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Results from 2+1 flavours of SLiNC fermions

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QCD results are presented for a 2+1 flavour fermion clover action (which we call the SLiNC action). A method of tuning the quark masses to their physical values is discussed. In this method the singlet quark mass is kept fixed, which solves the problem of different renormalisations (for singlet and non-singlet quark masses) occuring for non-chirally invariant lattice fermions. This procedure enables a wide range of quark masses to be probed, including the case with a heavy up-down quark mass and light strange quark mass. Preliminary results show the correct splittings for the baryon (octet and) decuplet spectrum.

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1. Introduction

There has been a steady progression of lattice results from a quenched sea to two-flavour and more recently $2+1$ flavour sea in an attempt to provide a more complete and quantitative description of hadronic phenomena. (By $2+1$ flavours we mean here 2 mass degenerate up-down, m_l, quarks and one strange, m_s, quark.) In this talk we shall consider a $2+1$ flavour clover action and discuss some ways of approaching in the m_l–m_s plane the physical point (m_l^*, m_s^*), where the natural starting point for these paths is an $SU_F(3)$ flavour symmetric point $m_l = m_s = m_{sym}^{(0)}$. For clover (i.e. non-chiral) fermions a problem arises because the measured pseudoscalar masses are proportional to the *renormalised* quark masses. The singlet, S, and nonsinglet, NS quark masses renormalise differently which means that the relation to the bare quark masses and hence κ, which is the adjustable simulation parameter, is more complicated [1]. Choosing the path such that the singlet quark mass is kept fixed provides an elegant solution to this problem. This procedure also has the advantage that it enables a wide range of quark masses to be probed (including the mass of the strange quark) and is thus particularly useful for strange quark physics and $SU_F(3)$–chiral perturbation theory, as the kaon mass is never larger than its physical value. (Note that this path choice although favourable for clover–type fermions is not restricted to them.) Indeed this includes the case with a heavy light quark mass and light strange quark mass. As a 'proof of concept' preliminary results given here show the correct splittings for the baryon (octet and) decuplet spectrum.

The particular clover action used here has a single iterated mild stout smearing for the hopping terms together with thin links for the clover term (this is in an attempt to ensure that the fermion matrix does not become too extended). Together with the (tree level) Symanzik improved gluon action this constitutes the Stout Link Non-perturbative Clover or SLiNC action for which the clover coefficient, c_{sw}, has recently been non-perturbatively (NP) determined, [2], using the Schrödinger Functional, or SF, formalism. Further details about the action may be found in this reference. Simulations have been performed using HMC with mass preconditioning for 2 mass-degenerate flavours and the rational approximation for the 1-flavour. Two programmes were used, a Fortran programme, [3], and also the Chroma programme, [4]. Quark mass degenerate runs (denoted by the subscript '*sym*') on $16^3 \times 32$ lattices followed by $24^3 \times 48$ lattices have located a suitable β-range, see Fig. 1. We see that there is good agreement between SF and pseudoscalar mass determinations of the critical value of κ_{sym}, $\kappa_{sym;c}$. The simulations reported here have been performed at $\beta = 5.50$ which gives $a \sim 0.08\,\mathrm{fm}$ (taking the scale $r_0 = 0.5\,\mathrm{fm}$ and a linear extrapolation of r_0/a to the chiral limit). We have furthermore checked that there is a distinct gap between the distribution of (the modulus of) the lowest eigenvalue and 0 indicating that the simulations are stable on present volumes. Further details of the results in this write-up will be given in [5].

2. Non-degenerate quark masses

As mentioned before the problem (at least for clover-like fermions, with no chiral symmetry) is that singlet and non-singlet quark mass can renormalise differently,

$$m_q^R = Z_m^{NS}(m_q - \overline{m}) + Z_m^S \overline{m}$$

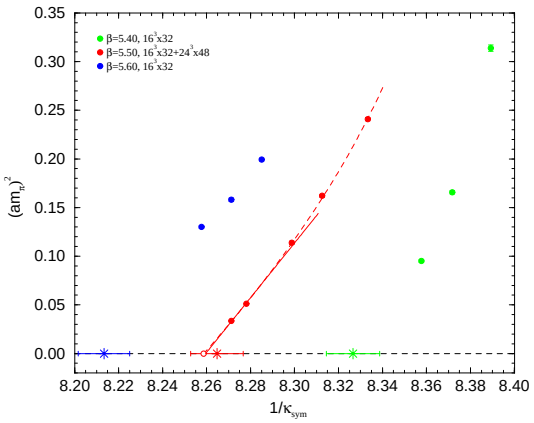

Figure 1: Results for the pseudoscalar mass against $1/\kappa_{sym}$ for $\beta = 5.40, 5.50, 5.60$. For $\beta = 5.50$ a linear fit (line) using the lightest three masses is made. For comparison a quadratic fit (dashed line) is also made to all the masses. The stars represent the SF determination of $\kappa_{sym;c}$; the open circle is the result from the linear fit.

$$= Z_m^{NS}(m_q + \alpha_Z \overline{m}) \quad \text{with} \quad \alpha_Z = \frac{Z_m^S - Z_m^{NS}}{Z_m^{NS}}, \tag{2.1}$$

with $q \in \{sym, l, s, v\}$ (also including possible different valence quarks to sea quarks) and

$$\overline{m} = \frac{1}{3}(2m_l + m_s). \tag{2.2}$$

(We are assuming here that the quadratic improvement terms, [6], are small, [5].) The bare quark mass in eq. (2.1) is defined by

$$am_q = \frac{1}{2}\left(\frac{1}{\kappa_q} - \frac{1}{\kappa_{sym;c}}\right). \tag{2.3}$$

Now if $m_{ps}^{q_1 q_2}(l,s)$ is the measured pseudoscalar mass (with quarks q_1, q_2) then we expect that

$$(am_K)^2 \equiv (am_{ps}^{q_1 q_2})^2 \propto am_{q_1}^R + am_{q_2}^R \propto am_{q_1} + am_{q_2} + 2\alpha_Z a\overline{m}. \tag{2.4}$$

In particular

$$(am_\pi)^2 \equiv (am_{ps}^{qq})^2 \propto 2(am_q + \alpha_Z a\overline{m}), \tag{2.5}$$

which is $\not\propto am_q$ unless $q = l = s = sym$ (i.e. $m_q = \overline{m}$). That α_Z is non-zero may be easily seen by considering partially quenched results (i.e. results for the pseudoscalar mass where the valence quark masses may differ from the sea quark masses). In Fig. 2 we show some partially quenched results and compare the extrapolation with the $SU_F(3)$-symmetric results. The lines clearly have different gradients. α_Z can be estimated as $(am_\pi)^2$ vanishes at κ_c^{pq} say (pq for 'partially quenched'), giving

$$\alpha_Z = -\frac{am_q|_{\kappa=\kappa_c^{pq}}}{a\overline{m}} = \frac{\left(\frac{1}{\kappa_{sym;c}} - \frac{1}{\kappa_c^{pq}}\right)}{\left(\frac{1}{\kappa_{sym}} - \frac{1}{\kappa_{sym;c}}\right)}, \tag{2.6}$$

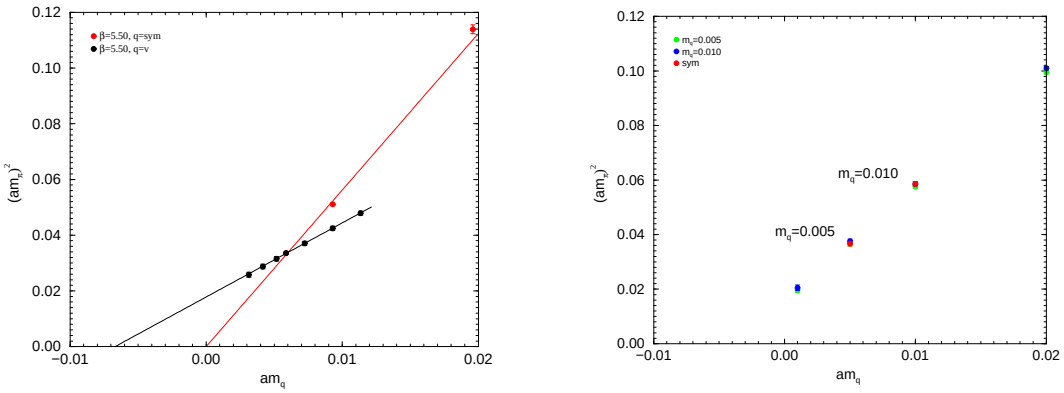

Figure 2: The left plot shows the $SU_F(3)$ symmetric pseudoscalar masses versus am_q (in red) together with the partially quenched results (in black) using the mass degenerate sea quark $\kappa_{sym} = 0.12090$. (This is the kappa value at the crossing point.) The right plot shows equivalent domain wall results from [7] table V.

(setting $l = s = sym$ in $\overline{m}$). This gives here $\alpha_Z \sim 1.2$ (but the determination is quite sensitive to small changes in $\kappa_{sym;c}$ and κ_c^{pq}). This is to be compared with domain-wall fermions, also shown in Fig. 2, where the results line up.

We wish to approach the physical point along some path in the m_l^R–m_s^R plane (m_q^R is considered as it is related to the measurable pseudoscalar mass) from an $SU_F(3)$ symmetric point ($m_l^R = m_s^R \equiv m_{sym}^{R(0)}$). This is depicted in Fig. 3. Two possibilities are $m_s^R = \text{const.} = m_s^{R*}$ (i.e. strange quark mass

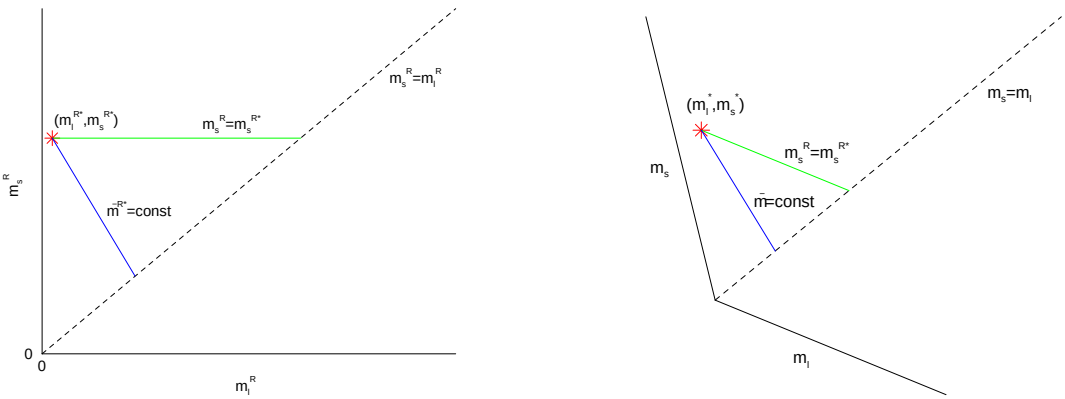

Figure 3: The left sketch shows the m_l^R–m_s^R plane. The physical quark masses are denoted by (m_l^{R*}, m_s^{R*}). The dashed diagonal line is the $SU_F(3)$-symmetric line. Two possible paths from this line are shown, $m_s^R = m_s^{R*}$ and $\overline{m}^R = \text{const.}$. The right sketch shows the equivalent results in the m_l–m_s plane.

being held constant) or $\overline{m}^R = \text{const.}$ (i.e. singlet quark mass being held constant). Note that the region covered is $m_l^R \geq 0$, $m_s^R \geq 0$. For simulations we need to translate this to unrenormalised quantities also shown in Fig. 3. Note that the physical domain is now $m_l \geq -(\frac{1}{3}\alpha_Z/(1+\frac{2}{3}\alpha_Z))m_s$, $m_s \geq -(\frac{2}{3}\alpha_Z/(1+\frac{1}{3}\alpha_Z))m_l$. While $m_s^R = \text{const.} = m_s^{R*}$ translated to bare quark masses now depends on the difficult-to-determine α_Z, the singlet quark mass $\overline{m}^R \propto \overline{m}$ or $m_s = (2m_l^* + m_s^*) - 2m_l$ is independent of the value of α_Z. This motivates the choice of this path. As $m_l \searrow m_l^*$ then $m_s \nearrow m_s^*$ i.e. the m_s–m_l splitting or m_K *increases* to its physical value. Other potential advantages include: the singlet quark mass is correct from the very beginning; flavour singlet quantities are flat at the

symmetric point – allowing simpler extrapolations; numerically the HMC cost change should be moderate along this path. (These points will be further discussed in [5].)

Of course practically we must now determine the initial $\kappa_{sym}^{(0)}$ to complete the relation between κ_s and κ_l,

$$\kappa_s = \frac{1}{\frac{3}{\kappa_{sym}^{(0)}} - \frac{2}{\kappa_l}}\,. \tag{2.7}$$

For our path choice, $\kappa_{sym}^{(0)}$ can be implicitly found by using eqs. (2.4), (2.5) together with a singlet scale X (so $X^* = X_{sym}^{(0)}$ here) to relate the known physical point to the initial symmetric point,

$$\frac{1}{c_X} \equiv \left.\frac{\frac{1}{3}(2m_K^2 + m_\pi^2)}{X^2}\right|^* \propto \overline{m}^{R*} = \overline{m}_{sym}^{R(0)} \propto \frac{(am_\pi^{(0)})^2}{(aX_{sym}^{(0)})^2}\,. \tag{2.8}$$

For X we have several choices. For example

- The centre of mass^2 of the octet baryons: $X_N^2 \equiv \frac{1}{3}(m_N^2 + m_\Sigma^2 + m_\Xi^2)$ (stable under strong interaction) giving $\frac{1}{3}(2m_K^2 + m_\pi^2)/\frac{1}{3}(m_N^2 + m_\Sigma^2 + m_\Xi^2)|^* = 0.169/1.34 = 1/7.93 = (am_\pi^{(0)})^2/(am_N^{(0)})^2$,
- Centre of mass^2 of the decuplet baryons: $X_\Delta^2 \equiv \frac{1}{3}(2m_\Delta^2 + m_\Omega^2)$ (which decay under strong interactions) giving $\frac{1}{3}(2m_K^2 + m_\pi^2)/\frac{1}{3}(2m_\Delta^2 + m_\Omega^2)|^* = 0.169/1.94 = 1/11.5 = (am_\pi^{(0)})^2/(am_\Delta^{(0)})^2$,
- A gluonic quantity $X_r^2 \equiv 1/r_0^2$ so $r_0^2(2m_K^2 + m_\pi^2)|^* = 0.169/0.395^2 = 1.083 = (r_0^{(0)}/a)^2(am_\pi^{(0)})^2$ ($r_0 = 0.5\,\text{fm}$).

Thus where the numerically determined $SU_F(3)$–symmetric line crosses with the line $(aX_{sym})^2 = c_X(am_\pi)^2$ gives our initial point. The results are shown in Fig. 4. We see that they are all consistent around the lightest pseudoscalar mass, namely for $\kappa_{sym}^{(0)} = 0.12090$ which we shall take as our starting value. From eq. (2.7) we now have a relation between κ_s and κ_l. After some experimentation we chose the κ_l, κ_s values given in Table 1. Note that is possible to choose κ_l, κ_s values (here

κ_l	κ_s	
0.12083	0.12104	$m_l > m_s$
0.12090	0.12090	$m_l = m_s$
0.12095	0.12080	$m_l < m_s$
0.12100	0.12070	$m_l < m_s$
0.12104	0.12062	$m_l < m_s$

Table 1: Present (κ_l, κ_s) values (simulated on $24^3 \times 48$ lattices).

$(0.12083, 0.12104)$) such that $m_l > m_s$. In this strange world we would expect to see an *inversion* of the particle spectrum, with, for example, the nucleon being the heaviest octet particle.

3. Hadron spectrum

As an example we now give some results in Fig. 5 for the baryon decuplet: Ω, Ξ^*, Σ^*, Δ. These are at present low statistics results and are meant to illustrate the 'proof of concept' only. Noteworthy is that the correct ordering of the particle spectrum has been achieved.

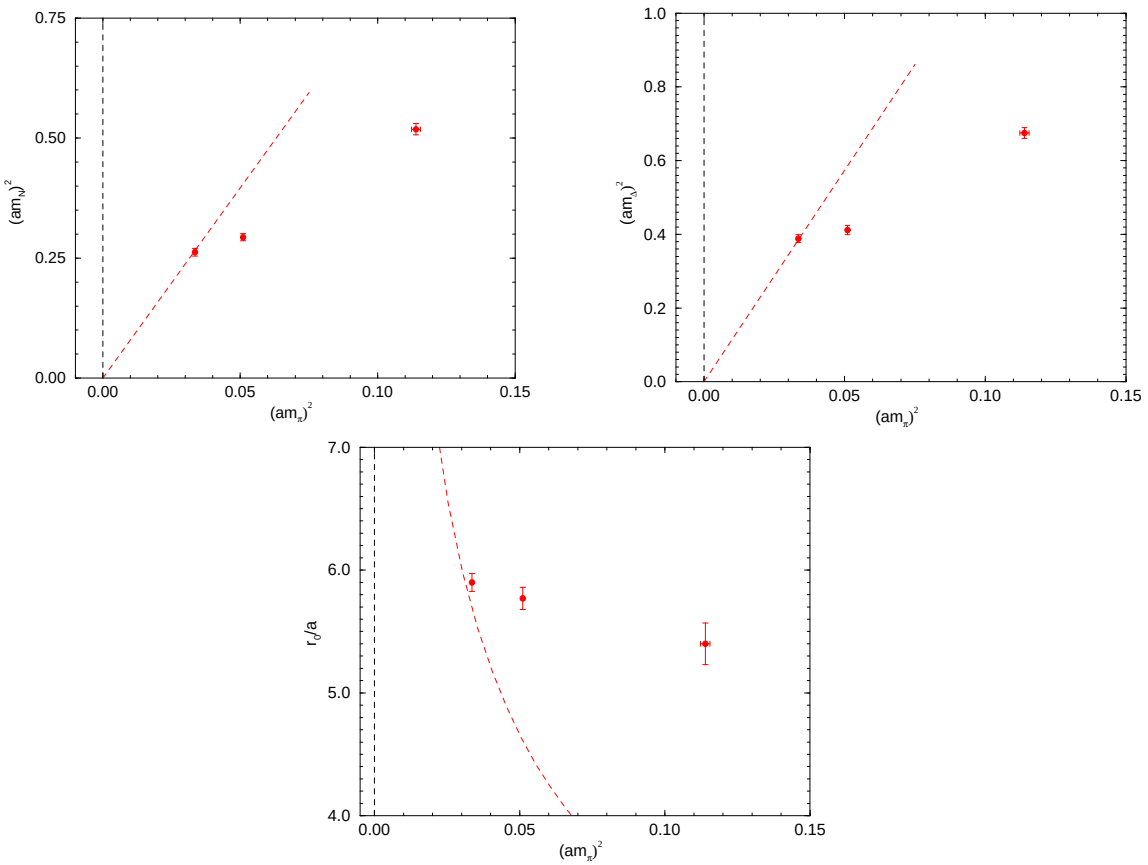

Figure 4: $(am_N)^2$ against $(am_\pi)^2$ (upper left picture) together with the line $(am_N)^2 = 7.93(am_\pi)^2$; $(am_\Delta)^2$ against $(am_\pi)^2$ (upper right picture) together with the line $(am_\Delta)^2 = 11.5(am_\pi)^2$; r_0/a against $(am_\pi)^2$ (lower picture) together with the line $(r_0/a)^2 = 1.083/(am_\pi)^2$. All data comes from the symmetric points.

4. Conclusions

For the NP $O(a)$-improved $2+1$ flavour clover action discussed here we have first found that there is consistency for the critical κ_{sym} between the SF approach and the determination from the vanishing of the hadron pseudoscalar mass. The quark mass renormalisation suggests that the simplest way of approaching the physical point is to hold the singlet quark mass fixed. Exploratory results for the hadron mass spectrum give splittings in the correct order (including *inversion* when $m_l > m_s$ i.e. we can simulate a strange world where, for example, the nucleon can decay). Further results will be published elsewhere, [5].

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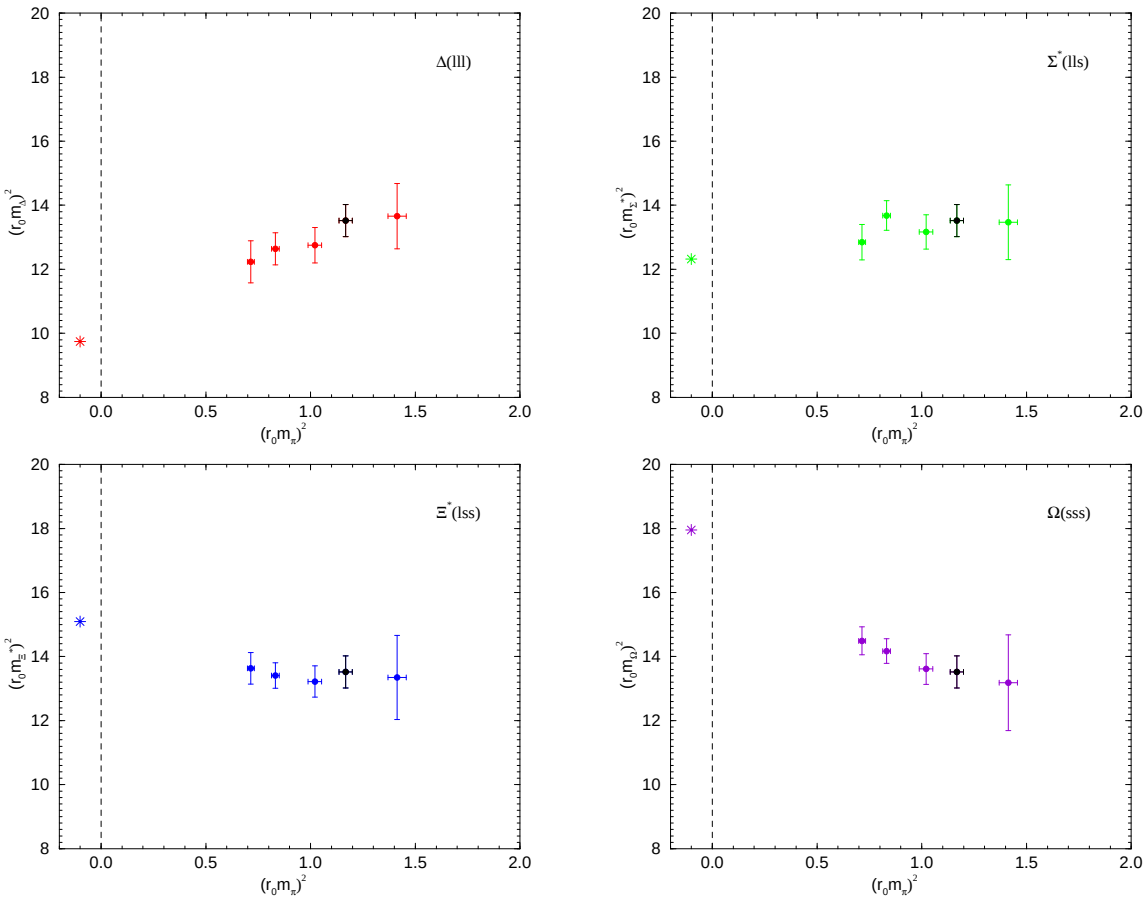

Figure 5: The decuplet baryons. The upper left picture shows the Δ, upper right Σ^*, lower left Ξ^* and lower right the Ω. The common $SU_F(3)$ symmetric quark mass values are shown in black. The experimental values are shown with stars.

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