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Broken R-Parity in the Sky and at the LHC

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Abstract

Supersymmetric extensions of the Standard Model with small R-parity and lepton number violating couplings are naturally consistent with primordial nucleosynthesis, thermal leptogenesis and gravitino dark matter. We consider supergravity models with universal boundary conditions at the grand unification scale, and scalar τ-lepton or bino-like neutralino as next-to-lightest superparticle (NLSP). Recent Fermi-LAT data on the isotropic diffuse gamma-ray flux yield a lower bound on the gravitino lifetime. Comparing two-body gravitino and neutralino decays we find a lower bound on a neutralino NLSP decay length, $c\tau_{\chi_1^0} \gtrsim 30$ cm. Together with gravitino and neutralino masses one obtains a microscopic determination of the Planck mass. For a $\widetilde{\tau}$-NLSP there exists no model-independent lower bound on the decay length. Here the strongest bound comes from the requirement that the cosmological baryon asymmetry is not washed out, which yields $c\tau_{\widetilde{\tau}_1} \gtrsim 4$ mm. However, without fine-tuning of parameters, one finds much larger decay lengths. For typical masses, $m_{3/2} \sim 100$ GeV and $m_{\rm NLSP} \sim 150$ GeV, the discovery of a photon line with an intensity close to the Fermi-LAT limit would imply a decay length $c\tau_{\rm NLSP}$ of several hundred meters, which can be measured at the LHC.

1 Introduction

Locally supersymmetric extensions of the Standard Model predict the existence of the gravitino, the gauge fermion of supergravity [1]. For some patterns of supersymmetry breaking, the gravitino is the lightest superparticle (LSP), and therefore a natural dark matter candidate [2]. Heavy unstable gravitinos may cause the 'gravitino problem' [3–5] for large reheating temperatures in the early universe. This is the case for thermal leptogenesis [6], where gravitino dark matter has become an attractive alternative [7] to the standard WIMP scenario [8].

Recently, it has been shown that models with small R-parity and lepton number breaking naturally yield a consistent cosmology incorporating primordial nucleosynthesis, leptogenesis and gravitino dark matter [9]. The gravitino is no longer stable, but its decays into Standard Model (SM) particles are doubly suppressed by the Planck mass and the small R-parity breaking parameter. Hence, its lifetime can exceed the age of the Universe by many orders of magnitude, and the gravitino remains a viable dark matter candidate [10].

Gravitino decays lead to characteristic signatures in high-energy cosmic rays, in particular to a diffuse gamma-ray flux [9–17]. The recent search of the Fermi-LAT collaboration for monochromatic photon lines [18] and the measurement of the diffuse gamma-ray flux up to photon energies of 100 GeV [19] severely constrain possible signals from decaying dark matter. In this paper we study the implications of this data for the decays of the next-to-lightest superparticle (NLSP) at the LHC, extending the estimates in [9].

We shall restrict our analysis to the simplest class of supergravity models with universal boundary conditions at the Grand Unification (GUT) scale, which lead to neutralino or $\widetilde{\tau}$-NLSP. Electroweak precision tests, thermal leptogenesis and gravitino dark matter together allow gravitino and NLSP masses in the range $m_{3/2} = 10 \ldots 500$ GeV and $m_{\rm NLSP} = 100 \ldots 500$ GeV [20]. Following [9], the breaking of R-parity is tied to the breaking of lepton number, which leads to a model with bilinear R-parity breaking [21,22]. The soft supersymmetry breaking terms are characteristic for gravity or gaugino mediation.

In order to establish the connection between the gamma-ray flux from gravitino decays and NLSP decays, one needs R-parity breaking matrix elements of neutral, charged and supercurrents. For the considered supergravity models we are able to obtain these matrix elements to good approximation analytically. This makes our results for the NLSP decay lengths rather transparent. As we shall see, the lower bound on the neutralino decay length is a direct consequence of the Fermi-LAT constraints on decaying dark matter. On the other hand, the lower bound on the $\widetilde{\tau}$-decay length is determined by the cosmological bounds on R-parity breaking couplings, which follow from the requirement that

the baryon asymmetry is not washed out [24, 25].

This paper is organized as follows. In Section 2 we discuss the general Lagrangian for R-parity breaking in a basis of scalar $SU(2)$ doublets where all bilinear mixing terms vanish. This leads to new Yukawa and gaugino couplings, some of which are proportional to the up-quark Yukawa couplings. Section 3 deals with the various supersymmetry, R-parity and lepton number breaking terms in the Lagrangian and the relations among them due to a $U(1)$ flavour symmetry of the considered model. The needed R-parity breaking matrix elements of neutral, charged and supercurrent are analytically calculated in Section 4, based on the diagonalization of the mass matrices which is discussed in detail in the appendix. The main results of the paper, the bounds on the NLSP decay lengths and the partial decay widths, are described in Section 5, followed by our conclusions in Section 6.

2 Bilinear R-Parity Breaking

Supersymmetric extensions of the Standard Model with bilinear R-parity breaking contain mass mixing terms between lepton and Higgs fields in the superpotential[1],

$$\Delta W = \mu_i H_u l_i \tag{1}$$

as well as the scalar potential induced by supersymmetry breaking,

$$-\Delta\mathcal{L} = B_i H_u \tilde{l}_i + m_{id}^2 \tilde{l}_i^\dagger H_d + \text{h.c.}\,. \tag{2}$$

These mixing terms, together with the R-parity conserving superpotential

$$W = \mu H_u H_d + h_{ij}^u q_i u_j^c H_u + h_{ij}^d d_i^c q_j H_d + h_{ij}^e l_i e_j^c H_d \ , \tag{3}$$

the scalar mass terms

$$\begin{aligned} -\mathcal{L}_{\rm M} =& m_u^2 H_u^\dagger H_u + m_d^2 H_d^\dagger H_d + (B H_u H_d + \text{h.c.}) \\ &+ \widetilde{m}_{li}^2 \tilde{l}_i^\dagger \tilde{l}_i + \widetilde{m}_{ei}^2 \tilde{e}_i^{c\dagger} \tilde{e}_i^c + \widetilde{m}_{qi}^2 \tilde{q}_i^\dagger \tilde{q}_i + \widetilde{m}_{ui}^2 \tilde{u}_i^{c\dagger} \tilde{u}_i^c + \widetilde{m}_{di}^2 \tilde{d}_i^{c\dagger} \tilde{d}_i^c \ , \end{aligned} \tag{4}$$

and the standard $SU(3) \times SU(2) \times U(1)_Y$ gauge interactions define the supersymmetric standard model with bilinear R-parity breaking. Note that the Higgs mass terms m_u^2 and m_d^2 contain the contributions from the superpotential (3) and the soft supersymmetry breaking terms. For simplicity, we have assumed flavour diagonal mass matrices in (4).

[1] Our notation for Higgs and matter superfields, scalars and left-handed fermions reads: $H_u = (H_u, h_u)$, $l_i = (\tilde{l}_i, l_i)$ etc.

For a generic choice of parameters the electroweak symmetry is broken by vacuum expectation values (VEVs) of all scalar $SU(2)$ doublets,

$$\langle H_u^0 \rangle = v_u \ , \quad \langle H_d^0 \rangle = v_d \ , \quad \langle \tilde{\nu}_i \rangle = v_i \ , \tag{5}$$

with[2]

$$\frac{v_u}{v_d} \equiv \tan\beta \ , \quad \widehat{\epsilon}_i \equiv \frac{v_i}{v_d} = \frac{B_i \tan\beta - m_{id}^2 - \mu\mu_i^*}{\widetilde{m}_{li}^2 + \frac{1}{2} m_Z^2 \cos 2\beta} \ , \tag{6}$$

where higher order terms in the R-parity breaking parameters have been neglected.

It is convenient to discuss the predictions of the model in a basis of $SU(2)$ doublets where the mass mixings μ_i, B_i and m_{id}^2 in Eqs. (1) and (2) are traded for R-parity breaking Yukawa coulings. This can easily be achieved by field redefinitions. First one rotates the superfields H_d and l_i,

$$H_d = H_d' - \epsilon_i l_i' \ , \quad l_i = l_i' + \epsilon_i H_d' \ , \quad \epsilon_i = \frac{\mu_i}{\mu} \ . \tag{7}$$

Then the bilinear term (1) vanishes for the new fields, i.e., $\mu_i' = 0$, and one obtains instead the cubic R-parity violating terms

$$\Delta W' = \frac{1}{2} \lambda_{ijk} l_i' e_j^c l_k' + \lambda_{ijk}' d_i^c q_j l_k' \ , \tag{8}$$

where

$$\lambda_{ijk} = -h_{ij}^e \epsilon_k + h_{kj}^e \epsilon_i \ , \quad \lambda_{ijk}' = -h_{ij}^d \epsilon_k \ . \tag{9}$$

The new R-parity breaking mass mixings are given by

$$B_i' = B_i - B\epsilon_i \ , \quad m_{id}^{2\prime} = m_{id}^2 + \epsilon_i (\widetilde{m}_{li}^2 - m_d^2) \ . \tag{10}$$

The corrections for R-parity conserving mass terms are negligable.

In a second step one can perform a non-supersymmetric rotation among all scalar $SU(2)$ doublets,

$$H_d' = H_d'' - \epsilon_i' \tilde{l}_i'' \ , \quad \varepsilon H_u^* = \varepsilon H_u'^* - \epsilon_i'' \tilde{l}_i'' \ , \quad \tilde{l}_i' = \tilde{l}_i'' + \epsilon_i' H_d'' + \epsilon_i'' \varepsilon H_u'^* \ , \tag{11}$$

where ε is the usual $SU(2)$ matrix, $\varepsilon = i\tau^2$. Choosing

$$\epsilon_i' = -\frac{B_i' B + m_{id}^{2\prime} (\widetilde{m}_{li}^2 - m_u^2)}{(\widetilde{m}_{li}^2 - m_u^2)(\widetilde{m}_{li}^2 - m_d^2) - B^2} \ , \tag{12}$$

$$\epsilon_i'' = \frac{B_i' (\widetilde{m}_{li}^2 - m_d^2) + B m_{id}^{2\prime}}{(\widetilde{m}_{li}^2 - m_u^2)(\widetilde{m}_{li}^2 - m_d^2) - B^2} \ , \tag{13}$$

[2]Note that our result for $\widehat{\epsilon}_i = v_i/v_d$ holds at all renormalization scales, contrary to different expressions used in the literature.

the $H_u\tilde{l}_i$ and $\tilde{l}^\dagger H_d$ mixing terms vanish in the new basis of doublets. According to (6) also the scalar lepton VEVs $\langle\tilde{\nu}_i\rangle$ vanish in this basis.

It is straightforward to work out the R-parity violating Yukawa couplings which are induced by the rotation (11). We are particularly interested in the terms containing one light superparticle, i.e, a scalar lepton, bino or wino. The corresponding couplings read, after dropping prime and double-prime superscripts on all fields[3],

$$\begin{aligned}-\Delta\mathcal{L} \supset\ & \frac{1}{2}\lambda_{ijk} l_i \tilde{e}^c_j l_k + \lambda'_{ijk} d^c_i q_j \tilde{l}_k + \hat{\lambda}_{ijk} l_i e^c_j \tilde{l}_k + \hat{\lambda}'_{ijk} q_i u^c_j \varepsilon \tilde{l}^*_k \\ & + h^e_{ij}(\epsilon'_i H_d + \epsilon''_i \varepsilon H^*_u) e^c_j h_d \\ & - \frac{g'}{\sqrt{2}}(\epsilon'_i H^\dagger_d - \epsilon''_i H^T_u \varepsilon) l_i b + \frac{g}{\sqrt{2}}(\epsilon'_i H^\dagger_d - \epsilon''_i H^T_u \varepsilon)\tau^I l_i w^I + \text{h.c.}\ , \end{aligned} \tag{14}$$

where the Yukawa couplings are given by

$$\lambda_{ijk} = -h^e_{ij}\epsilon_k + h^e_{kj}\epsilon_i\ , \quad \lambda'_{ijk} = -h^d_{ij}(\epsilon_k + \epsilon'_k)\ , \tag{15}$$

$$\hat{\lambda}_{ijk} = -h^e_{ij}(\epsilon_k + \epsilon'_k) + h^e_{kj}\epsilon_i\ , \quad \hat{\lambda}'_{ijk} = h^u_{ij}\epsilon''_k\ . \tag{16}$$

Since the field transformations are non-supersymmetric, the couplings λ_{ijk} and $\hat{\lambda}_{ijk}$ are no longer equal as in Eq. (9). Furthermore, a new coupling of right-handed up-quarks, $\hat{\lambda}'_{ijk}$, has been generated.

After electroweak symmetry breaking one obtains new mass mixings between higgsinos, gauginos and leptons,

$$-\Delta\mathcal{L}_M \supset\ m^e_{ij}\frac{\zeta_i}{c_\beta} e^c_j h_d - m_Z s_w \zeta^*_i \nu_i b + m_Z c_w \zeta^*_i \nu_i w^3 + \text{h.c.}\ , \tag{17}$$

where we have defined

$$\zeta_i = \frac{\epsilon'_i v_d + \epsilon''_i v_u}{v}\ , \quad v = \sqrt{v_u^2 + v_d^2}\ , \quad \frac{v_u}{v_d} = \tan\beta \equiv \frac{s_\beta}{c_\beta}\ , \tag{18}$$

$$m^e_{ij} = h^e_{ij} v_d\ , \quad m_Z = \frac{\sqrt{g^2 + g'^2}\, v}{\sqrt{2}}\ , \quad s_w = \frac{g'}{\sqrt{g^2 + g'^2}} = \sqrt{1 - c_w^2}\ . \tag{19}$$

Given the Yukawa couplings h^u_{ij}, h^d_{ij} and h^e_{ij}, the Lagrangian (14) predicts 108 R-parity breaking Yukawa couplings in terms of 9 independent parameters which may be chosen as

$$\mu_i\ ,\ B_i\ ,\ m^2_{id} \quad \text{or} \quad \epsilon_i\ ,\ \epsilon'_i\ ,\ \epsilon''_i\ . \tag{20}$$

These parameters determine lepton-gaugino mass mixings, lepton-slepton and quark-slepton Yukawa couplings, and therefore the low-energy phenomenology. The values of these parameters depend on the pattern of supersymmetry breaking and the flavour structure of the supersymmetric standard model.

[3]Our notation for gauge fields, field strengths and left-handed gauginos reads: B_μ, $B_{\mu\nu}$, b etc.

3 Spontaneous R-parity breaking

Let us now compute the parameters ϵ_i, ϵ_i' and ϵ_i'' in a specific example where the spontaneous breaking of R-parity is related to the spontaneous breaking of B-L, the difference of baryon and lepton number [9].

We consider a supersymmetric extension of the standard model with symmetry group

$$G = SU(3) \times SU(2) \times U(1)_Y \times U(1)_{B-L} \times U(1)_R \ . \tag{21}$$

In addition to three quark lepton generations and the Higgs fields H_u and H_d the model contains three right-handed neutrinos ν_i^c, two non-Abelian singlets N^c and N, which transform as ν^c and its complex conjugate, respectively, and three gauge singlets X, Φ and Z. The part of the superpotential responsible for neutrino masses has the usual form

$$W_\nu = h_{ij}^\nu l_i \nu_j^c H_u + \frac{1}{M_{\rm P}} h_{ij}^n \nu_i^c \nu_j^c N^2 \ , \tag{22}$$

where $M_{\rm P} = 2.4 \times 10^{18}$ GeV is the Planck mass. The expectation value of H_u generates Dirac neutrino masses, whereas the expectation value of the singlet Higgs field N generates the Majorana mass matrix of the right-handed neutrinos ν_i^c. The superpotential responsible for B-L breaking is chosen as

$$W_{B-L} = X(NN^c - \Phi^2) \ , \tag{23}$$

where unknown Yukawa couplings have been set equal to one. Φ plays the role of a spectator field, which will finally be replaced by its expectation value, $\langle\Phi\rangle = v_{B-L}$. Similarly, Z is a spectator field which breaks supersymmetry and $U(1)_R$, $\langle Z\rangle = F_Z\theta\theta$. The superpotential in Eqs. (22) and (23) is the most general one consistent with the R-charges listed in Table 1, up to nonrenormalizable terms which are irrelevant for our discussion.

The expectation value of Φ leads to the breaking of $B - L$,

$$\langle N\rangle = \langle N^c\rangle = \langle\Phi\rangle = v_{B-L} \ , \tag{24}$$

where the first equality is a consequence of the $U(1)_{B-L}$ D-term. This generates a Majorana mass matrix M for the right-handed neutrinos with three large eigenvalues

	Ψ	H_u	H_d	N	N^c	Φ	X	Z
R	1	0	0	0	-2	-1	4	0

Table 1: *R-charges of matter fields $\Psi = q, u^c, e^c, d^c, l, \nu^c$, Higgs fields and gauge singlets.*

$M_3 > M_2 > M_1$. If the largest eigenvalue of h^n is $\mathcal{O}(1)$, one has $M_3 \simeq v_{B-L}^2/M_{\rm P}$. Integrating out the heavy Majorana neutrinos one obtains the familiar dimension-5 seesaw operator which yields the light neutrino masses.

Since the field Φ carries R-charge -1, the VEV $\langle\Phi\rangle$ breaks R-parity, which is conserved by the VEV $\langle Z\rangle$. Thus, the breaking of $B-L$ is tied to the breaking of R-parity, which is then transmitted to the low-energy degrees of freedom via higher-dimensional operators in the superpotential and the Kähler potential. Bilinear R-parity breaking, as discussed in the previous section, is obtained from a correction to the Kähler potential,

$$\begin{aligned}\Delta K = &\frac{1}{{M_{\rm P}}^3}\left(a_i Z^\dagger \Phi^\dagger N^c H_u l_i + a_i' Z^\dagger \Phi N^\dagger H_u l_i\right)\\ &+\frac{1}{{M_{\rm P}}^4}\Big(b_i Z^\dagger Z \Phi^\dagger N^c H_u l_i + b_i' Z^\dagger Z \Phi N^\dagger H_u l_i\\ &\qquad +c_i Z^\dagger Z \Phi^\dagger N^c l_i^\dagger H_d + c_i' Z^\dagger Z \Phi N^\dagger l_i^\dagger H_d\Big) + \text{h.c.}\ . \end{aligned} \tag{25}$$

Replacing the spectator fields Z and Φ, as well as N^c and N by their expectation values, one obtains the correction to the superpotential

$$\Delta W = \mu_i H_u l_i\ ,$$

with

$$\mu_i = \sqrt{3}(a_i + a_i')m_{3/2}\Theta\ , \quad \Theta = \frac{v_{B-L}^2}{{M_{\rm P}}^2} \simeq \frac{M_3}{M_{\rm P}}\ , \tag{26}$$

where $m_{3/2} = F_Z/(\sqrt{3}M_{\rm P})$ is the gravitino mass. Note that Θ can be increased or decreased by including appropriate Yukawa couplings in Eqs. (22) and (23). The corresponding corrections to the scalar potential are given by

$$-\Delta\mathcal{L} = B_i H_u \tilde{l}_i + m_{id}^2 \tilde{l}_i^\dagger H_d + \text{h.c.}\ ,$$

where

$$B_i = 3(b_i + b_i')m_{3/2}^2\Theta\ , \quad m_{id}^2 = 3(c_i + c_i')m_{3/2}^2\Theta\ . \tag{27}$$

The corresponding R-parity conserving terms are generated by [26]

$$K \supset \frac{a_0}{M_{\rm P}} Z^\dagger H_u H_d + \frac{b_0}{{M_{\rm P}}^2} Z^\dagger Z H_u H_d + \text{h.c.}\ , \tag{28}$$

which yields

$$W \supset \mu H_u H_d\ , \qquad \mu = \sqrt{3}a_0 m_{3/2}\ , \tag{29}$$

$$-\mathcal{L} \supset B H_u H_d + \text{h.c.}\ , \qquad B = 3b_0 m_{3/2}^2\ . \tag{30}$$

Higher dimensional operators yield further R-parity violating couplings between scalars and fermions. However, the cubic couplings allowed by the symmetries of our

model are suppressed by one power of $M_{\rm P}$ compared to ordinary Yukawa couplings and cubic soft supersymmetry breaking terms. Note that the coefficients of the nonrenormalizable operators are free parameters, which are only fixed in specific models of supersymmetry breaking. In particular, one may have $\mu^2, \widetilde{m}_i^2 > m_{3/2}^2$ and hence a gravitino LSP. All parameters are defined at the GUT scale and have to be evolved to the electroweak scale by the renormalization group equations.

The phenomenological viability of the model depends on the size of R-parity breaking mass mixings and therefore on the scale v_{B-L} of R-parity breaking as well as the parameters $a_i \ldots c_i'$ in Eq. (25). Any model of flavour physics, which predicts Yukawa couplings, will generically also predict the parameters $a_i \ldots c_i'$. As a typical example, we use a model [27] for quark and lepton mass hierarchies based on a Froggatt-Nielsen $U(1)$ flavour symmetry, which is consistent with thermal leptogenesis and all contraints from flavour changing processes [28].

The mass hierarchy is generated by the expectation value of a singlet field ϕ with charge $Q_\phi = -1$ via nonrenormalizable interactions with a scale $\Lambda = \langle\phi\rangle/\eta > \Lambda_{GUT}$, $\eta \simeq 0.06$. The η-dependence of Yukawa couplings and bilinear mixing terms for multiplets ψ_i with charges Q_i is given by

$$h_{ij} \propto \eta^{Q_i+Q_j}, \quad \mu_i \propto \eta^{Q_i}, \quad B_i \propto \eta^{Q_i}, \quad m_{id}^2 \propto \eta^{Q_i} \ . \tag{31}$$

The charges Q_i for quarks, leptons, Higgs fields and singlets are listed in Table 2. The neutrino mass scale $m_\nu \simeq 0.01$ eV implies for the heaviest right-handed neutrinos $M_2 \sim M_3 \sim 10^{12}$ GeV. The corresponding scales for $B-L$ breaking and R-parity breaking are

$$v_{B-L} \simeq 10^{15} \text{ GeV} \ , \quad \Theta = \frac{v_{B-L}^2}{{M_{\rm P}}^2} \simeq 10^{-6} \ . \tag{32}$$

For the small R-parity breaking considered in this paper the neutrino masses are dominated by the conventional seesaw contribution [9].

The R-parity breaking parameters μ_i, B_i and m_{id}^2 strongly depend on the mechanism of supersymmetry breaking. In the example considered in this section all mass parameters are $\mathcal{O}(m_{3/2})$, which corresponds to gravity or gaugino mediation. From Eqs. (26),(27) and (31) one reads off

$$\mu_i = \hat{a}\eta^{Q_i} m_{3/2}\Theta \ , \quad B_i = \hat{b}\eta^{Q_i} m_{3/2}^2\Theta \ , \quad m_{id}^2 = \hat{c}\eta^{Q_i} m_{3/2}^2\Theta \ , \tag{33}$$

ψ_i	$\mathbf{10}_3$	$\mathbf{10}_2$	$\mathbf{10}_1$	$\mathbf{5}_3^*$	$\mathbf{5}_2^*$	$\mathbf{5}_1^*$	ν_3^c	ν_2^c	ν_1^c	H_u	H_d	Φ	X	Z
Q_i	0	1	2	1	1	2	0	0	1	0	0	0	0	0

Table 2: *Chiral $U(1)$ charges.* $\mathbf{10}_i = (q_i, u_i^c, e_i^c)$, $\mathbf{5} = (d_i^c, l_i)$, $i = 1 \ldots 3$.

with $\hat{a}, \hat{b}, \hat{c} = \mathcal{O}(1)$. Correspondingly, one obtains for ϵ-parameters (cf. (12),(13))

$$\epsilon_i = a\eta^{Q_i}\Theta \ , \quad \epsilon'_i = b\eta^{Q_i}\Theta \ , \quad \epsilon''_{id} = c\eta^{Q_i}\Theta \ , \tag{34}$$

with $a, b, c = \mathcal{O}(1)$. Our phenomenological analysis in Section 5.2 will be based on this parametrization of bilinear R-parity breaking.

Depending on the mechanism of supersymmetry breaking, the R-parity breaking soft terms may vanish at the GUT scale [22],

$$B_i(\Lambda_{\rm GUT}) = m_{id}^2(\Lambda_{\rm GUT}) = 0 \ . \tag{35}$$

Non-zero values of these parameters at the electroweak scale are then induced by radiative corrections. The renormalization group equations for the bilinear R-parity breaking mass terms read (cf. [22], $t = \ln\Lambda$):

$$16\pi^2\frac{d\mu_i}{dt} = 3\mu_i\left(h^u_{jk}h^{u*}_{jk} - \frac{1}{5}g_1^2 - g_2^2\right) + \mu_k h^e_{ij}h^{e*}_{kj} - \mu\left(\lambda_{ijk}h^{e*}_{kj} + 3\lambda'_{kji}h^{d*}_{kj}\right) \ , \tag{36}$$

$$\begin{aligned}16\pi^2\frac{dB_i}{dt} &= 3B_i\left(h^u_{jk}h^{u*}_{jk} - \frac{1}{5}g_1^2 - g_2^2\right) + 6\mu_i\left(\frac{1}{5}g_1^2M_1 + g_2^2M_2\right)\\ &\quad + B_k h^e_{ij}h^{e*}_{kj} - B\left(\lambda_{ijk}h^{e*}_{kj} + 3\lambda'_{kji}h^{d*}_{kj}\right) \ ,\end{aligned} \tag{37}$$

$$\begin{aligned}16\pi^2\frac{dm_{id}^2}{dt} &= \lambda^*_{kji}h^e_{kj}m_d^2 - m_{jd}^2h^e_{jk}h^{e*}_{ik} - 3\lambda'_{kji}h^d_{kj}m_d^2 + h^e_{jk}h^{e*}_{jk}m_{id}^2\\ &\quad + 3h^{d*}_{kj}h^d_{kj}m_{id}^2 + \widetilde{m}^2_{li}\lambda^*_{nki}h^e_{nk} - 3\widetilde{m}^2_{li}\lambda'^*_{nki}h^e_{nk}\\ &\quad + 2\lambda^*_{kji}\widetilde{m}^2_{lk}\lambda_{kj} + 2\lambda^*_{kji}h^e_{kj}\widetilde{m}^2_{ej} - 6\lambda'^*_{kji}h^d_{kj}\widetilde{m}^2_{dk} - 6\lambda'^*_{kji}\widetilde{m}^2_{qj}h^d_{kj} \ .\end{aligned} \tag{38}$$

In bilinear R-parity breaking, the R-parity violating Yukawa couplings vanish at the GUT scale. One-loop radiative corrections then yield for the soft terms at the electroweak scale (cf. Eqs. (36),(37); $\epsilon_i = \mu_i/\mu$):

$$B_i(\Lambda_{\rm EW}) = \frac{\mu_i}{16\pi^2}\left(\frac{6}{5}g'^2M_1 + 6g^2M_2\right)\ln\frac{\Lambda_{\rm GUT}}{\Lambda_{\rm EW}} \ , \quad m_{id}^2(\Lambda_{\rm EW}) = 0 \ . \tag{39}$$

This illustrates that the bilinear R-parity breaking terms μ_i^2, B_i and m_{id}^2 are not necessarily of the same order of magnitude at the electroweak scale.

4 Neutral, charged and supercurrents

In Section 2 we have discussed the R-parity breaking Yukawa couplings in our model. For a phenomenological analysis we also need the couplings of the gauge fields, i.e., photon, W-bosons and gravitino, to charged and neutral matter,

$$\mathcal{L} = -eJ_{e\mu}A^\mu - \frac{g}{c_w}J_{Z\mu}Z^\mu - \frac{g}{\sqrt{2}}J^-_\mu W^{+\mu} - \frac{g}{\sqrt{2}}J^+_\mu W^{-\mu} - \frac{1}{2M_{\rm P}}\overline{\psi}_\mu S^\mu \ . \tag{40}$$

The corresponding currents read

$$J_{e\mu} = \overline{w}^+\gamma_\mu w^+ - \overline{w}^-\gamma_\mu w^- - \overline{e}_i\gamma_\mu e_i + \overline{e}^c_i\gamma_\mu e^c_i - \overline{h}^-_d\gamma_\mu h^-_d + \overline{h}^+_u\gamma_\mu h^+_u \ , \tag{41}$$

$$J_{Z\mu} = \overline{w}^+\gamma_\mu w^+ - \overline{w}^-\gamma_\mu w^- + \frac{1}{2}\overline{\nu}_i\gamma_\mu\nu_i - \frac{1}{2}\overline{e}_i\gamma_\mu e_i + \frac{1}{2}\overline{h}^0_d\gamma_\mu h^0_d - \frac{1}{2}\overline{h}^-_d\gamma_\mu h^-_d + \frac{1}{2}\overline{h}^+_u\gamma_\mu h^+_u - \frac{1}{2}\overline{h}^0_u\gamma_\mu h^0_u - s^2_w J_{e\mu} \ , \tag{42}$$

$$J^-_\mu = \sqrt{2}\left(\overline{w}^3\gamma_\mu w^- - \overline{w}^+\gamma_\mu w^3\right) + \overline{\nu}_i\gamma_\mu e_i + \overline{h}^0_d\gamma_\mu h^-_d + \overline{h}^+_u\gamma_\mu h^0_u \ , \tag{43}$$

$$S^\mu = \frac{i}{4}\left[\gamma^\nu, \gamma^\rho\right]\gamma^\mu\left(\widetilde{b}B_{\nu\rho} + \widetilde{w}^I W^I_{\nu\rho}\right) + \ldots . \tag{44}$$

The gravitino and the gauginos are now Majorana fermions,

$$\widetilde{b} = b + b^c \ , \quad \widetilde{w}^I = w^I + w^{cI} \ , \tag{45}$$

where the superscript c denotes charge conjugation. In Eqs. (41) - (44) we have only listed contributions to the currents which will be relevant in our phenomenological analysis.

The R-parity breaking described in the previous section leads to mass mixings between the neutralinos b, w^3, h^0_u, h^0_d with the neutrinos ν_i, and the charginos w^+, h^+_u, w^-, h^-_d with the charged leptons e^c_i, e_i, respectively. The 7×7 neutralino mass matrix reads in the gauge eigenbasis

$$\mathcal{M}^N = \begin{pmatrix} M_1 & 0 & m_Z s_\beta s_w & -m_Z c_\beta s_w & -\zeta_i m_Z s_w \\ 0 & M_2 & -m_Z s_\beta c_w & m_Z c_\beta c_w & \zeta_i m_Z c_w \\ m_Z s_\beta s_w & -m_Z s_\beta c_w & 0 & -\mu & 0 \\ -m_Z c_\beta s_w & m_Z c_\beta c_w & -\mu & 0 & 0 \\ -\zeta_i m_Z s_w & \zeta_i m_Z c_w & 0 & 0 & 0 \end{pmatrix} , \tag{46}$$

where we have neglected neutrino masses. Correspondingly, the 5×5 chargino mass matrix which connects the states (w^-, h^-_d, e_i) and (w^+, h^+_u, e^c_i) is given by

$$\mathcal{M}^C = \begin{pmatrix} M_2 & m_Z s_\beta c_w & 0 & 0 & 0 \\ m_Z c_\beta c_w & \mu & \zeta_1 h^e_{11}\mu & \zeta_2 h^e_{22}\mu & \zeta_3 h^e_{33}\mu \\ \zeta_1 m_Z c_w & 0 & h^e_{11} v c_\beta & 0 & 0 \\ \zeta_2 m_Z c_w & 0 & 0 & h^e_{22} v c_\beta & 0 \\ \zeta_3 m_Z c_w & 0 & 0 & 0 & h^e_{33} v c_\beta \end{pmatrix} . \tag{47}$$

Note that all gaugino and higgsino mixings with neutrinos and charged leptons are parametrized by the three parameters ζ_i.

In the following section we shall need the couplings of gravitino, W- and Z-bosons to neutralino and chargino mass eigenstates. Since $\zeta_i \ll 1$, diagonalization of the mass

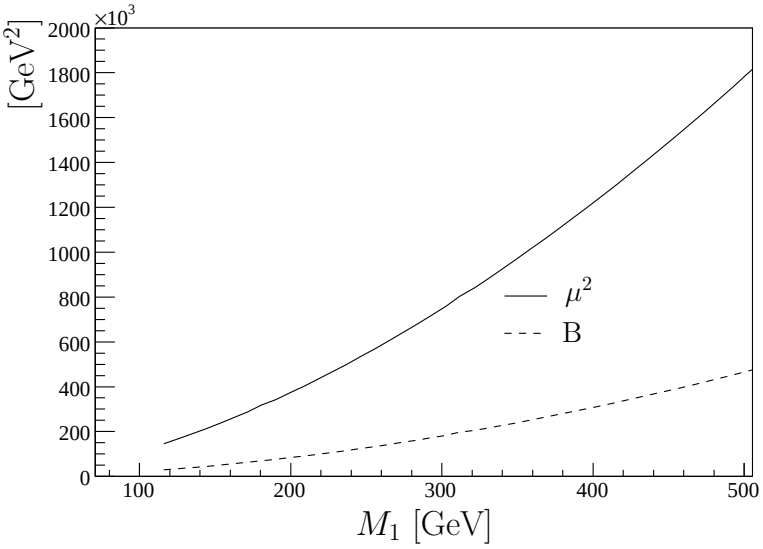

Figure 1: *The parameters μ and B of Eqs. (3) and (4), respectively, as functions of the bino mass M_1. The plot has been obtained by means of* `SOFTSUSY3.0` *[23].*

matrices to first order in ζ_i is obviously sufficient. We shall also consider supergravity models where the supersymmetry breaking parameters satisfy the inequalities (cf. Fig. 1)

$$m_Z < M_{1,2} < \mu \ . \tag{48}$$

The gaugino-higgsino mixings are $\mathcal{O}(m_Z/\mu)$, and therefore suppressed, and χ_1^0, the lightest neutralino, is bino-like.

The mass matrices $\mathcal{M}^N$ and $\mathcal{M}^C$ are diagonalized by unitary and bi-unitary transformations, respectively,

$$U^{(n)T}\mathcal{M}^N U^{(n)} = \mathcal{M}^N_{\rm diag} \ , \quad U^{(c)\dagger}\mathcal{M}^C \widetilde{U}^{(c)} = \mathcal{M}^C_{\rm diag} \ , \tag{49}$$

where $U^{(n)\dagger}U^{(n)} = U^{(c)\dagger}U^{(c)} = \widetilde{U}^{(c)\dagger}\widetilde{U}^{(c)} = \mathbf{1}$. These unitary transformations relate the neutral and charged gauge eigenstates to the mass eigenstates (χ_a^0, ν_i') $(a = 1, \ldots, 4)$ and (χ_α^-, e_i'), $(\chi_\alpha^+, e_i'^c)$ $(\alpha = 1, 2)$, respectively. Inserting these transformations in Eqs. (42) - (44) and dropping prime superscripts, one obtains neutral, charged and supercurrents in the mass eigenstate basis:

$$\begin{aligned} J_{Z\mu} =& \overline{\chi}_a^0 \gamma_\mu V_{ab}^{(\chi^0)} \chi_b^0 + \overline{\chi}_\alpha^- \gamma_\mu V_{\alpha\beta}^{(\chi^-)} \chi_\beta^- + \overline{\chi}_\alpha^+ \gamma_\mu V_{\alpha\beta}^{(\chi^+)} \chi_\beta^+ + \overline{\nu}_i \gamma_\mu V_{ij}^{(\nu)} \nu_i + \overline{e}_i \gamma_\mu V_{ij}^{(e)} e_i \\ & + \left(\overline{\chi}_a^0 \gamma_\mu V_{ai}^{(\chi,\nu)} \nu_i + \overline{\chi}_\alpha^- \gamma_\mu V_{\alpha i}^{(\chi^-,e)} e_i + \overline{\chi}_\alpha^+ \gamma_\mu V_{\alpha i}^{(\chi^+,e^c)} e_i^c + \text{h.c.} \right) \ , \end{aligned} \tag{50}$$

$$J_\mu^- = \overline{\chi}_a^0 \gamma_\mu V_{a\alpha}^{(\chi)} \chi_\alpha^- + \overline{\chi}_a^0 \gamma_\mu V_{ai}^{(\chi,e)} e_i + \overline{\nu}_i \gamma_\mu V_{i\alpha}^{(\nu,\chi)} \chi_\alpha^- + \overline{\nu}_i \gamma_\mu V_{ij}^{(\nu,e)} e_j \ , \tag{51}$$

$$S^\mu = \frac{i}{4} \left[\gamma^\nu, \gamma^\rho\right] \gamma^\mu \left(U_a^{(\widetilde{\gamma},\chi)} \chi_a^0 + U_a^{(\widetilde{\gamma},\chi)*} \chi_a^{0c} + U_i^{(\widetilde{\gamma},\nu)} \nu_i + U_i^{(\widetilde{\gamma},\nu)*} \nu_i^c \right) F_{\nu\rho} + \ldots , \tag{52}$$

where we have defined the photino matrix elements

$$U_a^{(\widetilde{\gamma},\chi)} = c_w U_a^{(b,\chi)} + s_w U_a^{(w,\chi)} \ , \quad U_i^{(\widetilde{\gamma},\nu)} = c_w U_i^{(b,\nu)} + s_w U_i^{(w,\nu)} \ . \tag{53}$$

In the appendix the unitary transformations between gauge and mass eigenstates and the resulting matrix elements of neutral and charged currents are given to next-to-leading order in m_Z/μ. As we shall see, that expansion converges remarkably well.

In the next section we shall need the couplings of the lightest neutralino χ_1^0 to charged leptons and neutrinos, and the coupling of the gravitino to photon and neutrino. From the formulae in appendix A one easily obtains[4] ($s_{2\beta} = 2 s_\beta c_\beta$)

$$V_{1i}^{(\chi,\nu)} = -\,\zeta_i \frac{m_Z s_w}{2M_1} \left(1 + \mathcal{O}\left(s_{2\beta}\frac{m_Z^2}{\mu^2}\right)\right) \ , \tag{54}$$

$$V_{1i}^{(\chi,e)} = -\,\zeta_i \frac{m_Z s_w}{M_1} \left(1 + \mathcal{O}\left(s_{2\beta}\frac{m_Z^2}{\mu^2}\right)\right) \ , \tag{55}$$

$$U_i^{(\tilde{\gamma},\nu)} = \zeta_i \frac{m_Z s_w c_w \,(M_2 - M_1)}{M_1 M_2} \left(1 + \mathcal{O}\left(s_{2\beta}\frac{m_Z^2}{\mu^2}\right)\right) \ . \tag{56}$$

Note that the charged and neutral current couplings agree up to the isospin factor at leading order in m_Z^2/μ^2, i.e., $V_{1i\ \mathrm{LO}}^{(\chi,\nu)} = V_{1i\ \mathrm{LO}}^{(\chi,e)}/2$. The mass of the lightest neutralino is given by

$$m_{\chi_1^0} = M_1 - \frac{m_Z^2\,(M_1 + \mu s_{2\beta})\, s_w^2}{\mu^2 - M_1^2} \left(1 + \mathcal{O}\left(\frac{m_Z^2}{\mu^2}\right)\right) \ . \tag{57}$$

We have numerically checked that varying M_1 between 120 and 500 GeV, the relative corrections in Eqs. (54) - (57) are less than 10%.

5 Fermi-LAT and the LHC

We are now ready to evaluate the implications of recent Fermi-LAT data [18, 19] and cosmological constraints [24, 25] for signatures of decaying dark matter at the LHC. We shall first discuss monochromatic gamma-rays produced by gravitino decays and then analyze the implications for a neutralino and a $\widetilde{\tau}$-NLSP, respectively.

In order to keep our analysis transparent we shall not study the most general parameter space of softly broken supersymmetry, but only consider two typical boundary conditions for the supersymmetry breaking parameters of the MSSM at the grand unification scale,

$$\text{(A)} \quad m_0 = m_{1/2}, \quad a_0 = 0, \quad \tan\beta = 10 \ , \tag{58}$$

[4]The matrix element $U_i^{(\tilde{\gamma},\nu)}$ agrees with the one used in [13, 29] for $M_2 - M_1 \ll M_1$.

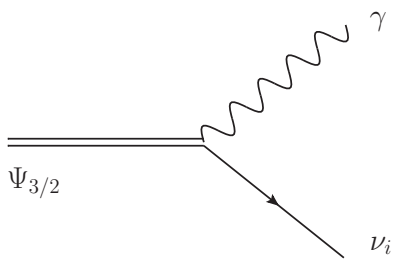

Figure 2: *Gravitino decay into photon and neutrino.*

with equal universal scalar and gaugino masses, m_0 and $m_{1/2}$, respectively; in this case a bino-like neutralino is the NLSP. The second boundary condition corresponds to no-scale models or gaugino mediation,

$$\text{(B)} \quad m_0 = 0, \quad m_{1/2}, \quad a_0 = 0, \quad \tan\beta = 10 \ , \tag{59}$$

which yields the right-handed stau as NLSP. In both cases, the trilinear scalar coupling a_0 is put to zero for simplicity. Choosing $\tan\beta = 10$ as a representative value of the Higgs vacuum expectation values, only the gaugino mass parameter $m_{1/2}$ remains as independent variable; the mass parameters μ and B are determined by requiring radiative electroweak symmetry breaking with the chosen ratio $\tan\beta$. For both boundary conditions (58) and (59), the gaugino masses at the electroweak scale satisfy the familiar relations

$$\frac{M_3}{M_1} \simeq 6.0 \ , \quad \frac{M_2}{M_1} \simeq 1.9 \ . \tag{60}$$

For the chosen supergravity models, consistency with electroweak precision tests, gravitino dark matter (GDM) and thermal leptogenesis leads to the following allowed mass ranges of gravitino and lightest neutralino [20],

$$10 \text{ GeV} < m_{3/2} < 500 \text{ GeV} \ , \quad 100 \text{ GeV} < m_{\chi_1^0} < 500 \text{ GeV} \ , \tag{61}$$

where we have used $m_{\chi_1^0} \simeq M_1$ (cf. (57)). Note that the masses M_1 and $m_{3/2}$ cannot be chosen independently. The GDM constraint implies that for a given gravitino mass the maximal bino mass is $M_1^{\max} \simeq 270 \text{ GeV}(m_{3/2}/100 \text{ GeV})^{1/2}$ [20].

Consider now the rate for gravitino decay into photon and neutrino[5] [10] (cf. Fig. 2),

$$\Gamma_{3/2}(\gamma\nu) = \frac{1}{32\pi} \sum_i |U_i^{(\widetilde{\gamma},\nu)}|^2 \frac{m_{3/2}^3}{{M_\text{P}}^2} \ , \tag{62}$$

[5] $\Gamma_{3/2}(\gamma\nu)$ denotes the sum of the decay rates into photon neutrino and photon antineutrino.

Inserting the matrix element (56) one obtains the gravitino lifetime

$$\tau_{3/2}(\gamma\nu) = \frac{32\sqrt{2}}{\alpha\zeta^2}\frac{G_F M_P^2}{m_{3/2}^3}\frac{M_1^2 M_2^2}{(M_2 - M_1)^2}\left(1 + \mathcal{O}\left(s_{2\beta}\frac{m_Z^2}{\mu^2}\right)\right) \ , \tag{63}$$

where α is the electromagnetic fine-structure constant, and we have defined

$$\zeta^2 = \sum_i \zeta_i^2 \ . \tag{64}$$

The corrections to the leading order expression in (63) are less than 10%. Using Eq. (60) and $M_{\rm P} = 2.4 \times 10^{18}$ GeV, one obtains

$$\tau_{3/2}(\gamma\nu) = 1 \times 10^{27}{\rm s}\left(\frac{\zeta}{10^{-7}}\right)^{-2}\left(\frac{M_1}{100\ {\rm GeV}}\right)^2\left(\frac{m_{3/2}}{10\ {\rm GeV}}\right)^{-3} \ . \tag{65}$$

Recent Fermi-LAT data yield for dark matter decaying into 2 photons the lower bound on the lifetime $\tau_{\rm DM}(\gamma\gamma) \gtrsim 1 \times 10^{29}$ s, which holds for photon energies in the range 30 GeV $< E_\gamma <$ 200 GeV [18]. For gravitino decays into photon and neutrino this implies

$$\tau_{3/2}(\gamma\nu) \gtrsim 5 \times 10^{28}\ {\rm s} \ , \quad 30\ {\rm GeV} < E_\gamma < 200\ {\rm GeV} \ . \tag{66}$$

Since according to the GDM constraint the largest allowed bino mass scales like $M_1^{\rm max} \propto m_{3/2}^{1/2}$, the largest lifetime (65), and therefore the most conservative bound on ζ, is obtained for the smallest value of $m_{3/2}$. For small gravitino masses, a rough lower bound on the lifetime can be obtained from the isotropic diffuse gamma-ray flux. The recent Fermi-LAT data give $E^2 dJ/dE|_{5\ {\rm GeV}} \simeq 3 \times 10^{-7}$ GeV (cm^2 s str)$^{-1}$ [19]. From the analysis in [12] one then obtains $\tau_{3/2} \gtrsim 10^{28}$ s.[6] Together with Eq. (65) one then obtains the approximate upper bound on the R-parity breaking parameter

$$\zeta \lesssim 3 \times 10^{-8} \ . \tag{67}$$

On the other hand, the observation of a photon line corresponding to a gravitino lifetime close to the present bound would determine the parameter ζ as[7]

$$\zeta_{\rm obs} = 10^{-9}\left(\frac{5 \times 10^{28}{\rm s}}{\tau_{3/2}(\gamma\nu)}\right)^{1/2}\left(\frac{M_1}{200\ {\rm GeV}}\right)\left(\frac{m_{3/2}}{100\ {\rm GeV}}\right)^{-3/2} \ . \tag{68}$$

Note the strong dependence of $\zeta_{\rm obs}$ on the gravitino mass. In (68) we have normalized these masses to central values suggested by thermal leptogenesis, electroweak precision tests and gravitino dark matter [20].

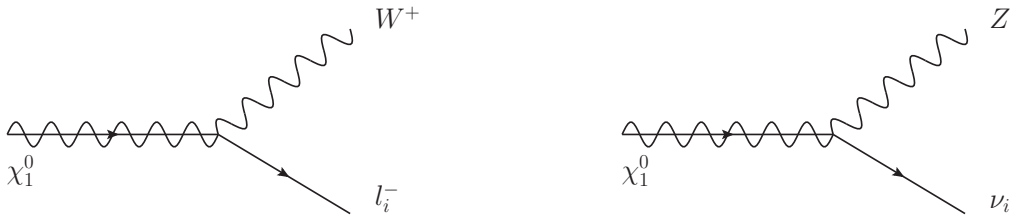

Figure 3: *Neutralino decays into charged lepton and W-boson, and neutrino and Z-boson.*

5.1 Neutralino NLSP

A neutralino NLSP heavier than 100 GeV dominantly decays into charged lepton and W-boson or neutrino and Z-boson [30] (cf. Fig. 3). The partial decay widths are given by

$$\Gamma\left(\chi_1^0 \to W^\pm l^\mp\right) = \frac{G_F m_{\chi_1^0}^3}{4\sqrt{2}\pi} \sum_i \left|V_{1i\ \mathrm{LO}}^{(\chi,e)}\right|^2 f_W(m_{\chi_1^0}) \left(1 + \mathcal{O}\left(s_{2\beta}\frac{m_Z^2}{\mu^2}\right)\right) , \tag{69}$$

$$\Gamma\left(\chi_1^0 \to Z\nu\right) = \frac{G_F m_{\chi_1^0}^3}{2\sqrt{2}\pi} \sum_i \left|V_{1i\ \mathrm{LO}}^{(\chi,\nu)}\right|^2 f_Z(m_{\chi_1^0}) \left(1 + \mathcal{O}\left(s_{2\beta}\frac{m_Z^2}{\mu^2}\right)\right) . \tag{70}$$

Here $V_{1i\ \mathrm{LO}}^{(\chi,e)}$ and $V_{1i\ \mathrm{LO}}^{(\chi,\nu)}$ are the charged and neutral current matrix elements at leading order, which are given in Eqs. (55) and (54), respectively, and

$$f_{W,Z}(m_{\chi_1^0}) = \left(1 - \frac{m_{W,Z}^2}{m_{\chi_1^0}^2}\right)^2 \left(1 + 2\frac{m_{W,Z}^2}{m_{\chi_1^0}^2}\right) \tag{71}$$

is a phase space factor which becomes important for neutralino masses close to the lower bound for $m_{\chi_1^0}$ of 100 GeV (cf. Fig. 4).

The total neutralino NLSP width is the sum

$$\Gamma_{\chi_1^0} = \Gamma(\chi_1^0 \to W^\pm l^\mp) + \Gamma(\chi_1^0 \to Z\nu) . \tag{72}$$

Using the matrix elements (54) and (55), one obtains the branching ratios

$$BR\left(\chi_1^0 \to W^\pm l^\mp\right) \simeq 2\ BR\left(\chi_1^0 \to Z\nu\right) . \tag{73}$$

Furthermore, the flavour structure of our model implies

$$BR\left(\chi_1^0 \to W^\pm \mu^\mp\right) \simeq BR\left(\chi_1^0 \to W^\pm \tau^\mp\right) . \tag{74}$$

[6]This lifetime is obtained by rescaling in Fig. 2 of [12] the signal by the factor 0.1.

[7]The results (67) and (68) are approximately consistent with the recent analysis [17].

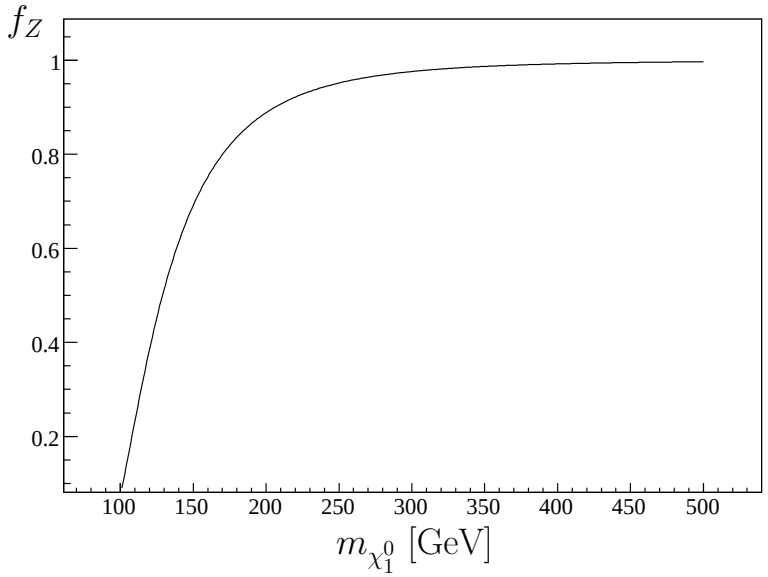

Figure 4: *Phase space suppression factor for neutralino decay to Z-boson and neutrino.*

Using the matrix elements (54), (55) and (56) for neutral, charged and supercurrent, respectively, one can express the neutralino lifetime directly in terms of the gravitino lifetime,

$$\begin{aligned}\tau_{\chi_1^0} =& \frac{c_w^2}{2\sqrt{2}} \frac{\left(M_2 - M_1\right)^2}{M_2^2} \frac{m_{3/2}^3}{G_F M_P^2 m_{\chi_1^0}^3} \tau_{3/2}(\gamma\nu) \\ & \times \left(2 f_W(m_{\chi_1^0}) + f_Z(m_{\chi_1^0})\right)^{-1} \left(1 + \mathcal{O}\left(s_{2\beta} \frac{m_Z^2}{\mu^2}\right)\right) \ . \end{aligned} \tag{75}$$

With the mass relations (57) and (60) one then obtains for the minimal neutralino decay length

$$\begin{aligned} c\tau_{\chi_1^0} \gtrsim \ & 80 \text{ cm} \left(\frac{m_{\chi_1^0}}{150\text{GeV}}\right)^{-3} \left(\frac{m_{3/2}}{10\text{GeV}}\right)^{3} \left(\frac{\tau_{3/2}(\gamma\nu)}{1 \times 10^{28} \text{ s}}\right) \\ & \times \left(2 f_W(m_{\chi_1^0}) + f_Z(m_{\chi_1^0})\right)^{-1} \left(1 + \mathcal{O}\left(s_{2\beta} \frac{m_Z^2}{\mu^2}\right)\right) \ . \end{aligned} \tag{76}$$

In Eqs. (75) and (76) the corrections to the leading order expressions are less than 10%. We emphasize again the strong dependence of this lower bound on the neutralino and gravitino masses. For instance, for a gravitino mass of 100 GeV and the Fermi-LAT bound $\tau_{3/2} \gtrsim 5 \times 10^{28}$ s, which applies for gravitino masses in the range 60 GeV $< m_{3/2} <$ 400 GeV, one obtains $c\tau_{\chi_1^0} \simeq 2$ km for a neutralino mass of 150 GeV. It is very interesting that such neutralino lifetimes are detectable at the LHC [31].

We conclude that, given the current bounds on the gravitino lifetime, a neutralino NLSP may still decay into gauge boson and lepton inside the detector, yielding a spectacular signature. However, for most of the parameter space a neutralino NLSP decays

outside the detector, leading to events indistinguishable from ordinary neutralino dark matter.

5.2 $\tilde{\tau}$-Lepton NLSP

Contrary to the neutralino NLSP decay, the R-parity violating decays of a $\widetilde{\tau}_1$-NLSP strongly depend on the flavour structure and the supersymmetry breaking parameters. The relative strength of the various decay modes becomes most transparent in the field basis where all bilinear R-parity breaking terms vanish, as discussed in Section 2. Since the R-parity breaking Yukawa couplings are proportional to the ordinary Yukawa couplings, decays into fermions of the second and third generation dominate. The leading partial decay widths of left- and right-handed $\widetilde{\tau}$-leptons are (cf. (14))

$$\Gamma_{\widetilde{\tau}_L}(\tau_R\nu) = \frac{1}{16\pi}\sum_i |\hat{\lambda}_{i33}|^2 m_{\widetilde{\tau}_L} \ , \tag{77}$$

$$\Gamma_{\widetilde{\tau}_L}(\bar{t}_L b_R) = \Gamma_{\widetilde{\tau}_L}(\bar{t}_L s_R) = \frac{3}{16\pi}|\lambda'_{333}|^2 m_{\widetilde{\tau}_L} \ , \tag{78}$$

$$\Gamma_{\widetilde{\tau}_L}(\bar{t}_R b_L) = \frac{3}{16\pi}|\hat{\lambda}'_{333}|^2 m_{\widetilde{\tau}_L} \ , \tag{79}$$

$$\Gamma_{\widetilde{\tau}_R}(\tau_L\nu) = \Gamma_{\widetilde{\tau}_R}(\mu_L\nu) = \frac{1}{16\pi}\sum_i |\lambda_{i33}|^2 m_{\widetilde{\tau}_R} \ . \tag{80}$$

In the flavour model discussed in Section 3, the order of magnitude of the various decay widths is determined by the power of the hierarchy parameter η ($\eta^2 \simeq 1/300$),

$$\begin{aligned}\Gamma_{\widetilde{\tau}_L}(\tau_R\nu) &\sim \Gamma_{\widetilde{\tau}_R}(\tau_L\nu) = \Gamma_{\widetilde{\tau}_R}(\mu_L\nu) \\ &\sim \Gamma_{\widetilde{\tau}_L}(\bar{t}_L b_R) \sim \Gamma_{\widetilde{\tau}_L}(\bar{t}_L s_R) \sim \eta^4\Theta^2 m_{\widetilde{\tau}} \ , \end{aligned} \tag{81}$$

$$\Gamma_{\widetilde{\tau}_L}(\bar{t}_R b_L) \sim \eta^2\Theta^2 m_{\widetilde{\tau}} \ . \tag{82}$$

The lightest mass eigenstate $\widetilde{\tau}_1$ is a linear combination of $\widetilde{\tau}_L$ and $\widetilde{\tau}_R$,

$$\widetilde{\tau}_1 = \sin\theta_\tau \widetilde{\tau}_L + \cos\theta_\tau \widetilde{\tau}_R \ . \tag{83}$$

From the above equations one obtains the $\widetilde{\tau}_1$-decay width

$$\Gamma_{\widetilde{\tau}_1} = \sin^2\theta_\tau \left(\Gamma_{\widetilde{\tau}_L}(\tau_R\nu) + 2\Gamma_{\widetilde{\tau}_L}(\bar{t}_L b_R) + \Gamma_{\widetilde{\tau}_L}(\bar{t}_R b_L)\right) + 2\cos^2\theta_\tau \Gamma_{\widetilde{\tau}_R}(\tau_L\nu) \ . \tag{84}$$

The total width is dominated by the contributions $\widetilde{\tau}_R \to \tau_L\nu, \mu_L\nu$ and $\widetilde{\tau}_L \to \bar{t}_R b_L$, respectively,

$$\Gamma_{\widetilde{\tau}_1} = \sin^2\theta_\tau \Gamma_{\widetilde{\tau}_L}(\bar{t}_R b_L) + 2\cos^2\theta_\tau \Gamma_{\widetilde{\tau}_R}(\tau_L\nu) \ , \tag{85}$$

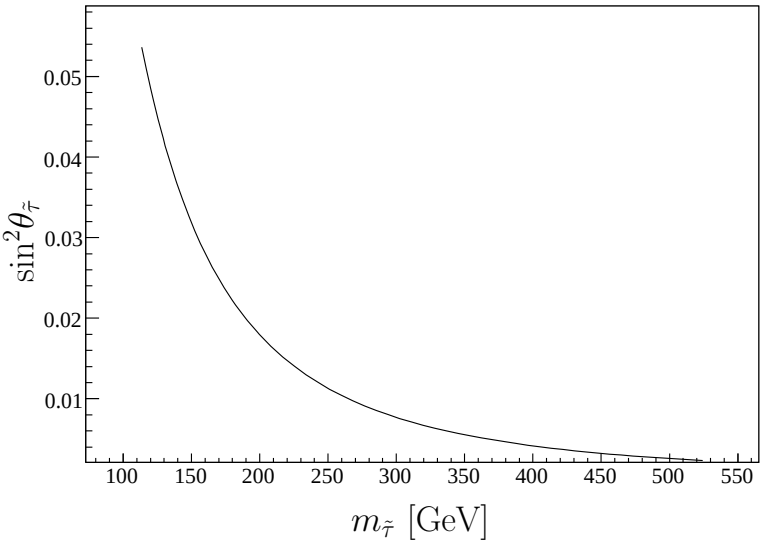

Figure 5: *$\widetilde{\tau}$-mixing angle:* $\sin^2(\theta_\tau)$ *as function of the lightest $\widetilde{\tau}$-mass* $m_{\widetilde{\tau}_1}$.

and it can be directly expressed in terms of the τ-lepton and top-quark masses,

$$\Gamma_{\widetilde{\tau}_1} = \frac{\epsilon^2}{16\pi v^2}\left(3m_t^2\sin^2\theta_\tau + 2m_\tau^2\tan^2\beta\cos^2\theta_\tau\right) m_{\widetilde{\tau}_1} \ , \tag{86}$$

where we have assumed

$$\epsilon_{2,3} = \epsilon'_{2,3} = \epsilon''_{2,3} \equiv \epsilon \ . \tag{87}$$

This corresponds to the parameter choice $a = b = c = 1$ in Eq. (34). Note that $\widetilde{\tau}_1$-decay width and branching ratios have a considerable uncertainty since these parameters depend on the unspecified mechanism of supersymmetry breaking. From Eqs. (18), (26) and $\eta \simeq 0.06$, one obtains for the R-parity breaking parameter

$$\epsilon \simeq \zeta \simeq \eta\Theta \simeq 6 \times 10^{-8} \ , \tag{88}$$

which is consistent with the present upper bound (67) within the theoretical uncertainties.

The dependence of the mixing angle θ_τ on $m_{\widetilde{\tau}_1}$ is shown in Fig. 5 for the boundary condition (59). For masses below the top-bottom threshold only leptonic $\widetilde{\tau}_1$-decays are possible. When the decay into top-bottom pairs becomes kinematically allowed, $\sin^2\theta_\tau$ is small. However, the suppression by a small mixing angle is compensated by the larger Yukawa coupling compared to the leptonic decay mode. This is a direct consequence of the couplings $\hat{\lambda}'$ which were not taken into account in previous analyses.

Due to the competition between mixing angle suppression and hierarchical Yukawa couplings, the top-bottom threshold is clearly visible in the $\widetilde{\tau}_1$-decay length as well as

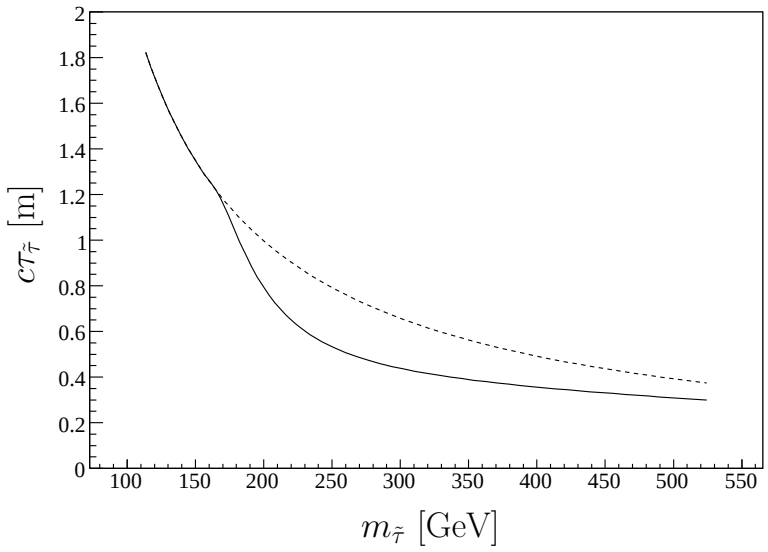

Figure 6: $\widetilde{\tau}_1$*-decay length as function of* $m_{\widetilde{\tau}_1}$*. Above the top-bottom threshold hadronic decays decrease the* $\widetilde{\tau}_1$*-lifetime.*

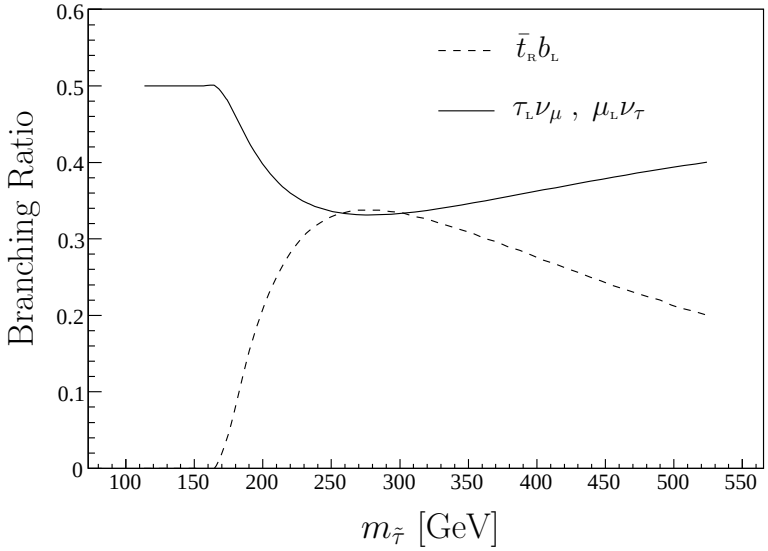

Figure 7: $\widetilde{\tau}_1$*-branching ratios as functions of* $m_{\widetilde{\tau}_1}$*. The dependence on the* $\widetilde{\tau}_1$*-mass is determined by the top-bottom threshold and the mass dependence of the* $\widetilde{\tau}_1$*-mixing angle.*

the branching ratios into leptons and heavy quarks. This is illustrated in Figs. 6 and 7, respectively, where these observables are plotted as functions of $m_{\widetilde{\tau}_1}$. Representative values of the $\widetilde{\tau}_1$-decay lengths below and above the top-bottom threshold are

$$m_{\widetilde{\tau}_1} < m_t + m_b : \quad c\tau_{\widetilde{\tau}_1}\Big|_{150\ \mathrm{GeV}} = 1.4\ \mathrm{m} \left(\frac{\epsilon}{5\times 10^{-8}}\right)^{-2} , \tag{89}$$

$$m_{\widetilde{\tau}_1} > m_t + m_b : \quad c\tau_{\widetilde{\tau}_1}\Big|_{250\ \mathrm{GeV}} = 0.6\ \mathrm{m} \left(\frac{\epsilon}{5\times 10^{-8}}\right)^{-2} . \tag{90}$$

Choosing for ϵ the representative value (68) from gravitino decay, $\epsilon = \zeta_{\mathrm{obs}} = 10^{-9}$, one obtains $c\tau_{\widetilde{\tau}_1} = 4$ km(1 km) for $m_{\widetilde{\tau}_1} = 150$ GeV(250 GeV). It is remarkable that such lifetimes can be measured at the LHC [31, 32].

Is it possible to avoid the severe constraint from gravitino decays on the $\widetilde{\tau}_1$-decay length? In principle, both observables are independent, and the unknown constants in the definition of ϵ, ϵ' and ϵ'' can be adjusted such that $\zeta = 0$. However, this corresponds to a strong fine-tuning, unrelated to an underlying symmetry. To illustrate this, consider the case where the soft R-parity breaking parameters vanish at the GUT scale, $B_i = m_{id}^2 = 0$, which was discussed in Section 3. In bilinear R-parity breaking, also the R-parity violating Yukawa couplings vanish at the GUT scale. With the one-loop radiative corrections at the electroweak scale (cf. (39); $\epsilon_i = \mu_i/\mu$),

$$B_i(\Lambda_{\mathrm{EW}}) = \frac{\epsilon_i \mu}{16\pi^2}\left(\frac{6}{5}g'^2 M_1 + 6g^2 M_2\right)\ln\frac{\Lambda_{\mathrm{GUT}}}{\Lambda_{\mathrm{EW}}} , \quad m_{id}^2(\Lambda_{\mathrm{EW}}) = 0 ,$$

and $M_{1,2} \sim \mu$, one reads off from Eqs. (10), (12) and (13)

$$\epsilon_i', \epsilon_i'' = \mathcal{O}(\epsilon_i) . \tag{91}$$

Hence, all R-parity breaking parameters are naturally of the same order, unless the fine-tuning also includes radiative corrections between the GUT scale and the electroweak scale.

Even if one accepts the fine-tuning $\zeta = 0$, one still has to satisfy the cosmological bounds on R-parity violating couplings, which yield $\epsilon_i = \mu_i/\mu \lesssim 10^{-6}$ [25]. In the flavour model discussed in Section 3 this corresponds to the choice $a = 20$ in Eq. (33). For the smaller $\widetilde{\tau}_1$-mass, which is preferred by electroweak precision tests, one then obtains the lower bound on the decay length

$$c\tau_{\widetilde{\tau}_1}\Big|_{150\ \mathrm{GeV}} \gtrsim 4\ \mathrm{mm} . \tag{92}$$

However, let us emphasize again that current constraints from Fermi-LAT on the diffuse gamma-ray spectrum indicate decay lengths several orders of magnitude larger.

5.3 Planck Mass Measurement

It has been pointed out in [9] that, in principle, one can determine the Planck mass from decay properties of a $\widetilde{\tau}$-NLSP together with the observation of a photon line in the diffuse gamma-ray flux, which is produced by gravitino decays. This is similar to the proposed microscopic determination of the Planck mass based on decays of very long lived $\widetilde{\tau}$-NLSP's in the case of a stable gravitino [33].

From our analysis of NLSP decays in this section it is clear that neutralino NLSP decays are particularly well suited for a measurement of the Planck mass, which does not require any additional assumptions. Eq. (75) implies ($G_F = \sqrt{2}/(4v^2)$),

$$\begin{aligned} M_{\rm P} =& c_w v \frac{M_2 - M_1}{M_2} \left(\frac{m_{3/2}}{m_{\chi_1^0}}\right)^{3/2} \left(\frac{\tau_{3/2}(\gamma\nu)}{\tau_{\chi_1^0}}\right)^{1/2} \\ & \times \left(2 f_W(m_{\chi_1^0}) + f_Z(m_{\chi_1^0})\right)^{-1/2} \left(1 + \mathcal{O}\left(s_{2\beta}\frac{m_Z^2}{\mu^2}\right)\right) \ . \end{aligned} \tag{93}$$

As expected, for gravitino and neutralino masses of the same order of magnitude, the ratio of the two-body lifetimes is determined by the ratio of the electroweak scale and the Planck mass,

$$\frac{\tau_{\chi_1^0}}{\tau_{3/2}(\gamma\nu)} \sim \frac{v^2}{{M_{\rm P}}^2} \ . \tag{94}$$

Quantitatively, using the relation (60) for the gaugino masses, one finally obtains ($v = 174$ GeV),

$$\begin{aligned} M_{\rm P} =& 3.6 \times 10^{18}\ {\rm GeV} \left(\frac{m_{3/2}}{m_{\chi_1^0}}\right)^{3/2} \left(\frac{\tau_{3/2}(\gamma\nu)}{10^{28}\ {\rm s}}\right)^{1/2} \left(\frac{\tau_{\chi_1^0}}{10^{-7}\ {\rm s}}\right)^{-1/2} \\ & \times \left(2 f_W(m_{\chi_1^0}) + f_Z(m_{\chi_1^0})\right)^{-1/2} \left(1 + \mathcal{O}\left(s_{2\beta}\frac{m_Z^2}{\mu^2}\right)\right) \ . \end{aligned} \tag{95}$$

It is remarkable that the observation of a photon line in the diffuse gamma-ray flux, together with a measurement of the neutralino lifetime at the LHC, can provide a microscopic determination of the Planck mass.

6 Summary and conclusions

We have studied a supersymmetric extension of the Standard Model with small R-parity breaking related to spontaneous $B - L$ breaking, which is consistent with primordial nucleosynthesis, thermal leptogenesis and gravitino dark matter. We have considered supergravity models with universal boundary conditions at the GUT scale, which lead to

scalar tau or bino-like neutralino as NLSP. Supersymmetry breaking terms have been introduced by means of higher-dimensional operators. The size of the soft terms corresponds to gravity or gaugino mediation.

We have analyzed our model, which represents a special case of bilinear R-parity breaking, in a basis of scalar $SU(2)$ doublets, where all bilinear terms vanish. In this basis one has R-parity violating Yukawa and gaugino couplings. They are given in terms of ordinary Yukawa couplings and 9 R-parity breaking parameters ϵ_i, ϵ'_i and ϵ''_i, $i = 1, ..., 3$, which are constrained by the flavour symmetry of the model. The R-parity violating couplings include terms proportional to the up-quark Yukawa couplings, which were not taken into accound in previous analyses.

The main goal of this paper are the quantitative connection between gravitino decays and NLSP decays, and the corresponding implications of recent Fermi-LAT data on the isotropic diffuse gamma-ray flux for superparticle decays at the LHC. To establish this connection one needs the relevant R-parity breaking matrix elements of neutral, charged and supercurrents. For the considered supergravity models these matrix elements can be obtained analytically to good approximation, since the diagonalization of the neutralino-neutrino and chargino-lepton mass matrices in powers of m_Z/μ converges well, as demonstrated in the appendix. The analytic expressions for the decay rates make the implications of the Fermi-LAT data for NLSP decays very transparent.

Our main quantitative results are the branching ratios for NLSP decays and the lower bounds on their decay lengths. For a neutralino NLSP with $m_{\chi_1^0} = 150$ GeV, the Fermi-LAT data yield the lower bound $c\tau_{\chi_1^0} \gtrsim 30$ cm. This bound does not depend on details of the superparticle mass spectrum or the flavour structure of the model. It directly follows from the comparison of two-particle gravitino and neutralino decays. On the contrary, there exists no model independent lower bound on the $\widetilde{\tau}_1$-decay length. The natural relation between gravitino and $\widetilde{\tau}$-decay widths can be avoided by fine-tuning. In this case the cosmological constraint that the baryon asymmetry is not washed out leads to the lower bound $c\tau_{\widetilde{\tau}_1} \gtrsim 4$ mm.

Without fine-tuning parameters the diffuse gamma-ray flux produced by gravitino decays constrains the lifetime of a neutralino as well as a $\widetilde{\tau}$-NLSP. For typical masses, $m_{3/2} \sim 100$ GeV and $m_{\rm NLSP} \sim 150$ GeV, the discovery of a photon line with an intensity close to the present Fermi-LAT limit would imply a decay length $c\tau_{\rm NLSP}$ of several hundered meters. This is a definite prediction of a class of supergravity models. It is very interesting that such lifetimes can be measured at the LHC [31, 32].

Finally, it is intriguing that the observation of a photon line in the diffuse gamma-ray flux, together with a measurement of the neutralino lifetime at the LHC, can yield a microscopic determination of the Planck mass, a crucial test of local supersymmetry.

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A Appendix: Gauge and mass eigenstates

A.1 Mass matrix diagonalization

The mass matrices $\mathcal{M}^N$ and $\mathcal{M}^C$ in the gauge eigenbasis were explicitly given in Eqs. (46) and (47), respectively,

$$\mathcal{M}^N = \begin{pmatrix} M_1 & 0 & m_Z s_\beta s_w & -m_Z c_\beta s_w & -\zeta_i m_Z s_w \\ 0 & M_2 & -m_Z s_\beta c_w & m_Z c_\beta c_w & \zeta_i m_Z c_w \\ m_Z s_\beta s_w & -m_Z s_\beta c_w & 0 & -\mu & 0 \\ -m_Z c_\beta s_w & m_Z c_\beta c_w & -\mu & 0 & 0 \\ -\zeta_i m_Z s_w & \zeta_i m_Z c_w & 0 & 0 & 0 \end{pmatrix},$$

$$\mathcal{M}^C = \begin{pmatrix} M_2 & m_Z s_\beta c_w & 0 & 0 & 0 \\ m_Z c_\beta c_w & \mu & \zeta_1 h^e_{11}\mu & \zeta_2 h^e_{22}\mu & \zeta_3 h^e_{33}\mu \\ \zeta_1 m_Z c_w & 0 & h^e_{11} v c_\beta & 0 & 0 \\ \zeta_2 m_Z c_w & 0 & 0 & h^e_{22} v c_\beta & 0 \\ \zeta_3 m_Z c_w & 0 & 0 & 0 & h^e_{33} v c_\beta \end{pmatrix}.$$

For non-vanishing R-parity breaking parameters ζ_i, $i = 1, \ldots, 3$, they induce a mixing between gauginos, Higgsinos and leptons,

$$\begin{aligned} -\mathcal{L} \supset & \frac{1}{2}\left(b, w^3, h_u^0, h_d^0, \nu_i\right) \mathcal{M}^N \left(b, w^3, h_u^0, h_d^0, \nu_i\right)^T \\ & + \left(\left(w^-, h_d^-, e_i\right) \mathcal{M}^C \left(w^+, h_u^+, e_i^c\right)^T + \text{h.c.}\right) . \end{aligned} \tag{A.1}$$

The matrices $\mathcal{M}^N$ and $\mathcal{M}^C$ are diagonalized by unitary and bi-unitary transformations, respectively,

$$U^{(n)T} \mathcal{M}^N U^{(n)} = \mathcal{M}^N_{\text{diag}} , \quad U^{(c)\dagger} \mathcal{M}^C \widetilde{U}^{(c)} = \mathcal{M}^C_{\text{diag}} , \tag{A.2}$$

where $U^{(n)\dagger} U^{(n)} = U^{(c)\dagger} U^{(c)} = \widetilde{U}^{(c)\dagger} \widetilde{U}^{(c)} = \mathbf{1}$. These unitary transformations relate the neutral and charged gauge eigenstates to the mass eigenstates (χ_a^0, ν_i') $(a = 1, \ldots, 4)$ and (χ_α^-, e_i'), $(\chi_\alpha^+, e_i'^c)$ $(\alpha = 1, 2)$, respectively.

In this work we consider the two boundary conditions (A) and (B), defined in Eqs. (58) and (59), respectively. The corresponding supergravity models satisfy the relation (48), $m_Z < M_{1,2} < \mu$, and in the regime 120 GeV $\lesssim M_1 \lesssim$ 500 GeV one finds $0.07 \lesssim m_Z/\mu \lesssim 0.25$. We diagonalized the above mass matrices to first order in the small parameters ζ_i and to second order in m_Z/μ. The size of the relative corrections given below has been calculated for the above parameter range using `SOFTSUSY3.0` [23]. As we shall see, the relative corrections are of order m_Z^2/μ^2, and the expansion converges well for most matrix elements.

The neutralino and neutrino mass eigenvalues are

$$m_{\chi_1^0} = M_1 - \frac{m_Z^2\,(M_1 + \mu s_{2\beta})\, s_w^2}{\mu^2 - M_1^2}\left(1 + \mathcal{O}\left(\frac{m_Z^2}{\mu^2}\right)\right)\,, \tag{A.3}$$

$$m_{\chi_2^0} = M_2 - \frac{m_Z^2\,(M_2 + \mu s_{2\beta})\, c_w^2}{\mu^2 - M_2^2}\left(1 + \mathcal{O}\left(\frac{m_Z^2}{\mu^2}\right)\right)\,, \tag{A.4}$$

$$m_{\chi_3^0} = \mu + \frac{m_Z^2\,(\mu - M_1 c_w^2 - M_2 s_w^2)\,(1 + s_{2\beta})}{2\,(\mu - M_1)\,(\mu - M_2)}\left(1 + \mathcal{O}\left(\frac{m_Z^2}{\mu^2}\right)\right)\,, \tag{A.5}$$

$$m_{\chi_4^0} = -\,\mu - \frac{m_Z^2\,(\mu + M_1 c_w^2 + M_2 s_w^2)\,(1 - s_{2\beta})}{2\,(\mu + M_1)\,(\mu + M_2)}\left(1 + \mathcal{O}\left(s_{2\beta}\frac{m_Z^2}{\mu^2}\right)\right)\,, \tag{A.6}$$

$$m_{\nu_i} = 0 + \mathcal{O}\left(\zeta^2 \frac{m_Z^2}{\mu^2}\right)\,. \tag{A.7}$$

We checked numerically that relative corrections $\mathcal{O}(m_Z^2/\mu^2)$ to the above neutralino masses are smaller than $0.05, 0.15, 0.10, 0.001$, for $m_{\chi_1^0}, \ldots, m_{\chi_4^0}$, respectively.

The chargino and lepton mass eigenvalues are

$$m_{\chi_1^\pm} = M_2 - \frac{{m_Z}^2(M_2 + \mu s_{2\beta})c_w^2}{2\left(\mu^2 - {M_2}^2\right)}\left(1 + \mathcal{O}\left(\frac{m_Z^2}{\mu^2}\right)\right)\,, \tag{A.8}$$

$$m_{\chi_2^\pm} = \mu + \frac{{m_Z}^2(\mu + M_2 s_{2\beta})c_w^2}{2\left(\mu^2 - {M_2}^2\right)}\left(1 + \mathcal{O}\left(\frac{m_Z^2}{\mu^2}\right)\right)\,, \tag{A.9}$$

$$m_{e_i'} = h_{ii}^e v c_\beta \left(1 + \mathcal{O}\left(\zeta^2 \frac{m_Z^2}{\mu^2}\right)\right)\,. \tag{A.10}$$

Here the relative corrections of $\mathcal{O}(m_Z^2/\mu^2)$ are numerically smaller than 5%.

The unitary matrix $U^{(n)}$ from Eq. (A.2) can be written as

$$U^{(n)} = \left(\begin{array}{c|c} U_{ab}^{(\chi^0)} & U_{ai}^{(\chi^0,\nu)} \\ \hline U_{ia}^{(\nu,\chi^0)} & U_{ij}^{(\nu)} \end{array}\right)\,, \tag{A.11}$$

with

$$U_{ab}^{(\chi^0)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$
$$+ \begin{pmatrix} -\frac{{m_Z}^2\left({M_1}^2+2\mu s_{2\beta}M_1+\mu^2\right)s_w^2}{2\left({M_1}^2-\mu^2\right)^2} & \frac{{m_Z}^2(M_2+\mu s_{2\beta})s_{2w}}{2(M_1-M_2)\left({M_2}^2-\mu^2\right)} & \frac{m_Z(c_\beta+s_\beta)s_w}{\sqrt{2}(M_1-\mu)} & \frac{m_Z(c_\beta-s_\beta)s_w}{\sqrt{2}(M_1+\mu)} \\ -\frac{{m_Z}^2(M_1+\mu s_{2\beta})s_{2w}}{2(M_1-M_2)\left({M_1}^2-\mu^2\right)} & -\frac{{m_Z}^2c_w^2\left({M_2}^2+2\mu s_{2\beta}M_2+\mu^2\right)}{2\left({M_2}^2-\mu^2\right)^2} & -\frac{m_Zc_w(c_\beta+s_\beta)}{\sqrt{2}(M_2-\mu)} & \frac{m_Zc_w(s_\beta-c_\beta)}{\sqrt{2}(M_2+\mu)} \\ \frac{m_Z(\mu c_\beta+M_1s_\beta)s_w}{{M_1}^2-\mu^2} & -\frac{m_Zc_w(\mu c_\beta+M_2s_\beta)}{{M_2}^2-\mu^2} & \frac{{m_Z}^2(c_\beta+s_\beta)}{\mu^2}x_1 & \frac{(c_\beta-s_\beta){m_Z}^2}{\mu^2}x_2 \\ -\frac{m_Z(M_1c_\beta+\mu s_\beta)s_w}{{M_1}^2-\mu^2} & \frac{m_Zc_w(M_2c_\beta+\mu s_\beta)}{{M_2}^2-\mu^2} & \frac{(c_\beta+s_\beta){m_Z}^2}{\mu^2}x_3 & \frac{(c_\beta-s_\beta){m_Z}^2}{\mu^2}x_4 \end{pmatrix}$$
$$\times \left(1+\mathcal{O}\left(\frac{m_Z^2}{\mu^2}\right)\right) \ , \tag{A.12}$$

where we used the abbreviations

$$x_1 = \frac{\mu}{4\sqrt{2}}\left(\frac{(M_2s_\beta-(M_2-2\mu)c_\beta)c_w^2}{(M_2-\mu)^2}+\frac{(M_1s_\beta-(M_1-2\mu)c_\beta)s_w^2}{(M_1-\mu)^2}\right) \ , \tag{A.13}$$

$$x_2 = \frac{\mu}{4\sqrt{2}}\left(-\frac{((M_2+2\mu)c_\beta+M_2s_\beta)c_w^2}{(M_2+\mu)^2}-\frac{((M_1+2\mu)c_\beta+M_1s_\beta)s_w^2}{(M_1+\mu)^2}\right) \ , \tag{A.14}$$

$$x_3 = \frac{\mu}{4\sqrt{2}}\left(\frac{((M_2-2\mu)s_\beta-M_2c_\beta)c_w^2}{(M_2-\mu)^2}+\frac{((M_1-2\mu)s_\beta-M_1c_\beta)s_w^2}{(M_1-\mu)^2}\right) \ , \tag{A.15}$$

$$x_4 = \frac{\mu}{4\sqrt{2}}\left(\frac{(M_2c_\beta+(M_2+2\mu)s_\beta)c_w^2}{(M_2+\mu)^2}+\frac{(M_1c_\beta+(M_1+2\mu)s_\beta)s_w^2}{(M_1+\mu)^2}\right) \ . \tag{A.16}$$

The numerical error of the matrix (A.12) in our parameter range of interest is smaller than 40% of the given NLO term. We do not discuss the slow convergence for this R-parity conserving sub-matrix further, since this is beyond the scope of our analysis.

Furthermore,

$$U_{ai}^{(\chi^0,\nu)} = \zeta_i \begin{pmatrix} s_w\frac{m_Z}{M_1} \\ -c_w\frac{m_Z}{M_2} \\ -\frac{{m_Z}^2c_\beta\left(M_1c_w^2+M_2s_w^2\right)}{M_1M_2\mu} \\ \frac{{m_Z}^2s_\beta\left(M_1c_w^2+M_2s_w^2\right)}{M_1M_2\mu} \end{pmatrix}\left(1+\mathcal{O}\left(s_{2\beta}\frac{m_Z^2}{\mu^2}\right)\right) \ , \tag{A.17}$$

$$U_{ia}^{(\nu,\chi^0)} = \zeta_i \begin{pmatrix} -s_w\frac{m_Z}{M_1} \\ c_w\frac{m_Z}{M_2} \\ \frac{{m_Z}^2\left(M_1c_w^2+M_2s_w^2-\mu\right)(c_\beta+s_\beta)}{\sqrt{2}(M_1-\mu)\mu(\mu-M_2)} \\ \frac{{m_Z}^2\left(M_1c_w^2+M_2s_w^2+\mu\right)(c_\beta-s_\beta)}{\sqrt{2}\mu(M_1+\mu)(M_2+\mu)} \end{pmatrix}\left(1+\mathcal{O}\left(\frac{m_Z^2}{\mu^2}\right)\right) \ , \tag{A.18}$$

$$U_{ij}^{(\nu)} = \delta_{ij}+\mathcal{O}\left(\zeta^2\frac{m_Z^2}{\mu^2}\right) \ . \tag{A.19}$$

The uncertainties in Eq. (A.17) evaluate numerically to less than 5%. For $U_{ia}^{(\nu,\chi^0)}$ they are less than $0.15, 0.10, 0.25, 0.25$, for $a = 1, \ldots, 4$, respectively.

The unitary matrices $U^{(c)}$ and $\widetilde{U}^{(c)}$ which diagonalize the matrix $\mathcal{M}^C$, cf. Eq. (A.2), can be denoted as

$$\widetilde{U}^{(c)} = \left(\begin{array}{c|c} \widetilde{U}_{\alpha\beta}^{(\chi^+)} & \widetilde{U}_{\alpha i}^{(\chi^+,e^c)} \\ \hline \widetilde{U}_{i\alpha}^{(e^c,\chi^+)} & \widetilde{U}_{ij}^{(e^c)} \end{array}\right) , \qquad U^{(c)} = \left(\begin{array}{c|c} U_{\alpha\beta}^{(\chi^-)} & U_{\alpha i}^{(\chi^-,e)} \\ \hline U_{i\alpha}^{(e,\chi^-)} & U_{ij}^{(e)} \end{array}\right) . \tag{A.20}$$

We find

$$\widetilde{U}_{\alpha\beta}^{(\chi^+)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -\frac{{m_Z}^2 c_w^2 (\mu c_\beta + M_2 s_\beta)^2}{2\left({M_2}^2 - \mu^2\right)^2} & -\frac{m_Z c_w (\mu c_\beta + M_2 s_\beta)}{{M_2}^2 - \mu^2} \\ \frac{m_Z c_w (\mu c_\beta + M_2 s_\beta)}{{M_2}^2 - \mu^2} & -\frac{{m_Z}^2 c_w^2 (\mu c_\beta + M_2 s_\beta)^2}{2\left({M_2}^2 - \mu^2\right)^2} \end{pmatrix} \left(1 + \mathcal{O}\left(\frac{m_Z^2}{\mu^2}\right)\right) , \tag{A.21}$$

$$\widetilde{U}_{ij}^{(e^c)} = \delta_{ij} + \mathcal{O}\left(\zeta^2\right) . \tag{A.22}$$

Numerically, the relative correction to the NLO contribution to $\widetilde{U}_{\alpha\beta}^{(\chi^+)}$ is less than 25%. The off-diagonal elements of the matrix $\widetilde{U}^{(c)}$ to leading order in h_{ii}^e are

$$\begin{aligned} \widetilde{U}_{\alpha i}^{(\chi^+,e^c)} =& -\zeta_i h_{ii}^e \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ & + \zeta_i h_{ii}^e \begin{pmatrix} \frac{m_Z c_w (M_2 s_\beta - v c_\beta)}{{M_2}^2} \\ \frac{{m_Z}^2 c_\beta c_w^2 (v\mu c_\beta + M_2 (v-\mu) s_\beta)}{{M_2}^2 \mu^2} \end{pmatrix} \left(1 + \mathcal{O}\begin{pmatrix} \frac{s_{2\beta} m_Z^2}{\mu^2} \\ \frac{m_Z^2}{\mu^2} \end{pmatrix}\right) , \end{aligned} \tag{A.23}$$

$$\begin{aligned} \widetilde{U}_{i\alpha}^{(e^c,\chi^+)} =& \zeta_i h_{ii}^e \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ & + \zeta_i h_{ii}^e \begin{pmatrix} \frac{m_Z c_w \left(M_2 s_\beta \mu^2 + \left({M_2}^2 (v+\mu) - v\mu^2\right) c_\beta\right)}{{M_2}^2 \left(M_2^2 - \mu^2\right)} \\ -\frac{{m_Z}^2 \mu^2 c_w^2 y}{2\left(\mu^2 - {M_2}^2\right)^2} \end{pmatrix} \left(1 + \mathcal{O}\left(\frac{m_Z^2}{\mu^2}\right)\right) , \end{aligned} \tag{A.24}$$

where

$$\begin{aligned} y =& \frac{1}{\mu^4} \left(v s_{2\beta} {M_2}^3 + \mu \left(2v {M_2}^2 + \mu^2 (\mu - 2v)\right) c_\beta^2 + \mu s_\beta \left(\mu \left(2\mu^2 - {M_2}^2\right) s_\beta \right.\right. \\ & \left.\left. - 2M_2 \left({M_2}^2 + (v - 2\mu)\mu\right) c_\beta\right)\right) . \end{aligned} \tag{A.25}$$

The numerical relative correction to the NLO term in $\widetilde{U}_{i\alpha}^{(e^c,\chi^+)}$ is smaller than $0.10, 0.15$ for $\alpha = 1, 2$, respectively. For $\widetilde{U}_{1i}^{(\chi^+,e^c)}$ it is smaller[8] than 1%, and smaller than 10% for $\widetilde{U}_{2i}^{(\chi^+,e^c)}$.

[8]The numerical calculation of the error reaches our numerical precision. The given value is calculated from the comparison with the analytical NNLO expression.

The block diagonal elements of the matrix $U^{(c)}$ are

$$U^{(\chi^-)}_{\alpha\beta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -\frac{{m_Z}^2 c_w^2 (M_2 c_\beta + \mu s_\beta)^2}{2\left({M_2}^2 - \mu^2\right)^2} & -\frac{m_Z c_w (M_2 c_\beta + \mu s_\beta)}{{M_2}^2 - \mu^2} \\ \frac{m_Z c_w (M_2 c_\beta + \mu s_\beta)}{{M_2}^2 - \mu^2} & -\frac{{m_Z}^2 c_w^2 (M_2 c_\beta + \mu s_\beta)^2}{2\left({M_2}^2 - \mu^2\right)^2} \end{pmatrix} \left(1 + \mathcal{O}\left(\frac{m_Z^2}{\mu^2}\right)\right) , \tag{A.26}$$

$$U^{(e)}_{ij} = \delta_{ij} + \mathcal{O}\left(\zeta^2\right) . \tag{A.27}$$

Numerically, the relative correction to the NLO contribution to $U^{(\chi^-)}_{\alpha\beta}$ is smaller than 20%. The off-diagonal elements of $U^{(c)}$ are

$$U^{(\chi^-,e)}_{\alpha i} = \zeta_i \begin{pmatrix} -\frac{m_Z c_w}{M_2} \\ \frac{{m_Z}^2 c_w^2 s_\beta}{M_2 \mu} \end{pmatrix} \left(1 + \mathcal{O}\left(\frac{s_{2\beta} m_Z^2}{\mu^2}\right)\right) , \tag{A.28}$$

$$U^{(e,\chi^-)}_{i\alpha} = \zeta_i \begin{pmatrix} \frac{m_Z c_w}{M_2} \\ \frac{{m_Z}^2 c_w^2 (\mu c_\beta + M_2 s_\beta)}{\mu^3 - {M_2}^2 \mu} \end{pmatrix} \left(1 + \mathcal{O}\left(\frac{s_{2\beta} m_Z^2}{\mu^2}\right)\right) . \tag{A.29}$$

Here we ignored corrections that are proportional to the Yukawa couplings h^e_{ii} or higher powers thereof. The numerical value of the higher order correction relative to the NLO term is smaller than 1% for $U^{(\chi^-,e)}_{\alpha i}$, smaller than 5% for $U^{(e,\chi^-)}_{i1}$, and smaller than 15% for $U^{(e,\chi^-)}_{i2}$.

A.2 The currents in mass eigenstate basis

The neutral and charged currents were given in Eqs. (50) and (51),

$$\begin{aligned} J_{Z\mu} =& \overline{\chi}^0_a \gamma_\mu V^{(\chi^0)}_{ab} \chi^0_b + \overline{\chi}^-_\alpha \gamma_\mu V^{(\chi^-)}_{\alpha\beta} \chi^-_\beta + \overline{\chi}^+_\alpha \gamma_\mu V^{(\chi^+)}_{\alpha\beta} \chi^+_\beta + \overline{\nu}_i \gamma_\mu V^{(\nu)}_{ij} \nu_i + \overline{e}_i \gamma_\mu V^{(e)}_{ij} e_i \\ & + \left(\overline{\chi}^0_a \gamma_\mu V^{(\chi,\nu)}_{ai} \nu_i + \overline{\chi}^-_\alpha \gamma_\mu V^{(\chi^-,e)}_{\alpha i} e_i + \overline{\chi}^+_\alpha \gamma_\mu V^{(\chi^+,e^c)}_{\alpha i} e^c_i + \text{h.c.}\right) , \\ J^-_\mu =& \overline{\chi}^0_a \gamma_\mu V^{(\chi)}_{a\alpha} \chi^-_\alpha + \overline{\chi}^0_a \gamma_\mu V^{(\chi,e)}_{ai} e_i + \overline{\nu}_i \gamma_\mu V^{(\nu,\chi)}_{i\alpha} \chi^-_\alpha + \overline{\nu}_i \gamma_\mu V^{(\nu,e)}_{ij} e_j . \end{aligned}$$

The CKM-like matrices $V^{(\chi^0)}_{ab}$, $V^{(\chi^-)}_{\alpha\beta}$, $V^{(\chi^+)}_{\alpha\beta}$, $V^{(\nu)}_{ij}$, $V^{(e)}_{ij}$, $V^{(\chi,\nu)}_{ai}$, $V^{(\chi^-,e)}_{\alpha i}$, $V^{(\chi^+,e^c)}_{\alpha i}$, $V^{(\chi)}_{a\alpha}$, $V^{(\chi,e)}_{ai}$, $V^{(\nu,\chi)}_{i\alpha}$, $V^{(\nu,e)}_{ij}$ follow from the currents in gauge eigenbasis, Eqs. (42) and (43), and the explicit matrices $U^{(n)}$, $U^{(c)}$ and $\widetilde{U}^{(c)}$. Here we focus on the matrices $V^{(\chi,\nu)}_{ai}$ and $V^{(\chi,e)}_{ai}$ since they determine the interactions of interest for this work. We find

$$V^{(\chi,\nu)}_{ai} = \zeta_i \begin{pmatrix} -\frac{s_w m_Z}{2M_1} \\ \frac{c_w m_Z}{2M_2} \\ \frac{{m_Z}^2 \mu v_1}{2\sqrt{2} M_1 (M_1 - \mu)(M_2 - \mu)} \\ \frac{{m_Z}^2 \mu v_2}{2\sqrt{2} M_1 (M_1 + \mu)(M_2 + \mu)} \end{pmatrix} \left(1 + \mathcal{O}\begin{pmatrix} \frac{s_{2\beta} m_Z^2}{\mu^2} \\ \frac{m_Z^2}{\mu^2} \\ \frac{m_Z^2}{\mu^2} \\ \frac{s_{2\beta} m_Z^2}{\mu^2} \end{pmatrix}\right) , \tag{A.30}$$

with abbreviations

$$\begin{aligned} v_1 =& \frac{1}{M_2\mu^2}\Big(M_1(M_1-\mu)((\mu-2M_2)c_\beta-\mu s_\beta)c_w^2 \\ &+M_2(M_2-\mu)((\mu-2M_1)c_\beta-\mu s_\beta)s_w^2\Big)\ , \end{aligned} \tag{A.31}$$

$$\begin{aligned} v_2 =& \frac{1}{M_2\mu^2}\Big(M_1(M_1+\mu)((2M_2+\mu)c_\beta+\mu s_\beta)c_w^2 \\ &+M_2(M_2+\mu)((2M_1+\mu)c_\beta+\mu s_\beta)s_w^2\Big)\ . \end{aligned} \tag{A.32}$$

Numerically, the relative errors are smaller than $0.10, 0.20, 0.15, 0.05$ for $a = 1, \ldots, 4$.

Finally,

$$V_{ai}^{(\chi,e)} = \zeta_i \begin{pmatrix} -\frac{s_w m_Z}{M_1} \\ -\frac{\left(\sqrt{2}-1\right)c_w m_Z}{M_2} \\ -\frac{m_Z^2\mu v_1}{M_2(M_1-\mu)(M_2-\mu)} \\ \frac{m_Z^2\mu v_2}{M_2(M_1+\mu)(M_2+\mu)} \end{pmatrix} \left(1+\mathcal{O}\begin{pmatrix} \frac{s_{2\beta}m_Z^2}{\mu^2} \\ \frac{m_Z^2}{\mu^2} \\ \frac{m_Z^2}{\mu^2} \\ \frac{m_Z^2}{\mu^2} \end{pmatrix}\right)\ , \tag{A.33}$$

with abbreviations

$$\begin{aligned} \widetilde{v}_1 =& \frac{1}{2\mu^2}\Big(c_\beta\Big((M_1-\mu)\left(\sqrt{2}M_2-2\mu\right)c_w^2+\sqrt{2}M_2(M_2-\mu)s_w^2\Big) \\ &+s_\beta\Big(\left(\sqrt{2}-2\right)(M_1-\mu)\mu c_w^2+\sqrt{2}M_2(M_2-\mu)s_w^2\Big)\Big)\ , \end{aligned} \tag{A.34}$$

$$\begin{aligned} \widetilde{v}_2 =& \frac{1}{2\mu^2}\Big(c_\beta\Big((M_1+\mu)\left(\sqrt{2}M_2+2\mu\right)c_w^2+\sqrt{2}M_2(M_2+\mu)s_w^2\Big) \\ &+s_\beta\Big(\left(\sqrt{2}-2\right)\mu(M_1+\mu)c_w^2-\sqrt{2}M_2(M_2+\mu)s_w^2\Big)\Big)\ . \end{aligned} \tag{A.35}$$

Here we again neglected corrections that involve the Yukawa couplings h_{ii}^e. The numerical corrections to the NLO contributions to $V_{ai}^{(\chi,e)}$ are smaller than $0.05, 0.15, 0.20$ for $a = 1, 2, 3$, respectively. For $a = 4$ we reach the limit of our numerical precision.

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