corresponding quarkonic operator matrix elements were calculated in Refs. [6, 13] to $O(a_s^2)$ and $O(a_s^2\varepsilon)$ in [14], respectively.

The renormalized gluonic operator matrix elements $A_{gq,Q}$ and $A_{gg,Q}$ to $O(a_s^2)$ are given by

$$\begin{aligned} A_{gq,Q} &= a_s^2\Bigg[\hat{A}^{(2)}_{gq,Q} + Z_{gq}^{-1,(2)}(N_f+1) - Z_{gq}^{-1,(2)}(N_f) \\ &\quad + \Big(\hat{A}^{(1)}_{gg,Q} + Z_{gg}^{-1,(1)}(N_f+1) - Z_{gg}^{-1,(1)}(N_f)\Big)\Gamma_{gq}^{-1,(1)}\Bigg] + O(a_s^3), \end{aligned} \tag{22}$$

$$\begin{aligned} A_{gg,Q} &= a_s\left[\hat{A}^{(1)}_{gg,Q} + Z_{gg}^{-1,(1)}(N_f+1) - Z_{gg}^{-1,(1)}(N_f)\right] \\ &\quad a_s^2\Bigg[\hat{A}^{(2)}_{gg,Q} + Z_{gg}^{-1,(2)}(N_f+1) - Z_{gg}^{-1,(2)}(N_f) + Z_{gg}^{-1,(1)}(N_f+1)\hat{A}^{(1)}_{gg,Q} \\ &\quad + Z_{gq}^{-1,(1)}(N_f+1)\hat{A}^{(1)}_{Qg} + \left[\hat{A}^{(1)}_{gg,Q} + Z_{gg}^{-1,(1)}(N_f+1) - Z_{gg}^{-1,(1)}(N_f)\right]\Gamma_{gg}^{-1,(1)}(N_f) \\ &\quad + \delta a_s \hat{A}^{(1)}_{gg,Q}\Bigg] + O(a_s^3). \end{aligned} \tag{23}$$

Here $\hat{A}_{ij}$ are the operator matrix elements after mass–renormalization has been carried out. The Z–factors $Z_{ij}(N_f)$ renormalize the ultraviolet singularities of the operators and $\Gamma_{ij}(N_f)$ remove

the collinear singularities, cf. [6,11,13,14]. The terms $Z^{-1}_{gq(g)}(N_f+1)$ are equal to

$$Z^{-1}_{gq}(N_f+1) = a_s\left[-\frac{1}{\varepsilon}\gamma^{(0)}_{gq}\right] + a_s^2\left[\frac{1}{\varepsilon}\left(-\frac{1}{2}\gamma^{(1)}_{gq} - \gamma^{(0)}_{gq}\delta a_s\right) + \frac{1}{\varepsilon^2}\left(\gamma^{(0)}_{gq}\beta_0 + \frac{1}{2}\gamma^{(0)}_{gq}\gamma^{(0)}_{qq} + \frac{1}{2}\gamma^{(0)}_{gq}\gamma^{(0)}_{gg}\right)\right] + O(a_s^3) \tag{24}$$

$$Z^{-1}_{gg}(N_f+1) = 1 + a_s\left[-\frac{1}{\varepsilon}\gamma^{(0)}_{gg}\right] + a_s^2\left[\frac{1}{\varepsilon}\left(-\frac{1}{2}\gamma^{(1)}_{gg} - \gamma^{(0)}_{gg}\delta a_s\right) + \frac{1}{\varepsilon^2}\left(\gamma^{(0)}_{gg}\beta_0 + \frac{1}{2}\gamma^{(0)}_{qg}\gamma^{(0)}_{qq} + \frac{1}{2}{\gamma^{(0)}_{gg}}^2\right)\right] + O(a_s^3)\ . \tag{25}$$

In Eqs. (24,25), $\gamma^{(l)}_{ij}$ are the $O(a_s^{l+1})$ anomalous dimensions and have to be taken - as well as β_0 - at N_f+1 flavors. We adopt the notation $\hat{\gamma}^{(l)}_{ij} = \gamma^{(l)}_{ij}(N_f+1) - \gamma^{(l)}_{ij}(N_f)$ and define for later use

$$f(\varepsilon) = \left(\frac{m_Q^2}{\mu^2}\right)^{\varepsilon/2} \exp\left[\sum_{k=2}^{\infty}\frac{\zeta_k}{k}\left(\frac{\varepsilon}{2}\right)^k\right]\ . \tag{26}$$

To the operator matrix element $\hat{A}^{(1)}_{gg,Q}$ necessarily only non-1PI diagrams contribute. The unrenormalized OME $\hat{A}^{(2)}_{gq,Q}$ is given by [7]

$$\hat{A}^{(2)}_{gq,Q} = \left(\frac{m_Q^2}{\mu^2}\right)^{\varepsilon}\left[\frac{2\beta_{0,Q}}{\varepsilon^2}\gamma^{(0)}_{gq} + \frac{\hat{\gamma}^{(1)}_{gq}}{2\varepsilon} + a^{(2)}_{gq,Q} + \overline{a}^{(2)}_{gq,Q}\varepsilon\right] + O(\varepsilon^2)\ . \tag{27}$$

The constant and $O(\varepsilon)$ contributions $a^{(2)}_{gq,Q}$ and $\overline{a}^{(2)}_{gq,Q}$ read

$$a^{(2)}_{gq,Q} = T_F C_F\left\{\frac{4}{3}\frac{N^2+N+2}{(N-1)N(N+1)}\left(2\zeta_2 + S_2 + S_1^2\right) - \frac{8}{9}\frac{8N^3+13N^2+27N+16}{(N-1)N(N+1)^2}S_1 + \frac{8}{27}\frac{P_1}{(N-1)N(N+1)^3}\right\}\ , \tag{28}$$

$$\overline{a}^{(2)}_{gq,Q} = T_F C_F\left\{\frac{2}{9}\frac{N^2+N+2}{(N-1)N(N+1)}\left(-2S_3 - 3S_2S_1 - S_1^3 + 4\zeta_3 - 6\zeta_2 S_1\right) + \frac{2}{9}\frac{8N^3+13N^2+27N+16}{(N-1)N(N+1)^2}\left(2\zeta_2 + S_2 + S_1^2\right) - \frac{4}{27}\frac{P_1 S_1}{(N-1)N(N+1)^3} + \frac{4}{81}\frac{P_2}{(N-1)N(N+1)^4}\right\}\ , \tag{29}$$

with

$$P_1 = 43N^4 + 105N^3 + 224N^2 + 230N + 86 \tag{30}$$

$$P_2 = 248N^5 + 863N^4 + 1927N^3 + 2582N^2 + 1820N + 496\ . \tag{31}$$

[7] In the following we drop the overall factor $[1+(-1)^N]/2$ in the operator matrix elements.

Here $S_{\vec{a}} \equiv S_{\vec{a}}(N)$ denote the (nested) harmonic sums, [21],

$$S_{b,\vec{a}}(N) = \sum_{k=1}^{N} \frac{(\mathrm{sign}(b))^k}{k^{|b|}} S_{\vec{a}}(k) \; . \tag{32}$$

The renormalized operator matrix element is given by

$$A_{gq,Q} = a_s^2 \left[\frac{\beta_{0,Q}\gamma_{gq}^{(0)}}{2} \ln^2\left(\frac{m_Q^2}{\mu^2}\right) + \frac{\hat{\gamma}_{gq}^{(1)}}{2} \ln\left(\frac{m_Q^2}{\mu^2}\right) + a_{gq,Q}^{(2)} - \frac{\beta_{0,Q}\gamma_{gq}^{(0)}}{2}\zeta_2 \right] + O(a_s^3) \; . \tag{33}$$

Here the anomalous dimensions $\gamma_{gq}^{(0,1)}$ and $\gamma_{gg}^{(0,1)}$ are

$$\gamma_{gq}^{(0)} = -4C_F \frac{N^2+N+2}{(N-1)N(N+1)} \; , \tag{34}$$

$$\gamma_{gg}^{(0)} = 8C_A \left[S_1 - 2\frac{N^2+N+1}{(N-1)N(N+1)(N+2)} \right] - 2\beta_0(N_f) \; , \tag{35}$$

$$\hat{\gamma}_{gq}^{(1)} = C_F T_F \left(-\frac{32}{3}\frac{N^2+N+2}{(N-1)N(N+1)} S_1 + \frac{32}{9}\frac{8N^3+13N^2+27N+16}{(N-1)N(N+1)^2} \right) \; , \tag{36}$$

$$\begin{aligned} \hat{\gamma}_{gg}^{(1)} = {} & 8C_F T_F \left[1 - \frac{4}{3}\frac{1}{N-1} + \frac{16}{N} - \frac{6}{N^2} + \frac{4}{N^3} - \frac{8}{N+1} - \frac{10}{(N+1)^2} + \frac{4}{(N+1)^3} - \frac{20}{3}\frac{1}{N+2} \right] \\ & + \frac{16}{3} C_A T_F \left[2 + \frac{23}{3}\frac{1}{N-1} - \frac{19}{3}\frac{1}{N} - \frac{2}{N^2} + \frac{19}{3}\frac{1}{N+1} - \frac{2}{(N+1)^2} - \frac{23}{3}\frac{1}{N+2} - \frac{10}{3} S_1 \right] \; , \end{aligned} \tag{37}$$

and $\hat{\gamma}_{gg}^{(0)} = (8/3)T_F$. A closer look at Eqs. (33,40) reveals, that the terms $\propto \zeta_2$ cancel. The coefficients of the un-renormalized OME $\hat{A}_{gg,Q}$ are given by

$$\hat{A}_{gg,Q}^{(1)} = -\frac{2\beta_{0,Q}}{\varepsilon} f(\varepsilon) \; , \tag{38}$$

$$\begin{aligned} \hat{A}_{gg,Q}^{(2)} = {} & \left(\frac{m_Q^2}{\mu^2}\right)^{\varepsilon} \left[\frac{1}{2\varepsilon^2} \left\{ \gamma_{gq}^{(0)} \hat{\gamma}_{qg}^{(0)} + 2\beta_{0,Q} \left(\gamma_{gg}^{(0)} + 2\beta_0 \right) \right\} + \frac{\hat{\gamma}_{gg}^{(1)}}{2\varepsilon} + a_{gg,Q}^{(2)} + \overline{a}_{gg,Q}^{(2)} \varepsilon \right] \\ & + \frac{4\beta_{0,Q}^2 f(\varepsilon)^2}{\varepsilon^2} + O(\varepsilon^2) \; . \end{aligned} \tag{39}$$

The constant and $O(\varepsilon)$ contributions $a_{gg,Q}^{(2)}$ and $\overline{a}_{gg,Q}^{(2)}$ are

$$\begin{aligned} a_{gg,Q}^{(2)} = {} & T_F C_A \left\{ -\frac{8}{3}\zeta_2 S_1 + \frac{16(N^2+N+1)\zeta_2}{3(N-1)N(N+1)(N+2)} - 4\frac{56N+47}{27(N+1)} S_1 \right. \\ & \left. + \frac{2P_3}{27(N-1)N^3(N+1)^3(N+2)} \right\} \\ & + T_F C_F \left\{ \frac{4(N^2+N+2)^2\zeta_2}{(N-1)N^2(N+1)^2(N+2)} - \frac{P_4}{(N-1)N^4(N+1)^4(N+2)} \right\} \; , \end{aligned} \tag{40}$$

$$\overline{a}_{gg,Q}^{(2)} = T_F C_A \Bigg\{ -\frac{8}{9}\zeta_3 S_1 - \frac{20}{9}\zeta_2 S_1 + \frac{16(N^2+N+1)}{9(N-1)N(N+1)(N+2)}\zeta_3 + \frac{2N+1}{3(N+1)}S_2 - \frac{S_1^2}{3(N+1)}$$
$$+\frac{4P_5\zeta_2}{9(N-1)N^2(N+1)^2(N+2)} - 2\frac{328N^4+256N^3-247N^2-175N+54}{81(N-1)N(N+1)^2}S_1$$
$$+\frac{P_6}{81(N-1)N^4(N+1)^4(N+2)}\Bigg\}$$

$$+T_F C_F \Bigg\{ \frac{4(N^2+N+2)^2\zeta_3}{3(N-1)N^2(N+1)^2(N+2)} + \frac{P_7\zeta_2}{(N-1)N^3(N+1)^3(N+2)}$$
$$+\frac{P_8}{4(N-1)N^5(N+1)^5(N+2)}\Bigg\}, \tag{41}$$

where

$$P_3 = 15N^8+60N^7+572N^6+1470N^5+2135N^4+1794N^3+722N^2-24N-72\ , \tag{42}$$
$$P_4 = 15N^{10}+75N^9+112N^8+14N^7-61N^6+107N^5+170N^4+36N^3$$
$$-36N^2-32N-16\ , \tag{43}$$
$$P_5 = 3N^6+9N^5+22N^4+29N^3+41N^2+28N+6\ , \tag{44}$$
$$P_6 = 3N^{10}+15N^9+3316N^8+12778N^7+22951N^6+23815N^5+14212N^4+3556N^3$$
$$-30N^2+288N+216\ , \tag{45}$$
$$P_7 = N^8+4N^7+8N^6+6N^5-3N^4-22N^3-10N^2-8N-8\ , \tag{46}$$
$$P_8 = 31N^{12}+186N^{11}+435N^{10}+438N^9-123N^8-1170N^7-1527N^6-654N^5$$
$$+88N^4-136N^2-96N-32\ . \tag{47}$$

The renormalized operator matrix element $A_{gg,Q}$ reads

$$A_{gg,Q} = a_s\frac{4}{3}T_F\ln\Big(\frac{m_Q^2}{\mu^2}\Big) + a_s^2\Bigg[\frac{1}{8}\Bigg\{2\beta_{0,Q}\Big(\gamma_{gg}^{(0)}+2\beta_0\Big)+\gamma_{gq}^{(0)}\hat{\gamma}_{qg}^{(0)}\Bigg\}\ln^2\Big(\frac{m_Q^2}{\mu^2}\Big) + \frac{\hat{\gamma}_{gg}^{(1)}}{2}\ln\Big(\frac{m_Q^2}{\mu^2}\Big)$$
$$+a_{gg,Q}^{(2)} - \frac{\zeta_2}{8}\Big[2\beta_{0,Q}\Big(\gamma_{gg}^{(0)}+2\beta_0\Big)+\gamma_{gq}^{(0)}\hat{\gamma}_{qg}^{(0)}\Big]\Bigg] + O(a_s^3)\ . \tag{48}$$

We agree with the results for $a_{gq,Q}^{(2)}$ and $a_{gg,Q}^{(2)}$ given in [11], which we presented in (28,40). The new terms $\overline{a}_{gq,Q}^{(2)}$ and $\overline{a}_{gg,Q}^{(2)}$, (29, 41), contribute to all OMEs $A_{ij}^{(3)}$ through renormalization. With respect to the mathematical structure, $a_{gq(gg),Q}^{(2)}$ and $\overline{a}_{gq(gg),Q}^{(2)}$, (28,40,29,41), belong to the class being observed for two–loop corrections before, [22]. In the present case even only single harmonic sums contribute. We checked our results for the moments $N = 2,\ldots,8$ using the code `MATAD`, [23]. An additional check is provided by the sum rules in Eqs. (10,11), which are fulfilled by the renormalized OMEs presented here and in Refs. [6, 11, 13]. Moreover, we observe that these rules are obeyed on the unrenormalized level as well, even up to $O(\varepsilon)$, [14].

To describe the evolution of the parton distributions, Eqs. (2–6), the OMEs (33,48) have to

be supplemented by the corresponding 1PR terms

$$A_{Qg}^{(2)} \to A_{Qg}^{(2)} + a_s^2 4\beta_{0,Q} T_F \frac{N^2+N+2}{N(N+1)(N+2)} \ln^2\left(\frac{\mu^2}{m_Q^2}\right) \;, \tag{49}$$

$$A_{gg,Q}^{(2)} \to A_{gg,Q}^{(2)} + a_s^2 \beta_{0,Q}^2 \ln^2\left(\frac{\mu^2}{m_Q^2}\right) \;. \tag{50}$$

Eqs. (49,50) agree with the results presented in Ref. [11]. In applying these parton densities in other hard scattering processes this modification also affects part of the massless hard scattering cross sections there, as outlined above.

4 Conclusions

We calculated the massive gluonic operator matrix elements $A_{gq,Q}$ and $A_{gg,Q}$, being required in the description of heavy flavor parton densities at scales sufficiently above threshold, to $O(a_s^2\varepsilon)$. We confirm previous results given in [11] for the constant terms and obtained newly the $O(\varepsilon)$ terms which enter the 3-loop corrections to A_{ij} via renormalization. We reminded of details of the charge renormalization and clarified that additional terms at $O(a_s^2)$ are to be included in the data analysis in the FFNS and VFNS using the $\overline{\rm MS}$ scheme.

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