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Metastable Electroweak Vacuum: Implications for Inflation

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Abstract

Within the Standard Model, the current Higgs and top quark data favor metastability of the electroweak vacuum, although the uncertainties are still significant. The true vacuum is many orders of magnitude deeper than ours and the barrier separating the two is tiny compared to the depth of the well. This raises a cosmological question: how did the Higgs field get trapped in the shallow minimum and why did it stay there during inflation? The Higgs initial conditions before inflation must be fine–tuned to about one part in 10^8 in order for the Higgs field to end up in the right vacuum. In this note, we show that these problems can be resolved if there is a small positive coupling between the Higgs and the inflaton.

The ATLAS and CMS experiments at the LHC have recently observed a particle whose properties are well consistent with those expected of the Standard Model Higgs boson. Its mass is determined to be

$$M_h = 126.0 \pm 0.4 \pm 0.4 \text{ GeV} \quad , \quad \text{ATLAS [1]}$$

$$M_h = 125.3 \pm 0.4 \pm 0.5 \text{ GeV} \quad , \quad \text{CMS [2]}$$

Such a light Higgs boson coupled with the recent Tevatron top quark mass determination

$$m_t^{\rm exp} = 173.2 \pm 0.9 \text{ GeV [3]}$$

favors metastability of the electroweak (EW) vacuum. Taking, for example, $M_h = 125$ GeV and $m_t = 173$ GeV, one finds that the quartic Higgs coupling turns negative at $\Lambda \sim 10^{10}$ GeV [4], indicating that the electroweak vacuum is not the ground state and therefore only metastable,

although its lifetime is greater than the age of the universe [5]. In fact, the two loop analysis of [4] finds that absolute stability is disfavored at 98% CL for $M_h < 126$ GeV, with the main uncertainty coming from the top mass determination. Although no conclusive statement can yet be made [6, 7, 8] as the uncertainties may be larger than those assumed in [4], this shows that the current data favor metastability of our vacuum.

Extrapolating the Standard Model all the way to the Planck scale, one would then conclude that the Higgs field is trapped in the false vacuum with a much larger energy density than that of the ground state and that the barrier separating the two is very small compared to the difference of the energy densities (Fig. 1):

$$\Lambda^4 \ll M_{\rm Pl}^4 \, . \tag{1}$$

Here we consider the large Higgs field regime $h \gg v$ ($v = 246$ GeV) such that [9]

$$V_{\rm Higgs}(h) \simeq \frac{1}{4}\lambda(h)\ h^4 \tag{2}$$

in the unitary gauge, where $\lambda_h(M_h) \simeq 0.13$. The coupling runs logarithmically with h and turns negative at the "instability scale" Λ. This raises the cosmological question: how did the universe end up in such an energetically disfavored state? For generic initial conditions $h \lesssim M_{\rm Pl}$ at the beginning of inflation [10, 11], the universe is overwhelmingly likely to evolve to the true ground state of the system. Not only would that lead to different physics, but would also be catastrophic since the latter is expected to have a negative Planck–scale energy density and even the (COBE–normalized) inflaton contribution $10^{-9} M_{\rm Pl}^4$ would not stop the gravitational collapse [12], before thermal effects get a chance to play any role. One therefore faces a fine–tuning problem: the initial value of the Higgs field must be extremely small, $h \lesssim \Lambda$, in Planck units. For $\Lambda \sim 10^{10}$ GeV, this constitutes a 1 in 10^8 tuning.[1]

Furthermore, even if the Higgs field starts at the origin, it will not necessarily remain there during inflation. Since it is effectively massless, its quantum fluctuations are of order the Hubble scale H. Therefore, for $H > \Lambda$ it is likely to end up in the wrong vacuum. For $H \ll \Lambda$, it will remain at the origin, yet this does not solve the problem of initial conditions.

One possibility to address this issue is based on the landscape idea, along the lines of Ref. [13]. Namely, the regions of multiverse with the "wrong" initial conditions collapse due to the AdS

[1] The Standard Model also suffers from another fine–tuning, namely, the hierarchy problem. It is however not directly related to the cosmological problem above since the latter has to do with a fine–tuning in the initial conditions.

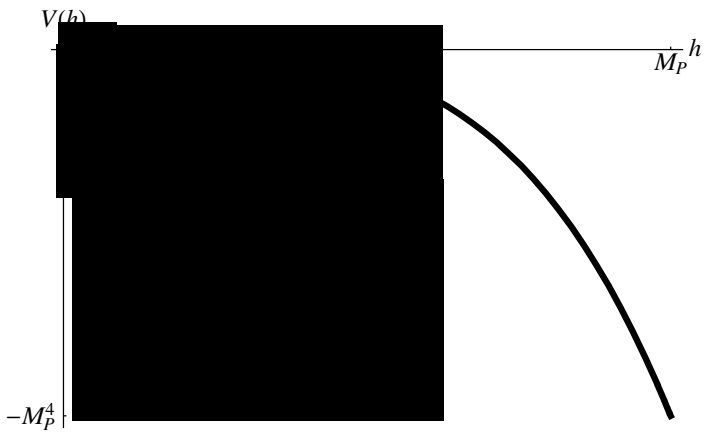

Figure 1: A *schematic* view of the Higgs potential ($\Lambda \sim 10^{10}$ GeV $\ll M_{\rm Pl}$).

instability such that all the remaining regions would have the Higgs field around the origin in field space. However, as shown in [13], those regions that survive (large–field) inflation would only allow for small curvature perturbations and the probability of generating the right amount of perturbations is exponentially small.

One may also declare that the Higgs field was prepared in a special state by unknown pre–inflationary dynamics, but this simply begs the question. Possible thermal effects would not do the job since at large h the fields which couple to the Higgs are heavy and not expected to be in thermal equilibrium.

It is worth noticing that the problem disappears altogether if one allows for physics beyond the Standard Model. For example, a tiny coupling of the Higgs to the hidden sector can stabilize the potential [14] and allow for "Higgs–portal" inflation [15]. We will however take a conservative view and assume that the SM, with the addition of an inflaton, is valid up to the Planck scale. The Higgs itself cannot play the role of an inflaton [16, 17, 18, 19] if the electroweak vacuum is metastable and the extra degree of freedom is necessary.

In this work, we show that the above problems can be resolved if there is a Higgs–inflaton coupling which drives the Higgs field to small values during inflation. Suppose the full scalar potential is given by

$$V = V_{\rm Higgs}(h) + V_{\rm cross}(h,\phi) + V_{\rm infl}(\phi)\ , \tag{3}$$

where ϕ is the inflaton. Then, the Higgs field evolves to the electroweak vacuum after inflation

if

$$h_{\rm end} \lesssim \Lambda \ , \tag{4}$$

where $h_{\rm end}$ is the Higgs field value at the end of inflation. This requirement constraints inflationary models and allowed Higgs–inflaton couplings. Restricting ourselves to sub–Planckian Higgs fields and using gauge invariance, we can expand

$$V_{\rm cross}(h,\phi) = h^2 f_1(\phi) + h^4 f_2(\phi) + ... \tag{5}$$

The desired effect of the cross term is to make the Higgs potential convex and induce a Higgs mass term above the Hubble scale H so that h would evolve to small values during inflation.

Consider the simplest case of a renormalizable Higgs–inflaton coupling (as in Higgs–portal models [20]) and the quadratic inflaton potential in chaotic inflation [11],

$$V_{\rm cross} = \frac{1}{2}\xi h^2\phi^2 \quad , \quad V_{\rm infl} = \frac{1}{2}m^2\phi^2 \ , \tag{6}$$

where ξ is positive. The first constraint is that the coupling ξ should not lead to large radiative corrections to the inflaton potential during the last 60 e–folds. The most important correction is of order (see e.g. [21])

$$\Delta V_{\rm infl} \simeq \frac{\xi^2}{64\pi^2}\ \phi^4\ \ln\frac{\xi\phi^2}{m^2} \ , \tag{7}$$

so for $m = 10^{-5}$ and $\phi \sim 10$ in $Planck$ units [22], the constraint is

$$\xi \lesssim 10^{-6} \ . \tag{8}$$

Next, the Higgs potential becomes dominated by the cross coupling at

$$\phi_0 > \sqrt{\frac{|\lambda|}{2\xi}}\ h_0 \sim 20 \ , \tag{9}$$

where we have taken $|\lambda| \simeq 10^{-1}$ and chosen the initial Higgs field value $h_0 = 0.1$ such that higher dimensional Higgs operators are unimportant. With these initial conditions, the effective Higgs mass squared is large and positive, and the field will naturally evolve to small values. Let us consider this process in more detail.

The evolution of the Higgs and inflaton fields is governed by

$$\begin{aligned} \ddot{h} + 3H\dot{h} + \frac{\partial V}{\partial h} &= 0 \ , \\ \ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} &= 0 \ , \end{aligned} \tag{10}$$

with

$$3H^2 = \frac{1}{2}\dot{h}^2 + \frac{1}{2}\dot{\phi}^2 + V \tag{11}$$

and

$$V \simeq \frac{1}{2}\xi h^2\phi^2 + \frac{1}{2}m^2\phi^2 \ . \tag{12}$$

Suppose that initially $\dot{h}$ and $\dot{\phi}$ are insignificant. Then the Hubble rate is dominated by the cross term, $H_0 \simeq \sqrt{\xi/6}\ \phi_0 h_0$, and the effective Higgs and inflaton masses satisfy

$$m_\phi \ll H_0 \ll m_h \ . \tag{13}$$

It is then clear that h will evolve quickly leading to a rapid decrease in the expansion rate, while the evolution of ϕ is "slow–roll". At the initial stage of inflation, h evolves according to

$$\ddot{h} + \sqrt{\frac{3}{2}}\ \sqrt{\dot{h}^2 + m_h^2 h^2}\ \dot{h} + m_h^2 h = 0 \ , \tag{14}$$

with $m_h^2 = \xi\phi_0^2$. The solution to this equation is known in the limit $m_h t \gg 1$ (see e.g. late time inflaton evolution [23]),

$$h \simeq C\ \frac{\cos m_h t}{m_h t} \ , \tag{15}$$

with order one C. Since $m_h \simeq 25 H_0$, the asymptotics $m_h t \gg 1$ is reached after a few Hubble times H_0^{-1}. Therefore, in about 10 Hubble times, the amplitude of the Higgs field decreases by more than an order of magnitude. From that point on, the quadratic potential $m^2\phi^2$ takes over the energy density and the usual slow roll inflation takes place. The expansion rate becomes approximately constant and the Higgs evolution is governed by

$$\ddot{h} + 3H\dot{h} + m_h^2 h = 0 \ , \tag{16}$$

with $H \simeq m\phi_0/\sqrt{6}$. Its solutions are $C_\pm \exp\big(-3/2\ H \pm \sqrt{9/4\ H^2 - m_h^2}\ \big)$. Since $m_h \gg H$, the Higgs field decays exponentially,

$$|h| \sim e^{-\frac{3}{2}Ht}|h(0)| \ . \tag{17}$$

Within about 20 e-folds, it will be of electroweak size (Fig. 2). On the other hand, the evolution of ϕ is "slow–roll" and not affected by h, as the latter makes a negligible contribution to the energy density. The total number of e-folds is given approximately by $1/4\phi_0^2 > 100$ for ϕ_0 satisfying (9). Finally, note that the Higgs "instability" scale during inflation becomes

$$\Lambda \simeq \sqrt{\frac{2\xi}{|\lambda|}}\ \phi \gg H \tag{18}$$

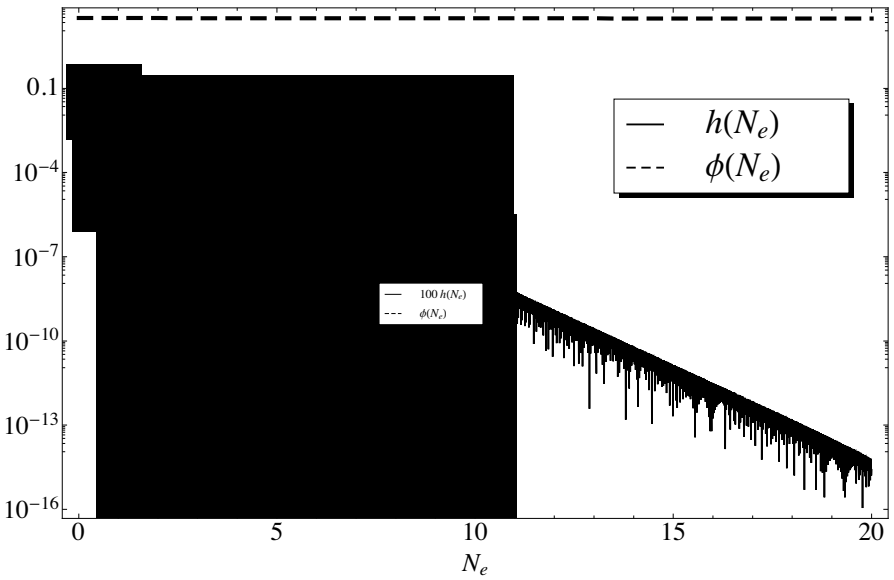

Figure 2: Evolution of the Higgs field (solid) and the inflaton (dashed) as a function of the number of e–folds N_e. The log–scale plot shows the absolute value of h which goes through zero during each oscillation, but gets cut off at a finite value for numerical reasons. The initial values are $\phi_0 = 32$, $h_0 = 0.1$ and $\xi = 10^{-6}$.

and the quantum fluctuations of the Higgs field are irrelevant.

We see that even in the simplest case of ϕ^2 inflation, the Higgs–inflaton coupling can stabilize the Higgs potential without spoiling the predictions for curvature perturbations. During inflation, the Higgs field evolves quickly to small values, yet the shape of the Higgs potential after inflation is unaffected since $\xi \ll 1$. The mechanism is operative in the following range:

$$10^{-10} \lesssim \xi \lesssim 10^{-6} \; . \tag{19}$$

The upper bound is dictated by the smallness of radiative corrections to the inflaton potential, while the lower bound comes from requiring fast Higgs evolution, $m_h \gtrsim H$. The latter is comparable to the limit on ξ imposed by the dominance of the classical roll of the inflaton over quantum fluctuations, $\phi_0 \lesssim 5/\sqrt{m}$ [21]. Note also that for $\xi < 10^{-8}$, the scalar potential is dominated by $m^2\phi^2$ and inflation is always "slow–roll".

The inflaton-Higgs coupling also provides the reheating mechanism through parametric resonance [24]. However, whether the reheating process is complete or not depends on the presence of other couplings which make the inflaton unstable. For instance, ϕ can couple to the right–

handed neutrinos[2] as $\phi\bar{N}N$, or have a trilinear coupling to the Higgs, ϕh^2. The reheating temperature is sensitive to such couplings and no model–independent prediction can be made. Unless it is exceedingly high (10^{15} GeV), the Higgs field will remain at small values throughout the reheating [13].

As seen from (9), there is no fundamental obstacle to increase the initial value of the Higgs field to Planckian values. In that case, however, calculability is lost due to higher order Higgs operators. It is also clear that the mechanism generalizes to other large–field inflationary potentials, as long as (9) is satisfied in the slow–roll region and $m_h \gtrsim H$.

The inflaton interactions may enjoy the shift symmetry which can justify the smallness of ξ and higher order operators [21]. In particular, small values of ξ are radiatively stable and not fine–tuned in the t'Hooft sense since setting $\xi = 0$ (and $m_\phi = 0$) makes the theory invariant under $\phi \to \phi +$ const, which is the usual shift symmetry of inflationary models. This is in contrast with the fine–tuning in the Higgs initial conditions, which is not justified by dynamics and symmetries.

The Higgs coupling to the inflaton obtained above is far too small to be probed at colliders. This applies to typical inflationary potentials, yet there is a notable exception. If one allows for a large non–minimal scalar coupling to gravity as in [16], ξ can be substantial. Suppose we add the term

$$\Delta\mathcal{L}/\sqrt{-g} \;=\; -\frac{1}{2}\kappa\phi^2 R\,, \tag{20}$$

where g is the determinant of the metric and R is the scalar curvature. Assume, for simplicity, that at large ϕ the scalar potential is dominated by

$$V \simeq \frac{1}{2}\xi h^2\phi^2 + \frac{1}{4}\lambda_\phi\phi^4\,. \tag{21}$$

Then, eliminating the non-minimal coupling to gravity by a conformal transformation, one finds that, for $\kappa\phi^2 \gg 1$, the scalar potential in the ϕ–direction is exponentially close to a flat one. Taking $\lambda_\phi \lesssim \mathcal{O}(1)$, the correct curvature perturbations are reproduced for $\kappa \sim 10^5$ [16]. The h–direction, on the other hand, is very steep with the effective mass of order $\sqrt{\xi/\kappa}$, while the Hubble rate is of order $\sqrt{\lambda_\phi}/\kappa$ (for details, see [15]). Thus, for $\xi \lesssim 0.1$, the Higgs will quickly evolve to small values, as before. Note that such values of ξ do not lead to significant quantum corrections to λ_h and the inflaton potential. In particular, the electroweak vacuum remains

[2]The right–handed neutrinos would be an important ingredient in the complete framework as they may constitute dark matter [25] and/or generate matter–antimatter asymmetry [26]. Note that their coupling to the Higgs does not improve the stability of the Higgs potential.

metastable. On the other hand, the Higgs coupling to the inflaton is similar in strength to the Higgs self–coupling, unlike in the previous scenarios. This is the familiar Higgs portal interaction [20], which, given a light enough inflaton, can potentially be probed at colliders. For example, the LHC already places some constraints on this scenario [27].

To conclude, we have argued that, in the Standard Model (which may include right–handed neutrinos), there is a fine–tuning problem with the initial conditions of the Higgs field at the beginning of inflation, if the electroweak vacuum is indeed metastable. Furthermore, the Higgs field is subject to large quantum fluctuations during inflation which can destabilize the EW vacuum. These problems can be circumvented by the presence of a small positive coupling of the Higgs to the inflaton, $\Delta V = \frac{1}{2}\xi h^2\phi^2$, as in Higgs portal models. In this case, the Higgs field is driven to small values during inflation, even if its initial value is close to the Planck scale. The important condition is that inflation be "large–field", while the specifics of the inflaton potential are not essential. The coupling ξ can be taken small enough not to affect the curvature perturbation predictions, in which case it does not change the shape of the Higgs potential after inflation, either. Finally, unlike a fine–tuning of the initial conditions, the smallness of the Higgs–inflaton coupling can in principle be justified by an approximate shift symmetry.

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