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New results for loop integrals: AMBRE, CSectors, hexagon

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We report on the three Mathematica packages `hexagon, CSectors, AMBRE`. They are useful for the evaluation of one- and two-loop Feynman integrals with a dependence on several kinematical scales. These integrals are typically needed for LHC and ILC applications, but also for higher order corrections at meson factories. `hexagon` is a new package for the tensor reduction of one-loop 5-point and 6-point functions with rank $R = 3$ and $R = 4$, respectively; `AMBRE` is a tool for derivations of Mellin-Barnes representations; `CSectors` is an interface for the package `sector_decomposition` and allows a convenient, direct evaluation of tensor Feynman integrals.

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1. Introduction

In recent years, we observe that higher energy, higher luminosity, higher precision, more massive particles at LHC and ILC, but also at low energy meson factories[1], lead to completely new demands on the efficient evaluation of Feynman diagrams, including:

- need of some 3-point 4-point two-loop diagrams, including double boxes,
- need of many n-point one-loop diagrams,

with massive and massless particles participating, leading to complicated many-scale problems.

A few of the approaches to answer the requests will be shortly introduced, concentrating on our own activities and on publicly available packages. They are devoted to the evaluation of L-loop n-point Feynman integrals of tensor rank R, with loop momenta k_l, with E external legs with momenta p_e, and with N internal lines with masses m_i and propagators $1/D_i$:

$$I^{\{\alpha\}} = \frac{e^{L\varepsilon\gamma_E}}{(i\pi^{d/2})^L} \int \frac{d^dk_1 \ldots d^dk_L \; N^{\alpha_1\cdots\alpha_R}}{D_1^{\nu_1}\ldots D_i^{\nu_i}\ldots D_N^{\nu_N}}, \tag{1.1}$$

$$D_i = \left(q_i^2 - m_i^2\right) = \left[\sum_{l=1}^{L} c_i^l k_l + \sum_{e=1}^{E} d_i^e p_e\right]^2 - m_i^2 \,, \tag{1.2}$$

where we call $d = 4-2\varepsilon$ the generic dimension and ν_i the index of the propagator. The numerator may contain a tensor structure, e.g.:

$$N = 1, \quad k_1^{\alpha_1}k_1^{\beta_1}, \quad k_1^{\alpha_1}\cdots k_R^{\alpha_R}. \tag{1.3}$$

There exists no general algorithm for the calculation of arbitrary Feynman integrals, but there exists a rising number of tools, and some of them are publicly available. In the next sections, I will comment on recent developments around three of them.

The Feynman integrals may be evaluated in quite different ways. One may derive for them (systems of coupled) difference or differential equations, or one may seek a (minimal) basis of (scalar) master integrals, and solve only the latter ones, etc. Often the Feynman parameter representation is useful, which replaces the d-dimensional momentum integrations by an appropriate number of parameter integrals. Feynman parameters are introduced by the representation:

$$\frac{1}{D_1^{\nu_1}D_2^{\nu_2}\ldots D_N^{\nu_N}} = \frac{\Gamma(\nu_1+\ldots+\nu_N)}{\Gamma(\nu_1)\ldots\Gamma(\nu_N)} \int_0^1 dx_1 \ldots \int_0^1 dx_N \frac{x_1^{\nu_1-1}\ldots x_N^{\nu_N-1}\delta(1-x_1\ldots-x_N)}{(x_1D_1+\ldots+x_ND_N)^{N_\nu}}, \tag{1.4}$$

with $N_\nu = \nu_1+\ldots\nu_N$. The denominator of $I^{\{\alpha\}}$ contains, after introduction of Feynman parameters x_i, the momentum dependent function m^2 with index-exponent N_ν:

$$(m^2)^{-(\nu_1+\ldots+\nu_N)} = (x_1D_1+\ldots+x_ND_N)^{-N_\nu} = (k_iM_{ij}k_j - 2Q_jk_j + J)^{-N_\nu}. \tag{1.5}$$

[1] For LHC and ILC, the statement is evident. At meson factories, we have in mind e.g. luminosity determination with Bhabha scattering at two-loop accuracy. For details on prospects at high-luminosity Φ- and B-factories, see e.g. the recent mini-review [1] and references quoted therein.

Here M is an (LxL)-matrix, $Q = Q(x_i, p_e)$ an L-vector and $J = J(x_i, m_i^2, p_{e_j} p_{e_l})$. The M, Q, J are linear in x_i. The momentum integration is now simple. Shift the momenta k such that m^2 has no linear term in $\bar{k}$, $k = \bar{k} + (M^{-1})Q$, and $m^2 = \bar{k}M\bar{k} - QM^{-1}Q + J$. For a scalar Feynman integral e.g. one gets:

$$I \; = \; (-1)^{N_\nu} e^{L\varepsilon\gamma_E} \frac{\Gamma\left(N_\nu - \frac{D}{2}L\right)}{\Gamma(\nu_1)\ldots\Gamma(\nu_N)} \int_0^1 \prod_{j=1}^N dx_j \, x_j^{\nu_j - 1} \delta\left(1 - \sum_{i=1}^N x_i\right) \frac{U_L(x)^{N_\nu - D(L+1)/2}}{F_L(x)^{N_\nu - DL/2}} \tag{1.6}$$

with

$$U_L(x) \; = \; (\det M), \tag{1.7}$$

$$F_L(x) \; = \; (\det M) \; \mu^2 \; = \; -(\det M) \, J + Q \, \tilde{M} \, Q. \tag{1.8}$$

For one-loop functions it is: $U_1(x) = \det M = 1 = \sum x_i$ and so U_1 'disappears'. Further, the construct $F_1(x) = -J + Q^2$ may be made bilinear in $x_i x_j$: $F_1(x) = -J(\sum x_i) + Q^2 = \sum A_{ij} x_i x_j$. For tensor Feynman integrals the expressions are a little more involved, but they have the same structure: sums of rationals in the x_i combined with non-integer powers of $U(x)$ and $F(x)$ [2, 3, 4].

2. AMBRE.m

There is an elegant approach to the x-integrations. In the $F(x)$-function (1.8) one may change sums of monomials in x into products (as often as necessary), by Mellin-Barnes transformations, e.g.:

$$\frac{1}{[A(s)x_1^{a_1} + B(t)x_1^{b_1}x_2^{b_2}]^a} = \frac{1}{2\pi i \, \Gamma(a)} \int_{-i\infty}^{i\infty} d\sigma [A(s)x_1^{a_1}]^\sigma [B(t)x_1^{b_1}x_2^{b_2}]^{a+\sigma} \, \Gamma(a+\sigma)\Gamma(-\sigma). \tag{2.1}$$

After this, one may perform the x-integrations:

$$\int_0^1 \prod_{j=1}^N dx_j \, x_j^{\alpha_j - 1} \, \delta\left(1 - \sum_{i=1}^N x_i\right) \; = \; \frac{\Gamma(\alpha_1) \, \cdots \, \Gamma(\alpha_N)}{\Gamma(\alpha_1 + \, \cdots \, + \alpha_N)}. \tag{2.2}$$

Let us look at an example, the integral `V6l4m1`, see Figure 1. [2] In a loop-by-loop approach, after the first momentum integration one gets here $U = 1$ and a first F-function (1.8), which depends yet on one internal momentum k_1:

```
f1 = m^2 [X[2]+X[3]+X[4]]^2 - s X[2]X[4] - PR[k1+p1,m] X[1]X[2]
                - PR[k1+p1+p2,0] X[2]X[3] - PR[k1-p2,m] X[1]X[4]
                     - PR[k1,0] X[3]X[4] ,
```

leading to a 7-dimensional MB-representation; after the second momentum integration, one has:

```
f2 = m^2 [X[2]+X[3]]^2 - s X[2]X[3] - s X[1]X[4] - 2s X[3]X[4],
```

[2] The naming convention follows [3].

leading to another 4-dimensional integral. After several applications of Barnes' first lemma, an 8-dimensional integral has to be treated.[3]

The package `AMBRE.m` is designed for a semi-automatic derivation of Mellin-Barnes (MB) representations for Feynman diagrams; for details and examples of use see the webpage `http://prac.us.edu.pl/~gluza/ambre/`. The package is also available from `http://projects.hepforge.org/mbtools/`. Version 1.0 is described in [4], the last released version is 1.2. We are releasing now version 2.0, which allows to construct MB-representations for two-loop *tensor* integrals. The package is yet restricted to the so-called loop-by-loop approach, which yields compact representations, but is known to potentially fail for non-planar topologies with several scales. An instructive example has been discussed in [5].

For one-scale problems, one may safely apply `AMBRE.m` to non-planar diagrams. For our example `V6l4m1`, one gets e.g. with the 8-dimensional MB-representation scetched above the following numerical output after running also `MB.m` [6] (see also the webpage `http://projects.hepforge.org/mbtools/`), at $s=-11$:

$$\texttt{V6l4m}\,(-s)^{2\varepsilon} = -0.0522082\,\frac{1}{\varepsilon} - 0.17002 + 0.25606\,\varepsilon + 4.67\,\varepsilon^2 + \mathcal{O}(\varepsilon^3). \tag{2.3}$$

In simpler problems, MB-representations are a good starting point for analytical solutions, typically by summing multiple sums of residua. Let us take as an example the diagram `F5l2m` appearing in the one-loop NLO process $gg \to q\bar{q}g$, shown in Figure 1, leading to (factorizing) two-fold infinite sums. The steps of evaluation follow closely [7, 8], and we reproduce here the result for the IR-divergent part in the infrared limit $t' = t \equiv t_m + 1$, with $m = 1$ and $t = (p_1+p_2)^2$, in closed form:

$$\texttt{F5l2m(IR)} = \frac{J_{-2}}{\varepsilon^2} + \frac{J_{-1}}{\varepsilon} + J_0, \tag{2.4}$$

$$J_{-2} = \frac{(-t_m)^{-2\varepsilon}}{s t_m^2} + \sum_{i=1,2} \frac{(v_i/s)^{-2\varepsilon}}{v_i} \frac{(-s)^{-2\varepsilon}}{2 s t_m}, \tag{2.5}$$

$$J_{-1} = \frac{(-t_m)^{-2\varepsilon}}{s t_m^2} \left[\log(-s) + \log(v_2/s) + \log(v_4/s)\right], \tag{2.6}$$

$$J_0 = -\frac{\pi^2}{3}\frac{1}{s t_m}\left(\frac{1}{v_2}+\frac{1}{v_4}\right) - \frac{\pi^2}{12}\frac{(-t_m)^{-2\varepsilon}}{s t_m^2} + \sum_{i=1,2} \frac{13\pi^2}{12} \frac{(v_i/s)^{-2\varepsilon}}{v_i} \frac{(-s)^{-2\varepsilon}}{2 s t_m}. \tag{2.7}$$

Whereever necessary, s has to be replaced by $s + i\varepsilon$. The term J_0 develops infrared endpoint singularities from the phase space integrations, due to the proportionality of $v_2 = p_2 p_3$ and $v_4 = p_4 p_3$ to the gluon momentum p_3.

Similarly, one gets for the QED pentagon `F5l3m`:

$$\texttt{F5l3m(IR)} = \frac{J_{-1}}{\varepsilon} + J_0, \tag{2.8}$$

$$J_{-1} = \sum_{i=1,2} \frac{1}{2s} \frac{(v_i/s)^{-2\varepsilon}}{v_i} (-s)^{-2\varepsilon} \sum_{n=0}^{\infty} \frac{(t)^n}{\binom{2n}{n}(2n+1)}, \tag{2.9}$$

[3] We made no attempt here to simplify the situation by any of the numerous tricks and reformulations etc. known to experts.

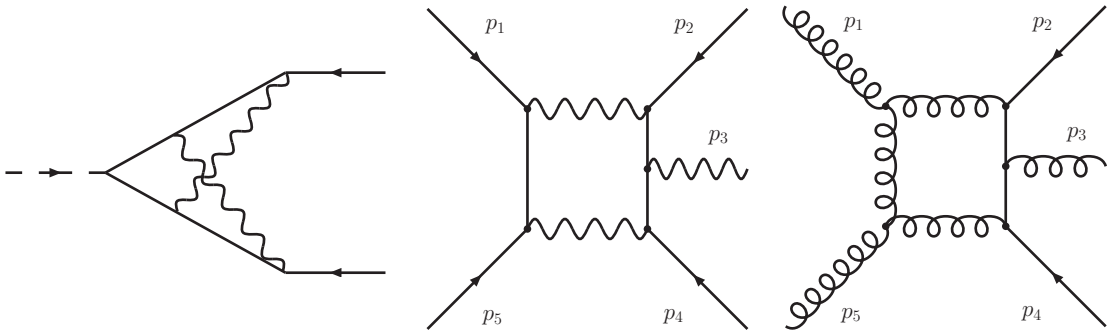

Figure 1: The master integrals `V6l4m1` and `F5l2m` and `F5l3m`.

$$J_0 = \sum_{i=1,2} \frac{1}{2s} \frac{(v_i/s)^{-2\varepsilon}}{v_i} (-s)^{-2\varepsilon} \sum_{n=0}^{\infty} \frac{(t)^n}{\binom{2n}{n}(2n+1)} \Big[-3S_1(n) + 2S_1(1+2n) \Big]. \quad (2.10)$$

The answer is less singular in ε, but more complicated. The inverse binomial sums with $S_1(n)$ and $S_1(2n+1)$ in J_0 are performed in [8]. The expression quoted there for J_0 contained instead of the factor $(v_i/s)^{-2\varepsilon}$ its expansion in ε, thus developing terms depending on powers of $\ln(v_i/s)$. This was a disadvantage notations for the subsequent regularization of the phase space integrals, so the present result is more appropriate.

3. CSectors.m

For Euclidean kinematics, the integrand for the multi-dimensional x-integrations is positive semi-definite. In numerical integrations, one has to separate the poles in $d-4$, and in doing so one has to avoid overlapping singularities. A method for that is sector decomposition.[4] The intention is to separate singular regions in different variables from each other, as is nicely demonstrated by an example borrowed from [11]:

$$\begin{aligned} I &= \int_0^1 dx \int_0^1 dy \frac{1}{x^{1+a\varepsilon} y^{b\varepsilon} [x+(1-x)y]} \\ &= \int_0^1 \frac{dx}{x^{1+(a+b)\varepsilon}} \int_0^1 \frac{dt}{t^{b\varepsilon}[1+(1-x)t]} + \int_0^1 \frac{dy}{y^{1+(a+b)\varepsilon}} \int_0^1 \frac{dt}{t^{1+a\varepsilon}[1+(1-y)t]}. \end{aligned} \quad (3.1)$$

At several occasions, we used the package `sector_decomposition` [10] (built on the C++ library GINAC [13]) for cross checks and felt a lack of simple treatment of Feynman integrals with numerators. For that reason, the interface `CSectors` was written; it will be made publicly available soon. The syntax is similar to that of `AMBRE`. The program input for the evaluation of the integral `V6l4m1` is simple; we again choose $m = 1, s = -11$, and the topology may be read from the arguments of propagator functions `PR`:

[4] There are quite a few recent papers on that, e.g. [9, 2, 10], and nice reviews are given in [11, 12].

```
<< CSectors.m

Options[DoSectors]
SetOptions[DoSectors, TempFileDelete -> False, SetStrategy -> C]

n1 = n2 = n3 = n4 = n5 = n6 = n7 = 1;
m = 1; s = -11;
invariants = {p1^2 -> m^2, p2^2 -> m^2, p1 p2 -> (s - 2 m^2)/2};

DoSectors[{1},
  {PR[k1,0,n1]       PR[k2,0,n2]       PR[k1+p1,m,n3]
   PR[k1+k2+p1,m,n5] PR[k1+k2-p2,m,n6] PR[k2-p2,m,n7]},
   {k2, k1}, invariants][-4, 2]
```

Here, the numerator is 1 (see the first argument $\{1\}$ of `DoSectors`), and the output contains the functions U_2 and F_2:

```
Using strategy C
U = x3 x4+x3 x5+x4 x5+x3 x6+x5 x6+x2 (x3+x4+x6)+x1 (x2+x4+x5+x6)

F = x1 x4^2+13 x1 x4 x5+x4^2 x5+x1 x5^2+x4 x5^2+13 x1 x4 x6
+2 x1 x5 x6+13 x4 x5 x6+x5^2 x6+x1 x6^2+x5 x6^2+x3^2 (x4+x5+x6)
+x2(x3^2+x4^2+13 x4 x6+x6^2+x3 (2 x4+13 x6))+x3 (x4^2+(x5+x6)^2
+x4 (2x5+13 x6))
```

Notice the presence of a U-function and the complexity of the F-function (compared to $U = 1$ and `f1` and `f2` in the loop-by-loop MB-approach) due to the non-sequential, direct performance of both momentum integrals at once. Both U and F are evidently positive semi-definite. The numerical result for the Feynman integral is:

$$\texttt{V6l4m1}(-s)^{2\varepsilon} = -0.052210\,\frac{1}{\varepsilon} - 0.17004 + 0.24634\,\varepsilon + 4.8773\,\varepsilon^2 + \mathcal{O}(\varepsilon^3). \tag{3.2}$$

The numbers may be compared to (2.3). We obtained a third numerical result, also by sector decomposition, with the Mathematica package `FIESTA` [14]:

$$\texttt{V6l4m1}(-s)^{2\varepsilon} = -0.052208\,\frac{1}{\varepsilon} - 0.17002 + 0.24622\,\varepsilon + 4.8746\,\varepsilon^2 + \mathcal{O}(\varepsilon^3). \tag{3.3}$$

The most accurate result can be obtained with an analytical representation based on harmonic polylogarithmic functions [15, 16] obtained by solving a system of differential equations [17]:

$$\texttt{V6l4m1}(-s)^{2\varepsilon} = -0.0522082\,\frac{1}{\varepsilon} - 0.170013 + 0.246253\,\varepsilon + 4.87500\,\varepsilon^2 + \mathcal{O}(\varepsilon^3). \tag{3.4}$$

All displayed digits are accurate here.

4. hexagon.m

The Mathematica package `hexagon` v. 1.0 (19 Sep 2008) was released quite recently [30 Sep 2008] at `http://www-zeuthen.desy.de/theory/research/CAS.html`. It may be used for the reduction of one-loop tensor 5- and 6-point Feynman integrals up to ranks $R = 4$ and $R = 3$, respectively, to scalar 3- and 4-point Feynman integrals. The latter have to be evaluated by some other package like `LoopTools` [18, 19] (case of only massive internal lines) or `QCDloops` [20] (general case); both these packages make also use of `FF` [21]. The formalism underlying this reduction and a short decsription as well as numerical examples may be found elsewhere [22, 23, 24, 25] so that we may hold this write-up short here. We only mention that it does not use Feynman parameters, but is based on recurrence relations with dimensional shifts [26]. In this approach, we have shown quite recently how to cancel explicitely and completely the inverse powers of the leading Gram determinant. `hexagon` was the first publicly available tensor reduction program for 5- and 6-point Feynman integrals with arbitrary internal masses. Now, also `GOLEM95` [27] became public, but in the released version it handles so far only massless internal particles.

5. Summary

We described new features of the packages `AMBRE` and `hexagon` and of the interface `CSectors`.

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