Dirac tensor with heavy photon

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Abstract

For the large-angles hard photon emission by initial leptons in process of high energy annihilation of $e^+e^- \to$ to hadrons the Dirac tensor is obtained, taking into account the lowest order radiative corrections. The case of large-angles emission of two hard photons by initial leptons is considered. This result is being completed by the kinematics case of collinear hard photons emission as well as soft virtual and real photons and can be used for construction of Monte-Carlo generators.

Key words: tensor, photon emission

1 Introduction

The problem of precise knowledge of the cross section of annihilation e^+e^- to hadrons caused by the long staying problem of theoretical estimation of muon anomalous magnetic moment $g-2$ [1]:

$$a_\mu^{\mathrm{hadr}} = \left(\frac{g-2}{2}\right)_\mu = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int\limits_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s}{s} R(s) K^{(1)}(s), \tag{1}$$
$$R(s) = \frac{\sigma^{e\bar{e}\to \mathrm{hadr}}}{\sigma^{e\bar{e}\to\mu\bar{\mu}}}, \quad K^{(1)}(s) = \int \mathrm{d}x \frac{x^2(1-x)}{x^2+\rho(1-x)}, \; \rho = \frac{s}{m_\mu^2}.$$

Extraction of cross-section $e^+e^- \to \gamma^* \to hadrons$ from experimental data is one of the main problem of modern experimental physics. The Monte-Carlo

Preprint submitted to Elsevier *13 December 2011*

programs creation which takes into account the emission of real photons by the initial leptons is the motivation of this paper.

Dirac tensor (cross-symmetry partner of Compton tensor) i.e. the bilinear combination of the currents of hard photon emission averaged on leptons spin states and summed on photon polarization states takes the contribution on Born level and the ones arising from 1-loop correction. Infrared divergences are parametrized by the introduction "photon mass" λ. In the final answer it is removed in a usual way by adding the contribution from additional soft photon emission.

We don't consider photon emission by the final charged particles as well as the effects of charge-add interference of emission of virtual or real photon emission from leptons and hadrons. So the Dirac tensor obtained in such way is universal.

The paper is organized as follow. In the part 2 we wrote down the contribution of Dirac tensor in terms only corrections associated with the positron legs. In part 3 we obtain the contribution arising from mass operator of positron and vertex function for the case when positron and photon are on mass shell. In the part 4 we consider the contribution from vertex function for the case of electron on mass shell and the box-type Feynman amplitude with electron, positron and one of photons on mass shell.

In section 5 we analyze the total result for Dirac tensor, adding the emission of additional soft photon contributions, which provide the infrared divergences free final result.

We put the form of hadronic tensor for several final states: $\gamma^* \to \pi^+\pi^-$, $\mu^+\mu^-$, $\rho^+\rho^-$. In Appendixes A and B the details of calculation are presented. In Appendix C the contribution to Dirac tensor for the case of two hard photon emission is given.

2 General analysis

The Born level matrix element of hard photon emission by initial leptons in process of annihilation e^+, e^- to hadrons through the single virtual photon intermediate state

$$e^+(p_+) + e^-(p_-) \to \gamma^*(q) + \gamma(k_1) \to \gamma(k_1) + h(q) \qquad (2)$$

have a form (see Fig. (1))

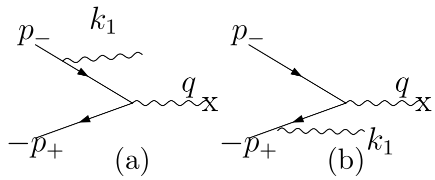

Fig. 1. Diagrams contributing in Born level.

$$M = \frac{(4\pi\alpha)^{3/2}}{q^2}\bar{v}(p_+)O_\rho^{(B)}u(p_-)H_\rho(q), \tag{3}$$
$$O_\rho^{(B)} = \gamma_\rho \frac{\hat{p}_- - \hat{k}_1}{-\chi_-}\hat{e} + \hat{e}\frac{-\hat{p}_+ + \hat{k}_1}{-\chi_+}\gamma_\rho,$$

where $\hat{e}(k_1)$ is polarization vector of the real photon. The $H_\rho(q)$ is the current describing the conversation of virtual photon whith momentum q to hadronic state. We will restrict ourselves by kinematics conditions of large-angles scattering.

$$s = 2p_+p_-, \quad \chi_\pm = 2k_1p_\pm, \quad k_1^2 = 0, \quad p_\pm^2 = m^2, \tag{4}$$
$$s - \chi_+ - \chi_- = q^2, \quad q^2 > 0, \quad \sim q^2 \sim \chi_+ \sim \chi_- \gg m^2.$$

In expressions below we put $m = 0$ everywhere except the denominators of loops integrals.

Cross section can be expressed in terms of the summed on spin states of the module of matrix element square:

$$\sum_{spin} |M|^2 = (4\pi\alpha)^3 \frac{4B_{\rho\rho_1}H_{\rho\rho_1}}{(q^2)^2},$$
$$B_{\rho\rho_1} = \frac{1}{4}\mathrm{Tr}\, \hat{p}_+ O_\rho \hat{p}_- \bar{O}_{\rho_1},$$
$$H_{\rho\rho_1} = \sum_{spin} H_\rho(q)H^*_{\rho_1}(q). \tag{5}$$

The differential cross-section can be written as:

$$\mathrm{d}\sigma = \frac{1}{8s}\sum |M|^2 \frac{\mathrm{d}^3k}{2\omega(2\pi)^3}\mathrm{d}\Gamma_f,$$
$$\mathrm{d}\Gamma_f = (2\pi)^4\delta^4\Big(p_+ + p_- - k_1 - \sum_f q_i\Big)\prod_f \frac{\mathrm{d}^3q_i}{2\varepsilon_i(2\pi)^3}. \tag{6}$$

For differential hard photon cross section we obtain

$$\frac{\omega\, \mathrm{d}\sigma^{e^+e^-\to\gamma X}}{\mathrm{d}^3k_1\mathrm{d}\Gamma_f} = \frac{2\alpha^3}{(q^2)^2} H_{\rho\rho_1} B_{\rho\rho_1}, \tag{7}$$

where for the case of Born level

$$B_{\rho\rho_1} = B^{(0)}_{\rho\rho_1} = B_g\tilde{g}_{\rho\rho_1} + B_{++}\tilde{p}_{+\rho}\tilde{p}_{-\rho_1} + B_{--}\tilde{p}_{-\rho}\tilde{p}_{-\rho_1} + B_{\pm}(\tilde{p}_{+\rho}\tilde{p}_{-\rho_1} + \tilde{p}_{+\rho_1}\tilde{p}_{-\rho}). \tag{8}$$

The quantities with the "tilde" are defined as

$$\tilde{g}_{\rho\rho_1} = g_{\rho\rho_1} - \frac{1}{q^2} q_\rho q_{\rho_1}, \tag{9}$$
$$\tilde{p}_{\pm\rho} = p_{\pm\rho} - q_\rho \frac{p_\pm q}{q^2}.$$

In the Born approximation (see Fig. (1)) we have

$$B_g = \frac{1}{\chi_+\chi_-}(2sq^2 + \chi_+^2 + \chi_-^2), \tag{10}$$
$$B_{++} = B_{--} = \frac{4q^2}{\chi_+\chi_-}, \quad B_\pm = 0.$$

For $q^2 = 0$ we reproduce the Dirac cross-section of $e^-e^+ \to \gamma\gamma$:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\mathcal{O}_1} = \frac{2\alpha^2}{s}\frac{\chi_+^2 + \chi_-^2}{\chi_+\chi_-}. \tag{11}$$

Below we concentrate on calculation of one-loop radiative correction to Dirac tensor is

$$B_{\rho\rho_1} \to B^{(0)}_{\rho\rho_1} + (\alpha/\pi)T^{(1)}_{\rho\rho_1}. \tag{12}$$

Let us now show that in considering of the corrections one can restrict only by half of full set of Feynman diagrams for considering process (2). So we put $O_\rho = O_\rho^- + O_\rho^+$ separating the contribution of emission from electron leg O_ρ^- and positron one O_ρ^+ (see Fig.(1) for the Born case and Fig.(2) for the 1-loop corrections).

One can show that using the cyclic property of the trace as well as the mirror property

$$\mathrm{Tr}\,\hat{a}_1\hat{a}_2\cdots\hat{a}_{2n} = \mathrm{Tr}\,\hat{a}_{2n}\cdots\hat{a}_2\hat{a}_1,$$

that the total contribution to the Dirac leptonic tensor can be written as:

$$\mathrm{Tr}\,\hat{p}_+ O^{(1)}_\rho \hat{p}_- \bar{O}^B_{\rho_1} + \mathrm{Tr}\,\hat{p}_+ O^B_\rho \hat{p}_- \bar{O}^{(1)}_{\rho_1}$$
$$= (1+\Delta_{\rho\rho_1})(1+\mathcal{P})\mathrm{Tr}\,\hat{p}_+ O^+_\rho \hat{p}_- \bar{O}^B_{\rho_1}. \quad (13)$$

Here the exchange operations acting as

$$\Delta_{\rho\rho_1} F_{\rho\rho_1} = F_{\rho_1\rho}, \quad (14)$$
$$\mathcal{P}F(p_+, p_-, k_1) = F(-p_-, -p_+, -k_1), \quad (15)$$
$$\mathcal{P}F(s, q^2, \chi_+, \chi_-) = F(s, q^2, \chi_-, \chi_+) \equiv \tilde{F}.$$

Here and below we imply the real part of leptonic tensor.

3 One-loop corrections

The virtual correction of lowest order is described by 8 Feynman diagrams shown on the Fig.(3).

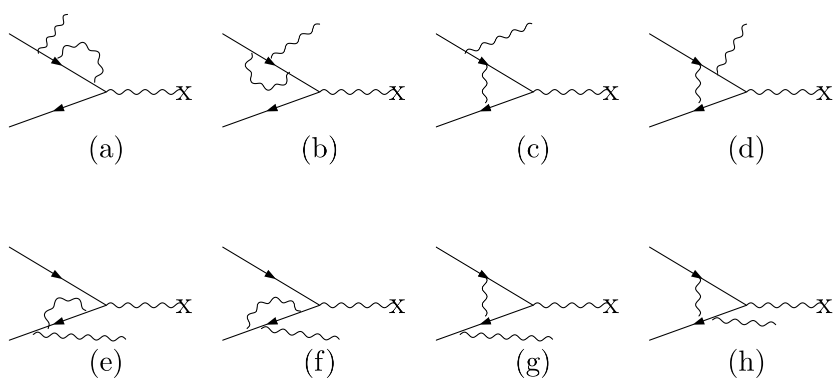

Fig. 2. Diagrams contributing in 1-loop level.

Let us distinguish contribution of FD Fig. (3(e-h)) to 3 classes

$$\mathrm{T}r\,\hat{p}_+ O^+_\rho p_- \bar{O}_\rho = T^{\mathrm{box}}_{\rho\rho_1} + T^{\mathrm{verx}}_{\rho\rho_1} + T^{\Sigma}_{\rho\rho_1} \quad (16)$$

with T^{box} and T^{verx} correspond to Fig. (3 (h,g)) and T^{Σ} to Fig. (3 e,f).

Consider first the contribution to matrix element arising from Feynman diagram Fig. 3 e,f.

Matrix element of FD Fig. (3 (e)) contain the mass operator of electron $\Sigma(\hat{p})$. In kinematics conditions of our problem ($\chi_+ \gg m^2$) we obtain [BLP]:

$$M_e = \frac{\alpha}{2\pi}(\frac{3}{2} + \frac{1}{2}l_+ - l_\lambda)\bar{v}(p_+)\hat{e}\Big(\frac{\hat{p}_+ + \hat{k}_1}{-\chi_+}\Big)\gamma_\rho u(p_-), \tag{17}$$

$$l_+ = \ln\frac{\chi_+}{m^2},\ l_\lambda = \ln\frac{m^2}{\lambda^2},$$

where λ is the so-called "photon mass".

Matrix elements of FD Fig. (3 (f)) contain the vertex function with real photon [3].

$$M_f = \frac{\alpha}{4\pi}\bar{v}(p_+)\int\frac{\mathrm{d}k\gamma^\lambda(-\hat{p}_+ - \hat{k})\hat{\mathrm{e}}(-\hat{p}_+ + \hat{k}_1 - \hat{k})\gamma^\lambda}{(0)(\bar{2})(q)} \tag{18}$$

$$\left(\frac{-\hat{p}_+ + \hat{k}_1}{-\chi_+}\right)\gamma_\rho u(p_-).$$

We use here the notations

$$\mathrm{d}k = \frac{\mathrm{d}^4k}{i\pi^2},\ (0) = k^2,\ (\bar{2}) = (p_+ + k)^2 - m^2,\ (q) = (-p_+ + k_1 - k)^2 - m^2. \tag{19}$$

Using the relevant loop integrals, obtained in [2] (see Appendix) we have a matrix elements of FD Fig. (3(f)) which contain the vertex function with real photon [3].

$$M_f = \frac{\alpha}{2\pi}\bar{v}(p_+)[-\frac{1}{\chi_+}(l_+ - \frac{1}{2})\hat{k}_1\hat{e}\hat{p}_+ \tag{20}$$

$$+\hat{e}(l_\lambda - \frac{1}{2}l_+ - \frac{3}{2})]\frac{-\hat{p}_+ + \hat{k}_1}{-\chi_+}\gamma_\rho u(p_-) \tag{21}$$

As a result we obtain the infra-red free and gauge-invariant expression:

$$M_e + M_f = \frac{\alpha}{\pi}\Phi_+\bar{v}(p_+)\hat{k}_1\hat{e}\gamma_\rho u(p_-),\ \Phi_+ = \frac{1}{2\chi_+}(l_+ - \frac{1}{2}). \tag{22}$$

Inserting this expression to the relevant part of O^+_ρ we obtain for $T^\Sigma_{\rho\rho_1}$

$$T^\Sigma_{\rho\rho_1} = \Phi_+ \mathrm{Tr}\,\hat{p}_+\hat{k}_1\gamma_\lambda\gamma_\rho\hat{p}_-\Big[\frac{1}{\chi_-}\gamma_\lambda(\hat{p}_- - \hat{k}_1)\gamma_{\rho_1} + \frac{1}{\chi_+}\gamma_{\rho_1}(-\hat{p}_+ + \hat{k}_1)\gamma_\lambda\Big], \tag{23}$$

where we used relation $k_{1\rho} = (p_+ + p_-)_\rho$ keeping mind the gauge invariance of hadronic tensor $q_\mu H_{\mu\nu} = 0$. Expression (23) leads to the follow form for contributions of diagramms Fig. (3(e,f)):

$$T^{\Sigma}_{\rho\rho_1} = 4\Phi_+[2p_{-\rho}p_{-\rho_1}(\frac{q^2}{\chi_-} - 1) + 2p_{-\rho}p_{+\rho_1}(\frac{s}{\chi_-} - 1) - (s - \chi_-)g_{\rho\rho_1}]. \quad (24)$$

Applying the operation $\Delta_{\rho\rho_1}$ and $\mathcal{P}$ we obtain the full result:

$$(1 + \Delta_{\rho\rho_1})(1 + \mathcal{P})T^{\Sigma}_{\rho\rho_1} = [2Q^{\Sigma}_{-}p_-p_- + 2\tilde{Q}^{\Sigma}_{-}p_+p_+ + (Q^{\Sigma}_{\pm} + \tilde{Q}^{\Sigma}_{\pm})(p_+p_- + p_-p_+) \quad (25)$$
$$+(2Q^{\Sigma}_{g} + 2\tilde{Q}^{\Sigma}_{g})g]_{\rho\rho_1},$$

where

$$Q^{\Sigma}_{-} = 8\Phi_+\frac{q^2 - \kappa_-}{\kappa_-};$$
$$Q^{\Sigma}_{\pm} = 8\Phi_+\frac{s - \kappa_-}{\kappa_-};$$
$$Q^{\Sigma}_{g} = 8\Phi_+(s - \kappa_-). \quad (26)$$

4 Vertex and box type diagram contributions

Contribution of Fig. (3(g,h)) can be written as

$$T^{\text{box}}_{\rho\rho_1} + T^{vert}_{\rho\rho_1} = \frac{\alpha}{\pi}\Big[\frac{S_1}{\chi_-} + \frac{S_2}{\chi_+} - \frac{C_1}{\chi_-\chi_+} - \frac{C_2}{\chi_+^2}\Big], \quad (27)$$

with

$$S_1 = \int \frac{\mathrm{d}k}{(0)(2)(\bar{2})(q)}\frac{1}{4}\mathrm{Tr}\,\hat{B}\hat{p}_-\gamma_\eta(\hat{p}_- - \hat{k}_1)\gamma_{\rho_1}, \quad (28)$$
$$S_2 = \int \frac{\mathrm{d}k}{(0)(2)(\bar{2})(q)}\frac{1}{4}\mathrm{Tr}\,\hat{B}\hat{p}_-\gamma_{\rho_1}(-\hat{p}_+ + \hat{k}_1)\gamma_\eta,$$
$$C_1 = \int \frac{\mathrm{d}k}{(0)(2)(q)}\frac{1}{4}\mathrm{Tr}\,\hat{V}\hat{p}_-\gamma_\eta(\hat{p}_- - \hat{k}_1)\gamma_{\rho_1},$$
$$C_2 = \int \frac{\mathrm{d}k}{(0)(2)(q)}\frac{1}{4}\mathrm{Tr}\,\hat{V}\hat{p}_-\gamma_{\rho_1}(-\hat{p}_+ + \hat{k}_1)\gamma_\eta$$
$$\hat{B} = \hat{p}_+\gamma_\lambda(-\hat{p}_+ - \hat{k})\gamma_\eta(-\hat{p}_+ + \hat{k}_1 - \hat{k})\gamma_\rho(\hat{p}_- - \hat{k})\gamma_\lambda,$$
$$\hat{V} = \hat{p}_+\gamma_\eta(-\hat{p}_+ + \hat{k}_1)\gamma_\lambda,(-\hat{p}_+ + \hat{k}_1 - \hat{k})\gamma_\rho(\hat{p}_- - \hat{k}_1)\gamma_\lambda.$$

Using the loop integrals listed in Appendix A we obtain:

$$(T^{\rm box}+T^{vert})_{\rho\rho_1} = A^+ p_{+\rho}p_{+\rho_1} + B^- p_{-\rho}p_{-\rho_1} + C^{\pm} p_{+\rho}p_{-\rho_1} + D^{\mp} p_-^{\rho} p_{+\rho_1} + E^g g_{\rho\rho_1}, \qquad (29)$$

with explicit expression for A-E given in Appendix B. Applying exchange operator $1+\Delta_{\rho\rho_1}$ we obtain

$$(1+\Delta_{\rho\rho_1})(T^{\rm box}+T^{\Sigma}+T^{\rm vert})_{\rho\rho_1} = 2A^+ p_{+\rho}p_{+\rho_1} + (2B^- + 2Q_-^{\Sigma})p_{-\rho}p_{-\rho_1} + (C^{\pm}+D^{\mp}+Q_{\pm}^{\Sigma})(p_{+\rho}p_{-\rho_1}+p_{-\rho}p_{+\rho_1}) + (2E^g + 2Q_g^{\Sigma})g^{\rho\rho_1}. \qquad (30)$$

Applying at least the operation $(1+\mathcal{P})$ (Eq. 16) we obtain the final expression for Dirac tensor with heavy photon for annihilation channel $B_{\rho\rho_1} = B^0_{\rho\rho_1} + \frac{\alpha}{\pi} T_{\rho\rho_1}$ with

$$T_{\rho\rho_1} = T_{--}\tilde{p}_{-\rho}\tilde{p}_{-\rho_1} + \tilde{T}_{--}\tilde{p}_{+\rho}\tilde{p}_{+\rho_1} + (\tilde{p}_{+\rho}\tilde{p}_{-\rho_1} + \tilde{p}_{-\rho}\tilde{p}_{+\rho_1})T_{+-} + T_g \tilde{g}_{\rho\rho_1}, \qquad (31)$$

with

$$\begin{aligned} T_{++} &= 2A^+ + 2\tilde{B}^- + 2\tilde{Q}_-^{\Sigma}, \\ T_{+-} &= C^{\pm} + D^{\mp} + \tilde{C}^{\pm} + \tilde{D}^{\mp} Q_{\pm}^{\Sigma} + \tilde{Q}_{\pm}^{\Sigma}, \\ T_g &= 2E^g + 2Q_g^{\Sigma} + 2\tilde{E}^g + 2\tilde{Q}_g^{\Sigma}. \end{aligned} \qquad (32)$$

These quantities contain the ultra-violet cut off logarithm $L = \ln\frac{\Lambda^2}{m^2}$ which is eliminated by standard regularization procedure [3] $L \to 2l_\lambda - 9/2$.

Besides it contain the large logarithm $l_s = \ln\frac{s}{m^2}$, infrared logarithm $l_\lambda = ln\frac{m^2}{\lambda^2}$.

Another kinds of logarithms:

$$l_q = \ln\frac{q^2}{m^2}, \qquad l_{\pm} = \ln\frac{\chi_{\pm}^2}{m^2}, \qquad (33)$$

enter into the final result as a following combinations:

$$l_{qs} = \ln\frac{q^2}{s}, \qquad l_{ps} = \ln\frac{\chi_+}{s}, \qquad l_{ms} = \ln\frac{\chi_-}{s}, \qquad (34)$$

which are of order of unity.

Here we restore the gauge-invariance by replacing $g_{\rho\rho_1} \to \tilde{g}_{\rho\rho_1}$, $p_{\pm\rho} \to \tilde{p}_{\pm\rho}$.

5 Discussion, ex-plicite form of tensor structures.

The infrared divergences constrained in contribution of virtual photon emission canceled when takes into account the emission of additional soft photon (center-of mass of e^+e^- initial is implied)

$$\mathrm{d}\sigma^{\gamma}_{soft} = \delta_{soft}\mathrm{d}\sigma_{\mathrm{B}} \tag{35}$$

$$\delta_{soft} = -\frac{4\pi\alpha}{16\pi^3}\int\frac{\mathrm{d}^3k}{w}(-\frac{p_-}{p_-k}+\frac{p_+}{p_+k})^2,\ w < \Delta\varepsilon \ll \sqrt{s}/2, \tag{36}$$

where $w = \sqrt{k^2+\lambda^2}$. Using the standard integrals we obtain

$$\delta_{soft} = -\frac{\alpha}{\pi}[(l_s-1)(l_\lambda + 2\ln\frac{\Delta E}{E}) + \frac{1}{2}l_s^2 - \frac{\pi^2}{3}]. \tag{37}$$

Dirac tensor have a form

$$B_{\rho\rho_1} = B^0_{\rho\rho_1}(1+\frac{\alpha}{\pi}(l_s-1)(\frac{3}{2}+2\ln\frac{\Delta E}{E})) + \frac{\alpha}{\pi}T^{(1)}_{\rho\rho_1}. \tag{38}$$

, Components of $T^{(1)}$ are free from infrared singularities and do not contain large logarithms. Explicit for of it is given below as a function of the center of mass scattering angle.

Similar properties have a cross-channel-Compton tensor with one real and another virtual (space-like) photon [4].

Hadronic tensor is the summed on spin states of bilinear combination of matrix elements $M_\rho M^*_{\rho_1}$, where the current M_ρ describes the conversion of heavy time-like photon to some set of hadrons. For the case of creation of a pair of charged pseudoscalar mesons ($\pi^+\pi^-$, K^+K^-, ...) we have

$$H^{p^+p_-}_{\rho\rho_1} = (p_+ - p_-)_\rho(p_+ - p_-)_{\rho_1},\ q = p_+ + p_-.$$

For conversion to pair of charged spin 1/2 fermions $\gamma \to \mu^+(p_+) + \mu^-(p_-)$ we have

$$H^{\mu^+\mu_-}_{\rho\rho_1} = 4[p_{+\rho}p_{-\rho_1} + p_{+\rho_1}p_{-\rho} - \frac{q^2}{2}g_{\rho\rho_1}]. \tag{39}$$

For creation of a pair of charged vectors mesons $\rho^+\rho^-$, $K^{*+}K^{*-}$ one obtaines

$$H^{p^+p_-}_{\rho\rho_1}(q_+,q_-)\approx q^2(8-2\eta)(g_{\rho\rho_1}-\frac{q_\rho q_{\rho_1}}{q^2}) \tag{40}$$

$$+(q_+-q_-)_\rho(q_+-q_-)_{\rho_1}(3-5\eta+\frac{9}{4}\eta^2),$$

$$\eta=\frac{q^2}{m_\rho^2},\quad q=q_{\rho+}+q_{\rho-}. \tag{41}$$

The gauge invariance requirement $H^h_{\rho\rho_1}(q)q_\rho=H^H_{\rho\rho_1}(q)q_\rho=0$ is fulfilled.

Acknowledgements

One f us (E.A. Kuraev) is grateful to Hamburg University, II Institute for Theoretical for warm hospitality in November 2011 where this work was done. As well he is grateful to RFBR 11-02-00112 and Belorussian 2011 year grants for financial support.

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Appendix A.One-loop Feynman integrals

In this section we perform the result of calculation one-loop Feynman diagramm which was calculated in [2] for the case of the absorption of virtual photon by electron from the pair created by the photon.

Here we consider the expressions for part of scalar,vector and tensor integrals, corresponding to the annihilation of electrons to virtual photon with emission additional real photon in subprocess

$$
\begin{aligned}
&e^-(p_-) + e^+(p_+) \to \gamma(p_1) + \gamma^*(q), \\
&p_\pm^2 = m^2, \quad p_1^2 = 0, s = 2p_+p_-, \quad \chi_\pm = 2p_1p_\pm \\
&s \sim q^2 \sim \chi_\pm \gg m^2, s - \vec{q}^{\ 2} = \chi_+ + \chi_-.
\end{aligned} \tag{42}
$$

The denominators of integrals defined as

$$
\begin{aligned}
(0) &= k^2 - \lambda^2, \\
(2) &= (p_- - k)^2 - m^2 + \mathrm{i}0 = k^2 - 2p_-k + \mathrm{i}0, \\
(\bar{2}) &= (-p_+ - k)^2 - m^2 + \mathrm{i}0, \\
(q) &= (p_1 - p_+ - k)^2 - m^2 + \mathrm{i}0.
\end{aligned} \tag{43}
$$

The four denominator scalar integral

$$
I_{02\bar{2}q} = \int \frac{\mathrm{d}^4k}{\mathrm{i}\pi^2} \frac{1}{(0)(2)(\bar{2})(q)} \tag{44}
$$

has the form

$$
I_{02\bar{2}q} = \frac{1}{s\chi_+}\left[l_q^2 - 2l_+l_s - l_sl_l + 2\mathrm{Li}_2\left(1 - \frac{q^2}{s}\right) - \frac{5\pi^2}{6}\right], \tag{45}
$$

where the logarithms was denoted in (33).

For the tree denominator scalar integrals

$$
I_{ijk} = \int \frac{\mathrm{d}^4k}{\mathrm{i}\pi^2} \frac{1}{(i)(j)(k)} \tag{46}
$$

we have following expressions

$$
\begin{aligned}
I_{0\bar{2}q} &= -\frac{1}{2\chi_+}\left[l_+^2 + \frac{2\pi^2}{3}\right], \qquad Re I_{02\bar{2}} = \frac{1}{2s}\left[l_s^2 + 2l_sl_l - \frac{4\pi^2}{3}\right], \\
Re I_{2\bar{2}q} &= -\frac{1}{2(s - q^2)}\left[l_q^2 - l_s^2\right], \\
I_{02q} &= \frac{1}{\chi_+ + q^2}\left[l_q(l_q - l_+) + \frac{1}{2}(l_q - l_+)^2 + 2\mathrm{Li}_2\left(1 + \frac{\chi_+}{q^2}\right) - \frac{3\pi^2}{2}\right].
\end{aligned} \tag{47}
$$

Two denominator scalar integrals are defined as $I_{ij} = \int \mathrm{d}^4k/\mathrm{i}\pi^2(i)(j)$. The explicit expressions for them are

$$
\begin{aligned}
&I_{02} = L + 1, && I_{2q} = L - l_q + 1, && I_{0q} = L - l_+ + 1, \\
&I_{0\bar{2}} = L + 1, && I_{2\bar{2}} = L - L_s + 1, && I_{\bar{2}q} = L - 1.
\end{aligned}
$$

The vector integrals can be defined as

$$I_r^\mu = \int \frac{\mathrm{d}^4 k k^\mu}{r} = a_r^+ q_+^\mu + a_r^- q_-^\mu + a_r^1 p_1^\mu \tag{48}$$

with $r = (ij), (ijk), (ijkl)$ where $i, j, k, l = (0), (2), (\bar{2}), (q)$.

For the vector integrals with two denominators we have (we put only nonzero coefficients)

$$\begin{aligned}
&Re\ a_{2q}^- = Re\ a_{2q}^1 = -Re\ a_{2q}^+ = \frac{1}{2}\Big(L - l_q + \frac{1}{2}\Big), && a_{0q}^1 = -a_{0q}^+ = \frac{1}{2}\Big(L - l_+ + \frac{1}{2}\Big),\\
&Re\ a_{2\bar{2}}^- = -Re\ a_{2\bar{2}}^+ = \frac{1}{2}\Big(L - l_s + \frac{1}{2}\Big), && a_{\bar{2}q}^1 = -\frac{1}{2} a_{\bar{2}q}^+ = \frac{1}{2}\Big(L - \frac{3}{2}\Big),\\
&a_{02}^- = \frac{1}{2}L - \frac{1}{4}, && a_{0\bar{2}}^+ = -\frac{1}{2}L + \frac{1}{4}
\end{aligned} \tag{49}$$

and the coefficients for the vector integrals with three denominators are

$$\begin{aligned}
&a_{02q}^- = \frac{1}{a}\Big(\chi_+ I_{02q} + \frac{2\chi_+}{a} l_+ + \frac{q^2 - \chi_+}{a} l_q\Big), && a_{02q}^+ = -a_{02q}^1 = \frac{1}{a}\Big(l_+ - l_q\Big),\\
&a_{0\bar{2}q}^1 = \frac{1}{\chi_+}\Big(-l_+ + 2\Big), && a = \chi_+ + q^2,\\
&a_{0\bar{2}q}^+ = -I_{0\bar{2}q} - \frac{1}{\chi_+} l_+, && a_{02\bar{2}}^- = -a_{02\bar{2}}^+ = \frac{1}{s} l_s,\\
&a_{2\bar{2}q}^- = \frac{1}{c}\Big(l_s - l_q\Big), && a_{2\bar{2}q}^+ = -I_{2\bar{2}q} + \frac{1}{c}\Big(l_s - l_q\Big),\\
&a_{2\bar{2}q}^1 = \frac{s}{c} I_{2\bar{2}q} + \frac{1}{c}\Big(-l_q + 2\Big) - \frac{2s}{c^2}\Big(l_s - l_q\Big), && c = s - q^2 = \chi_+ + \chi_-.
\end{aligned} \tag{50}$$

Finally, the coefficient of the vector integral with 4 denominators has the form

$$\begin{aligned}
&a^1 = \frac{s}{d}\Big(\chi_+ A + \chi_- B - sC\Big), && a^+ = \frac{\chi_-}{d}\Big(\chi_+ A - \chi_- B + sC\Big)\\
&a^- = \frac{\chi_+}{d}\Big(-\chi_+ A + \chi_- B + sC\Big), && d = 2s\chi_+\chi_-,\\
&A = I_{2\bar{2}q} - I_{0\bar{2}q}, && B = I_{02q} - I_{2\bar{2}q},\\
&C = I_{02q} - I_{02\bar{2}} - \chi_+ I_{02\bar{2}q}.
\end{aligned} \tag{51}$$

The second rank tensor integrals can be parameterized in the form

$$\begin{aligned}
I_r^{\mu\nu} = \int \frac{\mathrm{d}^4 k}{\mathrm{i}\pi^2}\frac{k_\mu k_\nu}{r} = \Big[&a_r^g g + a_r^{11} p_1 p_1 + a_r^{++} q_+ q_+ + a_r^{--} q_- q_- + a_r^{1+}(p_1 q_+ + q_+ p_1)\\
&+ a_r^{1-}(p_1 q_- + q_- p_1) + a_r^{+-}(q_+ q_- + q_- q_+)\Big]_{\mu\nu}.
\end{aligned} \tag{52}$$

The coefficients for tensor integral with four denominators are (we suppressed the index $02\bar{2}q$)

$$\begin{aligned}
a^{1+} &= \frac{1}{\chi_+}\Big(A_6 + A_7 - A_{10}\Big), & a^{+-} &= \frac{1}{s}\Big(A_2 + A_6 - A_{10}\Big),\\
a^{1-} &= \frac{1}{\chi_-}\Big(A_2 + A_7 - A_{10}\Big), & a^{11} &= \frac{1}{\chi_-}\Big(A_1 - sa^{1+}\Big),\\
a^{--} &= \frac{1}{s}\Big(A_5 - \chi_+ a^{1-}\Big), & a^{++} &= \frac{1}{s}\Big(A_3 - \chi_- a^{1+}\Big),\\
a^{g} &= \frac{1}{2}\Big(A_{10} - A_2 - \chi_+ a^{1+}\Big), & & \qquad (53)
\end{aligned}$$

with

$$\begin{aligned}
A_1 &= a^1_{2\bar{2}q} - a^1_{0\bar{2}q}, & A_6 &= a^+_{02q} - a^+_{2\bar{2}q},\\
A_2 &= a^-_{2\bar{2}q}, & A_7 &= a^1_{02q} - \chi_+ a^1,\\
A_3 &= a^+_{2\bar{2}q} - a^+_{0\bar{2}q}, & A_8 &= a^-_{02q} - a^-_{02\bar{2}} - \chi_+ a^-,\\
A_4 &= a^1_{02q} - a^1_{2\bar{2}q}, & A_9 &= a^+_{02q} - a^+_{02\bar{2}} - \chi_+ a^+,\\
A_5 &= a^-_{02q} - a^-_{2\bar{2}q}, & A_{10} &= I_{2\bar{2}q}. & & \qquad (54)
\end{aligned}$$

For the tensor integrals with three denominators $I^{\mu\nu}_{02q}$ we have coefficients

$$\begin{aligned}
a^g_{02q} &= \frac{1}{4}L + \frac{3}{8} - \frac{q^2}{4a}l_q - \frac{\chi_+}{a}l_+,\\
a^{+-}_{02q} &= -a^{1-}_{02q} = \frac{1}{2a}\left[\frac{\chi_+}{a}(l_+ - l_q) - 1\right],\\
a^{++}_{02q} &= a^{11}_{02q} = -a^{1+}_{02q} = \frac{1}{2a}(l_q - l_+),\\
a^{--}_{02q} &= \frac{1}{a^2}\left[\chi_+^2 I_{02q} + \frac{3\chi_+^2}{a}l_+ - \frac{(q^2)^2 + 4q^2\chi_+ - 3\chi_+^2}{2a}l_q - \frac{q^2 + 3\chi_+}{2}\right]. \qquad (55)
\end{aligned}$$

The coefficients entering into the tensor integral $I^{\mu\nu}_{02\bar{2}}$ are

$$a^g_{02\bar{2}} = \frac{1}{4}(L - l_s) + \frac{3}{8}, \quad a^{++}_{02\bar{2}} = a^{--}_{02\bar{2}} = \frac{1}{2s}(l_s - 1), \quad a^{+-}_{02\bar{2}} = -\frac{1}{2s}, \qquad (56)$$

and the coefficients for the tensor integral $I^{\mu\nu}_{0\bar{2}q}$ are

$$\begin{aligned}
a^g_{0\bar{2}q} &= \frac{1}{4}(L - l_+) + \frac{3}{8}, & a^{1+}_{0\bar{2}q} &= \frac{1}{\chi_+}\Big(l_+ - \frac{5}{2}\Big),\\
a^{11}_{0\bar{2}q} &= \frac{1}{2\chi_+}(-l_+ + 2), & a^{++}_{0\bar{2}q} &= I_{0\bar{2}q} + \frac{1}{2\chi_+}(3l_+ - 1). & & \qquad (57)
\end{aligned}$$

In the case of the tensor integral $I^{\mu\nu}_{2\bar{2}q}$ they have the form

$$a^{g}_{2\bar{2}q} = \frac{1}{2}\Big[\frac{1}{2}L + \frac{3}{4} - \frac{s}{2c}l_s + \frac{q^2}{2c}l_q\Big], \qquad a^{--}_{2\bar{2}q} = -\frac{1}{2c}(l_q - l_s), \tag{58}$$

$$a^{++}_{2\bar{2}q} = I_{2\bar{2}q} + \frac{3}{2c}(l_q - l_s), \qquad a^{+-}_{2\bar{2}q} = \frac{1}{2c}(l_q - l_s),$$

$$a^{1-}_{2\bar{2}q} = \frac{1}{c}\Big[-\frac{1}{2} + \frac{s}{2c}l_s - \frac{s}{2c}l_q\Big], \tag{59}$$

$$a^{1+}_{2\bar{2}q} = \frac{1}{c}\Big[-\frac{5}{2} - sI_{2\bar{2}q} + \frac{5s}{2c}l_s - \frac{2q^2+3s}{2c}l_q\Big],$$

$$a^{11}_{2\bar{2}q} = \frac{1}{c^2}\Big[4s - q^2 + s^2 I_{2\bar{2}q} - \frac{3s^2}{c}l_s + \frac{3s^2-(q^2)^2+4sq^2}{2c}l_q\Big]. \tag{60}$$

Appendix B.

Appendix C. Large-angles emission by the initial leptons masses

Cross section of 2 photon emission by the initial leptons masses

$$\mathrm{e}^+(p_+) + \mathrm{e}^-(p_-) \to \gamma(k_1) + \gamma(k_2) + \mathrm{hadr}(q) \tag{61}$$

have a form

$$\frac{\mathrm{d}\sigma^{2\gamma}}{\mathrm{d}\Gamma_h} = \frac{1}{2!}\frac{\alpha^4}{2\pi^2 s}\frac{H_{\rho\rho_1}O^{(2)}_{\rho\rho_1}}{(q^2)^2}\frac{\mathrm{d}^2k_1}{\omega_1}\frac{\mathrm{d}^2k_2}{\omega_2}, \quad \omega_1,\omega_2 < \Delta\varepsilon, \tag{62}$$

where the factor 1/2! takes into account the identity of final-state hard photons. The relevant contribution to lepton tensor is

$$Q^{(2)}_{\rho\rho_1} = \frac{1}{4}\mathrm{Tr}\, p_+ O^{\sigma\eta}_{12\rho} p_- \bar{O}^{\sigma\eta}_{12\rho}; \tag{63}$$

$$\begin{aligned} O^{\sigma\eta}_{12\rho} = {} & \gamma_\rho \frac{\hat{p}_- - \hat{k}_1 - \hat{k}_2}{d_{-12}}\Big(\gamma^\eta \frac{\hat{p}_- - \hat{k}_1}{d_{-1}}\gamma^\sigma + \gamma^\sigma \frac{\hat{p}_- - \hat{k}_2}{d_{-2}}\gamma^\eta\Big) \\ & + \Big(\gamma^\eta \frac{-\hat{p}_+ + \hat{k}_2}{d_{+2}}\gamma^\sigma + \gamma^\sigma \frac{-\hat{p}_+ + \hat{k}_1}{d_{+1}}\gamma^\eta\Big)\frac{-\hat{p}_+ + \hat{k}_1 + \hat{k}_2}{d_{+12}}\gamma_\rho \\ & + \frac{1}{d_{-1}d_{+2}}\gamma^\sigma(-\hat{p}_+ + \hat{k}_2)\gamma_\rho(\hat{p}_- - \hat{k}_1)\gamma^\eta \\ & + \frac{1}{d_{-2}d_{+1}}\gamma^\eta(-\hat{p}_+ + \hat{k}_1)\gamma_\rho(\hat{p}_- - \hat{k}_2)\gamma^\sigma, \end{aligned} \tag{64}$$

and

$$
\begin{aligned}
d_{-12} &= (p_- - k_1 - k_2)^2 - m^2; \\
d_{-1} &= (p_- - k_1)^2 - m^2; \quad d_{-2} = (p_- - k_2)^2 - m^2; \\
d_{+12} &= (-p_+ + k_1 + k_2)^2 - m^2; \\
d_{+1} &= (-p_+ + k_1)^2 - m^2; \quad d_{+2} = (p_+ + k_2)^2 - m^2.
\end{aligned} \tag{65}
$$

Tensor $Q^{(2)}_{\rho\rho_1}$ obey the gauge invariance $Q^{(2)}_{\rho\rho_1} q_\rho = Q^{(2)}_{\rho\rho_1} q_{\rho_1} = 0$ and can be put on the form

$$
\begin{aligned}
Q^{(2)}_{\rho\rho_1} = A_g \tilde{g}_{\rho\rho_1} + [A_- \tilde{p}_- \tilde{p}_- + A_+ \tilde{p}_+ \tilde{p}_+ + A_{11} \tilde{k}_1 \tilde{k}_1 + A_{+-}(\tilde{p}_+ \tilde{p}_- + \tilde{p}_- \tilde{p}_+) \\
+ A_{+1}(\tilde{p}_+ \tilde{k}_1 + \tilde{k}_1 \tilde{p}_+) + A_{-1}(\tilde{p}_- \tilde{k}_1 + \tilde{k}_1 \tilde{p}_-)]_{\rho\rho_1},
\end{aligned} \tag{66}
$$

coefficients A_i can be obtained in the standard way: constructing the values

$$
B_g,\ B_{11},\ B_{++},\ B_{--},\ ... = Q_{\rho\rho_1}[g_{\rho\rho_1},\ k_{1\rho} k_{1\rho_1},\ p_{+\rho} p_{+\rho_1},\ ...]
$$

and solving the set of 7 linear equations.