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Semileptonic form factors $D \to \pi, K$ and $B \to \pi, K$ from a fine lattice

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Abstract

We extract the form factors relevant for semileptonic decays of D and B mesons from a relativistic computation on a fine lattice in the quenched approximation. The lattice spacing is $a = 0.04$ fm (corresponding to $a^{-1} = 4.97$ GeV), which allows us to run very close to the physical B meson mass, and to reduce the systematic errors associated with the extrapolation in terms of a heavy quark expansion. For decays of D and D_s mesons, our results for the physical form factors at $q^2 = 0$ are as follows: $f_+^{D\to\pi}(0) = 0.74(6)(4)$, $f_+^{D\to K}(0) = 0.78(5)(4)$ and $f_+^{D_s\to K}(0) = 0.68(4)(3)$. Similarly, for B and B_s we find: $f_+^{B\to\pi}(0) = 0.27(7)(5)$, $f_+^{B\to K}(0) = 0.32(6)(6)$ and $f_+^{B_s\to K}(0) = 0.23(5)(4)$. We compare our results with other quenched and unquenched lattice calculations, as well as with light-cone sum rule predictions, finding good agreement.

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1 Introduction

Heavy meson decays are the main source of precision information on quark flavor mixing parameters in the Standard Model. The over-determination of the sides and the angles of the CKM unitarity triangle is the aim of an extensive experimental study: It addresses the question whether there is New Physics in flavor-changing processes and where it manifests itself. One of the sides of the unitarity triangle is given by the ratio $|V_{ub}/V_{cb}|$. V_{cb} is known to approximately 2% accuracy from $b \to c\ell\nu_\ell$ transitions [1, 2] whereas the present error on V_{ub} is much larger and there is also some tension between the determinations from inclusive and exclusive decay channels. Reduction of this error requires more experimental statistics but—even more so—an improvement of the theoretical prediction of the semileptonic spectra and decay widths.

This is the prime motivation for the study of semileptonic form factors of decays of a heavy meson $H = B, D$ into a light pseudoscalar meson $P = \pi, K$, which are usually defined as

$$\langle P(p)|V^\mu|H(p_H)\rangle = \frac{m_H^2 - m_P^2}{q^2} q^\mu f_0(q^2) + \left(p_H^\mu + p^\mu - \frac{m_H^2 - m_P^2}{q^2} q^\mu\right) f_+(q^2)\,. \tag{1}$$

Here $V^\mu = \overline{q}_2 \gamma^\mu q_1$ is the vector current in which q_1 (q_2) denotes a light (heavy) quark field; p (p_H) is the momentum of the light (heavy) meson with mass m_P (m_H), and $q := p_H - p$ is the four-momentum transfer. The $f_0(q^2)$ and $f_+(q^2)$ form factors are dimensionless, real functions of q^2 (in the physical region), which encode the strong interaction effects. They are subject to the kinematic constraint $f_+(0) = f_0(0)$.

In the approximation of massless leptons (which is highly accurate for $\ell = e$ or $\ell = \mu$), the differential decay rate for the $H \to P\ell\nu_\ell$ process involves $f_+(q^2)$ only:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \left|V_{q_2 q_1}\right|^2}{192\pi^3 m_H^3} \left[\left(m_H^2 + m_P^2 - q^2\right)^2 - 4 m_H^2 m_P^2\right]^{3/2} \left|f_+(q^2)\right|^2 . \tag{2}$$

Another motivation for our study is that $f_0(q^2)$ and $f_+(q^2)$ enter as ingredients in the analysis of nonleptonic two-body decays like $B \to \pi\pi$ and $B \to \pi K$ in the framework of QCD factorization [3, 4], with the objective to extract CP-violating effects and in particular the angle α of the CKM triangle. One issue that is especially important in this respect is the question of flavor SU(3) violation in the form factors of the decay $B \to \pi$ vs. the rare decay $B \to K$.

High-statistics unquenched lattice calculations of D-meson (and also B-meson) decay form factors in the kinematic region where the outgoing light hadron carries little energy (small recoil region) have been performed recently [5, 6, 7] and attracted a lot of attention. Direct simulations at large recoil, $q^2 \ll m_B^2$, with light hadrons carrying large momentum of order 2 GeV, prove to be difficult and require a very fine lattice which is so far not accessible in calculations with dynamical fermions. This problem is aggravated by the challenge to consider heavy quarks which either calls for using effective heavy quark theory methods or, again, a very fine lattice. In practice, one is forced to rely on extrapolations from larger momentum transfer q^2 and/or smaller heavy quark masses. Several extrapolation procedures have been suggested [8, 9, 10, 11, 12] that incorporate constraints from unitarity and the scaling laws in the heavy quark limit. Alternatively, B-meson form factors in the region of large recoil have been estimated using light-cone sum rules [13, 14] (for recent updates see [15, 16, 17, 18]).

In this paper we report on a quenched calculation of semileptonic $H \to P\ell\nu_\ell$ form factors with lattice spacing $a \sim 0.04$ fm using nonperturbatively $O(a)$ improved Wilson fermions and

$O(a)$ improved currents. On such a fine lattice a relativistic treatment of the c quark is justified and also the extrapolation to the physical b quark mass becomes much more reliable compared to similar calculations on coarser lattices. In addition, we can explore possible subtleties in approaching the continuum limit in form factor calculations: In our previous work [19] we did find indications for a substantial discretization error in the decay constants f_{D_s} etc.; similar conclusions have also been reached in Ref. [20]. This is particularly relevant in view of the claims of evidence for New Physics from comparison with recent dynamical simulations—see, e.g. Ref. [21] for a discussion.

On physical grounds, one may expect a nontrivial continuum limit because form factors at large momentum transfer are determined by the overlap of very specific kinematic regions in hadron wave functions (either soft end-point, or small transverse separation). The common wisdom that hadron structure is very "smooth"—and that numerical simulations on a coarse lattice could thus be sufficient to capture the continuum physics—may not work in this particular case. This can be especially important for SU(3) flavor-violating effects, which are of major interest for the phenomenology. Inclusion of dynamical fermions and the approach to the chiral limit are certainly also relevant problems, but not all issues can be addressed presently within one calculation.

This work should be viewed as a direct extension of the investigation of the APE collaboration in Ref. [22], who performed a quenched calculation with the same nonperturbatively $O(a)$ improved action and currents. Also their data analysis is similar. However, they use coarser lattices with $a \sim 0.07$ fm ($\beta = 6.2$). On the other hand, the spatial volume of their lattices is very close to ours ($L \sim 1.7$ fm). So the main difference lies in the lattice spacing, and a direct comparison of the results is possible yielding information on the size of lattice artefacts, while there is no need for us to perform simulations on a coarser lattice ourselves.

The presentation is organized as follows. Our strategy is discussed in detail in Sec. 2. It allows us to run fully relativistic simulations for values of m_H up to the vicinity of m_B: This is achieved by using a lattice characterized by a very fine spacing a. The extraction of physical quantities from our data and the final results with the associated error budget are presented in Sec. 3. The final Sec. 4 contains a summary and some concluding remarks. Some technical details and intermediate results of our calculation are shown in the Appendix. Preliminary results of this study have been presented in Refs. [23, 24].

2 Simulation details

The lattice study of heavy hadrons is an issue that involves some delicate technical aspects: The origin of the problem stems from the fact that, typically, the lattice cutoff is (much) smaller than the mass of the B meson.

Common strategies to solve this problem are based on heavy quark effective theory (HQET), i.e. expanding the relativistic theory in terms of m_Q^{-1}, where m_Q is the mass of the heavy quark. One can simulate in the static limit [25] or keep correction terms in the action to simulate at finite m_Q (non-relativistic QCD or NRQCD) [26]. These approaches have been employed effectively for studying B physics (see, for example, Refs. [27, 28, 29, 30]).

However, for smaller quark masses like the c quark in D mesons a large number of terms in the expansion must be included, making the simulations less attractive. The Fermilab group

developed an approach which interpolates between the heavy- and light-quark regimes [31]. The coefficients accompanying each term in the action are functions of the quark mass and in practice, normally, the lowest-level action is used. This corresponds to using the $O(a)$ improved relativistic action with a re-interpretation of the results. Except for HQET [32, 33], the associated renormalization constants for these approaches are only known perturbatively.

We reduce the uncertainties related to the extrapolation to the physical heavy meson mass by using lattices with a small spacing in conjunction with a non-perturbatively improved $O(a)$ relativistic quark action. This theoretically clean approach enables one to get sufficiently close to the mass of the physical B meson, so that the heavy-quark extrapolation is short-ranged. In addition, in the region of the D meson mass, the discretization errors are reduced to around 1%, see Section 3.1.

$L^3 \times T$	$40^3 \times 80$
β	6.6
lattice spacing a	0.04 fm
physical hypervolume	$1.6^3 \times 3.2$ fm^4
a^{-1}	4.97 GeV
# of configurations	114
$\kappa_{critical}$	0.135472(11)
κ_{heavy}	0.13, 0.129, 0.121, 0.115
κ_{light}	0.13519, 0.13498, 0.13472
m_P	526 MeV, 690 MeV, 856 MeV
c_{SW}	1.467
Z_V	0.8118
b_V	1.356
c_V	−0.0874

Table 1: Parameters of the lattice calculation (see the text for the definition of the various quantities).

Table 1 summarizes basic technical information about our study. We use the standard Wilson gauge action to generate quenched configurations with the coupling parameter $\beta = 6.6$. For this parameter choice, the lattice spacing in physical units determined from Ref. [34] using Sommer's parameter $r_0 = 0.5$ fm is $a = 0.04$ fm. Our calculation is based on the $O(a)$ improved clover formulation for the quark fields [35], with the nonperturbative value of the clover coefficient c_{SW} taken from Ref. [36]. We use $O(a)$ improved definitions of the vector currents in the form [37]

$$V_\mu = Z_V \left[1 + b_V \frac{am_{q_2} + am_{q_1}}{2}\right] (\overline{q}_2 \gamma_\mu q_1 + iac_V \partial_\nu \overline{q}_2 \sigma_{\mu\nu} q_1) \tag{3}$$

with $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. The renormalization factor Z_V, the improvement coefficient b_V as well as c_V are known nonperturbatively [38, 39, 40]. All statistical errors are evaluated through a bootstrap procedure with 500 bootstrap samples. We consider three hopping parameters corresponding to "light" quarks, κ_{light} (the corresponding masses of the light pseudoscalar meson states m_P are also given in Table 1), and four hopping parameters, κ_{heavy}, corresponding

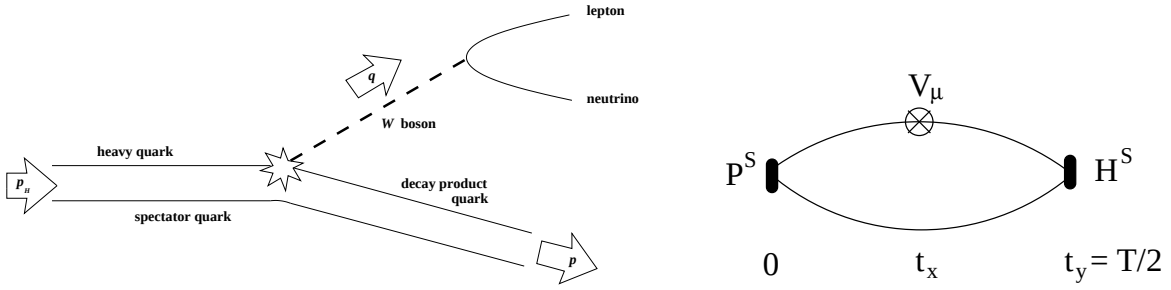

Figure 1: Diagram representing the semileptonic decay of a heavy-light pseudoscalar meson to a light pseudoscalar meson (left panel). A schematic representation of the corresponding three-point correlator calculated on the lattice is also shown (right panel).

to "heavy" quarks; in particular, $\kappa = 0.13498$ and $\kappa = 0.129$ are found to correspond to quark masses close to the physical strange and charm quark mass, respectively.

The extraction of the matrix element appearing in Eq. (1) from the lattice can be done by considering the large time behavior of three-point correlation functions $C_\mu^{(3)}(0, t_x, t_y)$ for a pseudoscalar light meson sink at time $t = 0$, a vector current at time t_x, and a pseudoscalar heavy-light meson source at time $t_y = T/2$ (see Fig. 1):

$$C_\mu^{(3)}(0, t_x, t_y) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}_H\cdot\vec{y}} e^{i\vec{q}\cdot\vec{x}} \langle H^S(\vec{y}, t_y) V_\mu(\vec{x}, t_x) P^S(0)\rangle \,. \tag{4}$$

Here, H^S and P^S are Jacobi-smeared operators of the form $\overline{q}_h\gamma_5 q_s$ and $\overline{q}_l\gamma_5 q_s$, respectively; q_h denotes the heavy quark, q_l is the decay-product quark, while q_s is the "spectator" quark.

For sufficiently large time separations (i.e. $0 \ll t_x \ll T/2$ or $T/2 \ll t_x \ll T$), $C_\mu^{(3)}(0, t_x, t_y)$ behaves as:

$$C_\mu^{(3)}(0, t_x, t_y) \longrightarrow \begin{cases} \frac{Z_H^S Z^S}{4E_H E} e^{-Et_x} e^{-E_H(t_y - t_x)} \ \langle H(p_H)|V_\mu|P(p)\rangle & \text{for } \ t_x < T/2 \\ \pm\frac{Z_H^S Z^S}{4E_H E} e^{-E(T-t_x)} e^{-E_H(t_x - t_y)} \ \langle H(p_H)|V_\mu|P(p)\rangle & \text{for } \ t_x > T/2 \end{cases} , \tag{5}$$

with $Z_H^S = |\langle 0|H^S|H(p_H)\rangle|$ and $Z^S = |\langle 0|P^S|P(p)\rangle|$, while E (E_H) denotes the energy of the light (heavy) meson. To extract the matrix elements we divide the three-point functions by the prefactors, which are extracted from fits to smeared-smeared two-point functions. The matrix element is then obtained by fitting this result to a constant, in an appropriate time range where a clear plateau forms (for example, for $12 \le t_x \le 28$).

We consider three-point functions associated with different combinations of the momenta p and p_H, which are listed in Table 2. In particular, we focus our attention onto three-momenta of modulus 0 and 1 [in units of $2\pi/(aL)$], since they yield the most precise signal, restricting ourselves to the cases where $\vec{p}$ and $\vec{p}_H$ lie in the same direction. Thus we measure directly 5 different values for the form factors, for every κ_{light} and κ_{heavy} combination. The full form factors can then be constructed from the data points obtained this way, by making an ansatz for the functional form of $f_0(q^2)$ and $f_+(q^2)$.

In the present work, we fit our data with the parametrization proposed by Bećirević and Kaidalov [8]:

$$f_0(q^2) = \frac{c_{BK}\cdot(1-\alpha)}{1 - \tilde{q}^2/\beta} \,, \qquad f_+(q^2) = \frac{c_{BK}\cdot(1-\alpha)}{(1-\tilde{q}^2)(1-\alpha\tilde{q}^2)}, \tag{6}$$

$\vec{p}_H$	$\vec{p}$	$\vec{q}$
$(0,0,0)$	$(1,0,0)$	$(-1,0,0)$
$(1,0,0)$	$(-1,0,0)$	$(2,0,0)$
$(0,0,0)$	$(0,0,0)$	$(0,0,0)$
$(1,0,0)$	$(0,0,0)$	$(1,0,0)$
$(1,0,0)$	$(1,0,0)$	$(0,0,0)$

Table 2: Momentum combinations considered in the analysis of the three-point functions, in units of $2\pi/(aL)$.

where $\tilde{q} := q/m_{H^\star}$, $m_{H^\star}$ being the mass of the lightest heavy-light vector meson.

The parametrization for the form factors given in Eq. (6) accounts for the basic properties that come from the heavy-quark scaling laws in the limits of large and small recoil and also satisfies the proportionality relation derived in Ref. [41]. It is also consistent with the trivial requirement that the l.h.s. of Eq. (1) be finite for vanishing momentum transfer, which implies $f_0(0) = f_+(0)$. The results that we obtained for the three parameters entering Eq. (6) from a simultaneous fit to f_0 and f_+ are presented in the Appendix.

Some alternative ansätze for the functional form of $f_+(q^2)$ were proposed in Refs. [15, 9, 10] and are discussed in Ref. [42]: They yield results essentially compatible with each other and with the Bećirević-Kaidalov parametrization Eq. (6). More recently, Bourrely, Caprini and Lellouch [12] discussed the representation of $f_+(q^2)$ as a (truncated) power series in terms of an auxiliary variable z. A similar parametrization has also been recently used by the Fermilab Lattice and MILC collaborations, see Refs. [43, 44] for a discussion.

3 Extraction of physical results

In order to extract physical results from our simulations, we follow a method analogous to Ref. [22]. We first perform a chiral extrapolation in the light quark masses. For a given quantity Φ [one of the BK parameters appearing in Eq. (6)], the extrapolation relevant for decays to a pion is performed as follows: We fit the results obtained at different values of the mass of the pseudoscalar state linearly in m_P^2,

$$\Phi = c_0 + c_1 \cdot m_P^2 \,, \tag{7}$$

and extrapolate to $m_P^2 = m_\pi^2$, where m_π is the mass of the physical pion. Examples of the extrapolations are shown in Figure 2 for the case of $\Phi = f_+(0)$, α and β at $\kappa_{heavy} = 0.115$. On the other hand, for decays to a kaon, we hold the hopping parameter of one of the two final quarks fixed to $\kappa = 0.13498$, which, for our configurations, corresponds to the physical strange quark at a high level of precision [19], and perform a short-ranged extrapolation of the curve obtained from the linear fit in m_P^2 to the square of the mass of the physical K meson.

Then we perform the interpolation to the physical c quark mass in terms of a heavy-quark expansion for the D (or D_s) meson decays, or the extrapolation to the physical b quark mass for the B (or B_s) meson. For our data, the extrapolation of the heavy quark mass to the physical b mass is short-ranged: for the heaviest $\kappa_{heavy} = 0.115$, it turns out that the inverse of the pseudoscalar meson mass (with the light quark mass already chirally extrapolated to its

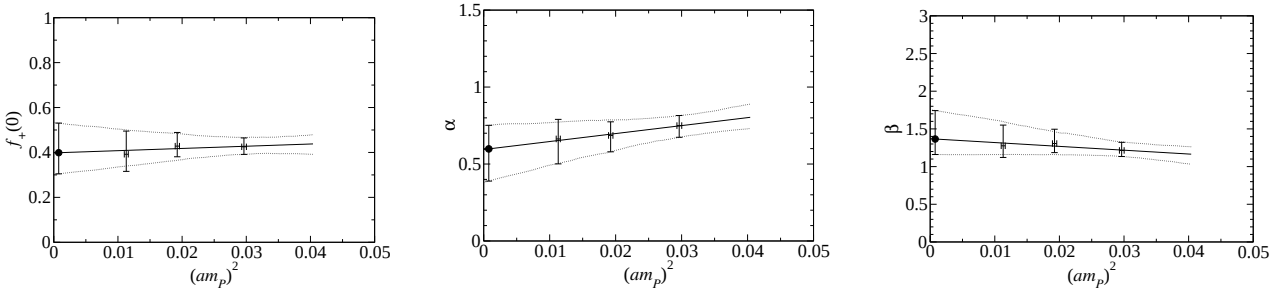

Figure 2: Extrapolation of the Bećirević–Kaidalov parameters to the chiral limit, for decays to a pion, at a fixed value $\kappa_{heavy} = 0.115$. The parameters obtained for $\kappa_{decay\,product} = \kappa_{spectator}$ are extrapolated linearly in m_P^2. The extrapolated values are shown as the full black dots.

Decay	fit	l_0	l_1	l_2	χ^2/d.o.f.
$B, D \to \pi$	linear	$4.1^{+1.3}_{-1.0}$	$-4.1^{+1.6}_{-2.3}$	–	0.1377/2
	quadratic	$5.1^{+2.9}_{-2.1}$	$-9.3^{+6.9}_{-9.6}$	$5.9^{+8.5}_{-5.9}$	0.021/1
$B, D \to K$	linear	$4.9^{+1.1}_{-0.9}$	$-5.4^{+1.5}_{-1.9}$	–	0.3247/2
	quadratic	$6.3^{+2.4}_{-1.9}$	$-12.2^{+6.3}_{-8.2}$	$7.7^{+7.3}_{-5.4}$	0.03813/1
$B_s, D_s \to K$	linear	$3.4^{+0.9}_{-0.8}$	$-2.9^{+1.3}_{-1.7}$	–	0.4025/2
	quadratic	$4.9^{+1.8}_{-1.3}$	$-11.0^{+4.6}_{-6.0}$	$9.7^{+5.4}_{-4.6}$	0.001888/1

Table 3: The coefficients obtained from the fits to $m_H^{3/2} f_+(0)$ in powers of m_H^{-1} according to Eq. (8) for different decays.

physical value) is about $m_H^{-1} = 0.243$ GeV^{-1}, to be compared with $m_B^{-1} = 0.189$ GeV^{-1} for the physical B meson. The extrapolation can be performed by taking advantage of the fact that, in the infinitely heavy quark limit, the Bećirević–Kaidalov parameters appearing in Eq. (6) enjoy certain scaling relations: $c_{BK}\sqrt{m_H}$, $(\beta-1)m_H$ and $(1-\alpha)m_H$ are expected to become constant in the $m_H \to \infty$ limit. For finite m_H, one can parametrize the scaling deviations in powers of m_H^{-1}:

$$\varphi = l_0 + l_1 \cdot m_H^{-1} + l_2 \cdot m_H^{-2} + \ldots \tag{8}$$

where $\varphi \in \{c_{BK}\sqrt{m_H},\ (\beta-1)\cdot m_H,\ (1-\alpha)\cdot m_H\}$. Note that, since $f_+(0) = c_{BK}\cdot(1-\alpha)$, one can also use $\varphi = f_+(0)\cdot m_H^{3/2}$—which was, in fact, our choice.

The extrapolation of $m_H^{3/2} f_+(0)$ is presented in Figure 3. The figure clearly shows the advantage of simulating on a fine lattice, which allows us to probe a mass range very close to the physical B meson mass. We compare the results obtained from an extrapolation to the inverse of the physical B meson mass using either a first- or a second-order polynomial in m_H^{-1} for the fit function, finding consistency (within error bars), for all decays. The corresponding fit results are listed in Table 3. In the following we refer to this first method as the "coefficient extrapolation" method.

An alternative method to extract the physical form factors from the lattice data was proposed by the UKQCD collaboration [45]. It consists of performing the chiral and heavy quark extrapolations at fixed $v\cdot p = (m_H^2 + m_P^2 - q^2)/(2m_H)$, where v is the four-velocity of the heavy meson and p is the four-momentum of the light meson. The following steps are performed:

1. fit of the form factors measured from the lattice simulations to the parametrization in

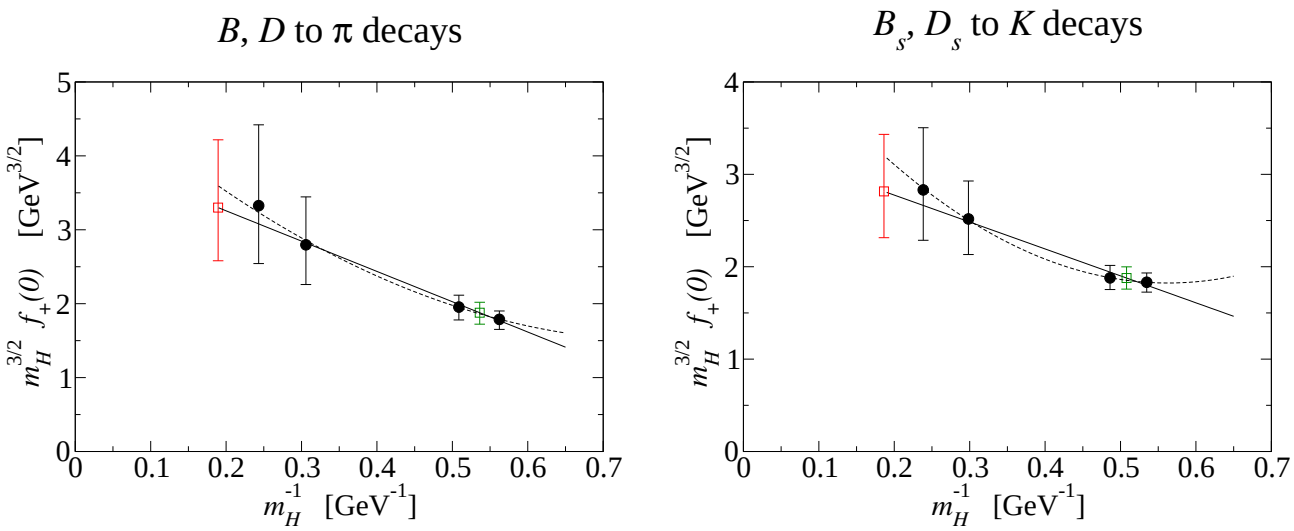

Figure 3: Left panel: The green (red) squares denote the interpolated (extrapolated) form factor $m_H^{3/2} f_+(0)$ to the physical D (B) meson, for a decay to a pion, using a linear fit in $1/m_H$ (solid line). A quadratic fit to the data is also shown (dashed line). Right panel: the results for the case of decay of a D_s (B_s) meson into a kaon.

Eq. (6);

2. interpolation of the form factors at given values of $v \cdot p$ within the range of simulated data;

3. chiral extrapolation of the points thus obtained, via a linear extrapolation in m_P^2 to either m_π^2 or m_K^2 (as described above);

4. linear or quadratic extrapolation in m_H^{-1} to the inverse of the physical heavy meson mass for the quantities:

$$\left[\frac{\alpha_s(m_B)}{\alpha_s(m_H)}\right]^{-\frac{\tilde{\gamma}_0}{2\beta_0}} f_0(v\cdot p)\sqrt{m_H}\,, \qquad \left[\frac{\alpha_s(m_B)}{\alpha_s(m_H)}\right]^{-\frac{\tilde{\gamma}_0}{2\beta_0}} \frac{f_+(v\cdot p)}{\sqrt{m_H}} \tag{9}$$

which enjoy scaling relations at fixed $v\cdot p$ [46, 47]. Here, β_0 is the first β-function coefficient, while $\tilde{\gamma}_0 = -4$ denotes the leading-order coefficient of the anomalous dimension for the vector current in HQET. It yields a (subleading) logarithmic dependence on m_H—see also Refs. [22, 45] for further details;

5. final fit of the points thus obtained to the parameterization in Eq. (6).

For comparison, we also calculate the physical form factors using this alternative approach, finding consistent results. This is illustrated in Table 4 which summarizes the results for $f_+(0)$ from both methods.

For our final results we take those obtained from the coefficient extrapolation method. We found this method to be superior in our case as the UKQCD method suffered from the fact that there was only a small region of overlap in the ranges of $v \cdot p$ for the form factors at different

κ_{light} and κ_{heavy}. In addition, since the data can be fitted with both a linear and quadratic function in m_H^{-1}, we use the linear fits for the central values and statistical errors and use the differences in the results from the linear and quadratic fit to estimate the systematic errors, as discussed in the next section. Our results for the form factors at finite q^2 are shown in Figs. 4 and 5.

	Coefficient extrapolation		UKQCD method	
Decay	linear in m_H^{-1}	quadratic in m_H^{-1}	linear in m_H^{-1}	quadratic in m_H^{-1}
$D \to \pi$	0.74^{+6}_{-6}	0.73^{+5}_{-6}	0.69^{+5}_{-5}	0.69^{+5}_{-6}
$D \to K$	0.78^{+5}_{-5}	0.77^{+5}_{-5}	0.75^{+4}_{-5}	0.75^{+4}_{-5}
$D_s \to K$	0.68^{+4}_{-4}	0.67^{+4}_{-4}	0.68^{+4}_{-4}	0.67^{+4}_{-4}
$B \to \pi$	0.27^{+8}_{-6}	0.30^{+11}_{-8}	0.29^{+13}_{-8}	0.31^{+15}_{-10}
$B \to K$	0.32^{+6}_{-5}	0.35^{+9}_{-8}	0.35^{+11}_{-8}	0.34^{+12}_{-9}
$B_s \to K$	0.23^{+5}_{-4}	0.26^{+7}_{-5}	0.23^{+6}_{-5}	0.27^{+8}_{-6}

Table 4: Final results for the physical values of the $f_+(0)$ form factor, for different decays, with statistical errors only. We compare the results obtained from the coefficient extrapolation and UKQCD methods as well as different truncations of the heavy quark expansion when extrapolating or interpolating in m_H^{-1}.

3.1 Systematic uncertainties

Systematic uncertainties affecting our lattice calculation include: the quenched approximation, the method to set the quark masses, the chiral extrapolation for the light quarks, discretization effects, the extrapolation (interpolation) of the heavy quark to the physical b (c) mass, finite volume effects, uncertainties in the renormalization coefficients, and effects related to the model dependence for $f_{0,+}(q^2)$. Let us now consider each source of error in turn.

Quenched approximation: the size of the error this approximation introduces is not known. However, one can take as an estimate the variation in the results if different quantities are used to set the scale. In the quenched approximation different determinations of the lattice spacing vary by approximately 10% [48]. By repeating the full analysis, we find that varying the lattice spacing by 10% induces an uncertainty of approximately 2% for the $D \to \pi$ decay, and of approximately 12% for $B \to \pi$.

Setting the quark masses: we use the κ values corresponding to the light (u/d) and strange quarks determined in Ref. [19]: $\kappa_l = 0.135456(10)$ and $\kappa_s = 0.134981(9)$. The uncertainty in these determinations leads to a very small uncertainty in the form factors. For the c and b quarks we do not quote the corresponding κ values. We interpolate (or extrapolate) our results directly to the physical masses of the pseudoscalar heavy-light states. The resulting uncertainty is determined by the statistical errors of the masses used for the interpolation (or extrapolation). The latter are found to contribute only a negligible amount to the overall systematic uncertainty.

Chiral extrapolation: the method we used to perform the chiral extrapolation of our simulation results is discussed above. Note that the use of a large lattice practically constrains us to use only a few and relatively large values for the light quark mass (so that the masses of our lightest pseudoscalar mesons are far from the physical pion mass). However, as the

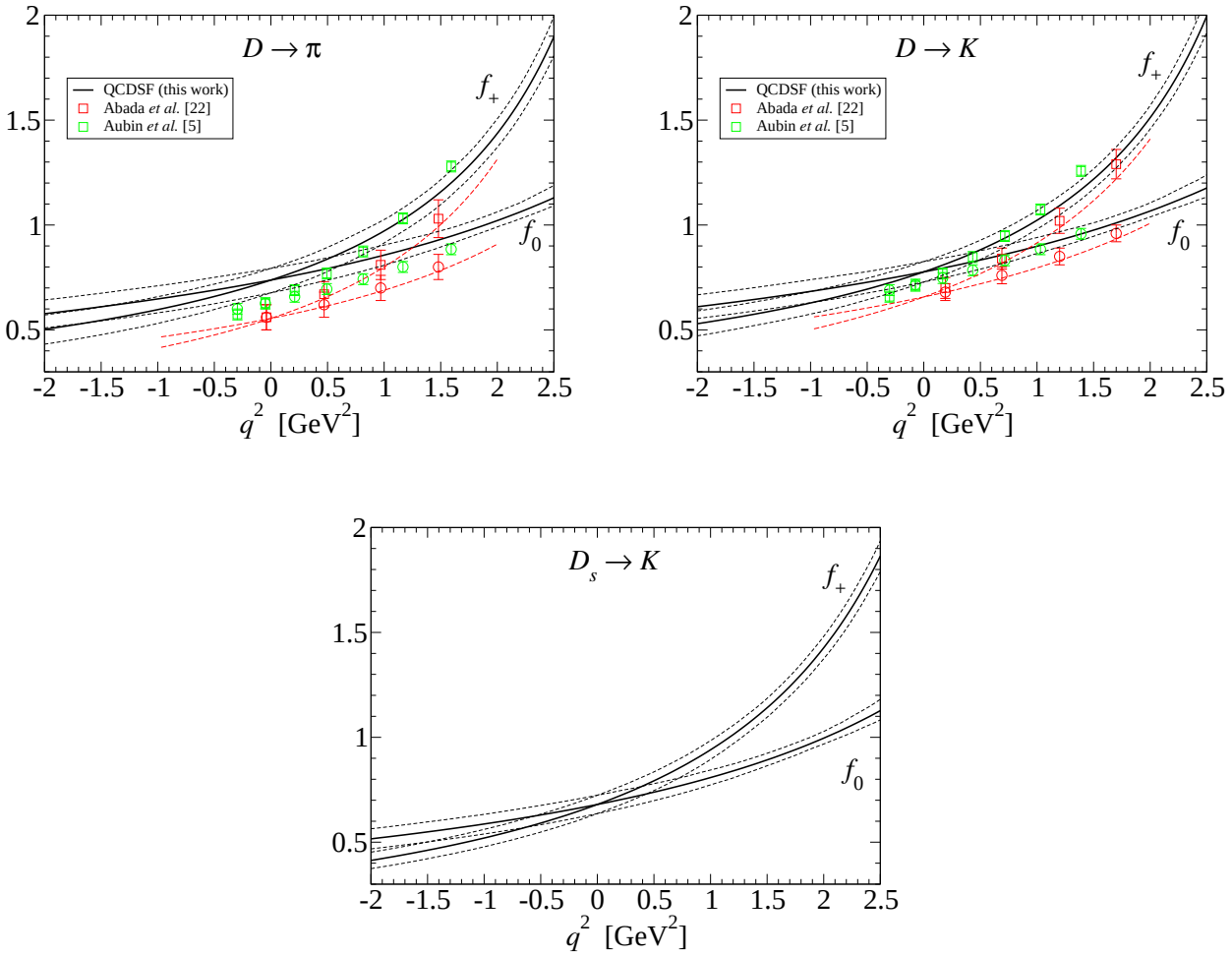

Figure 4: Physical form factors for D and D_s decays as a function of q^2 from this work and other quenched and dynamical studies. The solid black lines are the form factors obtained from the coefficient extrapolation method where Eq. (8) has been truncated at $O(m_H^{-1})$, while the dashed black lines indicate the error on the form factors. The range of $v \cdot p$ values achieved in our simulations approximately corresponds to $-1.5\ \mathrm{GeV}^2 \lesssim q^2 \lesssim 2\ \mathrm{GeV}^2$. The dashed red lines are the results for the coefficient extrapolation method from Ref. [22]. The open red squares and circles are their results obtained using the UKQCD method.

examples in Figure 2 show, the dependence of our results on the light quark mass is rather mild. So the size of the uncertainty arising from the chiral extrapolation though difficult to estimate is unlikely to be large.

Discretization effects: as it was already remarked above, the leading discretization effects in our calculation are reduced to $O(a^2)$; given that our lattice is very fine ($a = 0.04$ fm), the associated systematic error can be estimated to be of the order of 1% (10%) for the decays of charmed (beautiful) mesons [19].

Extrapolation/interpolation of the heavy quark: our data can be fitted to both a linear and quadratic function in m_H^{-1} with a reasonable χ^2. We use the results for the linear fit for our final results and the difference between the linear and quadratic fit as an indication of the systematic uncertainty. This leads to approximately a 1% uncertainty for D decays and

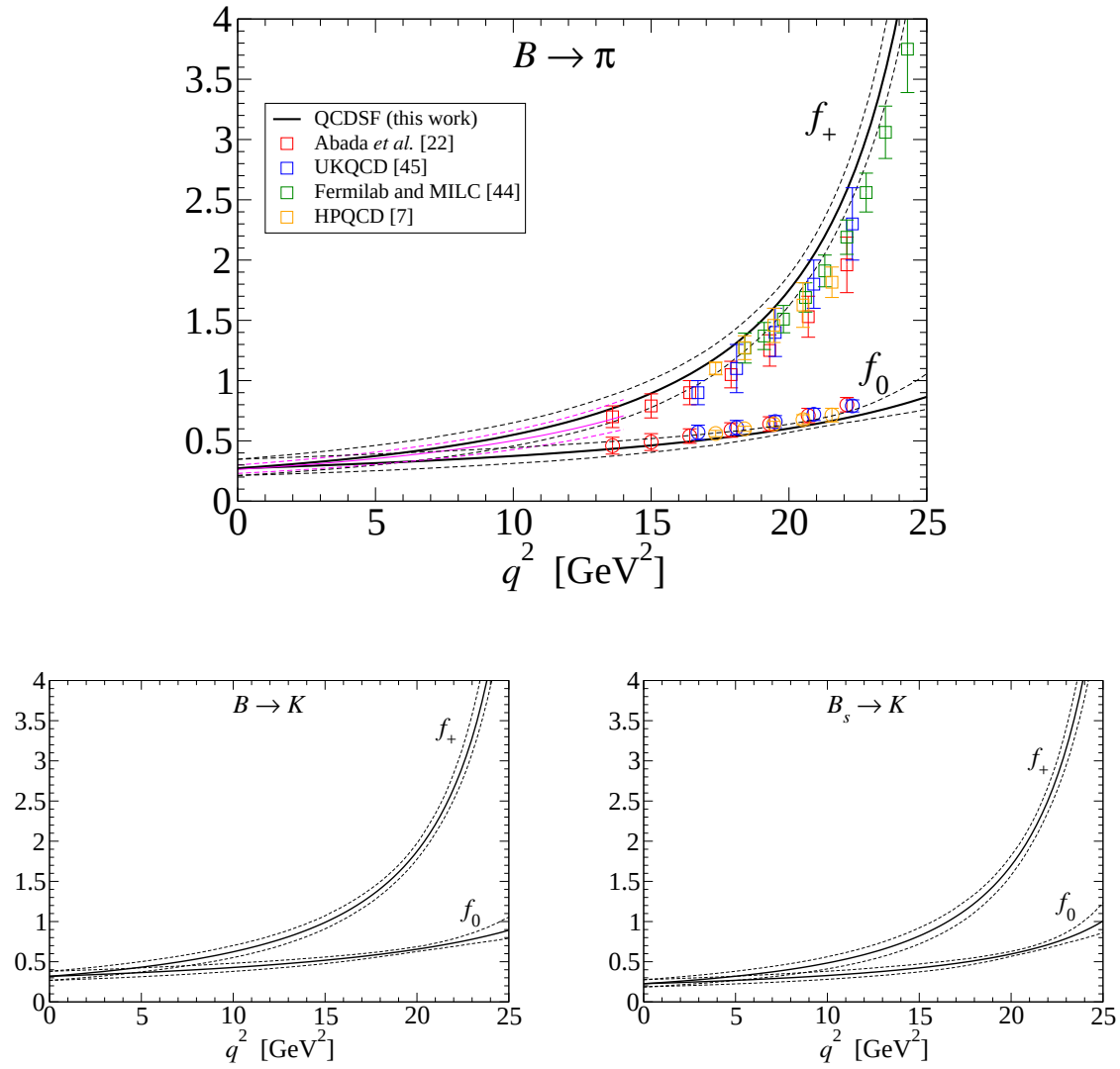

Figure 5: Same as in Fig. 4, but for B and B_s decays; in this case, the $v \cdot p$ values of our simulations are in the range 14 GeV$^2 \lesssim q^2 \lesssim$ 23 GeV^2. For $B \to \pi$, the dashed and solid magenta lines in the range $q^2 = 0 - 14$ GeV^2 indicate the prediction from light-cone sum rules [15, 16].

8% uncertainty for B decays.

Finite volume effects: for our calculation, finite-volume effects are not expected to be severe; in particular, the correlation length associated with the lightest pseudoscalar state that we simulated (for $\kappa_{light} = 0.13519$) corresponds to approximately 9 lattice spacings, which is more than four times shorter than the spatial extent of our lattice. Systematic infrared effects can thus be quantified around 1–2%. This is comparable with the estimate of Ref. [44], in which, using chiral perturbation theory [49, 50], the finite volume effects for their calculation with $2+1$ flavors of staggered quarks and values of $m_P L$ between 4 and 6 are estimated to be less than 1%.

Renormalization coefficients: the uncertainty associated with the Z_V coefficient, as determined in Ref. [38] for the quenched case, is about 0.5%. The same article also quotes a

1% uncertainty for b_V, which induces an error about 1% for decays of D mesons and about 3% for B mesons. Concerning c_V, a look at the results displayed in Fig. 2 of Ref. [39] would suggest that the relative error in the region of interest ($g_0^2 \simeq 0.91$) may be quite large, around 30%; however, it should be noted that c_V itself is a relatively small number, of the order of 9%, and the impact of the uncertainty on c_V on our results is about 1% (2%) for decays of D (respectively: B) mesons.

Model dependence: finally, the systematic effect related to the ansatz to parametrize the form factors was estimated in Ref. [42], through a comparison of different functional forms that satisfy analogous physical requirements. For the $B \to \pi$ decay, it turns out to be of the order of 2%.

Combining the systematic errors in quadrature we arrive at an overall error of 5% for D decays and about 18% for B decays.

3.2 Comparison with previous results

Our results can be compared to other lattice calculations of these quantities and also with results of light-cone sum rules (LCSR) [13, 14]. Table 5 summarizes the comparison for $f_+(0)$, while for finite q^2 the form factors from other studies are displayed in Figs. 4 and 5. In the following we discuss in detail the comparison with these works, highlighting the advantages and limitations of the different approaches, as well as the possible sources of discrepancies.

Our results can be closely compared with those obtained by the APE collaboration in Ref. [22], reporting a calculation very similar to ours. They worked in the quenched approximation, using the same non-pertubatively $O(a)$ improved action and currents and a similar analysis; on the other hand, their simulations were performed on a coarser lattice, with $\beta = 6.2$, yielding a lattice spacing $a = 0.07$ fm, or $a^{-1} \simeq 2.7$ GeV. The table and figures show that their values for the form factors lie around 3σ ($D \to \pi$) and 2.5σ ($D \to K$) below our results, in terms of the statistical errors, in the region of $q^2 = 0$. If we adjust the APE results to be consistent with setting the lattice spacing using r_0 instead of the mass of the K^* (used in Ref. [22]), the discrepancy reduces slightly, down to roughly 2.5σ ($D \to \pi$) and 2σ ($D \to K$). Assuming that $O(a^2)$ errors are the dominant source of the discrepancy, the difference in the results of the two studies is consistent with an upper limit on the discretization errors of approximately 0.08, or slightly above 1σ in our results for $f_+^{D\to\pi}(0)$ and 0.23 or $3-4\sigma$ in the APE results.

For B decays we are not able to make such a close comparison, because the study in Ref. [22] extrapolates to the B meson from results in the region of $1.7 - 2.6$ GeV for the heavy-light pseudoscalar meson mass. Although one would expect larger discretization effects for the B decay form factors, we find close agreement between our values and those from the APE collaboration. However, we should point out that any potential discrepancy may be masked by the long-ranged extrapolation in the heavy quark mass.

Several unquenched calculations have been performed recently, which are based on the MILC $N_f = 2+1$ dynamical rooted staggered fermions configurations [51]. Results are available from joint works from the Fermilab, MILC and HPQCD collaborations for D decays [5], and from Fermilab and MILC [52, 44] and (separately) HPQCD [7] for B decays. These results were obtained using the MILC "coarse" lattices with $a = 0.12$ fm for D decays and including a finer lattice with $a = 0.09$ fm for the B decays. While these lattices are much coarser than those used in both our and the APE study a detailed analysis of the chiral extrapolation was possible

through the use of 5 light quark masses for the 0.12 fm lattice (only two values were used for $a = 0.09$ fm).

The Fermilab, MILC and HPQCD joint work for $D \to \pi$ and $D \to K$ used an improved staggered quark action ("Asqtad") [51] for the light quarks and the Fermilab action for the heavy quark. To the order implemented in the study, the Fermilab action corresponds to a re-interpretation of the clover action. This approach can be used to simulate directly at the charm and bottom quark mass at the expense of more complicated discretisation effects. Discretisation errors arising from the final state energy (5%) and the heavy quark (7%) are estimated to lead to the largest systematic uncertainties in the calculation (compared to the 3% error from the chiral extrapolation). Given the coarseness of the lattice used, repeating the analysis on a much finer lattice would enable the estimates of the the discretisation errors to be confirmed. Overall, the results are consistent with ours, which suggests that the systematic effects due to the quenched approximation are not the dominant source of error.

For the decay $B \to \pi$, Fermilab and MILC used the same quark actions as for the study of D decays. Using the 5 light quark masses at $a = 0.12$ fm and 2 light quark masses at $a = 0.09$ fm they performed a joint continuum and chiral extrapolation which removed some of the discretisation effects. They estimated that a 3% discretisation error arising from the heavy quark remains after the extrapolation. The results at finite q^2 are compared with ours in Figure 5, with statistical and chiral extrapolation errors only (which cannot be separated). A value for $f_+(0)$ is not given in Ref. [44] which focuses on extracting $|V_{ub}|$ at finite q^2 using the parameterisation of Bourrely, Caprini and Lellouch [12]. However, an earlier analysis on the 0.12 fm lattices only was reported in Ref. [52]. Their result for $f_+(0)$ is given in Table 5.

HPQCD performed the calculations for the $B \to \pi$ decay on the MILC configurations using Asqtad light quarks and NRQCD for the b quark. Use of the latter enables direct simulations at the b quark mass. However, as NRQCD is an effective theory the continuum limit cannot be taken and scaling in the lattice results must be demonstrated at finite a. Results from the coarse lattice are shown in Figure 5, with statistical and chiral extrapolation errors only and for $f_+(0)$ in Table 5. A limited comparison of results on the finer lattice for one light quark mass did not indicate that the discretisation errors are large. The systematic errors are dominated by the estimated 9% uncertainty in the renormalisation factors which are calculated to 2 loops in perturbation theory.

The Fermilab-MILC and HPQCD results are consistent with each other to within 2σ and are also consistent with our results and those of the APE collaboration. As for the studies of D decays this suggests that quenching is not the dominant systematic error in the calculation of $B \to \pi$ decay. Similarly, unquenched results on finer lattices are needed to investigate the discretisation effects. Finally, note that in order not to overload Figures 4 and 5, we do not show the (older) quenched results of the Fermilab group [53]. For $B \to \pi$ decays these results are within the range of the other existing calculations, whereas for D-decays the form factors come out $10 - 20\%$ larger compared to most other calculations and also the new unquenched results obtained with similar methods.

A different type of comparison can be made with the estimates obtained in the framework of LCSR. This analytical approach is, to some extent, complementary to lattice calculations, since it allows one to calculate the form factors directly at large recoil, albeit with some assumptions. Figure 5 compares our extrapolation of the $f_+^{B\to\pi}(q^2)$ form factor in the region $q^2 < 12$ GeV^2 with the direct LCSR calculation [15, 16]. Their predictions are compatible with our results.

Similar consistency is found between lattice and LCSR calculations of $f_+(0)$, as seen in Table 5, for both B and D decays. Note that the uncertainty quoted for $f_+(0)$ for B decays is smaller than that for D meson decays, and comparable with the precision of the lattice results. However, while LCSR provides a systematic approach for calculating these quantities it is by definition approximate and the errors cannot be reduced below $10-15\%$, unlike the lattice approach, which is systematically improveable.

Decay	This work	Other results	Source	Method
$D \to \pi$	0.74(6)(4)	0.64(3)(6)	Fermilab-MILC-HPQCD [5]	$N_f = 2{+}1$ LQCD
		0.57(6)(1)	APE [22]	$N_f = 0$ LQCD
		0.65(11)	Khodjamirian *et al.* [54]	LCSR
		0.63(11)	Ball [55]	LCSR
$D \to K$	0.78(5)(4)	0.73(3)(7)	Fermilab-MILC-HPQCD [5]	$N_f = 2{+}1$ LQCD
		0.66(4)(1)	APE [22]	$N_f = 0$ LQCD
		0.78(11)	Khodjamirian *et al.* [54]	LCSR
		0.75(12)	Ball [55]	LCSR
$D_s \to K$	0.68(4)(3)			
$B \to \pi$	0.27(7)(5)	0.23(2)(3)	Fermilab-MILC [52]	$N_f = 2{+}1$ LQCD
		0.31(5)(4)	HPQCD [7]	$N_f = 2{+}1$ LQCD
		0.26(5)(4)	APE [22]	$N_f = 0$ LQCD
		0.258(31)	Ball and Zwicky [15]	LCSR
		0.26(4)	Duplančić *et al.* [16]	LCSR
		0.26(5)	Wu and Huang [18]	LCSR
$B \to K$	0.32(6)(6)	0.331(41)	Ball and Zwicky [15]	LCSR
		0.36(5)	Duplančić *et al.* [17]	LCSR
		0.33(8)	Wu and Huang [18]	LCSR
$B_s \to K$	0.23(5)(4)	0.30(4)	Duplančić *et al.* [17]	LCSR

Table 5: Comparison of the results for $f_+(0)$ of the present work with other calculations, obtained from lattice QCD (LQCD) simulations or from light-cone sum rules (LCSR) by various groups. Where two errors are quoted the first is statistical and the second is the combined systematic errors.

4 Conclusions

In this article we have presented a lattice QCD calculation of the form factors associated with semileptonic decays of heavy mesons.

We have performed a quenched calculation on a very fine lattice with $\beta = 6.6$ ($a = 0.04$ fm), which allows us to treat the D meson decays in a fully relativistic setup, and to get close to the region corresponding to the physical B meson mass. The importance of small lattice spacings for heavy-quark simulations has recently become clear in the context of the determination of f_{D_s}, the decay constant of the D_s meson. In spite of $O(a)$ improvement, a continuum extrapolation linear in a^2 seems to be reliable only for lattice spacings below about

0.07 fm in the quenched approximation [19, 20]. Depending on the particular improvement condition, even a non-monotonous a dependence can appear on coarser lattices.

In this work we have investigated to which extent the systematic effects caused by lattice discretization and long-ranged extrapolations to the physical heavy meson masses may influence the results of different lattice calculations in which all other sources of systematic errors are treated in a similar way. For these reasons, the results of our study can be directly compared with those by the APE collaboration in Ref. [22], which reports a very similar calculation on a coarser lattice at $\beta = 6.2$ ($a \simeq 0.07$ fm) with the same lattice action and currents. Adjusting the APE results so that they comply with our procedure for setting the physical value of the lattice spacing, we find quite large discrepancies of roughly 2.5σ ($D \to \pi$) and 2σ ($D \to K$). If we assume that $O(a^2)$ errors are the dominant source of this effect, the difference in the results of the two studies suggests an upper limit on the discretization errors of approximately 0.08 or slightly above 1σ in our numbers for $f_+^{D\to\pi}(0)$ and 0.23 or $3-4\sigma$ in the APE results.

It is, however, to be noted that the interpretation of this difference as a mere discretization error is somewhat more ambiguous than in the case of the decay constants considered in [19, 20], because the momentum transfer q^2 adds another parameter that has to be adjusted before the comparison can be attempted. The corresponding comparison for B decays can, in addition, be undermined by the long-ranged extrapolations in the heavy quark mass and/or q^2. These results suggest that, for high-precision phenomenological applications, completely reliable relativistic lattice calculations of these form factors could require even finer spacings, and that, for dynamical simulations at realistic pion masses, this goal might be difficult to achieve in the near future. While we believe that the progress in computational power will eventually allow one to realize this formidable task, it is fair to say that, for the moment, the less demanding approaches which interpolate between the D meson scale and non-relativistic results provide a valid alternative.

Finally, a few words are in order about the general perspective for calculations of the semileptonic form factors of heavy mesons. Form factors of B decays at small values of the relativistic momentum transfer q^2 involve a light meson with momentum up to 2.5 GeV in the final state and are very difficult to calculate on the lattice, mainly because no lattice effective field theory formulation is known for this kinematics that would allow for the consistent separation of the large scales of the order of the heavy quark mass, as implemented in the Soft-Collinear Effective Theory.

Thus one is left with two choices. The first one is to calculate the form factors at moderate recoil ($m_B^2 - q^2 \sim O(m_B \Lambda_{\rm QCD})$) using, e.g., the HQET or NRQCD expansion and then to extrapolate to large recoil ($m_B^2 - q^2 \sim O(m_B^2)$) guided by the dispersion relations. The advantage of this approach is that the calculations can be performed on relatively coarse and thus not very large (in lattice units) lattices. Therefore dynamical fermions may be included, high statistical accuracy can be achieved as well as a better control over the chiral extrapolation. The disadvantage is that a reliable extrapolation from the $q^2 > 12 - 15$ GeV^2 regime accessible in this method to $q^2 = 0$ may be subtle. However, this problem may be alleviated by a promising new approach, "moving NRQCD" [56], which formulates NRQCD in a reference frame where the heavy quark is moving with a velocity v. By giving the B meson significant spatial momentum, relatively low q^2 can be achieved for lower values of the final state momentum thus avoiding large discretisation effects.

For the particular case of the $B \to \pi$ semileptonic decay the problem of simulating at

large recoil can be avoided, at least in principle, since the shape of the form factor $f_+(q^2)$ can be extracted from the experimental data on the partial branching fraction in different q^2 bins, see, e.g., Ref. [57]. The normalization can then be fixed by comparison to lattice data in the $q^2 \sim 10 - 20$ GeV^2 range. This strategy (see Ref. [58] for a detailed discussion) is indeed promising and may lead to a considerable improvement in the accuracy of the $|V_{ub}|$ determination from exclusive B decays provided the combined data analysis using the full statistics of the BaBar and Belle experiments ($\sim 4 \cdot 10^8$ $\bar{B}B$ pairs) becomes available. However, for rare decays, such as $B \to K^*\gamma$, $B \to K^*\mu^+\mu^-$ etc., which are likely to take the central stage at LHCb and super-B factories, a similar strategy seems to be unfeasible.

The second choice are simulations with fully relativistic heavy quarks on very fine and large (in lattice units) lattices. This procedure is presently bound to the quenched approximation, but the benefit is that the extrapolation in the heavy quark mass and potentially also in q^2 is of much shorter range. It goes without saying that the inclusion of dynamical fermions and the approach to the chiral limit are also important problems, but presently it is impossible to address all relevant issues within one calculation.

In our opinion both methods are justified and we have chosen the second option in this paper. It turns out that our final results for, e.g., the $B \to \pi$ decays are consistent with determinations based on dynamical simulations and LCSR. This may indicate that the quenching effects are rather moderate. From our experience, the main problem that limits the usefulness of this approach is the construction of sources for the light hadrons which yield a good overlap with states of large momentum. It seems that the presently used sources are not good enough in this respect. Improved sources have to be developed if a similar calculation is attempted on a larger scale in the future.

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A Simulation results

Fig. 6 shows a subset of the form factors $f_0(q^2)$ and $f_+(q^2)$ that we extracted from our simulations, for different combinations of the κ values for the heavy and spectator quarks, with $\kappa_{decay\ product} = 0.13472$. In Table 6 we present our results for the fits to the simulation data with the Bećirević–Kaidalov parameterization [8] according to Eq. (6). This parameterization uses the vector meson mass $m_{H^\star}$. Our results for $m_{H^\star}$ in lattice units are shown in Table 7.

$\kappa_{dec.\ prod.}$	κ_{heavy}	$\kappa_{spect.}$	$c_{BK}\cdot(1-\alpha)$	$1/\beta$	α	χ^2/d.o.f.
0.13519	0.13	0.13519	0.775^{+39}_{-45}	0.580^{+85}_{-79}	0.01^{+11}_{-10}	0.77
0.13519	0.129	0.13519	0.724^{+49}_{-47}	0.604^{+95}_{-90}	0.09^{+12}_{-12}	1.08
0.13519	0.121	0.13519	0.484^{+88}_{-72}	0.73^{+11}_{-12}	0.49^{+15}_{-16}	0.66
0.13519	0.115	0.13519	0.39^{+10}_{-8}	0.78^{+11}_{-14}	0.66^{+13}_{-16}	0.45
0.13519	0.13	0.13498	0.742^{+32}_{-37}	0.658^{+79}_{-75}	0.229^{+88}_{-81}	1.35
0.13519	0.129	0.13498	0.674^{+37}_{-35}	0.714^{+81}_{-85}	0.337^{+91}_{-91}	1.51
0.13519	0.121	0.13498	0.442^{+61}_{-52}	0.808^{+87}_{-96}	0.65^{+11}_{-11}	0.71
0.13519	0.115	0.13498	0.364^{+68}_{-52}	0.84^{+8}_{-10}	0.76^{+9}_{-11}	0.36
0.13519	0.13	0.13472	0.750^{+29}_{-30}	0.670^{+77}_{-71}	0.338^{+75}_{-74}	1.05
0.13519	0.129	0.13472	0.685^{+34}_{-29}	0.730^{+70}_{-82}	0.437^{+76}_{-77}	0.99
0.13519	0.121	0.13472	0.456^{+49}_{-42}	0.821^{+78}_{-89}	0.709^{+86}_{-98}	0.39
0.13519	0.115	0.13472	0.387^{+59}_{-46}	0.829^{+76}_{-88}	0.787^{+78}_{-98}	0.21
0.13498	0.13	0.13519	0.783^{+33}_{-35}	0.587^{+78}_{-68}	0.065^{+94}_{-83}	0.87
0.13498	0.129	0.13519	0.740^{+41}_{-38}	0.596^{+85}_{-70}	0.12^{+10}_{-9}	1.20
0.13498	0.121	0.13519	0.536^{+71}_{-65}	0.67^{+11}_{-12}	0.44^{+14}_{-14}	0.78
0.13498	0.115	0.13519	0.419^{+84}_{-67}	0.76^{+10}_{-12}	0.64^{+12}_{-15}	0.61
0.13498	0.13	0.13498	0.781^{+30}_{-28}	0.613^{+63}_{-61}	0.200^{+67}_{-66}	1.03
0.13498	0.129	0.13498	0.735^{+34}_{-29}	0.627^{+67}_{-66}	0.256^{+71}_{-72}	1.24
0.13498	0.121	0.13498	0.512^{+54}_{-46}	0.723^{+77}_{-91}	0.557^{+92}_{-98}	0.56
0.13498	0.115	0.13498	0.428^{+60}_{-47}	0.765^{+77}_{-97}	0.69^{+9}_{-11}	0.36
0.13498	0.13	0.13472	0.779^{+28}_{-26}	0.664^{+62}_{-61}	0.325^{+62}_{-62}	1.18
0.13498	0.129	0.13472	0.726^{+27}_{-27}	0.690^{+61}_{-63}	0.386^{+62}_{-65}	1.03
0.13498	0.121	0.13472	0.484^{+41}_{-36}	0.803^{+65}_{-72}	0.675^{+73}_{-85}	0.48
0.13498	0.115	0.13472	0.407^{+46}_{-38}	0.822^{+65}_{-75}	0.766^{+70}_{-83}	0.54
0.13472	0.13	0.13519	0.821^{+32}_{-31}	0.565^{+74}_{-61}	0.053^{+80}_{-70}	0.82
0.13472	0.129	0.13519	0.781^{+34}_{-33}	0.560^{+78}_{-65}	0.094^{+90}_{-77}	0.97
0.13472	0.121	0.13519	0.565^{+60}_{-54}	0.66^{+11}_{-10}	0.42^{+14}_{-12}	0.80
0.13472	0.115	0.13519	0.472^{+78}_{-62}	0.72^{+11}_{-12}	0.59^{+13}_{-13}	0.69
0.13472	0.13	0.13498	0.802^{+28}_{-26}	0.620^{+60}_{-59}	0.197^{+62}_{-58}	0.86
0.13472	0.129	0.13498	0.762^{+30}_{-27}	0.625^{+63}_{-59}	0.244^{+68}_{-65}	1.15
0.13472	0.121	0.13498	0.530^{+43}_{-42}	0.734^{+78}_{-76}	0.549^{+95}_{-88}	0.68
0.13472	0.115	0.13498	0.450^{+50}_{-46}	0.764^{+77}_{-83}	0.665^{+89}_{-92}	0.59
0.13472	0.13	0.13472	0.803^{+26}_{-22}	0.648^{+50}_{-53}	0.290^{+52}_{-53}	1.01
0.13472	0.129	0.13472	0.751^{+27}_{-23}	0.673^{+53}_{-53}	0.353^{+58}_{-59}	0.80
0.13472	0.121	0.13472	0.524^{+35}_{-31}	0.768^{+57}_{-66}	0.616^{+69}_{-71}	0.79
0.13472	0.115	0.13472	0.425^{+39}_{-33}	0.823^{+58}_{-67}	0.749^{+65}_{-75}	1.07

Table 6: Results of the fits of the lattice data with Eq. (6). Note that at vanishing q^2 one has $f_+(0) = f_0(0)$, which is given by $c_{BK}\cdot(1-\alpha)$ (fourth column of this Table).

The parameters $f_+(0)$, α and β are then extrapolated to the light-quark mass as described in Section 3 and illustrated in Fig. 2. Finally, the parameters describing the physical form factors are obtained through extrapolation of these values to the physical B meson mass (or through interpolation to the physical D meson mass), according to the heavy-quark scaling laws, see

κ_{heavy}	κ_{light}	am_H	$am_{H^\star}$
0.13	0.13519	0.3681^{+13}_{-11}	0.3949^{+18}_{-16}
0.13	0.13498	0.3762^{+13}_{-10}	0.4017^{+16}_{-14}
0.13	0.13472	0.3867^{+11}_{-10}	0.4110^{+14}_{-14}
0.129	0.13519	0.4060^{+13}_{-11}	0.4300^{+17}_{-15}
0.129	0.13498	0.4138^{+13}_{-11}	0.4366^{+16}_{-15}
0.129	0.13472	0.4240^{+12}_{-10}	0.4458^{+14}_{-14}
0.121	0.13519	0.6672^{+16}_{-13}	0.6804^{+20}_{-17}
0.121	0.13498	0.6743^{+14}_{-12}	0.6868^{+17}_{-16}
0.121	0.13472	0.6836^{+12}_{-12}	0.6956^{+15}_{-14}
0.115	0.13519	0.8369^{+17}_{-14}	0.8460^{+20}_{-18}
0.115	0.13498	0.8437^{+15}_{-13}	0.8523^{+17}_{-16}
0.115	0.13472	0.8527^{+13}_{-12}	0.8611^{+15}_{-14}

Table 7: Masses of the heavy-light pseudoscalar and vector states, H and $H^\star$, respectively, in lattice units, for the different $(\kappa_{heavy}, \kappa_{light})$ combinations.

Fig. 7.

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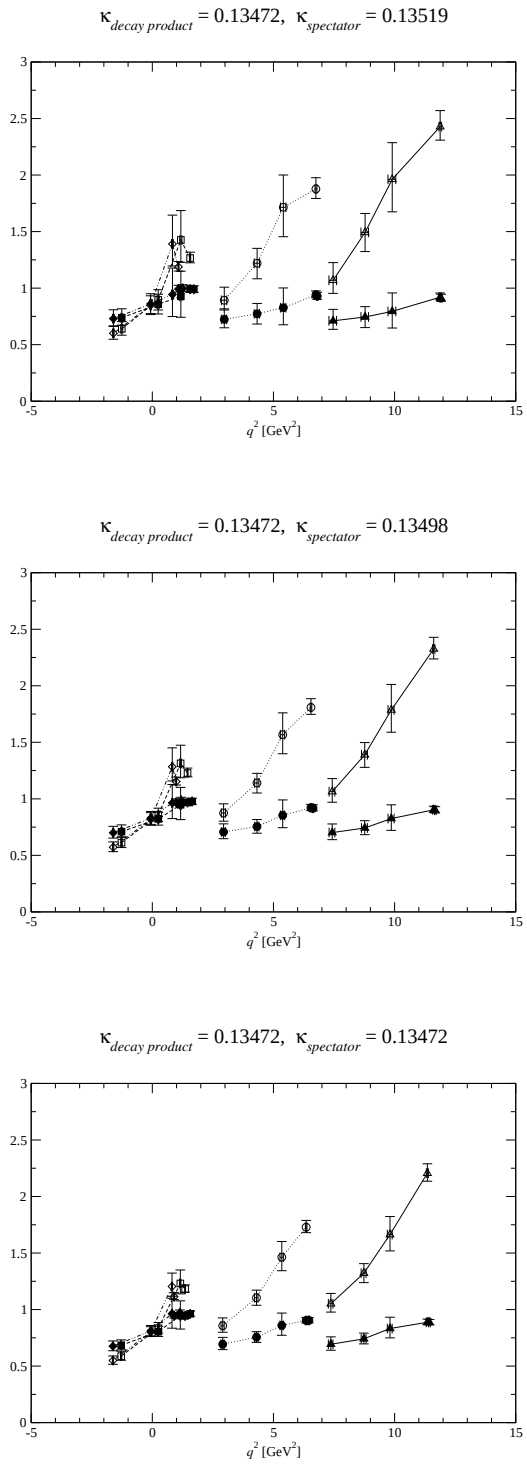

Figure 6: Sample of form factors directly measured in our simulations. The three panels show the results obtained for $\kappa_{decay\,product} = 0.13472$, and for different values of $\kappa_{spectator}$. In each plot, the results for f_0 (denoted by full symbols) and for f_+ (empty symbols) are plotted against the square of the transferred momentum q^2. The results for different values of κ_{heavy} are displayed using different symbols: diamonds ($\kappa_{heavy} = 0.13$), squares ($\kappa_{heavy} = 0.129$), circles ($\kappa_{heavy} = 0.121$) and triangles ($\kappa_{heavy} = 0.115$).

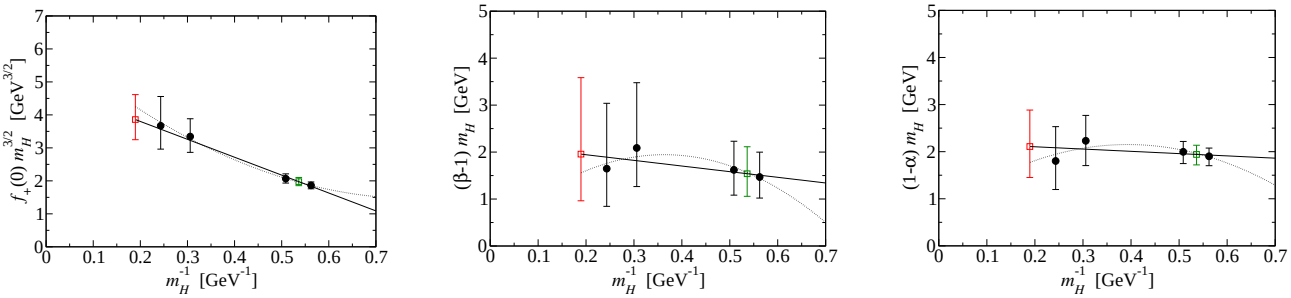

Figure 7: Extrapolation and interpolation of the (chirally extrapolated) Bećirević–Kaidalov parameters in the heavy quark mass. The plots show the results obtained for the combinations $f_+(0) \cdot m_H^{3/2}$ (l.h.s. panel), $(\beta - 1)m_H$ (central panel) and $(1 - \alpha)m_H$ (r.h.s. panel), for decays of a B (red squares) or of a D meson (green squares) to a kaon. The results are obtained using linear fits in m_H^{-1} (solid lines) according to Eq. (8); for comparison, the curves resulting from fits to quadratic order in m_H^{-1} (dotted lines) are also shown.