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Higher-order predictions for splitting functions and coefficient functions from physical evolution kernels

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We have studied the physical evolution kernels for nine non-singlet observables in deep-inelastic scattering (DIS), semi-inclusive e^+e^- annihilation and the Drell-Yan (DY) process, and for the flavour-singlet case of the photon- and heavy-top Higgs-exchange structure functions (F_2, F_ϕ) in DIS. All known contributions to these kernels show an only single-logarithmic large-x enhancement at all powers of $(1-x)$. Conjecturing that this behaviour persists to (all) higher orders, we have predicted the highest three (DY: two) double logarithms of the higher-order non-singlet coefficient functions and of the four-loop singlet splitting functions. The coefficient-function predictions can be written as exponentiations of $1/N$-suppressed contributions in Mellin-N space which, however, are less predictive than the well-known exponentiation of the $\ln^k N$ terms.

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1. Introduction: hard lepton-hadron processes in perturbative QCD

We are interested in the structure functions in deep-inelastic scattering (DIS), the corresponding fragmentation functions in semi-inclusive e^+e^- annihilation (SIA), and the cross section $\frac{1}{\sigma_0}\,d\sigma/dM_{ll}^2$ for Drell-Yan (DY) lepton-pair production in hadron-hadron collisions (see Ref. [1] for a detailed introduction). These one-scale observables, here denoted by $F_a(x,Q^2)$, are generically given by

$$F_a(x,Q^2) \;=\; \bigl[C_{a,i\{j\}}(\alpha_s(\mu^2),\mu^2/Q^2)\otimes f_i^h(\mu^2)\{\otimes f_j^{h'}(\mu^2)\}\bigr](x) + \mathcal{O}(1/Q^{(2)})\;. \tag{1.1}$$

Here Q^2 denotes the physical hard scale (e.g., $Q^2=M_{ll}^2$ for the DY case), and x the corresponding scaling variable. μ represents the $\overline{\rm MS}$ renormalization and factorization scale (there is no need to keep them different here), and $\otimes$ stands for the Mellin convolution. The parts of Eq. (1.1) in curly brackets only apply to the DY case, and summation over i {and j} is understood.

At $\mu^2=Q^2$ the expansion of the coefficient functions C_a in powers of the strong coupling α_s is

$$C_{a,i}(x,\alpha_s) \;=\; (1-\delta_{aL})\,\delta_{iq}\,\delta(1-x) + a_s c_{a,i}^{(1)}(x) + a_s^2 c_{a,i}^{(2)}(x) + a_s^3 c_{a,i}^{(3)}(x) + \ldots \tag{1.2}$$

with $a_s\equiv\alpha_s/(4\pi)$. As indicated by the first term of the r.h.s., of all cases we consider here only the longitudinal coefficient functions in DIS and SIA vanish at order α_s^0. The (spacelike) parton and (timelike) fragmentation distributions f_i^h are, of course, non-perturbative quantities. However their scale dependence is calculable perturbatively via the renormalization-group evolution equations

$$\frac{d}{d\ln\mu^2}\,f_i(x,\mu^2) \;=\; [P_{ik}^{S,T}(\alpha_s(\mu^2))\otimes f_k(\mu^2)](x)\;\;,\quad P(x,\alpha_s) \;=\; \textstyle\sum_{l=0} a_s^{l+1}P^{(l)}(x)\;. \tag{1.3}$$

Except for F_L in DIS and SIA, the terms up to $c_a^{(n)}(x)$ and $P^{(n)}(x)$ in Eqs. (1.2) and (1.3) define the N^{n}LO approximations to Eqs. (1.1). Precise predictions including a sound numerical uncertainty estimate require, at least, calculations at the next-to-next-to-leading order (NNLO $\equiv$ N^{2}LO). The same order is usually required for deducing structural features such as the ones discussed below.

The NNLO coefficient functions for the quantities mentioned above Eq. (1.1), with the exception of $c_L^{(3)}(x)$ in SIA, have been obtained in Refs. [2–7] (the latter two new articles deal with the only theoretically relevant Higgs-exchange structure function F_ϕ in the heavy-top limit). The NNLO spacelike (S) splitting functions in Eq. (1.3) are fully known from Refs. [8], while for the timelike (T) case only the diagonal quantities $P_{qq,gg}^{(2)T}(x)$ have been derived so far [9]. At N^{3}LO only the coefficient functions for the structure functions $F_{1,2,3,\phi}$ are available at this point [6, 10].

2. $\ln(1-x)$ contributions to the splitting functions and coefficient functions

From order α_s^2 the quark coefficient functions in Eq. (1.2) and quark-quark splitting functions in Eq. (1.3) need to be decomposed into (large-x dominant) non-singlet and (suppressed) pure-singlet contributions. The non-singlet splitting functions receive an only single-logarithmic (SL) higher-order enhancement, and that only in terms relatively suppressed by $(1-x)^{k\geq 2}$ [8, 9, 11, 12]

$$P_{\rm ns}^{(l)}(x) \;=\; A_{l+1}(1-x)_+^{-1} + B_{l+1}\,\delta(1-x) + C_{l+1}\ln(1-x) + \mathcal{O}\bigl((1-x)^{k\geq 1}\ln^l(1-x)\bigr)\;. \tag{2.1}$$

Also the $C_F=0$ part of the gluon-gluon splitting functions is of this form [6]. The corresponding (pure-)singlet splitting functions include double-logarithmic (DL) contributions

$$P^{(l)}_{\rm ps,gg|_{C_F}} \,/\, P^{(l)}_{\rm qg,gq} : \;\; \text{terms up to} \;\; (1-x)\ln^{2l-1}(1-x) \,/\, \ln^{2l}(1-x)\;. \tag{2.2}$$

The non-singlet coefficient functions for the structure functions $F_{1,2,3}$, the (transverse, angle-integrated and asymmetric) fragmentation functions $F_{T,I,A}$ and the quark-antiquark annihilation DY cross section $F_{\rm DY}$, on the other hand, show a DL enhancement already (but not only) at the $(1-x)^{-1}_+$ plus-distribution level, i.e,

$$c^{(l)}_{a,\rm ns} : \;\; \text{terms up to} \;\; (1-x)^{-1}\ln^{2l-1}(1-x)\;. \tag{2.3}$$

The highest $(1-x)^{-1}$ logarithms are resummed by the threshold exponentiation [13], with the exponents now known to next-to-next-to-next-to-leading logarithmic ($\rm N^3LL$) accuracy (up to the numerically irrelevant four-loop cusp anomalous dimension A_4 in Eq. (2.1)), see Refs. [14–16]. The leading contributions for the corresponding longitudinal DIS and SIA coefficient functions are down by a factor $(1-x)$ and one power of $\ln(1-x)$ w.r.t. Eq. (2.3). Despite a recently renewed interest in such terms which behave as $N^{-1}\ln^k N$ in Mellin space, see, e.g., Refs. [17], corresponding resummations have not been derived so far for these contributions to the coefficient functions.

In the flavour-singlet sector we will confine ourselves to the DIS cases of F_2 and F_ϕ [6, 7] in the present contribution. The gluon coefficient function for F_ϕ is of the form (2.3), while the 'off-diagonal' (see section 5) quantities are also double-logarithmic but suppressed by $(1-x)$,

$$c^{(l)}_{2,g/\phi,q} : \;\; \text{terms up to} \;\; \ln^{2l-1}(1-x)\;. \tag{2.4}$$

3. Non-singlet physical kernels and coefficient-function predictions

We now switch to moment space (and often suppress the Mellin variable N), which considerably simplifies the following calculations by turning the convolutions in Eqs. (1.1) and (1.3) into simple products. The resulting manipulations of harmonic sums [18] and harmonic polylogarithms [19] have been mostly carried out in FORM 3 and TFORM [20].

The non-singlet physical evolution kernels K_a for the DIS and SIA cases are constructed by

$$\begin{aligned}\frac{dF_a}{d\ln Q^2} \;&=\; \frac{d}{d\ln Q^2}\,(C_a\,q) \;=\; \frac{dC_a}{d\ln Q^2}\,q + C_a P\,q \;=\; \Big(\beta(a_s)\,\frac{dC_a}{da_s} + C_a P\Big)\,C_a^{-1}F_a \\ &=\; \Big(P_a + \beta(a_s)\,\frac{d\ln C_a}{da_s}\Big)\,F_a \;=\; K_a F_a \;\equiv\; \textstyle\sum_{l=0} a_s^{l+1} K_{a,l}\,F_a \end{aligned} \tag{3.1}$$

for $\mu^2 = Q^2$ (the additional terms for $\mu^2 \neq Q^2$ can be readily reconstructed), where $\beta(a_s)$ is the usual beta function of QCD, $\beta(a_s) = -a_s^2\beta_0 - a_s^3\beta_1 - \ldots$ with $\beta_0 = 11/3\,C_A - 1/3\,n_f$ etc, and n_f is the number of effectively massless flavours. For $a \neq L$ Eq. (1.2) leads to the expansion

$$K_a \;=\; a_s P_{a,0} + \textstyle\sum_{l=1} a_s^{l+1}\big(P_{a,l} - \sum_{k=0}^{l-1}\,\beta_k\,\tilde{c}_{a,l-k}\big) \tag{3.2}$$

with

$$\begin{aligned} \tilde{c}_{a,1} \;&=\; c_{a,1}\;, & \tilde{c}_{a,3} \;&=\; 3c_{a,3} - 3c_{a,2}\,c_{a,1} + c_{a,1}^3 \\ \tilde{c}_{a,2} \;&=\; 2c_{a,2} - c_{a,1}^2\;, & \tilde{c}_{a,4} \;&=\; 4c_{a,4} - 4c_{a,3}\,c_{a,1} - 2c_{a,2}^2 + 4c_{a,2}\,c_{a,1}^2 - c_{a,1}^4\;, \;\ldots\;. \end{aligned} \tag{3.3}$$

The structure for the DY case is the same except for $P_{a,n} \equiv P^{(n)}_{a,\rm ns} \to 2P_{a,n}$ in Eq. (3.2).

The threshold resummation of these coefficient functions [13], again for $a \neq L$, is given by

$$C_a(N,\alpha_s) \;=\; g_0(a_s)\,\exp\{Lg_1(a_sL)+g_2(a_sL)+\ldots\} \,+\, \mathcal{O}(1/N) \tag{3.4}$$

with $L \equiv \ln N$. Due to the logarithmic derivative in the second line of Eq. (3.1), the exponentiation (3.4) guarantees a single-logarithmic large-N/large-x enhancement of the physical kernels [21],

$$K_a(N,\alpha_s) \;=\; -\sum_{l=1} A_l\, a_s^l L + \beta(a_s)\,\frac{d}{da_s}\,\{Lg_1(a_sL)+g_2(a_sL)+\ldots\} \,+\, \ldots \;. \tag{3.5}$$

We are now ready to present the first crucial observation: all considered non-singlet kernels K_a (including $a = L$ in DIS and SIA) are single-log enhanced to all orders in N^{-1} or $(1-x)$ [12,22]. Switching back to x-space, the universal $a \neq L$ leading-logarithmic terms in DIS (upper sign) and SIA (lower sign) to N^{3}LO read, with $p_{\rm qq}(x) = (1-x)_+^{-1} - 1 - x$,

$$\begin{aligned}
K_{a,0}(x) \;&=\; 2C_F\,p_{\rm qq}(x) + 3C_F\,\delta(1-x) \\
K_{a,1}(x) \;&=\; \ln(1-x)\,p_{\rm qq}(x)\,\left[-2C_F\beta_0 \mp 8C_F^2\ln x\right] \\
K_{a,2}(x) \;&=\; \ln^2(1-x)\,p_{\rm qq}(x)\,\left[\,2C_F\beta_0^2 \pm 12C_F^2\,\beta_0\ln x + 16C_F^3\ln^2 x\right] \\
K_{a,3}(x) \;&=\; \ln^3(1-x)\,p_{\rm qq}(x)\left[-2C_F\beta_0^3 \mp 44/3\,C_F^2\,\beta_0^2\ln x - 32C_F^3\,\beta_0\ln^2 x + \xi_{P_3}\,C_F^4\ln^3 x\right] \;,
\end{aligned} \tag{3.6}$$

where ξ_{P_3} is the unknown four-loop SL coefficient in Eq. (2.1). For DIS the N^{3}LO relation is based on Refs. [10], while for SIA we have used incomplete but sufficient analytic-continuation results presented in Ref. [12] where also the DY relations analogous to Eqs. (3.6), known only to NNLO, can be found. The first terms on the right-hand-sides include the leading large-n_f terms for which Eqs. (3.6) can be generalized to all orders in DIS, using the $C_{2,\rm ns}$ results of Ref. [23].

It is now rather obvious to conjecture that the physical evolution kernels receive only SL contributions to all orders in α_s at all powers of n_f. This implies an exponentiation (see section 4) of the coefficient functions beyond the $(1-x)_+^{-1}$ terms. The emergence of the resulting fourth-order predictions can be illustrated by recalling the last relation written out in Eq. (3.3),

$$\underbrace{\tilde{c}_{a,4}}_{\rm SL} \;=\; \underbrace{4c_{a,4}}_{\rm DL,\,new}\;\underbrace{-\,4c_{a,3}\,c_{a,1} - 2c_{a,2}^2 + 4c_{a,2}\,c_{a,1}^2 - c_{a,1}^4}_{\rm DL,\,known\ for\ DIS/SIA}\;. \tag{3.7}$$

I.e., the $\ln^{7,6,5}(1-x)$ DL fourth-order contributions for $F_{1,2,3}$ and $F_{T,I,A}$ in Eq. (1.2) need to cancel the corresponding terms from the known lower-order coefficient functions at all orders in $(1-x)$, and consequently can be predicted from those results. We do not have the space here to give an explicit example of these predictions and their numerical size, but refer the reader to Ref. [12]. Note, however, that our results explain an old observation [25] for the highest $1/N$ logarithms.

Due to the universality of the leading terms in Eqs. (3.6), also for $c_{L,\rm ns}^{(l>3)}$ in DIS and SIA the coefficients of the three highest logarithms are predicted, by the respective differences $K_2 - K_1$ and $K_I - K_T$. The agreement of these predictions with those obtained from the quite different kernels K_L [22] – the above differences are of the order $(1-x)^0$, while the leading large-x terms of K_L are of the form $(1-x)_+^{-1}$ – provides a quite non-trivial check of the above conjecture. On the other hand, only two logarithms can be predicted completely at this point at the third and all higher orders for the DY case [12], as the corresponding coefficient function is only known to order α_s^2 [5].

4. All-order exponentiation of the $1/N$ non-singlet coefficient functions

The subleading $1/N$ contributions to the non-singlet coefficient functions for $F_{1,2,3}$, $F_{T,I,A}$ and $F_{\rm DY}$ can be cast in an all-order form analogous to (if unavoidably less compact than) Eq. (3.4),

$$C_a - C_a\big|_{N^0L^k} \;=\; \frac{1}{N}\left(\left[d_{a,1}^{(1)}L + d_{a,0}^{(1)}\right]a_{\rm s} + \left[d_{a,1}^{(2)}L + d_{a,0}^{(2)}\right]a_{\rm s}^2 + \ldots\right)\exp\left\{Lh_1(a_{\rm s}L) + h_2(a_{\rm s}L) + \ldots\right\}\,. \tag{4.1}$$

The exponentiation functions are defined by the series $h_k(a_{\rm s}L) = \sum_{k=1} h_{kn}(a_{\rm s}L)^n$ with $L \equiv \ln N$. Their coefficients for DIS/SIA (given by the upper/lower sign in Eqs. (4.2) and (4.3)) relative to the corresponding coefficients for the N^0L^k soft-gluon exponentiation are given by

$$\begin{aligned} h_{1k} \;&=\; g_{1k}\,, & h_{22} \;&=\; g_{22} + \frac{5}{24}\beta_0^2 \pm \frac{17}{9}\beta_0 C_F - 18C_F^2 \\ h_{21} \;&=\; g_{21} + \frac{1}{2}\beta_0 \pm 6C_F\,, & h_{23} \;&=\; g_{23} + \frac{1}{8}\beta_0^3 \pm \left(\frac{\xi_{\rm K_4}}{8} - \frac{53}{18}\right)\beta_0^2 C_F - \frac{34}{3}\beta_0 C_F^2 \pm 72C_F^3\,. \end{aligned} \tag{4.2}$$

Note that only the $C_F\beta_0^l$ and $C_F^2\beta_0^{l-1}$ terms of $K_{a,l}$ in Eqs. (3.6) are relevant at this order in $1/N$. $\xi_{\rm K_4}$ is the corresponding subleading large-n_f coefficient at the fourth order, the calculation of which should become possible in the not too distant future. Also the first term of h_3 in Eq. (4.1) is known for DIS and SIA but non-universal, as the effects of F_L set in at this point. See again Ref. [12] for these results as well as the prefactor coefficients in Eq. (4.1) and all corresponding DY results.

The corresponding exponentiation for the longitudinal structure function and fragmentation function is given by [22] (see also Ref. [24])

$$C_L^{(\pm)}(N) \;=\; N^{-1}(d_{L,1}^{(\pm)}a_{\rm s} + d_{L,2}^{(\pm)}a_{\rm s}^2 + \ldots)\exp\left\{Lh_{L,1}(a_{\rm s}L) + h_{L,2}(a_{\rm s}L) + \ldots\right\} + \mathscr{O}(N^{-2})\,, \tag{4.3}$$

where the following coefficients can be determined from the third-order result of Refs. [3, 12]:

$$\begin{aligned} h_{L,11} \;&=\; 2C_F\,, \quad h_{L,12} \;=\; \frac{2}{3}\beta_0 C_F\,, \quad h_{L,13} \;=\; \frac{1}{3}\beta_0^2 C_F \\ h_{L,21} \;&=\; \beta_0 + 4\gamma_e C_F - C_F + (4 - 4\zeta_2)(C_A - 2C_F) \\ h_{L,22} \;&=\; \frac{1}{2}(\beta_0 h_{21} + A_2) - 8(C_A - 2C_F)^2(1 - 3\zeta_2 + \zeta_3 + \zeta_2^2)\,. \end{aligned} \tag{4.4}$$

Both these exponentiations have far less predictive power than their N^0L^k counterparts [13–16] where, e.g., unlike in Eqs. (4.2) and (4.4), no other new coefficient enters g_{22} besides the two-loop cusp anomalous dimension A_2. A full NLL accuracy, i.e., a complete determination of the function $h_2(a_{\rm s}L)$ may be feasible for Eq. (4.1). On the other hand, the corresponding leading coefficients for h_3 [12] and the results for $h_{L,2}$ in Eq. (4.4) indicate a major, possibly insurmountable obstacle on the way to full NNLL and NLL accuracy for the quantities F_a ($a \neq L$) and F_L, respectively.

5. The singlet evolution of F_2 and F_ϕ and splitting-function predictions

DIS via the exchange of a scalar ϕ directly coupling only to gluons (like the Higgs boson in the heavy-top limit [26]), is an ideal complement to the standard structure function F_2. The evolution kernels for the resulting system of observables are as in the first line of Eq. (3.1), but with

$$F \;=\; \begin{pmatrix} F_2 \\ F_\phi \end{pmatrix}\,, \quad C \;=\; \begin{pmatrix} C_{2,q} & C_{2,g} \\ C_{\phi,q} & C_{\phi,g} \end{pmatrix}\,, \quad K \;=\; \begin{pmatrix} K_{22} & K_{2\phi} \\ K_{\phi 2} & K_{\phi\phi} \end{pmatrix} \tag{5.1}$$

and the splitting-functions matrix P_{ij}. This system has first been discussed at NLO in Ref. [1] (it may also be interesting to study other systems such as (F_2, F_L) [27] and corresponding SIA cases). Instead of the second line of Eq. (3.1), we now have (with $[C,P]$ denoting the matrix commutator)

$$\frac{dF}{d\ln Q^2} \;=\; \Big(\underbrace{\beta(a_s)\frac{d\ln C}{da_s}}_{\text{DL (ns + ps)}} + \underbrace{[C,P]\,C^{-1}+P}_{\text{DL (singlet only)}} \Big) F \;=\; K F \;. \tag{5.2}$$

As far as they are completely known now, i.e., at NLO and NNLO, also the matrix entries of K show an only single-logarithmic enhancement at all powers of $(1-x)$, $K_{ab}^{(n)} \sim \ln^n(1-x) + \ldots$. Moreover, the leading-log contributions to $K_{22/\phi\phi}^{(n)}$ are the same as in the non-singlet quark-case and the very closely related $C_F = 0$ gluon case [6]. Conjecturing that this behaviour holds also at N^{3}LO, the highest three logarithms of the unknown four-loop splitting functions,

$$\ln^{6,5,4}(1-x) \;\text{ of }\; P_{\rm qg,gq}^{(3)} \quad\text{and}\quad \ln^{5,4,3}(1-x) \;\text{ of }\; P_{\rm ps,gg}^{(3)}\big|_{C_F} \tag{5.3}$$

can be predicted from the known [6, 10] three-loop coefficient functions for F_2 and F_ϕ at all orders in $(1-x)$. For example, the leading $(1-x)^0$ part of the N^{3}LO gluon-quark splitting function reads

$$\begin{aligned} P_{\rm qg}^{(3)}(x) \;=\; & \ln^6(1-x)\cdot 0 \;+\; \ln^5(1-x)\Big[\frac{22}{27}C_{AF}^3 n_f - \frac{14}{27}C_{AF}^2 C_F n_f - \frac{4}{27}C_{AF}^2 n_f^2\Big] \\ & + \ln^4(1-x)\Big[\Big(\frac{293}{27} - \frac{80}{9}\zeta_2\Big)C_{AF}^3 n_f + \Big(\frac{4477}{162} - 8\zeta_2\Big)C_{AF}^2 C_F n_f - \frac{13}{81}C_{AF}C_F^2 n_f \\ & \quad - \frac{116}{81}C_{AF}^2 n_f^2 + \frac{17}{81}C_{AF}C_F n_f^2 - \frac{4}{81}C_{AF} n_f^3\Big] \;+\; \mathcal{O}\left(\ln^3(1-x)\right) \end{aligned} \tag{5.4}$$

with $C_{AF} \equiv C_A - C_F$. The vanishing of the leading $\ln^6(1-x)$ term is due to an accidental cancellation of positive and negative contributions to its coefficient. On the other hand, the colour factors of the DL terms in Eq. (5.4) follow the same pattern as the corresponding lower-order contributions: all DL terms vanish for $C_A = C_F$ (part of the supersymmetric limit), with the leading terms of $P_{\rm qg}^{(l)}$ being of the form $n_f C_{AF}^l$, the next-to-leading logarithms $n_f\{C_F, n_f\}C_{AF}^{l-1}$ etc. Rather non-trivially, this pattern is predicted to hold for all four-loop singlet splitting functions at all orders in $(1-x)$ [6].

Unlike in the non-singlet case, there is no direct all-order generalization here, as the cancellation of the DL contributions in Eq. (5.2) involves the corresponding terms (2.2) and (2.4) of the N^{n}LO splitting functions and coefficient functions which are both unknown at $n \geq 4$. One may try a simultaneous extraction of at least the leading logarithms of both quantities, but it turns out that the only single-logarithmic enhancement of the physical kernel does not quite provide enough constraints, even if the colour structure of the previous paragraph is assumed in addition.

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