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Study of the Higgs-Yukawa theory in the strong-Yukawa coupling regime *

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In this article, we present an ongoing lattice study of the Higgs-Yukawa model, in the regime of strong-Yukawa coupling, using overlap fermions. We investigated the phase structure in this regime by computing the Higgs vacuum expectation value, and by exploring the finite-size scaling behaviour of the susceptibility corresponding to the magnetisation. Our preliminary results indicate the existence of a second-order phase transition when the Yukawa coupling becomes large enough, at which the Higgs vacuum expectation value vanishes and the susceptibility diverges.

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1. Introduction

In recent years, there have been interests in the possible existence of heavy extra-generation fermions (mass $\geq$ 600 GeV) beyond the standard model (SM). Such heavy fermions are a consequence of strong-Yukawa couplings. Their presence in nature remains to be examined by experimental data that will be collected at the LHC. An important consequence of a 4th fermion generation is the substantial enhancement the amount of CP violation [1]. Large bare values of the Yukawa coupling may also lead to the formation of bound states which can replace the role of the Higgs boson in unitarising the WW scattering process [2, 3, 4]. Such a scenario is clearly of nonperturbative nature and motivates the use of lattice field theory as a first-principle and nonperturbative tool for this research avenue. Lattice investigations at small and moderate values of the bare Yukawa coupling showed [5] that the lower Higgs boson mass bound is strongly affected by the presence of a heavy 4th fermion generation when compared to results using a physical value of the top quark mass [6, 7]. Still, in these simulations no signs of bound states were observed, as expected.

However, lattice simulations have also revealed the existence of an interesting phase structure of the model at large values of the Yukawa coupling [8, 9, 10, 11, 12, 13, 14, 15]. These simulations, performed around 1990, were lacking an exact chiral symmetry on the lattice and the results of these works are therefore not easy to interpret and to connect to the SM. Recently, exact lattice chiral symmetry [16] were established and, in fact, lattice simulations employing this lattice chiral symmetry confirmed the phase structure at strong bare Yukawa coupling [17, 18] as found in the earlier studies.

In this work we further explore the phase structure of a lattice chirally invariant Higgs-Yukawa model at large values of the Yukawa coupling. Our main aim is to start a systematic investigation, whether the phase transitions between the symmetric phase with vanishing vacuum expectation value (VEV) $v = 0$ and the broken phase with $v > 0$ are governed by critical exponents that differ from the (Gaussian) one of the SM. This is a highly non-trivial question. In particular, it can be shown that the lattice Higgs-Yukawa model in the limit of infinite bare Yukawa coupling reduces to a pure scalar non-linear σ-model [8, 9, 10, 17, 18] with again Gaussian critical exponents. Hence, the most interesting and important question is, whether at large but finite value of the Yukawa-coupling a new and non-trivial universality class emerges. We emphasise that a quantitative determination of critical exponents of the phase transitions in the strong Yukawa coupling region using chiral invariant lattice fermions was never attempted before. The above potential of nonperturbative physics at large Yukawa couplings motivates clearly such an investigation.

2. Simulation details

The discretisation of the scalar field theory leads to the action (with lattice spacing a set to 1)

$$S_\varphi = -\sum_{x,\mu} \varphi_x^\alpha \varphi_{x+\hat{\mu}}^\alpha + \sum_x \left[\frac{1}{2}(2d + m_0^2)\varphi_x^\alpha \varphi_x^\alpha + \frac{1}{4}\lambda_0 (\varphi_x^\alpha \varphi_x^\alpha)^2 \right], \tag{2.1}$$

where α labels the four components of the scalar fields, d is the number of the space-time dimensions, m_0 is the bare mass and λ_0 is the bare quartic self-coupling. For practical lattice simulations, it is convenient to perform the change of variables

$$\varphi = \sqrt{2\kappa}\phi, \;\; m_0^2 = \frac{1-2\hat{\lambda}-2d\kappa}{\kappa}, \;\; \lambda_0 = \frac{\hat{\lambda}}{\kappa^2}, \tag{2.2}$$

where κ is the hopping parameter. This renders the lattice scalar field theory to the Ising form

$$S_\phi = -2\kappa \sum_{x,\mu} \phi_x^\alpha \phi_{x+\hat{\mu}}^\alpha + \sum_x \left[\phi_x^\alpha \phi_x^\alpha + \hat{\lambda}\,(\phi_x^\alpha \phi_x^\alpha - 1)^2 \right], \tag{2.3}$$

which is more suitable for exploring the phase structure.

In this work, we use the overlap operator $\mathscr{D}^{(\mathrm{ov})}$ in the lattice fermion action

$$S_F = \bar{\Psi}\mathscr{M}\Psi, \text{ where } \mathscr{M} = \mathscr{D}^{(\mathrm{ov})} + P_+ \, \Phi^\dagger \;\; \mathrm{diag}(y_{t'}, y_{b'}) \, \hat{P}_+ + P_- \;\; \mathrm{diag}(y_{t'}, y_{b'}) \, \Phi \, \hat{P}_-, \tag{2.4}$$

with

$$\Psi = \begin{pmatrix} t' \\ b' \end{pmatrix}, \text{ and } \Phi = \begin{pmatrix} \phi^2 + i\phi^1 \\ \phi^0 - i\phi^3 \end{pmatrix}. \tag{2.5}$$

The chiral projectors are defined as

$$P_\pm = \frac{1 \pm \gamma_5}{2}, \;\; \hat{P}_\pm = \frac{1 \pm \hat{\gamma}_5}{2}, \;\; \hat{\gamma}_5 = \gamma_5 \left(1 - \frac{1}{\rho} \mathscr{D}^{(\mathrm{ov})} \right), \tag{2.6}$$

where ρ is the radius of the circle of eigenvalues in the complex plane of the free overlap operator. We also set $y_{t'} = y_{b'} = y$, to ensure that the fermion determinants are positive definite.

Our simulations have been performed at two κ values, $\kappa = 0.00$ and 0.06, with the bare Yukawa coupling y in the range between 14 and 25. The bare scalar quartic coupling $\hat{\lambda}$ is fixed to infinity, which results in the largest possible Higgs mass [5, 6, 7]. We already accumulated data with reasonable statistics for the volumes $8^3 \times 16$, $12^3 \times 24$ and $16^3 \times 32$. In generating dynamical scalar field configurations, we use the polynomial HMC (pHMC) algorithm [19], treating the weight factor as an observable [20]. We have found that high degrees of polynomials (~ 180 for $16^3 \times 32$ lattices) are necessary in order to obtain reasonable statistical accuracy for the weight factor. For each set of the bare couplings, we carry out 1000 pHMC trajectories to precondition the fermion matrix and to thermalise the simulation. Our measurements are then performed on ~ 2000 thermalised trajectories.

3. The scalar vacuum expectation value

We first measure the scalar VEV to probe the phase structure. Our simulations have been performed without external sources, therefore a naive computation of the VEV would always lead to vanishing

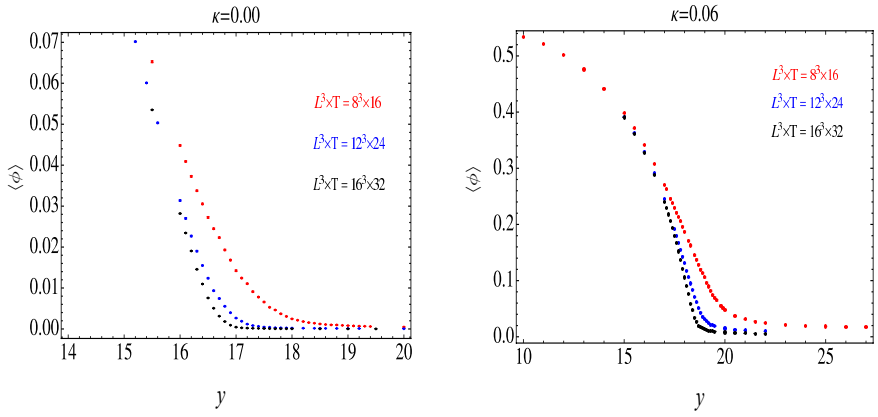

Figure 1: Bare scalar VEV at two κ values with $\hat{\lambda} = \infty$.

results in finite volume. In this work, we follow the procedure in Refs. [21, 22, 23] to project the scalar fields on the direction of magnetisation

$$m = \frac{1}{V_4}\left(\sum_{\alpha,x} |\phi_x^\alpha|^2\right)^{1/2} \quad (V_4 \text{ is the } 4-\text{dimensional volume}), \tag{3.1}$$

then compute the VEV. This "projected" VEV coincides with the scalar VEV in the infinite-volume limit, and its introduction in finite volume is equivalent to coupling the scalar fields to external sources [23].

The results for the bare VEV, computed from the above projection procedure, are shown in Fig. 1[1]. It is clear that there is a phase transition when y becomes large, at which the system enters a symmetric phase. These plots also indicate that the value of $y = y_{\text{crit}}$ at which the phase transition occurs grows with κ. This agrees with the qualitative predictions from the strong-coupling expansion [25], the large-N_f expansion [17] and an exploratory numerical study using overlap fermions [18].

4. Finite-size scaling of the magnetisation susceptibility

In order to determine the order of the phase transition observed in the last section, we investigate the finite-size scaling behaviour of the susceptibility corresponding to the magnetisation. It is defined as

$$\chi = V_4\left(\langle m^2\rangle - \langle m\rangle\langle m\rangle\right), \tag{4.1}$$

where V_4 and m are defined in Eq. (3.1). This quantity diverges at the critical points in the infinite-volume limit. Finite-size effects result in the crossover from this bulk behaviour to the finite-volume scaling behaviour in lattice calculations. In the vicinity of a would-be second-order phase

[1]We are currently computing the Goldstone wavefunction renormalisation to obtain the renormalised Higgs VEV [24].

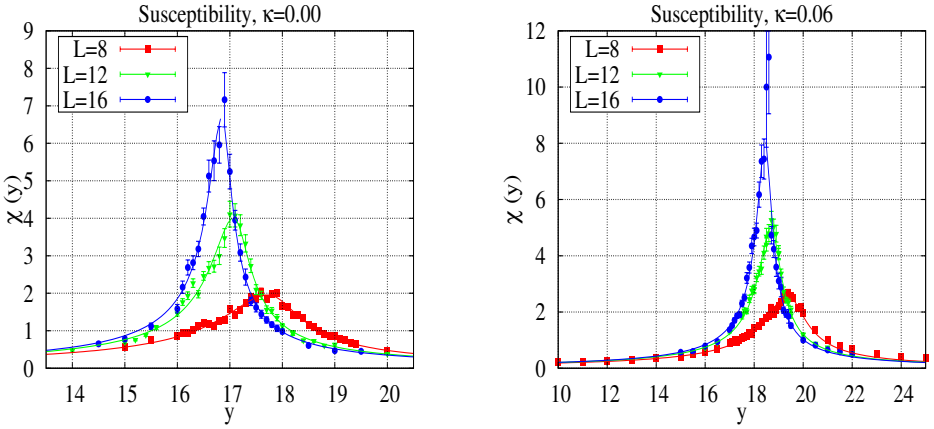

Figure 2: Susceptibility at two κ values at various volumes with $\hat{\lambda} = \infty$.

transition, the solution of the renormalisation group equation (RGE) in finite volume predicts [26]

$$\chi L_s^{-\gamma/\nu} = g(tL_s^{1/\nu}), \text{ with } t = (y/(y_{\text{crit}} - A_4/L_s^b) - 1), \tag{4.2}$$

where g is a universal function, L_s is the spatial volume, y_{crit} is the critical Yukawa coupling at which the phase transition occurs in the infinite-volume limit, A_4 is a phenomenological parameter, ν and γ are the universal critical exponents (anomalous dimensions), and b is the shift exponent [27].

For each of the two κ values in our simulations, we perform a simultaneous fit of the data for the susceptibility at all volumes, to the partly-empirical formula [28]

$$\chi = A_1 \left\{ L_s^{-2/\nu} + A_{2,3} \left(y - y_{\text{crit}} - A_4/L_s^b \right)^2 \right\}^{-\gamma/2}, \tag{4.3}$$

where $A_{1,2,3,4}$ are unknown phenomenological coefficients. They are determined, together with ν, γ, y_{crit} and b, from the fits. For our best procedure, the fit ranges of y are $(14.5, 19.5)$ for $\kappa = 0$, and $(14, 22)$ for $\kappa = 0.06$. The extracted y_{crit}, γ, ν and b are presented in Table 1. For comparison with the scaling behaviour in the regime where the Yukawa couplings are weak and perturbative, we also list the predictions from the mean-field calculation in the $O(4)$ scalar model. We use these fit results to obtain the susceptibility as a function of y according to Eq. (4.3). This is plotted with our data points in Fig. 2. We also use the same fit results to construct $\chi L_s^{-\gamma/\nu}$ and $tL_s^{1/\nu}$, to examine the finite-size scaling behaviour of Eq. (4.2). The outcome of this test is shown in Fig. 3.

Our data, as presented in Figs. 2 and 3, establish evidence for the existence of a second-order phase transition in the strong-Yukawa coupling regime. To further investigate the nature of this strong-Yukawa symmetric phase, we have to perform more detailed studies on the critical exponents and the spectrum. As demonstrated by the results collected in Table 1, γ is almost consistent with the corresponding mean-field prediction for the $O(4)$ scalar model, while ν exhibits a more significant deviation from it. By varying the fit ranges in y in a reasonable interval for the above finite-size scaling analysis, the shifts in y_{crit} and the critical exponent γ are not statistically distinguishable.

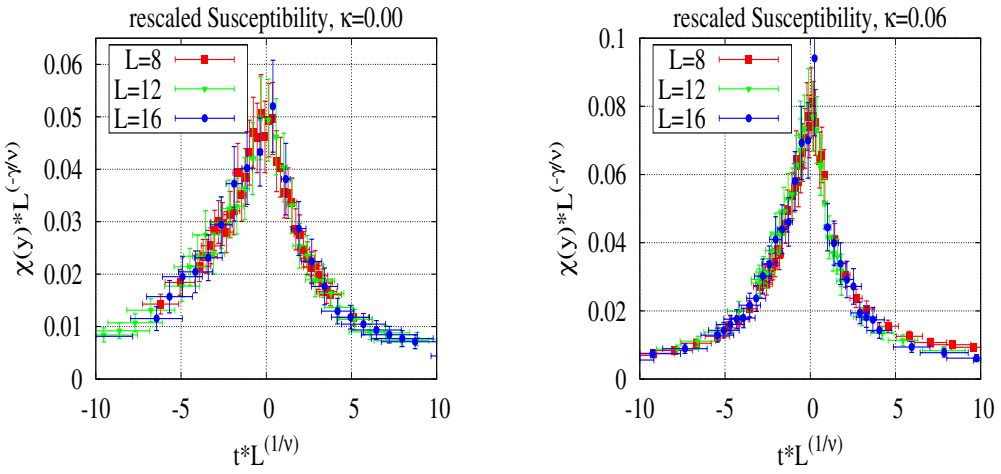

Figure 3: The scaling behaviour of susceptibility at two κ values with $\hat{\lambda} = \infty$.

On the other hand, ν can change by $\sim 11\%$ and become almost consistent with the value in the $O(4)$ scalar model. We have also tried incorporating logarithmic volume effects in Eq. (4.3) by replacing the coefficient A_4 with $A_4[1 + c\ \mathrm{Log}(L_s)]$ (c is an unknown parameter). This procedure leads to no statistically significant variation of the critical exponents, ν and γ.

5. Summary and outlook

In this article, we present an ongoing numerical investigation of the phase structure of the strong-Yukawa model on the lattice. Using overlap fermions which respect exact lattice chiral symmetry, and performing simulations at various volumes, enable us to explore the details of the phase structure. From our computation of the scalar VEV and the study of the finite-size scaling behaviour of the magnetisation susceptibility, we obtain strong evidence that there exists a symmetric phase in the strong-Yukawa coupling regime, and that the transition between this phase and the broken phase is of second-order nature. At the moment, we cannot determine if the critical exponents for this phase transition are different from those in the weak-Yukawa regime.

We are currently generating large lattices ($24^3 \times 48$) which will allow us to have more precise extraction of the critical exponents. These lattices will also enable us to control the infinite-volume extrapolations in our future calculation for spectral quantities.

	$\kappa = 0.00$	$\kappa = 0.06$	O(4) scalar model
y_{crit}	16.57 ± 0.06	18.11 ± 0.06	N/A
γ	1.02 ± 0.02	1.08 ± 0.01	1
ν	0.57 ± 0.03	0.66 ± 0.02	0.5
b	2.05 ± 0.20	2.04 ± 0.20	N/A

Table 1: The critical Yukawa coupling y_{crit}, the critical exponents γ, ν, and the shift exponent b determined from the best fits to the finite-volume scaling function. The errors are statistical only. Predictions from the mean-field calculation in the $O(4)$ scalar model are also listed.

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