DESY 09-125
SFB/CPP-09-77

arXiv:0909.1487v1 [hep-ph] 8 Sep 2009

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2- and 3-Loop Heavy Flavor Corrections to Transversity

Johannes Blümlein[1], Sebastian Klein[1] and Beat Tödtli[1]

1- Deutsches Elektronen-Synchrotron DESY, Zeuthen
Platanenallee 6 - 15738 Zeuthen, Germany

We calculate the two- and three-loop massive operator matrix elements (OMEs) contributing to the heavy flavor Wilson coefficients of transversity. We obtain the complete result for the two-loop OMEs and compute the first thirteen Mellin moments at three-loop order. As a by-product of the calculation, the moments $N = 1$ to 13 of the complete two-loop and the T_F-part of the three-loop transversity anomalous dimension are obtained.

1 Framework

The transversity distribution belongs to the three twist-2 parton distribution functions (PDFs), together with those for unpolarized and polarized deep-inelastic scattering. It is a flavor non-singlet, chiral-odd distribution and can be measured in semi-inclusive deep-inelastic scattering (SIDIS) and via the polarized Drell-Yan process. [a] Different experiments perform transversity measurements at the moment, cf. Refs. [2]. Recently, a first phenomenological parameterization has been given for the transversity up- and down-quark distributions in Ref. [3], the moments of which are in qualitative agreement with first lattice calculations [4].

For semi-inclusive deeply inelastic charged lepton-nucleon scattering $lN \to l'h + X$ the scattering cross section is given by

$$\frac{d^3\sigma^{\rm SIDIS}}{dxdydz} = \frac{4\pi\alpha_{\rm em}^2 s}{Q^4} \sum_{a=q,\overline{q}} e_a^2 x \Biggl\{ \frac{1}{2}\left[1+(1-y)^2\right] F_a(x,Q^2) D_a(z,Q^2) - (1-y)|\mathbf{S}_\perp||\mathbf{S}_{h\perp}| \cos\left(\phi_S + \phi_{S_h}\right) \Delta_T F_a(x,Q^2) \Delta_T D_a(z,Q^2) \Biggr\}, \quad (1)$$

after the $\mathbf{P}_{h\perp}$-integration has been performed, [1]. We consider, for definiteness, only scattering cross sections free of $\mathbf{k}_\perp$- effects to refer to twist–2 quantities. x and y denote the Bjorken variables, z the fragmentation variable, $Q^2 = -q^2$ the space-like 4–momentum transfer, $\alpha_{\rm em}$ the fine structure constant, e_a the quark charge, and s the cms energy squared. $\mathbf{S}_\perp$ and $\mathbf{S}_{h\perp}$ are the transverse spin vectors of the incoming nucleon N and the measured hadron h. $F_a(z,Q^2), \Delta_T F_a(z,Q^2)$ and $D_a(z,Q^2), \Delta_T D_a(z,Q^2)$ denote the unpolarized and transversity structure- and fragmentation functions, respectively. The angles ϕ_{S,S_h} are measured in the plane transverse to the $\gamma^* N$ axis between the x-axis and the respective vector. In process (1) the spin of the *transversely* polarized hadron h has to be measured.

The transversity distribution may also be measured in the transversely polarized Drell-Yan processes. In Mellin space the scattering cross section is given by, [5],

$$\frac{d\Delta_T\sigma^{\rm DY}}{d\phi} = \frac{\alpha_{\rm em}^2}{9s} \cos(2\phi) \Delta_T H(N,M^2) \cdot \Delta_T C_q^{\rm DY}(N,M^2) \quad (2)$$

[a] For a review see Ref. [1].

where N denotes the Mellin variable and ϕ is the azimuthal angle of one of the final state leptons $l^{\pm}$ relative to the axis defined by the transverse polarizations.

$$\Delta_T H(N,Q^2) = \sum_q e_q^2 \left[\Delta_T q_1(N,Q^2)\Delta_T \overline{q}_2(N,Q^2) + \Delta_T \overline{q}_1(N,Q^2)\Delta_T q_2(N,Q^2)\right]$$

is a combination of transversity parton distributions for the incoming light (anti-)quarks, and $\Delta_T C_q^{\rm DY}(N,M^2)$ denotes the Wilson coefficient of the Drell-Yan process, with M^2 the invariant mass of the produced lepton pair.

Like in the case of unpolarized and polarized deep-inelastic processes transversity receives heavy flavor corrections in higher orders in QCD. These are given by the corresponding heavy flavor Wilson coefficients. As for other non-singlet quantities [6, 7], these corrections start at $O(a_s^2)$, with $a_s = \alpha_s/(4\pi)$. In SIDIS one can tag $Q\bar{Q}$-production in the same way as in the deep-inelastic process, [8]. A measurement is possible in high luminosity experiments. In the Drell-Yan process, on the other hand, heavy flavor contributions emerge inclusively since there the final-state l^+l^--pairs are measured in the first place. The calculation of the heavy quark Wilson coefficients for $Q^2 \gg m^2$ proceeds in the same way as in unpolarized and polarized deep-inelastic scattering [6, 7, 9, 10]

The complete Wilson coefficients for transversity can be decomposed into a light- and a heavy quark contribution

$$C_q^{\rm TR}\left(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) = C_q^{\rm TR,light}\left(x, \frac{Q^2}{\mu^2}\right) + H_q^{\rm TR}\left(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) \; . \tag{3}$$

As shown in [6], the heavy quark Wilson coefficient for hard processes factorizes into the light quark Wilson coefficients and the massive operator matrix element $A_{qq,Q}^{\rm TR}$ at large enough scales $Q^2 \gg m^2$. We apply this to the heavy flavor Wilson coefficient for transversity $H_q^{\rm TR}$

$$\begin{aligned} H_q^{\rm TR}\left(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) &= C_q^{\rm TR,light}\left(x, \frac{Q^2}{\mu^2}\right) \otimes A_{qq,Q}^{\rm TR}\left(x, \frac{m^2}{\mu^2}\right) \\ &= a_s^2 \left[\Delta_T A_{qq,Q}^{(2),\rm NS,TR}(N_f+1) + \Delta_T \hat{C}_q^{(2)}(N_f)\right] + a_s^3 \left[\Delta_T A_{qq,Q}^{(3),\rm NS,TR}(N_f+1)\right. \\ &+ \left. \Delta_T A_{qq,Q}^{(2),\rm NS,TR}(N_f+1) \otimes \Delta_T C_q^{(1)}(N_f+1) + \hat{C}_q^{(3)}(N_f)\right] \; . \end{aligned} \tag{4}$$

The aim of this article is to present a computation of the renormalized two- and three-loop heavy-flavor operator matrix elements contributing to transversity. Details of the calculation are given in Ref. [11]. The operator matrix element $\langle q | O^{\rm NS,TR} | q \rangle$ is given by a two-point Green's function containing a closed loop of a heavy quark Q and external massless quarks q. The local operator is given by

$$O_{F,a;\mu\mu_1\ldots\mu_n}^{\rm NS,TR} = i^n \mathbf{S}\left[\overline{\psi}\gamma_5\sigma_{\mu\mu_1} D_{\mu_2}\ldots D_{\mu_n}\frac{\lambda_a}{2}\psi\right] - \text{trace terms} \; , \tag{5}$$

cf. [12]. Here $\mathbf{S}$ denotes symmetrization of the Lorentz indices, $\sigma_{\mu\nu} = (i/2)\left[\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu\right]$, and D_μ is the covariant derivative. The Green's function in $D = 4+\varepsilon$ dimensions obeys the following vector decomposition

$$\begin{aligned} \hat{G}_{\mu,q,Q}^{ij,\rm TR,NS} = J_N \langle q | O_{F,a;\mu\mu_1\ldots\mu_n}^{\rm NS,TR} | q \rangle = \delta_{ij} \Bigg\{ &\Delta_\rho \sigma^{\mu\rho} \hat{\hat{A}}_{qq,Q}^{\rm TR,NS}\left(\frac{\hat{m}^2}{\mu^2}, \varepsilon, N\right) + c_1 \Delta^\mu \\ &+ c_2 p^\mu + c_3 \gamma^\mu \not{p} + c_4 \not{\Delta} \not{p} \Delta^\mu + c_5 \not{\Delta} \not{p} p^\mu \Bigg\} (\Delta\cdot p)^{N-1} \end{aligned} \tag{6}$$

contracting the OME with a source term $J_N = \Delta^{\mu_1} \dots \Delta^{\mu_N}$, with $\Delta^2 = 0$, with p the parton momentum. It determines the un-renormalized massive OME

$$\hat{A}_{qq,Q}^{\rm TR,NS}\left(\frac{\hat{m}^2}{\mu^2},\varepsilon,N\right) = \frac{-i\,\delta^{ij}}{4N_c\,(\Delta\cdot p)^{N+1}\,(D-2)}\Biggl\{ {\rm Tr}\left[\not{\Delta}\not{p} p^\mu \hat{G}^{ij,{\rm TR,NS}}_{\mu,q,Q}\right] - \Delta\cdot p\,{\rm Tr}\left[p^\mu \hat{G}^{ij,{\rm TR,NS}}_{\mu,q,Q}\right] + i\Delta\cdot p\,{\rm Tr}\left[p^\mu \hat{G}^{ij,{\rm TR,NS}}_{\mu,q,Q}\right]\Biggr\}. \quad (7)$$

A total of 129 diagrams contribute, which were generated using `QGRAF` [13]. These were projected onto $\hat{\hat{A}}^{TR,NS}_{qq,Q}$, cf. [9], using codes written in `FORM` [14]. After tensor reduction, the loop integrals are of the tadpole-type, since the single external quark is on-shell and massless. The integrals were then evaluated using `MATAD` [15]. The renormalization of the OMEs is described in Ref. [9].

After mass- and charge renormalization one obtains the massive OMEs in the on-mass-shell scheme, cf. [9],

$$\Delta_T A_{qq,Q}^{(2),{\rm NS},\overline{\rm MS}} = \frac{\beta_{0,Q}\gamma_{qq}^{(0),{\rm TR}}}{4}\ln^2\left(\frac{m^2}{\mu^2}\right) + \frac{\hat{\gamma}_{qq}^{(1),{\rm TR}}}{2}\ln\left(\frac{m^2}{\mu^2}\right) + a_{qq,Q}^{(2),{\rm TR}} - \frac{\beta_{0,Q}\gamma_{qq}^{(0),{\rm TR}}}{4}\zeta_2\,, \quad (8)$$

$$\begin{aligned}\Delta_T A_{qq,Q}^{(3),{\rm NS},\overline{\rm MS}} =\;& -\frac{\gamma_{qq}^{(0),{\rm TR}}\beta_{0,Q}}{6}\left(\beta_0+2\beta_{0,Q}\right)\ln^3\left(\frac{m^2}{\mu^2}\right) + \frac{1}{4}\Biggl\{2\gamma_{qq}^{(1),{\rm TR}}\beta_{0,Q} \\ & -2\hat{\gamma}_{qq}^{(1),{\rm TR}}\left(\beta_0+\beta_{0,Q}\right)+\beta_{1,Q}\gamma_{qq}^{(0),{\rm TR}}\Biggr\}\ln^2\left(\frac{m^2}{\mu^2}\right) + \frac{1}{2}\Biggl\{\hat{\gamma}_{qq}^{(2),{\rm TR}} \\ & -\left(4a_{qq,Q}^{(2),{\rm TR}} - \zeta_2\beta_{0,Q}\gamma_{qq}^{(0),{\rm TR}}\right)\left(\beta_0+\beta_{0,Q}\right)+\gamma_{qq}^{(0),{\rm TR}}\beta_{1,Q}^{(1)}\Biggr\}\ln\left(\frac{m^2}{\mu^2}\right) \\ & +4\overline{a}_{qq,Q}^{(2),{\rm TR}}(\beta_0+\beta_{0,Q}) - \gamma_{qq}^{(0)}\beta_{1,Q}^{(2)} - \frac{\gamma_{qq}^{(0),{\rm TR}}\beta_0\beta_{0,Q}\zeta_3}{6} - \frac{\gamma_{qq}^{(1),{\rm TR}}\beta_{0,Q}\zeta_2}{4} \\ & +2\delta m_1^{(1)}\beta_{0,Q}\gamma_{qq}^{(0),{\rm TR}} + \delta m_1^{(0)}\hat{\gamma}_{qq}^{(1),{\rm TR}} + 2\delta m_1^{(-1)}a_{qq,Q}^{(2),{\rm TR}} + a_{qq,Q}^{(3),{\rm TR}}\,, \end{aligned} \quad (9)$$

at 2– and 3–loops. Here, ζ_k denotes the Riemann ζ-function at $\gamma_{qq}^{(l),{\rm TR}}$ are the transversity anomalous dimensions for $l = 0, 1, 2$ in LO [16], NLO [5,17], and NNLO [18], with $\hat{f}(N_f) = f(N_f+1) - f(N_f)$. For the other quantities we refer to [9]. The new terms being calculated are $a_{qq,Q}^{(2),{\rm TR}}(N)$, $\overline{a}_{qq,Q}^{(2),{\rm TR}}(N)$ and $a_{qq,Q}^{(3),{\rm TR}}(N)$, and for the higher values of N, $\hat{\gamma}_{qq}^{(2),{\rm TR}}(N)$.

2 Results

2.1 Massive Operator Matrix Elements

At $O(a_s^2)$ the massive operator matrix elements for transversity $\Delta_T A_{qq,Q}^{(2),{\rm NS},\overline{\rm MS}}$ are obtained for general values of N, cf. Eq. (8). The un-renormalized OME is computed to $O(\varepsilon)$ to also extract the functions $\overline{a}_{qq,Q}^{{\rm TR},(2)}(N)$. The new terms at 2–loops are $a_{qq,Q}^{{\rm TR},(2)}$ and $\bar{a}_{qq,Q}^{{\rm TR},(2)}$, cf.

Eqs. (8, 9):

$$a_{qq,Q}^{\mathrm{TR},(2)}(N) = C_F T_F \left\{ -\frac{8}{3} S_3 + \frac{40}{9} S_2 - \left[\frac{224}{27} + \frac{8}{3} \zeta_2 \right] S_1 + 2\zeta_2 + \frac{\left(24 + 73\,N + 73\,N^2\right)}{18N\,(1+N)} \right\} \tag{10}$$

$$\begin{aligned} \overline{a}_{qq,Q}^{\mathrm{TR},(2)}(N) = C_F T_F \Bigg\{ & - \left[\frac{656}{81} + \frac{20}{9}\,\zeta_2 + \frac{8}{9}\,\zeta_3 \right] S_1 + \left[\frac{112}{27} + \frac{4}{3}\,\zeta_2 \right] S_2 - \frac{20}{9}\,S_3 \\ & + \frac{4}{3}\,S_4 + \frac{1}{6}\,\zeta_2 + \frac{2}{3}\,\zeta_3 + \frac{\left(-144 - 48\,N + 757\,N^2 + 1034\,N^3 + 517\,N^4\right)}{216 N^2\,(1+N)^2} \Bigg\}\,, \end{aligned} \tag{11}$$

with $S_k \equiv S_k\,(N)$ the single harmonic sums.

At $O(a_s^3)$ the moments $N = 1$ to 13 were computed for the massive OMEs, as e.g.

$$\begin{aligned} \Delta_T\ A_{qq,Q}^{(3),\mathrm{NS},\overline{\mathrm{MS}}}(13) \quad = \quad & C_F T_F \Bigg\{ \Big(\frac{1751446}{110565} C_A - \frac{7005784}{1216215} T_F (N_f + 2) \Big) \ln^3\Big(\frac{m^2}{\mu^2}\Big) \\ & + \Big(- \frac{20032048197492631}{2193567563187000} C_F - \frac{137401473299}{8027019000} C_A - \frac{93611152819}{3652293645} T_F \Big) \ln^2\Big(\frac{m^2}{\mu^2}\Big) \\ & + \Big[\Big(\frac{1705832327329042449983}{263491335690022440000} + \frac{7005784}{45045} \zeta_3 \Big) C_F + \Big(\frac{3385454488248191237}{65807026895610000} \\ & - \frac{7005784}{45045} \zeta_3 \Big) C_A - \frac{458114791076413771}{6580702689561000} N_f T_F - \frac{217179304}{3648645} T_F \Big] \ln\Big(\frac{m^2}{\mu^2}\Big) \\ & + \Big(- \frac{7005784}{135135} B_4 + \frac{3502892}{15015} \zeta_4 - \frac{81735983092}{243486243} \zeta_3 \\ & + \frac{5537627829952273383742505 2493}{12208080565190119690080000 0} \Big) C_F + \Big(\frac{3502892}{135135} B_4 - \frac{3502892}{15015} \zeta_4 \\ & + \frac{4061479439}{12162150} \zeta_3 - \frac{3486896974743882556775647}{12935029206601101600000} \Big) C_A \\ & + \Big(- \frac{2799227526321603558606 97}{3557133031815302940000} + \frac{56046272}{1216215} \zeta_3 \Big) T_F N_f \\ & + \Big(\frac{2915265503027600701553 03}{7114266063630605880000} - \frac{14011568}{173745} \zeta_3 \Big) T_F \Bigg\}\,, \end{aligned} \tag{12}$$

where

$$B_4 = -4\zeta_2 \ln^2(2) + \frac{2}{3} \ln^4(2) - \frac{13}{2} \zeta_4 + 16 \mathrm{Li}_4\left(\frac{1}{2}\right)\,.$$

Like for the massive OMEs in case of unpolarized deep-inelastic scattering, the structure of $\Delta_T\ A_{qq,Q}^{(3),\mathrm{NS},\overline{\mathrm{MS}}}(N)$ is widely known for general values of N, except the contributions due to the finite part $a_{qq,Q}^{(3),NS}$ and the 3-loop anomalous dimension $\hat{\gamma}_{qq}^{(2),\mathrm{TR}}(N)$. One notices the cancellation of all ζ_2 terms in $\Delta_T\ A_{qq,Q}^{(3),\mathrm{NS},\overline{\mathrm{MS}}}(N)$ after renormalization.

2.2 Anomalous Dimensions

The transversity anomalous dimension is given by

$$\gamma_{qq}^{\rm TR}\left(N, a_s\right) = \sum_{i=1}^{\infty} a_s^i \gamma_{qq}^{(i),{\rm TR}}\left(N\right). \tag{13}$$

From Eq. (9) one may determine the complete 2-loop anomalous dimension [5,17] and the T_F-part of the 3-loop anomalous dimension [18]. We agree with the results given in [5,17] and confirm the T_F-contributions for the moments $N = 1$ to 8 given in Refs. [18]. Furthermore, we obtain $\hat{\gamma}_{qq}^{(3),{\rm TR}} = \gamma_{qq}^{(3),{\rm TR}}(N_f + 1) - \gamma_{qq}^{(3),{\rm TR}}(N_f)$ newly for $N = 9$ to 13, as e.g.

$$\begin{aligned}\hat{\gamma}_{qq}^{(3),{\rm TR}}\left(N = 13\right) = &- C_F T_F \left[\frac{3671331901540714157 0017}{1317456678450112200 00} C_F - \frac{14011568}{45045}\left(C_F - C_A\right)\zeta_3\right.\\ &\left. + \frac{66409807459266571}{3290351344780500} T_F(1 + 2N_f) + \frac{6571493644375020121}{65807026895610000} C_A\right].\end{aligned}$$

2.3 A Remark on the Soffer Bound

If the Soffer inequality [19]

$$\left|\Delta_T f(x,Q^2)\right| \leq \frac{1}{2}\left[f(x,Q^2) + \Delta f(x,Q^2)\right] \tag{14}$$

holds for the non-perturbative PDFs in Eq. (14) one may check its generalization from $f_i \to F_i$ for the corresponding structure functions. This includes the non-singlet evolution operator (Eq. (6), Ref. [20]) and the heavy flavor Wilson coefficient. At perturbative scales, it holds for the evolution operator [11], generalizing a result from [5] for the moments $N = 1$ to 13 at 3–loops. For the heavy quark Wilson coefficients in SIDIS we only know the massive OMEs so far. As shown in Ref. [11], a final conclusion can only be drawn knowing the yet undetermined massless Wilson coefficients. Here the difference $[A_{qq,Q}^{\rm V} - A_{qq,Q}^{\rm TR}](x)$ of the massive OMEs, shows a sign change to negative values for Q^2/m^2 in the physical range. For large scales $Q^2/m^2 \gg 1$ positive values are obtained.

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