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Super-Higgs in Superspace

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Abstract

We determine the effective gravitational couplings in superspace whose components reproduce the supergravity Higgs effect for the constrained Goldstino multiplet. It reproduces the known Gravitino sector whilst constraining the off-shell completion. We show that these components arise by computing the effective action. This may be useful for phenomenological studies and model building: We give an example of its application to multiple Goldstini.

1 Introduction

The spontaneous breakdown of gobal supersymmetry generates a massless Goldstino [1], which is well described by the Akulov-Volkov (A-V) effective action [2]. When supersymmetry is made local, the Gravitino "eats" the Goldstino of the A-V action to become massive: The super-Higgs mechanism [3].

In terms of superfields, the constrained Goldstino multiplet X_{NL}, of Komargodski and Seiberg [4] is equivalent to the A-V formulation. It is therefore natural to extend the description of supergravity with this multiplet, in superspace, to one that can reproduce the super-Higgs mechanism. In this paper we write down the most minimal

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set of *new* terms in superspace that incorporate both supergravity and the Goldstino multiplet in order to reproduce the super-Higgs mechanism of [3, 5] at lowest order in M_{Pl}.

In writing down an effective action we should expect that these terms are generated radiatively. To check this we explicitly compute the effective action in components and demonstrate that the necessary terms do indeed appear. What is perhaps surprising is that these terms arise from an interplay between the gravitational cosmological constant and the Goldstino multiplet. We then set the gravitational cosmological constant to cancel the F-term of the Goldstino, a prerequisitive for the super-Higgs mechanism.

Whilst our effective superspace action does not have the full complexity of supergravity, including complicated expressions involving the Kähler metric, we gain a simpler more transparent construction that constrains the off-shell completion. This may be useful for analysing Goldstino couplings to $U(1)_R$ currents and may be naturally extended to the use of $S^{\alpha\dot{\alpha}}$ and $R^{\alpha\dot{\alpha}}$ current multiplets [6]. It may also offer an in principle phenomenological way to determine the correct supercurrent multiplet that describes nature. Furthermore, this setup allows for straightforward analysis of models with many Goldstini [7] and also with Pseudo-Goldstini [8].

To find these *new* superspace terms that we must add, we take suitable couplings of the Goldstino multiplet X_{NL}, the supercurrent multiplet $\mathcal{J}^{\alpha\dot{\alpha}}$, the Graviton multiplet $H^{\alpha\dot{\alpha}}$, M_{Pl} and the cosmological constant term C, constrained by dimensional analysis. For a theory without matter the procedure is straightforward. For a theory with matter, to demonstrate the full shift of the super-Higgs mechanism, the Goldberger-Treiman relation(s) should appear explicitly. This component is related to various non derivative couplings of superfields to the Goldstino multiplet and are then accounted for [9]. We then demonstrate the super-Higgs mechanism with matter supercurrents.

The outline of this paper is as follows: In the next section we will review the constrained Goldstino multiplet X_{NL} and its relation to the Ferrara-Zumino (F-Z) [10] supercurrent multiplet $\mathcal{J}^{\alpha\dot{\alpha}}$. We then compute the effective action in components that reproduces the terms necessary for the super-Higgs mechanism. Next we promote the component terms to a full superspace effective action, by coupling the Goldstino multiplet to the supercurrent multiplet and determine that the superspace formulation of these new terms correctly reproduces the components of the super-Higgs mechanism. In an appendix we include a review of [5], which is the component formulation of the super-Higgs mechanism. We adopt two-component spinor notation throughout.

2 Non-Linear SUSY coupled to Supergravity

To describe a Goldstino supermultiplet we may start from a left handed chiral superfield $\bar{D}_{\dot{\alpha}}X = 0$ and apply the constraint $X^2 = 0$. The solution to this equation is given by the nonlinear Goldstino multiplet [4, 11]

$$X_{NL} = \frac{G^2}{2F} + \sqrt{2}\theta G + \theta^2 F. \tag{2.1}$$

The field G^α is the Goldstino and F the Auxiliary field. f is the scale of supersymmetry breaking. In general one must integrate out F, which may be complicated to do in practice, but will lead as $\langle F \rangle = f + ...$, where the ellipses may often be ignored due to terms with higher derivatives. The Goldstino multiplet also satisfies the conservation equation [10]

$$\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X. \tag{2.2}$$

where $\mathcal{J}_{\alpha\dot{\alpha}} = -2\sigma^\mu_{\alpha\dot{\alpha}}J_\mu$ is the Ferrara-Zumino supercurrent multiplet, which is a real linear multiplet: $D^2J = \bar{D}^2J = 0$. In components it is given by

$$\begin{aligned} \mathcal{J}_\mu &= j_\mu + \theta^\alpha(S_{\mu\alpha} + \frac{1}{3}\sigma_{\mu\alpha\dot{\alpha}}\bar{\sigma}^{\nu\dot{\alpha}\beta}S_{\nu\beta}) + \frac{i}{2}\theta^2\partial_\mu(\bar{x} + \bar{C}) + \bar{\theta}_{\dot{\alpha}}(\bar{S}^{\dot{\alpha}}_\mu + \frac{1}{3}(\epsilon^{\dot{\alpha}\dot{\gamma}}\bar{S}_{\nu\dot{\beta}}\bar{\sigma}^{\nu\dot{\beta}\alpha}\sigma_{\mu\alpha\dot{\gamma}})) \\ &- \frac{i}{2}\bar{\theta}^2\partial_\mu(x + C) + \theta^\alpha\sigma^\nu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}(2T_{\nu\mu} - \frac{2}{3}\eta_{\mu\nu}T - \frac{1}{4}\epsilon_{\nu\mu\rho\sigma}\partial^{[\rho}j^{\sigma]}) + ... \end{aligned} \tag{2.3}$$

J_μ is the R-current, S^α_μ the supercurrent, $T_{\mu\nu}$ the stress energy tensor and we have made explicit the presence of a complex constant C in the definition of the scalar component x, this will play the part of a cosmological constant as shown in the remainder of the paper. The ellipses may be determined from a shift from $y^\mu = x^\mu - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\beta}}$. Our metric conventions are mostly minus, $\eta_{\mu\nu} = (1, -1, -1, -1)$. The supersymmetry current algebra is

$$\{\bar{Q}_{\dot{\alpha}}, S_{\alpha\mu}\} = \sigma^\nu_{\alpha\dot{\alpha}}(2T_{\mu\nu} + i\partial_\nu j_\mu - i\eta_{\mu\nu}\partial^\lambda j_\lambda - \frac{1}{4}\epsilon_{\nu\mu\rho\lambda}\partial^\rho j^\lambda) \tag{2.4}$$

$$\{Q_\beta, S_{\mu\alpha}\} = 2i\epsilon_{\lambda\beta}(\sigma_{\mu\rho})^\lambda_\alpha\partial^\rho(\bar{x} + \bar{C}) \tag{2.5}$$

where the first term of 2.4 is the conserved symmetric energy tensor and the remaining terms are Schwinger Terms that vanish in the vacuum. It is straightforward to use the definition of the Supercharge, $Q_\alpha = \int d^3xS^0_\alpha$, to relate this expression to the super algebra

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}}P_\mu. \tag{2.6}$$

The simplest non linear Goldstino superfield X_{NL} action that breaks supersymmetry spontaneously is given by

$$\mathcal{L} = \int d^4\theta X^\dagger_{NL}X_{NL} + \int d^2\theta f^\dagger X_{NL} + \int d^2\bar{\theta} f X^\dagger_{NL}. \tag{2.7}$$

As shown in [4] this action, in the absence of other couplings involving the Auxiliary field, is equivalent to the full A-V action in components

$$\mathcal{L}_{AV} = -|f|^2 + i\partial_\mu \bar{G}\bar{\sigma}^\mu G + \frac{1}{4|f|^2}\bar{G}^2\partial^2 G^2 - \frac{1}{16|f|^6}G^2\bar{G}^2\partial^2 G^2\partial^2\bar{G}^2. \tag{2.8}$$

Let us for the moment focus on only the first two terms, which comprise the A-V action, ignoring terms with a higher number of derivatives. The relevant terms in the A-V action are the supersymmetry breaking term and the Goldstino kinetic terms. The supersymmetry breaking term $-|f|^2$ in the action is a cosmological constant. The minimum of the scalar potential is $V_{\min} = +|f|^2$ which is positive definite, and we see that global supersymmetry is broken. The supergravity action will also generate a cosmological constant, but of opposite sign. If these are made to be equal, the overall cosmological constant vanishes. This will appear shortly.

We introduce the linear supergravity action which provides kinetic terms for the Gravitino. The supergravity fields are embedded in a real vector superfield. In Wess-Zumino gauge the components are given by

$$\begin{aligned} H_\mu &= \theta^\alpha\sigma^\nu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}(h_{\mu\nu} - \eta_{\mu\nu}h) + \bar{\theta}^2\theta^\alpha(\psi_{\mu\alpha} + \sigma_{\mu\alpha\dot{\alpha}}\bar{\sigma}^{\rho\dot{\alpha}\beta}\psi_{\rho\beta}) \\ &+ \frac{i}{2\sqrt{3}}\theta^2 M_\mu - \frac{i}{2\sqrt{3}}\bar{\theta}^2 M^\dagger_\mu + \theta^2\bar{\theta}_{\dot{\alpha}}(\bar{\psi}^{\dot{\alpha}}_\mu + \bar{\sigma}^{\dot{\alpha}\alpha}_\mu\sigma^\rho_{\alpha\dot{\beta}}\bar{\psi}^{\dot{\beta}}_\rho) - \frac{1}{2}\theta^2\bar{\theta}^2 A_\mu \end{aligned} \tag{2.9}$$

where $H_\mu = \frac{1}{4}\bar{\sigma}^{\dot{\alpha}\alpha}_\mu H_{\alpha\dot{\alpha}}$, $h_{\mu\nu}$ is the linear Graviton, ψ^α_μ is the Gravitino and M_μ, A_μ are Auxiliary fields. The kinetic terms of the supergravity action are given by [11]

$$-\int d^4\theta H^{\alpha\dot{\alpha}}E^{FZ}_{\alpha\dot{\alpha}} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\psi_{\mu\alpha}\bar{\sigma}^{\dot{\alpha}\alpha}_\nu\partial_\rho\bar{\psi}_{\sigma\dot{\alpha}} - \bar{\psi}_{\mu\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}_\nu\partial_\rho\psi_{\sigma\alpha}) - \frac{1}{3}|\partial_\mu M^\mu|^2 + \frac{1}{3}A_\mu A^\mu ... \tag{2.10}$$

The ellipses include a linearised Graviton kinetic term. $E^{FZ}_{\alpha\dot{\alpha}}$ is defined as

$$E^{FZ}_{\alpha\dot{\alpha}} = \bar{D}_{\dot{\tau}}D^2\bar{D}^{\dot{\tau}}H_{\alpha\dot{\alpha}} + \bar{D}_{\dot{\tau}}D^2\bar{D}_{\dot{\alpha}}H^{\dot{\tau}}_\alpha + D^\gamma\bar{D}^2 D_\alpha H_{\gamma\dot{\alpha}} - 2\partial_{\alpha\dot{\alpha}}\partial^{\gamma\dot{\tau}}H_{\gamma\dot{\tau}}. \tag{2.11}$$

This gives a kinetic term for the Graviton and Gravitino but the remaining fields are Auxilliary and therefore not dynamical. The Auxiliary field A_μ integrates out to give $A_\mu = \frac{j_\mu}{\bar{M}_{Pl}} + ...$, with the ellipses denoting higher order terms in $1/F$ and $1/\bar{M}_{Pl}$. The complex field M_μ plays a role in generating a cosmological constant once we weakly couple the supercurrent mutliplet to linear supergravity.

Now that we have introduced the Goldstino and supergravity actions, we would like to couple the supercurrent multiplet to a linear supergravity muliplet [11]

$$\begin{aligned} \frac{1}{\bar{M}_{Pl}}\int d^4\theta\mathcal{J}_{\alpha\dot{\alpha}}H^{\alpha\dot{\alpha}} &= \frac{1}{2\bar{M}_{Pl}}(h_{\mu\nu}T^{\mu\nu} + \psi_\mu S^\mu + \bar{\psi}_\mu\bar{S}^\mu - j^\mu A_\mu) \\ &- \frac{1}{4\sqrt{3}\bar{M}_{Pl}}\partial_\mu M^\mu(x + C) - \frac{1}{4\sqrt{3}\bar{M}_{Pl}}\partial_\mu M^{\dagger\mu}(\bar{x} + \bar{C}) \end{aligned} \tag{2.12}$$

As we have integrated by parts on $\partial_\mu C$ there is also a total derivative term. Also $\bar{M}_{pl} = M_{pl}/8\pi$ is the reduced Planck mass, $M_{pl} = (8\pi G)^{-\frac{1}{2}}$ where G is the Gravitational constant. It is useful here to recall that the supersymmetry current multiplet $J^{\alpha\dot{\alpha}}$ contains at order θ^β a contribution from fermionic matter and a term proportional to the supersymmetry breaking F term,

$$S^\mu = S^\mu_{\text{matter}} + i\sqrt{2}F\sigma^\mu_{\alpha\dot{\alpha}}\bar{G}^{\dot{\alpha}} \quad , \quad \bar{S}^{\mu\dot{\alpha}} = \bar{S}^\mu_{\text{matter}} + i\sqrt{2}F^\dagger\bar{\sigma}^{\mu\dot{\alpha}\alpha}G_\alpha. \tag{2.13}$$

3 Super-Higgs as an Effective Action

Before we write down a superspace effective action, whose components reproduce the super-Higgs mechanism, it is worthwhile asking if one can indeed generate the necessary terms radiatively. In this section we focus on the component formalism and explicitly compute the relevant terms in the effective action. We shall find that the necessary terms all arise from couplings to the cosmological constant terms that we introduced in our definition of the supercurrent multiplet.

3.1 The cosmological constant

First we focus on the cosmological constant. Varying the combination of 2.12 and 2.10 with respect to M^μ (treating M_μ and $M^\dagger_\mu$ as independent fields) we find that

$$\partial_\nu\left(\frac{1}{3}\partial_\mu M^{\dagger\mu} + \frac{\sqrt{3}}{12\bar{M}_{Pl}}(x+C)\right) = 0 \tag{3.1}$$

which has as a solution

$$M^* = \partial_\mu M^{\mu\dagger} = -\frac{\sqrt{3}}{4\bar{M}_{Pl}}(x+C) + D \tag{3.2}$$

where D is an arbitrary mass dimension 2 integration constant [12] which for the purposes of our discussion can be set to zero. Substituting this result back in the original action, and integrating out A^μ, gives the terms

$$\mathcal{L}_{\text{aux}} = \frac{3(x+C)(\bar{x}+\bar{C})}{16\bar{M}^2_{Pl}} - \frac{j_\mu j^\mu}{48\bar{M}^2_{Pl}}. \tag{3.3}$$

We can see that this mechanism provides a non-vanishing contribution to the vacuum energy density of the form

$$\rho_{\text{Vac}} = -\frac{3|C|^2}{16\bar{M}^2_{Pl}} - \frac{3\langle x\bar{x}\rangle}{16\bar{M}^2_{Pl}} - \frac{3C\langle\bar{x}\rangle}{16\bar{M}^2_{Pl}} - \frac{3\bar{C}\langle x\rangle}{16\bar{M}^2_{Pl}} \tag{3.4}$$

which upon using $\langle x\rangle = \langle\bar{x}\rangle = \langle x\bar{x}\rangle = 0$ simplifies to,

$$\rho_{\rm Vac} = -\frac{3|C|^2}{16\bar{M}_{Pl}^2}. \tag{3.5}$$

Taking the cosmological constant $|f|^2$ from the Goldstino action we may cancel the overall cosmological constant by setting

$$\frac{3|C|^2}{16\bar{M}_{Pl}^2} = |f|^2, \tag{3.6}$$

and hence

$$C = \frac{4}{\sqrt{3}}\bar{M}_{Pl} f. \tag{3.7}$$

It has been shown in [3] that whether one generates a Gravitino mass, or not, depends on whether these terms cancel, which is related to whether we are in Minkowski background or anti de Sitter space. We take them to exactly cancel such that the massive Gravitino does appear.

3.2 The component terms

Starting from Eqn. (2.12) it is straightforward to see that the effective action generated from linear supergravity contains

$$\mathcal{L}_{\rm eff} \supset \frac{i}{8M_{pl}^2}(\tilde{G}_{3/2}(0)\psi_\mu^\alpha(\sigma^{\mu\nu})_\alpha^{\ \beta}\psi_{\beta\nu}+\tilde{\bar{G}}_{3/2}(0)\bar{\psi}_{\dot{\alpha}\mu}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\ \dot{\beta}}\bar{\psi}_\nu^{\dot{\beta}}+S_\mu^\alpha\tilde{D}^{\mu\nu}_{\alpha\beta}(0)S^{\nu\beta}+\bar{S}^\mu_{\dot{\alpha}}\tilde{\bar{D}}^{\dot{\alpha}\dot{\beta}}_{\mu\nu}(0)S^\nu_{\dot{\beta}}). \tag{3.8}$$

We define the time ordered current correlators

$$\langle S^\mu_\alpha(p)S^\nu_\beta(-p)\rangle = \tilde{\Pi}^{\mu\nu}_{\alpha\beta}\tilde{G}_{3/2}(p^2) \tag{3.9}$$

$$\langle S^\mu_\alpha(x)S^\nu_\beta(0)\rangle = \Pi^{\mu\nu}_{\alpha\beta}G_{3/2}(x^2) \tag{3.10}$$

which are related by a Fourier Transform. Additionally we define the two point function

$$\langle\psi^\mu_\alpha(x)\psi^\nu_\beta(y)\rangle = \int\frac{d^4p}{(2\pi)^4}\tilde{D}^{\mu\nu}_{\alpha\beta}(p^2)e^{-ik.(x-y)} \tag{3.11}$$

where we take that the two point function has the form

$$\tilde{D}^{\mu\nu}_{\alpha\beta}(p^2) = -\frac{1}{3}\frac{im_{3/2}\eta^{\mu\nu}\epsilon_{\alpha\beta}}{p^2-m_{3/2}^2}. \tag{3.12}$$

The mass pole

$$m_{3/2} = \frac{i\tilde{G}_{3/2}(0)}{8M_{pl}^2} \tag{3.13}$$

is a consequence of a standard geometric sum of mass insertions. For the model outlined in this paper, as will be demonstrated below,

$$\tilde{G}_{3/2}(0) = 2C \quad \text{and} \quad \tilde{D}^{\mu\nu}_{\alpha\beta}(0) = \frac{1}{3}\frac{i\eta^{\mu\nu}\epsilon_{\alpha\beta}}{m_{3/2}}, \tag{3.14}$$

such that one recovers the effective contributions

$$\mathcal{L}_{\text{eff}} = -\frac{if}{\sqrt{3}\bar{M}_{Pl}}\left(\psi^{\alpha}_{\mu}(\sigma^{\mu\nu})^{\beta}_{\alpha}\psi_{\nu\beta} + \bar{\psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}}\bar{\psi}^{\dot{\beta}}_{\nu}\right) - m_{3/2}G^{\alpha}G_{\alpha} - m^{\dagger}_{3/2}G_{\dot{\alpha}}G^{\dot{\alpha}}$$
$$-\frac{i}{2\sqrt{6}\bar{M}_{Pl}}G^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{S}^{\dot{\alpha}}_{\mu\text{matter}} - \frac{i}{2\sqrt{6}\bar{M}_{Pl}}\bar{G}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}S_{\alpha\mu\text{matter}}. \tag{3.15}$$

From these we see that the cosmological constant factor $\frac{C}{4\bar{M}^2_{Pl}}$ determines the gravitino mass $m_{3/2}$. It is only after we require the vanishing cosmological constant, in the form of 3.6, that we find the relation $m_{3/2} = \frac{f}{\sqrt{3}\bar{M}_{Pl}}$. These are all the necessary pieces for the super-Higgs mechansim, as will be generated in the next section. The local supersymmetry transformations are also *modified* by the presence of a non vanishing C, these become

$$\delta\psi_{\mu\alpha} = -\left(2\bar{M}_{Pl}\partial_{\mu}\epsilon_{\alpha} + i\frac{C}{4\bar{M}_{Pl}}\sigma_{\mu\alpha\dot{\alpha}}\epsilon^{\dot{\alpha}}\right). \tag{3.16}$$

3.2.1 Computing the current correlator

Here we demonstrate the evaluation of the time ordered current correlator appearing in 3.14

$$\langle S^{\mu}_{\alpha}(x)S^{\nu}_{\beta}(y)\rangle = \langle T[S^{\mu}_{\alpha}(x)S^{\nu}_{\beta}(y)]\rangle\,. \tag{3.17}$$

We wish to write this correlator in terms of the super algebra. To do this we take first the definition

$$\langle T[S^{\mu}_{\alpha}(x)S^{\nu}_{\beta}(y)]\rangle = \theta(x^0-y^0)\,\langle S^{\mu}_{\alpha}(x)S^{\nu}_{\beta}(y)\rangle - \theta(y^0-x^0)\,\langle S^{\nu}_{\beta}(y)S^{\mu}_{\alpha}(x)\rangle\,, \tag{3.18}$$

where $\theta(x^0-y^0)$ is the Heaviside function and the minus sign between the two terms on the right hand side is standard for fermionic operators. Using the standard manipulation

$$\partial_{\mu}\,\langle T[S^{\mu}_{\alpha}(x)S^{\nu}_{\beta}(y)]\rangle = \delta(x^0-y^0)\,\langle\{S^{0}_{\alpha}(x), S^{\nu}_{\beta}(y)\}\rangle + \langle T[\partial_{\mu}S^{\mu}_{\beta}(x)S^{\nu}_{\alpha}(y)]\rangle \tag{3.19}$$

and then integrating by parts

$$-x^{\rho}\partial_{\mu}\,\langle T[S^{\mu}_{\alpha}(x)S^{\nu}_{\beta}(y)]\rangle = (\partial_{\mu}x^{\rho})\,\langle T[S^{\mu}_{\alpha}(x)S^{\nu}_{\beta}(y)]\rangle \tag{3.20}$$

one obtains

$$\langle T[S^{\rho}_{\alpha}(x)S^{\nu}_{\beta}(y)]\rangle = -x^{\rho}\left(\delta(x^0-y^0)\,\langle\{S^{0}_{\alpha}(x), S^{\nu}_{\beta}(y)\}\rangle + \langle T[\partial_{\mu}S^{\mu}_{\beta}(x)S^{\nu}_{\alpha}(y)]\rangle\right). \tag{3.21}$$

Using $\partial_{\mu}S^{\mu} = 0$ to remove the second term and then taking the integral gives

$$\int d^4y\,\langle T[S^{\rho}_{\alpha}(x)S^{\nu}_{\beta}(y)]\rangle = -x^{\rho}\,\langle\{Q_{\alpha}(x), S^{\nu}_{\beta}(y)\}\rangle\,. \tag{3.22}$$

Inserting

$$\{Q_{\beta}, S_{\mu\alpha}\} = 2i\epsilon_{\lambda\beta}(\sigma_{\mu\rho})^{\lambda}_{\alpha}\partial^{\rho}(\bar{x}+\bar{C}) \tag{3.23}$$

integrating by parts, we find

$$\int d^4y \, \langle T[S^\rho_\alpha(x) S^\nu_\beta(y)]\rangle = (\partial^\sigma x^\rho) 2i\epsilon_{\lambda\alpha}(\sigma_{\mu\sigma})^\lambda_\beta(\bar{x} + \bar{C}). \tag{3.24}$$

Thus we see that the constant C appears in the current correlator.

Digressing slightly, we make the comment that both f^2 and C are contained within the super algebra and at this point are both free parameters. However the requirement of vanishing overall cosmological constant suggests that there is really only one free parameter. It is therefore tempting to suggest that whatever mechanism gives rise to supersymmetry breaking should generate both terms. We hope to have more to say on this in the future.

4 The Superspace Terms

In the previous section we demonstrated that certain terms needed for the super-Higgs mechanism are generated in an effective action that combines linear supergravity and the Goldstino multiplet. In this section we will promote the effective terms in components to full superspace terms in the action. We will demonstrate that the components combined do indeed satisfy the super-Higgs mechanism. Our starting point of this superspace effective action is to assume all the relevant pieces of section 2. In particular we also assume the vanishing overall cosmological constant Eqn. (3.6). Next we introduce a *new* term, we add to the supergravity action

$$-\frac{m_{3/2}}{64}\int d^4\theta \left(\frac{X_{NL}}{f} + \frac{X^\dagger_{NL}}{f^\dagger}\right)\left[\bar{D}_{\dot{\alpha}} D_\alpha H^{\beta\dot{\beta}} \bar{D}^{\dot{\alpha}} D^\alpha H_{\beta\dot{\beta}} - \frac{4}{3}(\bar{D}_{\dot{\alpha}} D_\alpha H^{\alpha\dot{\alpha}})^2\right] \tag{4.1}$$

which in components gives a Gravitino self coupling

$$-i\left(m_{3/2}\psi^\alpha_\mu(\sigma^{\mu\nu})^\beta_\alpha\psi_{\nu\beta} + m^\dagger_{3/2}\bar{\psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}}\bar{\psi}^{\dot{\beta}}_\nu\right) + ... \tag{4.2}$$

The second superspace term was introduced to remove terms of the form ψ^2. The ellipses denote some higher order terms or derivative couplings, which we will ignore. Different off-shell completions will generate different couplings at this order, including couplings between the Goldstino and the off-shell supergravity fields, which may be interesting to catalogue but deviate too far from the present discussion.

Next we introduce the *new* superspace term

$$\frac{9m_{3/2}}{1568}\int d^4\theta \left(\frac{X_{NL}}{f} + \frac{X^\dagger_{NL}}{f^\dagger}\right)\frac{\mathcal{J}_{\alpha\dot{\alpha}}\mathcal{J}^{\alpha\dot{\alpha}}}{|f|^2}, \tag{4.3}$$

which introduces the Goldstino couplings

$$\frac{i}{2\sqrt{6}\bar{M}_{Pl}}(G^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{S}^{\dot{\alpha}}_{\mu\text{matter}} + \bar{G}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}S_{\alpha\mu\text{matter}}) - m_{3/2}\bar{G}_{\dot{\alpha}}\bar{G}^{\dot{\alpha}} - m^\dagger_{3/2}G^\alpha G_\alpha. \tag{4.4}$$

There are also some higher order terms, mostly suppressed by higher orders in $1/F$. Collecting the relevant terms one obtains

$$\mathcal{L} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\psi_{\mu\alpha}\bar{\sigma}_{\nu}^{\dot{\alpha}\alpha}\partial_{\rho}\bar{\psi}_{\sigma\dot{\alpha}} - \bar{\psi}_{\mu\dot{\alpha}}\bar{\sigma}_{\nu}^{\dot{\alpha}\alpha}\partial_{\rho}\psi_{\sigma\alpha}) - i\left(m_{3/2}\psi_{\mu}^{\alpha}(\sigma^{\mu\nu})_{\alpha}^{\beta}\psi_{\nu\beta} + m_{3/2}^{\dagger}\bar{\psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})_{\dot{\beta}}^{\dot{\alpha}}\bar{\psi}_{\nu}^{\dot{\beta}}\right)$$
$$+\frac{i}{2\sqrt{6}\bar{M}_{Pl}}(G^{\alpha}\sigma_{\alpha\dot{\alpha}}^{\mu}\bar{S}_{\mu\mathrm{matter}}^{\dot{\alpha}} + \bar{G}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}S_{\alpha\mu\mathrm{matter}}) - m_{3/2}\bar{G}_{\dot{\alpha}}\bar{G}^{\dot{\alpha}} - m_{3/2}^{\dagger}G^{\alpha}G_{\alpha}$$
$$+\frac{1}{2\bar{M}_{Pl}}(\psi_{\mu}S^{\mu} + \bar{\psi}_{\mu}\bar{S}^{\mu}) + \frac{1}{2}iG^{\alpha}\sigma_{\alpha\dot{\alpha}}^{\mu}\partial_{\mu}\bar{G}^{\dot{\alpha}} + \frac{1}{2}i\bar{G}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}\partial_{\mu}G_{\alpha} + \ldots \tag{4.5}$$

4.1 The case of no matter

We consider first the case of vanishing matter contributions, $S_{\alpha\mathrm{matter}}^{\mu} = 0$. The component Lagrangian Eqn. (4.5) reduces to

$$\mathcal{L} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\psi_{\mu\alpha}\bar{\sigma}_{\nu}^{\dot{\alpha}\alpha}\partial_{\rho}\bar{\psi}_{\sigma\dot{\alpha}} - \bar{\psi}_{\mu\dot{\alpha}}\bar{\sigma}_{\nu}^{\dot{\alpha}\alpha}\partial_{\rho}\psi_{\sigma\alpha}) - i\left(m_{3/2}\psi_{\mu}^{\alpha}(\sigma^{\mu\nu})_{\alpha}^{\beta}\psi_{\nu\beta} + m_{3/2}^{\dagger}\bar{\psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})_{\dot{\beta}}^{\dot{\alpha}}\bar{\psi}_{\nu}^{\dot{\beta}}\right)$$
$$-m_{3/2}\bar{G}_{\dot{\alpha}}\bar{G}^{\dot{\alpha}} - m_{3/2}^{\dagger}G^{\alpha}G_{\alpha} + \frac{i}{\sqrt{2}\bar{M}_{Pl}}(F\psi_{\mu}^{\alpha}\sigma_{\alpha\dot{\alpha}}^{\mu}\bar{G}^{\dot{\alpha}} + F^{\dagger}\bar{\psi}_{\mu\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}G_{\alpha})$$
$$+\frac{1}{2}iG^{\alpha}\sigma_{\alpha\dot{\alpha}}^{\mu}\partial_{\mu}\bar{G}^{\dot{\alpha}} + \frac{1}{2}i\bar{G}_{\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}\partial_{\mu}G_{\alpha} + \ldots \tag{4.6}$$

In this case, we can realise the super-Higgs mechanism by applying the shifts

$$\Psi_{\mu\alpha} = \psi_{\mu\alpha} - \frac{i}{\sqrt{6}}\sigma_{\mu\alpha\dot{\alpha}}\bar{G}^{\dot{\alpha}} - \sqrt{\frac{2}{3}}\frac{\partial_{\mu}G_{\alpha}}{m_{\frac{3}{2}}} \quad , \quad \bar{\Psi}_{\mu}^{\dot{\alpha}} = \bar{\psi}_{\mu}^{\dot{\alpha}} - \frac{i}{\sqrt{6}}\bar{\sigma}_{\mu}^{\dot{\alpha}\alpha}G_{\alpha} - \sqrt{\frac{2}{3}}\frac{\partial_{\mu}\bar{G}^{\dot{\alpha}}}{m_{\frac{3}{2}}^{\dagger}}, \tag{4.7}$$

then one reproduces

$$\mathcal{L} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\Psi_{\mu}^{\alpha}\sigma_{\nu\alpha\dot{\alpha}}\partial_{\rho}\bar{\Psi}_{\sigma}^{\dot{\alpha}} - \bar{\Psi}_{\mu\dot{\alpha}}\bar{\sigma}_{\nu}^{\dot{\alpha}\alpha}\partial_{\rho}\Psi_{\sigma\alpha}) - i\left(m_{3/2}\Psi_{\mu}^{\alpha}(\sigma^{\mu\nu})_{\alpha}^{\beta}\Psi_{\nu\beta} + m_{3/2}^{\dagger}\bar{\Psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})_{\dot{\beta}}^{\dot{\alpha}}\bar{\Psi}_{\nu}^{\dot{\beta}}\right)$$

the Lagrangian of a massive Gravitino Weyl spinor Ψ_{μ}^{α}.

4.2 Coupling to matter

To couple the massive Gravitino to the matter supercurrent requires that all components of the shift of Eqn. (4.7) couple to the matter supercurrent. After integration by parts, one of these shifted terms is the Goldberger Treiman relation. These terms are the most useful for phenomenology [13]. In the superspace formalism, the Goldberger Treiman terms appear in components in their non derivative form [8,9] from superspace terms such as

$$\int d^2\theta \frac{m_{\lambda}}{2f}X_{NL}W^{\alpha}W_{\alpha} + \int d^2\bar{\theta}\frac{m_{\lambda}^{\dagger}}{2f^{\dagger}}X_{NL}^{\dagger}\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}} \tag{4.8}$$

and

$$\int d^4\theta \frac{m_0^2}{|f|^2}X_{NL}^{\dagger}X_{NL}\Phi^{\dagger}\Phi \tag{4.9}$$

where Φ represents some generic matter chiral superfield and W^α is the superfield strength tensor. m_λ and m_0 are the soft supersymmetry breaking masses of Gauginos and Scalars. Expanding out these terms in superspace, first give the soft breaking mass terms and at linear order in the Goldstino, give the Golberger Treiman relations. After collecting the components and use of the equation of motion, this will supply

$$\frac{G^\alpha}{2f}\partial_\mu S^\mu_{\alpha\text{matter}} + \frac{\bar{G}_{\dot{\alpha}}}{2f^\dagger}\partial_\mu \bar{S}^{\mu\dot{\alpha}}_{\text{matter}}. \tag{4.10}$$

Including these terms with the action given by Eqn. (4.5) and applying the shifts of Eqn. (4.7) we find

$$\mathcal{L} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\Psi^\alpha_\mu \sigma_{\nu\alpha\dot{\alpha}}\partial_\rho \bar{\Psi}^{\dot{\alpha}}_\sigma - \bar{\Psi}_{\mu\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}_\nu \partial_\rho \Psi_{\sigma\alpha}) - i\left(m_{3/2}\Psi^\alpha_\mu(\sigma^{\mu\nu})^\beta_\alpha \Psi_{\nu\beta} + m^\dagger_{3/2}\bar{\Psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}}\bar{\Psi}^{\dot{\beta}}_\nu\right)$$

$$+\frac{1}{2\bar{M}_{Pl}}(\Psi^\alpha_\mu(S^\mu_\alpha)_{\text{matter}} + \bar{\Psi}_{\mu\dot{\alpha}}(S^{\mu\dot{\alpha}})_{\text{matter}}) + ... \tag{4.11}$$

The Lagrangian of a massive Weyl Gravitino coupled to matter.

5 Applications

So far what we have written is rather abstract. In this section we wish to demonstrate some of the uses that an effective action of the super-Higgs mechanism in superspace may have. To demonstrate this we first write down the effective action which, in components, will reproduce the linearised action that correctly describes the theory of multiple Goldstini [7].

5.1 Goldstini

As a simple application, we would like to write a superspace action whose components naturally reproduce the effect of multiple supersymmetry breaking sectors, and hence, multiple Goldstini. We use an index i running from 1 to N, to label the supersymmetry breaking sectors, with F_i F-terms and X_i chiral superfields. This is of course not the mass basis. The index i is implicitly summed. We introduce the N Goldstino multiplets

$$\int d^4\theta X_{iNL}X^\dagger_{iNL} + \left(\int d^2\theta f^\dagger_i X_{iNL} + \int d^2\bar{\theta} f_i X^\dagger_{iNL}\right). \tag{5.1}$$

It should be noted that f^2_{eff} is the sum of the f^2_i terms. In the effective action the Goldstini self couplings are dependent on C. Once the overall cosmological constant is assumed to vanish $C = \frac{4}{\sqrt{3}}f^2_{eff}\bar{M}_{Pl}$, this sets $m_{3/2} = f_{eff}/\sqrt{3}M_{pl}$, this also fixes the masses of the other η_i Goldstini which are proportional to C (and not on their respective F_i as one might naively think from an effective theory approach). That it is the vanishing of the overall cosmological constant that sets $m_a = 2m_{3/2}$ seems to be the most intuitive argument for the Goldstini mass.

We add the terms necessary to generate the relevant Goldstino self coupling terms

$$\frac{9}{1568\sqrt{3}\bar{M}_{pl}}\int d^4\theta \left(\frac{f_{\text{eff}}^\dagger}{f_i(f_i^\dagger)^2}X_{iNL}+\frac{f_{\text{eff}}}{f_i^\dagger(f_i)^2}X_{iNL}^\dagger\right)J_{i\alpha\dot{\alpha}}J_i^{\alpha\dot{\alpha}} \tag{5.2}$$

where $f_iX_i|_{\theta^2}=\sum_i f_i^2=f_{eff}^2$ and we have chosen the coefficients such that $m_{3/2}=f_{eff}/\sqrt{3}M_{pl}$.

The last step is to introduce the single Gravitino self coupling

$$-\frac{1}{64\sqrt{3}\bar{M}_{pl}}\int d^4\theta \left(\frac{f_i^\dagger X_{iNL}}{f_{\text{eff}}^\dagger}+\frac{f_iX_{iNL}^\dagger}{f_{\text{eff}}}\right)\left[\bar{D}_{\dot{\alpha}}D_\alpha H^{\beta\dot{\beta}}\bar{D}^{\dot{\alpha}}D^\alpha H_{\beta\dot{\beta}}-\frac{4}{3}(\bar{D}_{\dot{\alpha}}D_\alpha H^{\alpha\dot{\alpha}})^2\right]. \tag{5.3}$$

This action, with Eqn. (2.10) and Eqn. (2.12) and the obvious generalisations of Eqn. (4.8) with Eqn. (4.9), will in the mass basis reproduce both the super-Higgs mechanism for the true Goldstino and generate $m_i=2m_{3/2}$ for the uneaten Goldstini.

6 Discussion

In this paper we have written an effective action, in superspace, that manifestly respects global supersymmetry and whose components reproduce the super-Higgs mechanism. The coefficients of these superspace terms appear to be chosen by hand to reproduce the necessary components. We demonstrate that these coefficients arise from an effective action. After using this choice of coefficients, the components respect the necessary modified local supersymmetry transformations. It is perhaps unfortunate that local supersymmetry does not seem to fix the coefficients at the level of superfields but suggest that we should interpret our action as an effective one, in any case.

Still we think this setup is useful as it achieves the super-Higgs mechanism of the Goldstino multiplet and more interestingly, through the use of the supercurrent multiplet. Additionally we have outlined how an effective action may be written that reproduces the results of multiple Goldstini. [7].

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A The Super-Higgs Mechanism: A Review

In this section we review the super-Higgs mechanism following closely the appendix of [5]. In two component spinor notation, if supersymmetry is broken by an F term vacuum expectation value, then the Goldstino must transform as $\delta_\epsilon\chi=\sqrt{2}F\epsilon$ and

$\delta_\epsilon \bar{\chi} = \sqrt{2} F^\dagger \bar{\epsilon}$ where $(\epsilon, \bar{\epsilon})$ are supersymmetry transformation parameters and $\sqrt{2}$ is a convention. Treating supersymmetry as a global symmetry, Noether's theorem leads to a conserved supercurrent

$$\delta\mathcal{L} = (\partial_\mu \epsilon^\alpha) S^\mu_\alpha + (\partial_\mu \bar{\epsilon}_{\dot{\alpha}}) \bar{S}^{\mu\dot{\alpha}} = 0. \tag{A.1}$$

Integrating by parts one finds the variation of the action

$$\delta\mathcal{S} = -\int d^4x \left[\epsilon^\alpha (\partial_\mu S^\mu_\alpha) + \bar{\epsilon}_{\dot{\alpha}} (\partial_\mu \bar{S}^{\mu\dot{\alpha}})\right] = 0. \tag{A.2}$$

The action may be determined

$$S_{GT} = \int d^4x \left[\frac{\chi^\alpha}{\sqrt{2}F} \partial_\mu S^\mu_\alpha + \frac{\bar{\chi}_{\dot{\alpha}}}{\sqrt{2}F^\dagger} \partial_\mu \bar{S}^{\mu\dot{\alpha}}\right]. \tag{A.3}$$

This gives the familiar Goldberger Treiman relation and a kinetic term for the Goldstino. The supercurrent should contain general matter contributions and a term proportional to the vev:

$$S^\mu_\alpha = S^\mu_{\alpha\mathrm{matter}} + i\sqrt{2}F \sigma^\mu_{\alpha\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} \quad , \quad \bar{S}^{\mu\dot{\alpha}} = \bar{S}^{\mu\dot{\alpha}}_{\mathrm{matter}} + i\sqrt{2}F^\dagger \bar{\sigma}^{\mu\dot{\alpha}\alpha} \chi_\alpha. \tag{A.4}$$

Invariance of the action under supersymmetry transformations implies the canonically normalised Goldstino kinetic terms

$$\frac{1}{2} i \chi^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \bar{\chi}^{\dot{\alpha}} + \frac{1}{2} i \bar{\chi}_{\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \partial_\mu \chi_\alpha, \tag{A.5}$$

these being related to each other by an integration by parts. The reader can verify that varying the above kinetic contribution with respect to χ^α and $\bar{\chi}^{\dot{\alpha}}$ independently will lead to the constraints $\partial_\mu S^{\mu\alpha} = 0$ and $\partial_\mu \bar{S}^{\mu\dot{\alpha}} = 0$, as required in A.2, thus avoiding double counting the kinetic terms as would result by substituting A.4 in A.3 directly.

We now introduce the Gravitino

$$S_{kin} = -\frac{1}{2} \int d^4x \epsilon^{\mu\nu\rho\sigma} (\psi^\alpha_\mu \sigma_{\nu\alpha\dot{\alpha}} \partial_\rho \bar{\psi}^{\dot{\alpha}}_\sigma - \bar{\psi}_{\mu\dot{\alpha}} \bar{\sigma}^{\dot{\alpha}\alpha}_\nu \partial_\rho \psi_{\sigma\alpha}) \tag{A.6}$$

and consider weakly gauging gravity by the introduction of a Gravitino that couples to the supercurrent

$$S_{\mathrm{int1}} = \int d^4x \frac{1}{2\bar{M}_{Pl}} [\psi^\alpha_\mu S^\mu_\alpha + \bar{\psi}_{\dot{\alpha}\mu} \bar{S}^{\dot{\alpha}\mu}]. \tag{A.7}$$

This term naturally leads to

$$\frac{iF}{\sqrt{2}\bar{M}_{Pl}} \psi^\alpha_\mu \sigma^\mu_{\alpha\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} + \frac{iF^\dagger}{\sqrt{2}\bar{M}_{Pl}} \bar{\psi}_{\mu\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \chi_\alpha, \tag{A.8}$$

Additionally one must introduce the term

$$\mathcal{S}_{\mathrm{int2}} = \int d^4x \frac{i}{2\sqrt{6}\bar{M}_{Pl}} [\chi^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \bar{S}^{\dot{\alpha}}_\mu + \bar{\chi}_{\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} S_{\alpha\mu}], \tag{A.9}$$

to obtain

$$-\frac{F^\dagger}{\sqrt{3}\bar{M}_{Pl}}\chi^\alpha\chi_\alpha - \frac{F}{\sqrt{3}\bar{M}_{Pl}}\bar{\chi}_{\dot\alpha}\bar{\chi}^{\dot\alpha}. \tag{A.10}$$

which is the first source of what will later become the Gravitino mass of the super-Higgs mechanism. In addition, it generates a non derivative coupling between the Goldstino and the supercurrent

$$\frac{i}{2\sqrt{6}\bar{M}_{Pl}}\bar{\chi}^{\dot\alpha}\sigma^\mu_{\alpha\dot\alpha}(S^\alpha_\mu)_{\text{matter}} + \frac{i}{2\sqrt{6}\bar{M}_{Pl}}\chi^\alpha\sigma^\mu_{\alpha\dot\alpha}(\bar{S}^{\dot\alpha}_\mu)_{\text{matter}}. \tag{A.11}$$

To preserve local supersymmetry invariance under the *modified* [3] transformations

$$\begin{aligned} \delta\psi_{\mu\alpha} &= -\bar{M}_{Pl}\left(2\partial_\mu\epsilon_\alpha + im_{\frac{3}{2}}\sigma_{\mu\alpha\dot\alpha}\epsilon^{\dot\alpha}\right) && \text{(A.12)} \\ \delta\chi_\alpha &= \sqrt{2}F\epsilon_\alpha && \text{(A.13)} \end{aligned}$$

one must add a Gravitino self-coupling term

$$S_{m_{3/2}} = -i\int d^4x\left(m_{3/2}\psi^\alpha_\mu(\sigma^{\mu\nu})_\alpha^\beta\psi_{\nu\beta} + m^\dagger_{3/2}\bar{\psi}_{\mu\dot\alpha}(\bar{\sigma}^{\mu\nu})^{\dot\alpha}_{\dot\beta}\bar{\psi}^{\dot\beta}_\nu\right), \tag{A.14}$$

provided that

$$m_{3/2} = \frac{F}{\sqrt{3}\bar{M}_{Pl}}. \tag{A.15}$$

Gathering all the terms together the overall Lagrangian is therefore

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\psi^\alpha_\mu\sigma_{\nu\alpha\dot\alpha}\partial_\rho\bar{\psi}^{\dot\alpha}_\sigma - \bar{\psi}_{\mu\dot\alpha}\bar{\sigma}^{\dot\alpha\alpha}_\nu\partial_\rho\psi_{\sigma\alpha}) - i\left(m_{3/2}\psi^\alpha_\mu(\sigma^{\mu\nu})^\beta_\alpha\psi_{\nu\beta} + m^\dagger_{3/2}\bar{\psi}_{\mu\dot\alpha}(\bar{\sigma}^{\mu\nu})^{\dot\alpha}_{\dot\beta}\bar{\psi}^{\dot\beta}_\nu\right) \\ &+\frac{i}{2}\chi^\alpha\partial_\mu\sigma^\mu_{\alpha\dot\alpha}\bar{\chi}^{\dot\alpha} + \frac{i}{2}\bar{\chi}_{\dot\alpha}\partial_\mu\bar{\sigma}^{\dot\alpha\alpha}\chi_\alpha - \frac{F^\dagger}{\sqrt{3}\bar{M}_{Pl}}\chi^\alpha\chi_\alpha - \frac{F}{\sqrt{3}\bar{M}_{Pl}}\bar{\chi}_{\dot\alpha}\bar{\chi}^{\dot\alpha} \\ &+\frac{1}{2\sqrt{6}\bar{M}_{Pl}}i\sigma^\mu_{\alpha\dot\alpha}\bar{\chi}^{\dot\alpha}(S^\alpha_\mu)_{\text{matter}} + \frac{1}{2\sqrt{6}\bar{M}_{Pl}}i\chi^\alpha\sigma^\mu_{\alpha\dot\alpha}(\bar{S}^{\dot\alpha}_\mu)_{\text{matter}} \\ &+\frac{iF}{\sqrt{2}\bar{M}_{Pl}}\psi^\alpha_\mu\sigma^\mu_{\alpha\dot\alpha}\bar{\chi}^{\dot\alpha} + \frac{iF^\dagger}{\sqrt{2}\bar{M}_{Pl}}\bar{\psi}_{\mu\dot\alpha}\bar{\sigma}^{\mu\dot\alpha\alpha}\chi_\alpha \\ &+\frac{1}{2\bar{M}_{Pl}}(\psi^\alpha_\mu(S^\mu_\alpha)_{\text{matter}} + \bar{\psi}_{\mu\dot\alpha}(S^{\mu\dot\alpha})_{\text{matter}}) \\ &+\frac{\chi^\alpha}{\sqrt{2}F}\partial_\mu S^\mu_{\alpha\text{matter}} + \frac{\bar{\chi}_{\dot\alpha}}{\sqrt{2}F^\dagger}\partial_\mu\bar{S}^{\mu\dot\alpha}_{\text{matter}}.\end{aligned} \tag{A.16}$$

The super-Higgs mechanism is realised by applying the shift

$$\begin{aligned} \Psi_{\mu\alpha} &\to \psi_{\mu\alpha} - \frac{i}{\sqrt{6}}\sigma_{\mu\alpha\dot\alpha}\bar{\chi}^{\dot\alpha} - \sqrt{\frac{2}{3}}\frac{1}{m_{\frac{3}{2}}}\partial_\mu\chi_\alpha && \text{(A.17)} \\ \bar{\Psi}^{\dot\alpha}_\mu &\to \bar{\psi}^{\dot\alpha}_\mu - \frac{i}{\sqrt{6}}\bar{\sigma}^{\alpha\dot\alpha}_\mu\chi_\alpha - \sqrt{\frac{2}{3}}\frac{1}{m^\dagger_{\frac{3}{2}}}\partial_\mu\bar{\chi}^{\dot\alpha} && \text{(A.18)} \end{aligned}$$

so that the Gravitino eats the Goldstino degrees of freedom and the Lagrangian becomes that of the massive Gravitino coupled to matter

$$\mathcal{L} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\Psi_\mu^\alpha \sigma_{\nu\alpha\dot{\alpha}}\partial_\rho \bar{\Psi}_\sigma^{\dot{\alpha}} - \bar{\Psi}_{\mu\dot{\alpha}}\bar{\sigma}_\nu^{\dot{\alpha}\alpha}\partial_\rho\Psi_{\sigma\alpha}) - i\left(m_{3/2}\Psi_\mu^\alpha(\sigma^{\mu\nu})_\alpha^\beta\Psi_{\nu\beta} + m_{3/2}^\dagger\bar{\Psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})_{\dot{\beta}}^{\dot{\alpha}}\bar{\Psi}_\nu^{\dot{\beta}}\right) + \frac{1}{2\bar{M}_{Pl}}(\Psi_\mu^\alpha(S_\alpha^\mu)_{\rm matter} + \bar{\Psi}_{\mu\dot{\alpha}}(S^{\mu\dot{\alpha}})_{\rm matter}), \tag{A.19}$$

with the Gravitino now carrying the right degrees of freedom for the self-interaction term to be correctly identified with the Gravitino mass.

This review of the super-Higgs mechanism generates the same Lagrangian as that of Deser-Zumino [3], which is a combination of the A-V action [2] plus linear supergravity, with one addtion: In their paper they additionally comment on the two cosmological constants $-\frac{f^2}{2}e + ce$, which are set to cancel. where $e \equiv det(e_\mu^a)$ and e_μ^a is the Vielbein of the Graviton. In this review and in [5] this is implicitly assumed.

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