arXiv:1011.2630v2 [hep-ph] 27 Jan 2011

DESY 10-198

CP violation with Higgs–dependent Yukawa couplings

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Abstract

The framework of Higgs–dependent Yukawa couplings allows one to eliminate small couplings from the Standard Model, which can be tested at the LHC. In this work, I study the conditions for CP violation to occur in such models. I identify a class of weak basis invariants controlling CP violation. The invariant measure of CP violation is found to be more than 10 orders of magnitude greater than that in the Standard Model, which can be sufficient for successful electroweak baryogenesis.

1 Introduction

The flavor puzzle of the Standard Model (SM) remains one of the outstanding issues in modern particle physics. The observed hierarchy of the fermion masses is not explained in the SM, but instead parametrized in terms of small Yukawa couplings. One possibility that has been put forward independently in [1] and [2] is that the Yukawa couplings are effective couplings dominated by higher dimensional operators involving the Higgs field. This eliminates small fundamental couplings in favor of $\mathcal{O}(1)$ parameters. The smallness of the fermion masses is then due to the smallness of the Higgs vacuum expectation value (VEV) compared to the new physics scale (of order TeV). This idea can be tested at the LHC by measuring the Higgs decay branching ratios.

In the present work, I study the conditions for CP violation to occur in this framework. It is helpful to formulate the problem in a basis-independent way, making use of CP violating basis invariants which generalize the Jarlskog invariant [3–5]. This approach has been employed in various new physics models, including the seesaw [6], supersymmetric [7] and 2 Higgs doublet models [8]. The basis independent formulation allows one to obtain an invariant measure of CP violation, which can be relevant to electroweak (EW) baryogenesis (see [9] for a review). Indeed, it is the smallness of the Jarlskog invariant (and the Higgs sector constraints) that makes electroweak baryogenesis essentially impossible in the SM. The structure of the invariants with Higgs-dependent Yukawa couplings is very different from that of the Standard Model and bears some similarity to the supersymmetric case [7]. It is therefore plausible that successful EW baryogenesis can be achieved.

2 Higgs-dependent Yukawa couplings

The main assumption of our framework is that the Yukawa couplings are functions of the Higgs field. They are effective quantities which can be expanded in powers of $H^\dagger H/M^2$ with M being the cutoff of the effective theory,

$$Y_{ij}(H) = c_{ij}^{(0)} + c_{ij}^{(1)} \frac{H^\dagger H}{M^2} + ... \tag{1}$$

In the Standard Model, many of the Yukawa couplings are very small, down to 10^{-5}. It is therefore plausible that the higher order terms are comparable or even dominant. A particularly interesting possibility would be to have no small fundamental couplings at all. For that the above expansion has to start with some non-zero power of $H^\dagger H/M^2$. That is, all $c_{ij}^{(n)}$ vanish until some integer n_{ij},

$$Y_{ij}^{u,d}(H) = c_{ij}^{u,d} \left(\frac{H^\dagger H}{M^2}\right)^{n_{ij}^{u,d}} . \tag{2}$$

This can happen due to symmetries of the UV completion of our effective theory. Such symmetries are particularly easy to realize in supersymmetric (or 2 Higgs doublet model) extensions of the Standard Model, where $H^\dagger H$ is replaced by $H_1 H_2$. The latter can have a (possibly discrete) charge *à la* Froggatt-Nielsen [10] such that the vanishing of some $c_{ij}^{(n)}$ is dictated by charge conservation. In the SM case, the analog would be some non-abelian symmetry acting on $H^\dagger H$.[1]

In this framework, the smallness of the Yukawa couplings is explained by the smallness of the Higgs VEV compared to the new physics scale M,

$$\epsilon \equiv \frac{\langle H^\dagger H \rangle}{M^2} \simeq \frac{1}{60} \,, \tag{3}$$

with the numerical value being fixed by $\epsilon = m_b/m_t$. In terms of ϵ, the Yukawa matrices are expressed as [2]

$$Y^d \sim \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon^1 & \epsilon^1 \end{pmatrix} \quad , \quad Y^u \sim \begin{pmatrix} \epsilon^3 & \epsilon^1 & \epsilon^1 \\ \epsilon^3 & \epsilon^1 & \epsilon^1 \\ \epsilon^2 & \epsilon^0 & \epsilon^0 \end{pmatrix} \quad . \tag{4}$$

This texture reproduces the observed quark masses and mixings. The new feature is that the couplings to the physical Higgs boson are modified dramatically. The relevant Lagrangian is given by

$$-\mathcal{L} = Y_{ij}^u(H)\ \bar{q}_{Li} u_{Rj} H^c + Y_{ij}^d(H)\ \bar{q}_{Li} d_{Rj} H + \text{h.c.} \,, \tag{5}$$

where $H^c = i\sigma_2 H^*$ and $Y_{ij}^{u,d}(H)$ are given by Eq. (2). The quark couplings to the physical Higgs increase by a factor $2n_{ij}+1$ compared to that of the SM,

$$y_{ij}^{u,d} = (2n_{ij}^{u,d}+1)\big(y_{ij}^{u,d}\big)_{\rm SM} \,, \tag{6}$$

where $\big(y_{ij}^{u,d}\big)_{\rm SM} = m_{ij}^{u,d}/(\sqrt{2}v)$ and the integers $n_{ij}^{u,d}$ can be read off from the texture (4). As a result, the Higgs decay rate into quarks increases by a significant factor ranging from 9 for the bottom quark to 49 for up- and down- quarks leading to observable effects at the LHC.

Since the mass matrices and the physical Higgs couplings differ by a flavor-dependent factor, they cannot be diagonalized in the same basis and Higgs-mediated FCNC are induced. These however are suppressed by the quark masses and, for the texture (4), satisfy the experimental bounds (apart from ϵ_K which sets a mild constraint on a CP phase). On the other hand, the flavor changing effects involving the top quark are significant and can be observed at the LHC [2].

[1] In general, one may separate the (low energy) Froggatt-Nielsen field and the Higgs (see, e.g. [11]). In this case, the breaking of the Froggatt-Nielsen symmetry *a priori* has no relation to the electroweak scale.

Along with extra flavor violation, this framework brings in additional sources of CP violation. The extra CP phases reside in the quark couplings to the physical Higgs boson and can be relevant to baryogenesis. In what follows, I study the conditions for CP violation and construct the corresponding CP violating weak basis invariants.

3 CP violating invariants with 2 quark species

Consider a system of 2 quark species, say a top quark and a charm quark. We have two relevant flavor objects: the mass matrix and the matrix of the physical Higgs couplings. These are proportional to

$$Y_{ij} \quad , \quad \tilde{Y}_{ij} \equiv N_{ij} Y_{ij} \quad , \tag{7}$$

respectively, with integer $N_{ij} = 2n_{ij} + 1$. This corresponds to a special ("symmetric") basis in which $\mathrm{Arg} Y_{ij} = \mathrm{Arg} \tilde{Y}_{ij}$. Under quark basis transformations Y and $\tilde{Y}$ transform as

$$\begin{aligned} Y &\to U_L^\dagger \; Y \; U_R \; , \\ \tilde{Y} &\to U_L^\dagger \; \tilde{Y} \; U_R \; , \end{aligned} \tag{8}$$

where U_L, U_R are unitary matrices. It is clear that, in general, these matrices are not diagonal in the same basis. This provides us with sources for FCNC and CP violation.

It is easy to see that CP violation originates from a single CP phase. Indeed, 3 complex phases in Y and $\tilde{Y}$ can be eliminated by a quark phase redefinition (8) with

$$\begin{aligned} U_L &= \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}) \; , \\ U_R &= \mathrm{diag}(e^{i\beta_1}, e^{i\beta_2}) \; . \end{aligned} \tag{9}$$

In other words, CP violation is sourced by a reparametrization invariant combination

$$\mathrm{Im}\Big(\; Y_{11} Y_{22} Y_{12}^* Y_{21}^* \; \Big) \; . \tag{10}$$

This quantity induces CP phases in the couplings of the quark mass eigenstates to the physical Higgs. In the mass eigenstate basis,

$$Y \to \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix} \quad , \quad \tilde{Y} \to \begin{pmatrix} \tilde{y}_{11} & \tilde{y}_{12} \\ \tilde{y}_{21} & \tilde{y}_{22} \end{pmatrix} \; , \tag{11}$$

with positive y_1 and y_2 (proportional to the quark masses), the matrix of the Higgs couplings has 3 CP phases: Arg $\tilde{y}_{11}$, Arg $\tilde{y}_{22}$ and Arg $\tilde{y}_{12}\tilde{y}_{21}$. Note that since this basis is only defined up to a phase transformation $U_L = U_R = \mathrm{diag}(e^{i\delta_1}, e^{i\delta_2})$, the physical phases must be invariant under this residual symmetry.

The presence of CP violation in the theory can be formulated in a basis independent way. For that one needs a quantity which is invariant under the U(2)×U(2) transformations (8) and odd under the CP transformation

$$Y \xrightarrow{\text{CP}} Y^* \ . \tag{12}$$

Such invariants can be constructed systematically by forming an object that transforms under one of the U(2)'s and taking a trace.[2] For example, $Y\tilde{Y}^\dagger$ and $YY^\dagger$ transform under U_L only. Then, a trace of an anti-hermitian matrix formed out of these objects will have the required properties. The simplest non-zero invariants are

$$\begin{aligned} &\mathrm{Tr}\big[A^2 - \text{h.c.}\big] \ , \\ &\mathrm{Tr}\big[AB - \text{h.c.}\big] \ , \end{aligned} \tag{13}$$

where $A \equiv Y\tilde{Y}^\dagger$ and $B \equiv YY^\dagger$. Note that the invariant $\mathrm{Tr}\big[A - \text{h.c.}\big]$ vanishes identically for real N_{ij}. In terms of Y_{ij} and N_{ij}, these invariants can be expressed as

$$\begin{aligned} \mathrm{Im\ Tr}\big[A^2\big] &= 2(N_{12}N_{21} - N_{11}N_{22})\ \mathrm{Im}\Big(\ Y_{11}Y_{22}Y_{12}^*Y_{21}^*\ \Big) \ , \\ \mathrm{Im\ Tr}\big[AB\big] &= (N_{12} + N_{21} - N_{11} - N_{22})\ \mathrm{Im}\Big(\ Y_{11}Y_{22}Y_{12}^*Y_{21}^*\ \Big) \ . \end{aligned} \tag{14}$$

Note the appearance of the reparametrization invariant quantity $\mathrm{Im}\big(\ Y_{11}Y_{22}Y_{12}^*Y_{21}^*\ \big)$. If it is zero, all CP odd invariants vanish. Since there is only one independent CP phase, the vanishing of one CP odd invariant in the non-degenerate case guarantees that there is no CP violation.

In the degenerate case, i.e. when there are special relations among N_{ij}'s or eigenvalues, the situation is more subtle. For example, $\mathrm{Im\ Tr}\big[A^2\big]$ vanishes if $\mathrm{Det}\ N = 0$. Yet, $\mathrm{Im\ Tr}\big[AB\big]$ can be non-zero. However, if both vanish, no CP violation is possible. To see this, note that $\mathrm{Det}\ N = 0$ means that the columns (or rows) of N are linearly dependent, which in conjunction with $N_{12} + N_{21} - N_{11} - N_{22} = 0$ implies that N has the form

$$N = \begin{pmatrix} N_1 & N_2 \\ N_1 & N_2 \end{pmatrix} \ , \tag{15}$$

up to a transposition. Then, $\tilde{Y}$ factorizes as

$$\tilde{Y} = Y\ \mathrm{diag}(N_1, N_2) \ . \tag{16}$$

[2] This is analogous to constructing gauge invariant operators [12]. For instance, in the SM the flavor group $\mathrm{U}(3)_L \times \mathrm{U}(3)_{R_u} \times \mathrm{U}(3)_{R_d}$ can be gauged with Y^u, Y^d transforming as bifundamentals. A choice of Y^u, Y^d breaks this symmetry *à la* Higgs, with 26 degrees of freedom being eaten by the $\mathrm{SU}(3)^3 \times \mathrm{U}(1)^2$ gauge bosons (one U(1) is decoupled). The remaining 10 represent the observable masses, mixing angles and the CP phase. This also gives the dimension of the moduli space in the corresponding SUSY gauge theory.

In the mass eigenstate basis (11), it has the form

$$\tilde{Y} \;\rightarrow\; \mathrm{diag}(y_1, y_2)\; U_R^\dagger\, \mathrm{diag}(N_1, N_2)\; U_R \;. \tag{17}$$

Since $U_R^\dagger \mathrm{diag}(N_1, N_2) U_R$ is hermitian and $y_{1,2}$ real, the only phase of the resulting matrix can be removed by phase redefinition with $U_L = U_R = \mathrm{diag}(e^{i\delta_1}, e^{i\delta_2})$. In other words, Arg $\tilde{y}_{11}$, Arg $\tilde{y}_{22}$ and Arg $\tilde{y}_{12}\tilde{y}_{21}$ all vanish. Since the flavor objects are real in this basis, all possible CP violating invariants vanish.

The reparametrization invariant $\mathrm{Im}\big(\; Y_{11}Y_{22}Y_{12}^*Y_{21}^* \;\big)$ can vanish due to hidden symmetries. In the mass eigenstate basis, it can be written as

$$\mathrm{Im}\big(\; Y_{11}Y_{22}Y_{12}^*Y_{21}^* \;\big) = y_1 y_2 (y_1^2 - y_2^2)\; \mathrm{Im}\big(U_{L_{11}}^* U_{L_{12}} U_{R_{11}} U_{R_{12}}^*\big) \;, \tag{18}$$

where $y_{1,2}$ are the eigenvalues of Y and $U_L^\dagger Y U_R = \mathrm{diag}(y_1, y_2)$. It is then clear that there can be no CP violation if there is a massless eigenstate or degenerate spectrum. In the latter case, there is an extra U(2) symmetry which eliminates the CP phase. Similarly, there is an extra U(1) associated with phase redefinition of the massless state. This is qualitatively different from CP violation in the Standard Model. Recall that only in the degenerate (and not in the massless) case can one rotate away the CKM phase. This has to do with the fact that CP violation in the SM is associated with the relative phases in $Y^u Y^{u\dagger}$ and $Y^d Y^{d\dagger}$ which both transform under U_L, whereas in our case CP violation is due to the phases between Y and $\tilde{Y}$ which transform under biunitary transformations U_L and U_R.

4 Generalizations

4.1 3 flavor case

Although CP violation comes predominantly from the mixing of 2 flavor states, it is instructive to consider the 3 flavor case. The Yukawa matrix has 9 phases, 5 of which can be eliminated by quark phase redefinitions leaving 4 physical phases. These can be chosen as

$$\mathrm{Arg}\big(\; Y_{ij}Y_{i+1,j+1}Y_{i+1,j}^*Y_{i,j+1}^* \;\big) \tag{19}$$

with $i, j = 1, 2$. The corresponding weak basis invariants can be taken to be

$$\begin{aligned} &\mathrm{Tr}\big[A^k - \mathrm{h.c.}\big] \;, \\ &\mathrm{Tr}\big[A^l B^m - \mathrm{h.c.}\big] \;, \end{aligned} \tag{20}$$

with $A \equiv Y\tilde{Y}^\dagger$ and $B \equiv YY^\dagger$ and integer k, l, m ($k > 1$). In the non-degenerate case, the vanishing of 4 independent invariants would ensure absence of CP violation.[3]

[3]The resulting equations are non-linear in CP phases and may have spurious solutions for special values of the mixing angles. Here we ignore this possibility (for a related discussion, see [13]).

The degenerate case is rather complicated. For a special class of N-matrices, no CP violation is possible. The analog of Eq. (15) is

$$N = \begin{pmatrix} N_1 & N_2 & N_2 \\ N_1 & N_2 & N_2 \\ N_1 & N_2 & N_2 \end{pmatrix} , \tag{21}$$

up to a transposition and permutations of the columns. In this case, $\tilde{Y}$ has the following form in the mass eigenstate basis:

$$\tilde{Y} \;\rightarrow\; \mathrm{diag}(y_1, y_2, y_3)\; U_R^\dagger\; \mathrm{diag}(N_1, N_2, N_2)\; U_R \; . \tag{22}$$

The only reparametrization invariant phase of the hermitian matrix $U_R^\dagger \mathrm{diag}(N_1, N_2, N_2) U_R$ can be removed due to the U(2) symmetry of the lower 2×2 block. Note that it is not sufficient to have a rank 1 structure and 2 columns of N must be identical to ensure absence of CP violation. Unlike in the 2×2 case, it is not clear what is the minimal set of CP odd invariants, vanishing of which would ensure absence of CP violation since the resulting equations are highly non-linear in N_{ij}.

4.2 Inclusion of up- and down-sectors

4.2.1 2 generations

A different class of CP violating phases result from an interplay of the up- and down-sectors with the symmetry group $\mathrm{U}(2)_L \times \mathrm{U}(2)_{R_u} \times \mathrm{U}(2)_{R_d}$. In our framework, we have 4 flavor objects $Y^u, \tilde{Y}^u, Y^d, \tilde{Y}^d$ with the following transformation properties :

$$\begin{aligned} &Y^u \rightarrow U_L^\dagger \; Y^u \; U_{R_u} \quad , \quad \tilde{Y}^u \rightarrow U_L^\dagger \; \tilde{Y}^u \; U_{R_u} \; , \\ &Y^d \rightarrow U_L^\dagger \; Y^d \; U_{R_d} \quad , \quad \tilde{Y}^d \rightarrow U_L^\dagger \; \tilde{Y}^d \; U_{R_d} \; , \end{aligned} \tag{23}$$

as required by the $\mathrm{SU}(2)_L$ symmetry. Out of these matrices one can form various objects that transform under one of the symmetries. For instance, $Y^{u\dagger} Y^u$, $Y^{u\dagger} \tilde{Y}^u$ and $\tilde{Y}^{u\dagger} \tilde{Y}^u$ all transform under U_{R_u}. Their misalignment results in the CP phase studied in the previous section. On the other hand, quantities transforming under U_L involve both up- and down- sectors: $Y^u Y^{u\dagger}$, $Y^d Y^{d\dagger}$, etc. They are responsible for the extra CP phases.

Consider our "symmetric" basis (7). 5 out of 8 phases in Y^u, Y^d can be eliminated by

$$U_L = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}) \; , \; U_{R_u} = \mathrm{diag}(e^{i\beta_{1u}}, e^{i\beta_{2u}}) \; , \; U_{R_d} = \mathrm{diag}(e^{i\beta_{1d}}, e^{i\beta_{2d}}) \; . \tag{24}$$

The 3 physical phases can be chosen as

$$\phi_u \;\; = \;\; \mathrm{Arg}\Big(\; Y^u_{11} Y^u_{22} Y^{u*}_{12} Y^{u*}_{21} \; \Big) \; ,$$

$$\begin{aligned}\phi_d &= \mathrm{Arg}\Big(Y^d_{11}Y^d_{22}Y^{d*}_{12}Y^{d*}_{21} \Big) , \\ \phi &= \mathrm{Arg}\Big(Y^u_{11}Y^{u*}_{21}Y^{d*}_{11}Y^d_{21} \Big) . \end{aligned} \tag{25}$$

The phase ϕ is a new object resulting from a misalignment of the two sectors. The corresponding CP violating basis invariants (in a non-degenerate case) are

$$\begin{aligned}&\mathrm{Tr}\big[(Y^u\tilde{Y}^{u\dagger})^2 - \mathrm{h.c.}\big] , \\ &\mathrm{Tr}\big[(Y^d\tilde{Y}^{d\dagger})^2 - \mathrm{h.c.}\big] , \\ &\mathrm{Tr}\big[Y^uY^{u\dagger}, \tilde{Y}^u\tilde{Y}^{u\dagger}, Y^dY^{d\dagger}\big] , \end{aligned} \tag{26}$$

where $[A, B, C]$ denotes a completely antisymmetric product of A, B and C. While the first two invariants are proportional to $\sin\phi_u$ and $\sin\phi_d$, the last one is sensitive to $\sin\phi$. Invariants of this type have appeared before in the context of supersymmetry [7]. Note that there is no Jarlskog-type invariant since $\mathrm{Tr}[Y^uY^{u\dagger}, Y^dY^{d\dagger}]^3 = 0$ for 2 generations.

An explicit calculation gives

$$\begin{aligned}\mathrm{Tr}\big[Y^uY^{u\dagger}, \tilde{Y}^u\tilde{Y}^{u\dagger}, Y^dY^{d\dagger}\big] &= ia\sin\phi_u + ib\sin\phi + ic\sin(\phi+\phi_d) \\ &+ id\sin(\phi-\phi_u) + ie\sin(\phi+\phi_d-\phi_u) , \end{aligned} \tag{27}$$

where

$$\begin{aligned}a &= 6f(Y^d)\ (N_{12}N_{22} - N_{11}N_{21})\ \big|Y^u_{11}Y^u_{12}Y^u_{21}Y^u_{22}\big| , \\ b &= 6\Big(f(Y^u)N_{11}N_{21} - f(\tilde{Y}^u)\Big)\ \big|Y^u_{11}Y^u_{21}Y^d_{11}Y^d_{21}\big| , \\ c &= 6\Big(f(Y^u)N_{11}N_{21} - f(\tilde{Y}^u)\Big)\ \big|Y^u_{11}Y^u_{21}Y^d_{12}Y^d_{22}\big| , \\ d &= 6\Big(f(Y^u)N_{12}N_{22} - f(\tilde{Y}^u)\Big)\ \big|Y^u_{12}Y^u_{22}Y^d_{11}Y^d_{21}\big| , \\ e &= 6\Big(f(Y^u)N_{12}N_{22} - f(\tilde{Y}^u)\Big)\ \big|Y^u_{12}Y^u_{22}Y^d_{12}Y^d_{22}\big| , \end{aligned}$$

and $f(Y)$ is defined by

$$f(Y) \equiv |Y_{11}|^2 + |Y_{12}|^2 - |Y_{21}|^2 - |Y_{22}|^2 . \tag{28}$$

We see that this invariant is controlled by $\sin\phi$. In the non-degenerate case, the vanishing of the 3 invariants (26) implies absence of CP violation. The first 2 invariants are proportional to $\sin\phi_u$ and $\sin\phi_d$, respectively, while for $\phi_u = \phi_d = 0$, the last invariant is proportional to $\sin\phi$.

CP violation in this system exists even if the N-matrix has the degenerate form (15) in both sectors, as long $N_1 \neq N_2$. In this case, $\mathrm{Tr}\big[Y^uY^{u\dagger}, \tilde{Y}^u\tilde{Y}^{u\dagger}, Y^dY^{d\dagger}\big]$ generally does not vanish. There is no CP violation if N has equal matrix elements, as it should be, since all the coefficients a to e vanish.

It is instructive to consider the above invariant in the mass eigenstate basis. Diagonalizing $Y^u Y^{u\dagger} \to \mathrm{diag}(y_1^{u2}, y_2^{u2})$ and parametrizing $\tilde{Y}^u \tilde{Y}^{u\dagger} = U \mathrm{diag}(\tilde{y}_1^{u2}, \tilde{y}_2^{u2}) U^\dagger$, $Y^d Y^{d\dagger} = V \mathrm{diag}(y_1^{d2}, y_2^{d2}) V^\dagger$ in this basis, we have

$$\mathrm{Tr}\big[Y^u Y^{u\dagger}, \tilde{Y}^u \tilde{Y}^{u\dagger}, Y^d Y^{d\dagger}\big] = 6i(y_1^{u2} - y_2^{u2})(\tilde{y}_1^{u2} - \tilde{y}_2^{u2})(y_1^{d2} - y_2^{d2}) \mathrm{Im}\big(U_{11} U_{21}^* V_{11}^* V_{21}\big) \ . \quad (29)$$

More generally, for hermitian A, B and C, the invariant $\mathrm{Tr}[A, B, C]$ is proportional to the sine of $\mathrm{Arg}(B_{12} C_{12}^*)$, which is the only reparametrization invariant phase in this basis (the basis is defined up to $U_L = \mathrm{diag}(e^{i\delta_1}, e^{i\delta_2})$). If the invariant vanishes, the phase is zero (or can be rotated away) and no other CP violating invariant out of A, B and C can be constructed.

Consider the degenerate case. The above invariant vanishes if there are degenerate eigenvalues. In this case, the residual symmetry is U(2) instead of U(1) and two matrices can be diagonalized simultaneously. Therefore, all objects can be made real in this basis and all CP violating invariants vanish. Another possible degeneracy lies in the N-matrix. If all matrix elements of N are the same, two matrices can be diagonalized simultaneously and no CP violation occurs. This is not generally the case for N-matrices of the form (15) with $N_1 \neq N_2$, and there is CP violation.

We have so far considered the system of $Y^u Y^{u\dagger}, \tilde{Y}^u \tilde{Y}^{u\dagger}$ and $Y^d Y^{d\dagger}$. The discussion can be repeated for other choices of the 3 hermitian objects, including $\tilde{Y}^d \tilde{Y}^{d\dagger}$. It is easy to see that if both Y^u (or $\tilde{Y}^u$) and Y^d (or $\tilde{Y}^d$) have degenerate eigenvalues, no CP violation is possible. This is also the case when N in the up sector has identical matrix elements and, at the same time, N in the down sector has the same property.

4.2.2 3 generations

The generalization to the case of 3 generations is straightforward. The symmetry group is $\mathrm{U}(3)_L \times \mathrm{U}(3)_{R_u} \times \mathrm{U}(3)_{R_d}$. Out of 18 phases of $Y_{u,d}$, 3×3-1=8 can be eliminated, leaving 10 physical. 4+4=8 of them have the form (19), while the last two are analogs of ϕ in (25),

$$\mathrm{Arg}\Big(\ Y_{11}^u Y_{21}^{u*} Y_{11}^{d*} Y_{21}^d \ \Big) \quad , \quad \mathrm{Arg}\Big(\ Y_{22}^u Y_{32}^{u*} Y_{22}^{d*} Y_{32}^d \ \Big) \ . \quad (30)$$

In addition to the invariants (20) in each sector, one can take further two of the form

$$\mathrm{Tr}\big[A^k, B^l, C^m\big] \quad (31)$$

as the invariants sensitive to the above 2 phases. Here k, l, m are integer and A, B, C are hermitian matrices from the list $\{Y^u Y^{u\dagger}, \tilde{Y}^u \tilde{Y}^{u\dagger}, Y^d Y^{d\dagger}, \tilde{Y}^d \tilde{Y}^{d\dagger}\}$.[4] This completes the list of 10 relevant CP violating invariants in the non-degenerate case.

[4] One can form further hermitian matrices, but this list would suffice in the non-degenerate case.

Some discussion of the degenerate case can be found in Ref. [7]. Clearly, CP violation exists even if the N-matrices have identical elements. In this case there is a CKM phase and the Jarlskog invariant replaces (31). Also, there have to be many degenerate eigenvalues to eliminate CP violation. For example, even if Y^u and Y^d have all degenerate eigenvalues, one can still form the Jarlskog-type invariant $\mathrm{Tr}[\tilde{Y}^u\tilde{Y}^{u\dagger}, \tilde{Y}^d\tilde{Y}^{d\dagger}]^3$, which would not vanish in general. A complete study of the degenerate case is beyond the scope of this work.

5 Applications

In the Standard Model, CP violation is controlled by the Jarlskog invariant of order 12 in quark masses,

$$\begin{aligned} \mathrm{Im\ Tr}[Y^u Y^{u\dagger}, Y^d Y^{d\dagger}]^3 &= (y_u^2 - y_c^2)(y_c^2 - y_t^2)(y_t^2 - y_u^2)(y_d^2 - y_s^2)(y_s^2 - y_b^2)(y_b^2 - y_d^2) \\ &\times \ J \ \sim \ 10^{-22} \ , \end{aligned} \tag{32}$$

where $J \sim 10^{-5}$ is a combination of the CKM matrix entries. Its smallness is due to the fact that one must have at least 3 generations and both up- and down-sectors in order to have CP violation. In the context of electroweak baryogenesis, the Jarlskog invariant appears in the calculation of the CP asymmetry [14], with the factor $v^{12}/T^{12} \sim 1$, where $v = 174$ GeV and T is the temperature of the electroweak transition. One of the problems with the SM baryogenesis is that it is very difficult if not impossible to generate the observed baryon asymmetry $\eta \sim 10^{-10}$ out of such a small number (see however [15]).

If one allows for Higgs–dependent Yukawa couplings, the situation changes dramatically. CP violation exists already for 2 generations within a single (up- or down-) sector. The corresponding weak basis invariant is of order 4 in quark masses. Taking as an example a system of a top and an up quark, the invariant is given by

$$\begin{aligned} \mathrm{Im\ Tr}\left[\left(Y^u \tilde{Y}^{u\dagger}\right)^2\right] &= y_t y_u (y_u^2 - y_t^2) \\ &\times \ 2\ \mathrm{Det}N\ \mathrm{Im}\left(U^*_{L_{11}} U_{L_{12}} U_{R_{11}} U^*_{R_{12}}\right) \sim 10^{-9}\ \sin\delta \ , \end{aligned} \tag{33}$$

where δ is the relevant CP phase. It is more than 10 orders of magnitude larger than the Jarlskog invariant which is likely to be sufficient to generate the required baryon asymmetry.

Let us elaborate on the above calculation. To obtain this number, it is necessary to assume a specific Yukawa texture. Inspection of the texture (4) shows that there is no CP violation in the $\{t, c\}$ system because it falls into the "degenerate" category (15). However, in the $\{t, u\}$ system CP violation exists. The relevant 2×2 Yukawa texture and the N-matrix are

$$Y^u \sim \begin{pmatrix} \epsilon^3 & \epsilon^1 \\ \epsilon^2 & \epsilon^0 \end{pmatrix} \quad , \quad N = \begin{pmatrix} 7 & 3 \\ 5 & 1 \end{pmatrix} \quad . \tag{34}$$

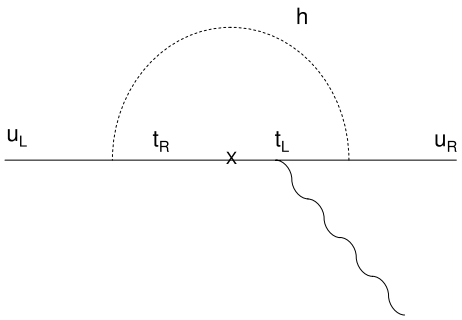

Figure 1: The leading contribution to the neutron EDM.

In the mass eigenstate basis, the matrix of the Higgs couplings $\tilde{Y}^u$ is not diagonal and contains complex phases. The diagonalizing matrices have the form

$$V_L \sim \begin{pmatrix} 1 & -\epsilon \\ \epsilon & 1 \end{pmatrix} \quad , \quad V_R \sim \begin{pmatrix} 1 & -\epsilon^2 \\ \epsilon^2 & 1 \end{pmatrix} \quad , \tag{35}$$

such that

$$\tilde{Y}^u \sim \begin{pmatrix} \epsilon^3 & \epsilon^1 \\ \epsilon^2 & \epsilon^0 \end{pmatrix} \tag{36}$$

in the mass eigenstate basis. Its off-diagonal elements carry order one phases. The combination $\mathrm{Im}\left(U^*_{L11} U_{L12} U_{R11} U^*_{R12}\right)$ is of order ϵ^3, which gives the estimate (33). Of course, the main difference between Eqs. (32) and (33) is the absence of the large quark mass suppression in the latter, which is independent of the Yukawa texture.

The increase in the amount of CP violation does not come for free. The same effect generates the neutron EDM at one loop. This places a constraint on the relevant CP phase. The leading contribution comes from the flavor off-diagonal $t-u$ interactions (Fig. 1), which induce the neutron EDM at order ϵ^3. A simple estimate shows that the reparametrization invariant CP phase $\mathrm{Arg}\left(\tilde{Y}^u_{12}\tilde{Y}^u_{21}\right)$ has to be smaller than 10^{-1}. (Of course, the estimate depends on the "order 1" coefficients in the texture and reducing the off-diagonal entries helps relax the bound). As a result, the phase δ in Eq.(33) cannot be greater than 10^{-1}. Nevertheless, the value of the invariant is still sufficiently large to be compatible with the observed baryon asymmetry. A similar in spirit study of EDMs versus EW baryogenesis can be found in [16].

One can also use other CP-violating invariants involving both up- and down- sectors. Consider 2 heavy generations. According to Eq.(29),

$$\mathrm{Im\ Tr}\left[Y^u Y^{u\dagger}, \tilde{Y}^u \tilde{Y}^{u\dagger}, Y^d Y^{d\dagger}\right] \sim y_t^4 y_b^2\ U_{21} V_{21}\ \ \sin\phi\ . \tag{37}$$

For our texture (4), this is of order $\epsilon^4 \sin\phi \sim 10^{-7} \sin\phi$. Note that CP violation exists in this system despite the degenerate N-matrix for the $t-c$ block. The corresponding CP phase ϕ is essentially unconstrained because the FCNC bounds from the heavy

quark systems are satisfied for any phase, while the EDM contribution comes at two loops. It is interesting that rephasing invariance requires interference with the SM contribution mediated by the W boson. In the mass eigenstate basis $Y^u = \mathrm{diag}(y_t, y_c)$, $Y^d = \mathrm{diag}(y_b, y_s)$, the CKM phase convention eliminates the residual phase symmetry $U_L^u = U_R^u$ and $U_L^d = U_R^d$. For example, consider the $t - c$ flavor change. While the Higgs exchange generates operators like $(\bar{t}_L c_R)^2$, the W exchange generates $(\bar{t}_L c_L)^2$. The physical phase between them is fixed by requiring real masses and real W-vertices. For the light generations, an analogous physical phase is constrained by ϵ_K [2].

An insufficient amount of CP violation is not the only obstacle for baryogenesis in the SM. The other problem is that it fails to provide a sufficiently strong first order phase transition, which would only be possible for an unacceptably light Higgs, $m_h < 72$ GeV. As a result, the baryon asymmetry is erased by the sphaleron processes. This statement is no longer true if there is a dimension six operator [17–19]

$$\Delta V = \frac{1}{\Lambda^2} \left(H^\dagger H - v^2\right)^3, \tag{38}$$

with $\Lambda \sim 1$ TeV. This operator changes the relation between the strength of the EW phase transition and the Higgs mass such that EW baryogenesis becomes possible. In our framework, such an operator is expected to be generated by integrating out TeV mass states, for instance, gauge singlets. The details depend on a particular UV completion of our effective theory, but the presence of the above operator is "decoupled" from the CP and flavor physics, and can safely be assumed.

To summarize, it appears that the framework of Higgs-dependent Yukawa couplings has the necessary ingredients to address the problem of baryogenesis. In particular, the Higgs interactions with the top and up quarks contain a sufficient amount of CP violation. A detailed study will be presented elsewhere.

6 Conclusion

The framework of Higgs-dependent Yukawa couplings allows one to eliminate small fundamental couplings from the Standard Model. In this work, I have analyzed the conditions for CP violation to occur in such a setup. In particular, I have identified a class of basis invariants responsible for CP violation. Unlike in the Standard Model, the CP symmetry can already be violated in a system of 2 quark species. The invariant measure of CP violation is found to be more than 10 orders of magnitude greater than that in the Standard Model. It is therefore plausible that this framework contains a sufficient amount of CP violation for successful electroweak baryogenesis.

Acknowledgements. I am grateful to L. Lebedeva for enlightening discussions and motivation.

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