MS-TP-13-07
DESY 13-065

Towards the spectrum of low-lying particles in supersymmetric Yang-Mills theory

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April 07, 2013

arXiv:1304.2168v1 [hep-lat] 8 Apr 2013

We present the current results of our simulations of $\mathcal{N} = 1$ supersymmetric Yang-Mills theory on a lattice. The masses of the gluino-glue particle, the a–η', the a–f_0 meson, and the scalar glueball are obtained at finer lattice spacing than before, and extrapolations towards vanishing gluino mass are made. The calculations employ different levels of stout smearing. The statistical accuracy as well as the control of finite size effects and lattice artefacts are better than in previous investigations. Taking the statistical and systematic uncertainties into account, the extrapolations towards vanishing gluino mass of the masses of the fermionic and bosonic states in our present calculations are consistent with the formation of degenerate supermultiplets.

1 Introduction

$\mathcal{N} = 1$ supersymmetric Yang-Mills theory (SYM) is a theory of gluons, described by non-Abelian gauge theory, and their superpartners, the gluinos, which are spin 1/2 Majorana fermions in the adjoint representation of the gauge group. The on-shell Lagrangian of SYM in Minkowski space is

$$\mathcal{L} = \mathrm{tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{2}\bar{\lambda}\gamma^{\mu}D_{\mu}\lambda - \frac{m_g}{2}\bar{\lambda}\lambda\right], \tag{1}$$

where $F_{\mu\nu}$ is the non-Abelian field strength formed out of the gluon fields $A_{\mu}(x)$, $\lambda(x)$ is the gluino field, and D_{μ} denotes the gauge covariant derivative in the adjoint representation. The supersymmetry of the theory is broken softly by the gluino mass term.

In several aspects the model is similar to QCD [1]. The importance of related theories in extensions of the Standard Model and the connection to QCD [2] is a main motivation for the recent interest in these models. SYM is asymptotically free and is assumed to show confinement. Thus gluons and gluinos are not particle states in the physical Hilbert space, which instead contains bound states of gluons and gluinos. These bound states must form degenerate supermultiplets, if supersymmetry is unbroken. The determination of the masses of bound states is a non-perturbative problem. Previous work of our group has been dedicated to the calculation of the low-lying masses of bound states in SYM by means of numerical simulations on a space-time lattice [3, 4]. The present article is a continuation of our preparatory work, which should be referred to for more details and references [5].

On the lattice, supersymmetry is generically broken [6]. In SYM a fine-tuning of the bare gluino mass parameter in the continuum limit is enough to approach the symmetries of the continuum theory [7, 8]. These symmetries include (spontaneously broken) chiral symmetry and supersymmetry. The theoretical prediction of the existence of a supersymmetric chiral continuum limit needs to be confronted with the numerical lattice simulations.

A necessary condition for the restoration of supersymmetry in the continuum limit is the degeneracy of fermionic and bosonic masses. From low energy effective theories, predictions have been made for two low-lying supermultiplets [9, 10]. Each multiplet consists of a scalar, a pseudoscalar, and a fermionic spin 1/2 particle. The lighter multiplet consists of a 0^{++} and a 0^{-+} glueball and a gluino-glue state. The gluino-glue is an exotic spin 1/2 Majorana fermion, which can be created by the operator

$$\tilde{O}_{g\tilde{g}} = \sum_{\mu\nu} \sigma_{\mu\nu}\mathrm{tr}\left[F^{\mu\nu}\lambda\right], \tag{2}$$

with $\sigma_{\mu\nu} = \frac{1}{2}\left[\gamma_{\mu}, \gamma_{\nu}\right]$. The heavier multiplet is built from the scalar meson a–f_0, represented by $\bar{\lambda}\lambda$, the pseudo-scalar meson a–η', represented by $\bar{\lambda}\gamma_5\lambda$, and a gluino-glue state.

In our previous work [3, 4] the expected degeneracy of the fermionic and bosonic masses was not observed. The gluino-glue appeared heavier than its lightest possible superpartners. In that work, however, the masses were obtained at a fixed lattice spacing, and the influence of finite volume effects was not known with sufficient accuracy. In order to obtain results relevant for the presumed supersymmetric continuum limit, the chiral limit, the continuum limit, and the infinite volume limit have to be extrapolated from the simulations.

In our preparatory work [5] we have made a detailed analysis of finite volume effects, and observed supersymmetry breaking effects due to the finite spatial extent of the lattice. The

results of these calculations allow to estimate the lattice sizes necessary for neglecting them. Moreover, simulations at a lattice spacing smaller than before indicate that the influence of the finite lattice spacing is larger than expected, and could be the cause of the apparent supersymmetry breaking.

In the present article we conclude our investigations of the finer lattice spacing, taking into account also different levels of stout smearing. The masses of the gluino-glue particle, the a–η', the a–f_0 meson, and the scalar glueball are obtained with the high statistics that turned out to be necessary, and extrapolations towards vanishing gluino mass are made. The statistical accuracy as well as the control of finite size effects and lattice artefacts is better than in all the previous investigations.

2 Numerical simulations

In our numerical simulations of SYM, the Euclidean version of the model is formulated on a space-time lattice with an action proposed by [7]. We are considering the case of gauge group SU(2). The gauge field dynamics is defined by the tree-level Symanzik improved plaquette action. The inverse bare gauge coupling in the current simulations was $\beta = 1.75$. The results will be compared to the previous ones at $\beta = 1.6$ [3, 4]. The gluinos are described by Wilson fermions in the adjoint representation. The Wilson-Dirac operator

$$(\mathrm{D_w})_{x,a,\alpha;y,b,\beta} = \delta_{xy}\delta_{ab}\delta_{\alpha\beta} - \kappa \sum_{\mu=1}^{4} \Big[(1-\gamma_\mu)_{\alpha\beta}(V_\mu(x))_{ab}\delta_{x+\mu,y} + (1+\gamma_\mu)_{\alpha\beta}(V_\mu^\dagger(x-\mu))_{ab}\delta_{x-\mu,y}\Big] \quad (3)$$

contains stout smeared [11] gauge links $V_\mu(x)$ in the adjoint representation. The hopping parameter κ is related to the bare gluino mass via $\kappa = 1/(2m_g + 8)$. The recovery of both supersymmetry and chiral $\mathrm{U}(1)_R$ symmetry in the continuum limit requires to tune the hopping parameter to the point $\kappa_c(\beta)$, where the renormalised gluino mass vanishes [7, 8]. In practice, this is achieved by monitoring the mass of the unphysical adjoint pion (a–π), which can be defined in a partially quenched setup. The adjoint pion mass, which is expected to vanish at κ_c, can can be numerically obtained relatively easily. Its correlator is defined as the connected contribution of the correlator for the a–η' meson. The consistency of the tuning according to $m_{\mathrm{a}\text{–}\pi}$ with the supersymmetric Ward identities was shown in [3]. Further details can be found in [5].

The configurations have been obtained by updating with a two-step polynomial hybrid Monte Carlo (PHMC) algorithm [12, 3]. Near κ_c the occurrence of low eigenvalues of the Hermitian Wilson-Dirac operator makes it necessary to introduce correction factors to the polynomial approximation in the PHMC algorithm. These have been obtained, when necessary, from the correct fermionic contribution of the lowest eigenvalues. In contrast to the theory in the continuum, the lattice theory has a (mild) sign problem. The Pfaffian obtained by the integration of the Majorana fermions can sometimes have a negative sign [13]. When necessary, we included this sign in the reweighting. To reduce the statistical errors we have chosen the parameters of our present simulations such that the reweighting with correction factors and Pfaffian signs is not relevant for the results.

On the basis of our investigation of finite volume effects [5], the lattice sizes have been chosen such that finite volume effects can be neglected in comparison with statistical errors.

For a comparison of dimensionful quantities we use the Sommer parameter r_0, obtained from the static quark potential. For illustration, QCD units are being used by setting the Sommer parameter to $r_0 = 0.5\,\mathrm{fm}$. We have always taken the value of r_0 obtained by an extrapolation to κ_c of the data obtained at $\kappa > \kappa_c$. In QCD units the lattice spacing is $a = 0.058\,\mathrm{fm}$ at $\beta = 1.75$, and $a = 0.088\,\mathrm{fm}$ at $\beta = 1.6$ for the runs with one level of stout smearing. A convenient substitute for the gluino mass is the squared adjoint pion mass in physical units, $(r_0 m_{\text{a-}\pi})^2$.

The parameter sets, underlying our calculations at $\beta = 1.75$, are summarised in Tab. 1.

κ	l_s	$L^3 \times T$	$(r_0 m_{\text{a-}\pi})^2$	N_{conf}	τ_P
0.1490	1	$24^3 \times 48$	4.60(22)	9350	1.5(1)
0.1490	1	$32^3 \times 64$	4.63(20)	4864	1.5(1)
0.1492	1	$24^3 \times 48$	3.34(16)	13017	5(1)
0.1492	1	$32^3 \times 64$	3.37(15)	6207	1.5(1)
0.14925	1	$24^3 \times 48$	2.92(17)	7224	5(1)
0.1493	1	$24^3 \times 48$	2.27(32)	10280	5(1)
0.1494	1	$32^3 \times 64$	2.09(12)	5466	1.5(1)
0.1495	1	$32^3 \times 64$	1.36(10)	2007	1.5(1)
0.1350	3	$24^3 \times 64$	12.77(27)	10056	1.5(1)
0.1355	3	$24^3 \times 64$	9.71(22)	10311	1.5(1)
0.1360	3	$24^3 \times 64$	6.68(17)	10023	1.5(1)
0.1365	3	$24^3 \times 64$	3.785(92)	9667	1.5(1)
0.1368	3	$24^3 \times 64$	2.09(12)	10188	1.5(1)
0.1368	3	$32^3 \times 64$	2.176(61)	3813	1.5(1)

Table 1: Summary of the simulation parameters. l_s is the level of stout smearing in the Wilson-Dirac operator. The value of r_0/a used in $(r_0 m_{\text{a-}\pi})^2$ is obtained from the extrapolation to the chiral limit. Its value is $r_0/a = 9.02(18)$ for $l_s = 1$, and $r_0/a = 8.663(81)$ for $l_s = 3$. N_{conf} is the number of configurations in the measurements of the pion mass. The number of reweighting factors smaller than 0.98 is around 3% at $\kappa = 0.1495$ and around 1% at $\kappa = 0.1494$ ($l_s = 1$; $32^3 \times 64$). In all other runs the reweighting is not taken into account because the number of negative Pfaffians is always below 1%. τ_P is the integrated autocorrelation time.

3 Bound state masses

We have investigated the masses of the gluino-glue particle, the a–η' and the a–f_0 mesons, and the scalar glueball. The masses of the bound states are obtained from fits to the corresponding correlation functions. In case of the a–π and the gluino-glue the correlation functions yield rather good fits for a number of fit intervals. By means of a histogram method [14] reliable mass estimates for these particles could be obtained. In case of the mesons a–η' and a–f_0 the mass estimate is based on a single optimally selected interval in

Euclidean time. This interval is fixed to the region where a plateau of the effective mass is observed. A second estimate is taken from a fit interval of the same length, but shifted by one positive unit in Euclidean time. The difference of these two masses provides an estimate of the systematic error of the procedure. Further details of the histogram method and the fit procedure are explained in [5]. The fit procedure for the glueball is detailed below. The results for the masses are collected in Table 2.

κ	l_s	L	$r_0 m_{\mathrm{a}-\pi}$	$r_0 m_{\mathrm{a}-\eta'}$	$r_0 m_{\mathrm{a}-f_0}$	$r_0 m_{\tilde{g}g}$	$r_0 m_{0^{++}}$
0.1490	1	24	2.145(51)	3.39(21)(10)	3.60(36)(39)	3.65(27)	4.39(26)(25)
0.1490	1	32	2.151(46)	3.14(16)(12)	5.34(72)(97)	3.44(25)	4.26(43)(50)
0.1492	1	24	1.829(43)	2.92(17)(21)	2.53(31)(51)	3.36(16)	3.78(26)(31)
0.1492	1	32	1.835(41)	2.84(15)(21)	3.58(43)(44)	3.25(19)	3.84(32)(35)
0.14925	1	24	1.710(49)	2.78(20)(21)	3.55(65)(17)	2.97(16)	2.72(29)(41)
0.1493	1	24	1.51(11)	2.82(25)(14)	2.24(49)(28)	2.86(23)	3.16(30)(88)
0.1494	1	32	1.447(42)	2.70(23)(22)	3.94(47)(35)	3.17(19)	4.36(38)(29)
0.1495	1	32	1.167(44)	2.52(34)(21)	1.50(80)(22)	2.85(21)	4.15(45)(63)
0.1350	3	24	3.574(38)	4.113(91)(16)	4.58(51)(30)	4.77(15)	4.609(320)(77)
0.1355	3	24	3.117(35)	3.779(97)(170)	4.18(40)(26)	4.21(27)	4.27(28)(13)
0.1360	3	24	2.584(32)	3.319(88)(160)	3.47(30)(42)	4.10(18)	3.32(22)(48)
0.1365	3	24	1.946(24)	2.92(13)(17)	3.488(270)(47)	3.47(14)	3.92(25)(16)
0.1368	3	24	1.445(42)	2.30(24)(27)	3.08(23)(16)	3.143(91)	3.49(21)(37)
0.1368	3	32	1.475(21)	2.36(17)(22)	2.76(42)(33)	3.09(10)	2.31(21)(58)

Table 2: The masses of the relevant bound states in units of the Sommer scale r_0. For the mesons a–η' and a–f_0 and for the glueball 0^{++} the indicated error is given as (statistical error)(systematic error of the plateau estimation). The a–π and gluino-glue ($\tilde{g}g$) mass is obtained with the histogram method and already includes an estimate of the systematic error.

The extrapolation to the chiral limit is obtained from a linear fit of the masses as a function of the squared adjoint pion mass. Up to lattice artifacts and finite size effects this limit coincides with the supersymmetric limit, where the particle masses should be grouped in the predicted multiplets.

For the lattices with spatial extent $L = 24$ our statistics is much higher than for $L = 32$. An extent of $L = 24$ is also sufficiently large in view of finite size effects. Therefore we have decided to include only the $L = 24$ data in the extrapolations. Adding the $L = 32$ data would lead to small changes only.

The best accuracy can be obtained for the mass of the gluino-glue particle. Its correlator has been obtained using a combination of APE and Jacobi smearing. The gluino-glue mass is shown in Fig. 1 as a function of the squared adjoint pion mass, together with the extrapolation to the chiral limit. In all the figures the error bars include both the statistical and systematic errors.

The correlators for the a–η' and the a–f_0 bosons contain connected and disconnected contributions. The disconnected parts have been calculated using the stochastic estimator method [15], taking into account the exact contribution of the 100 lowest eigenmodes of the

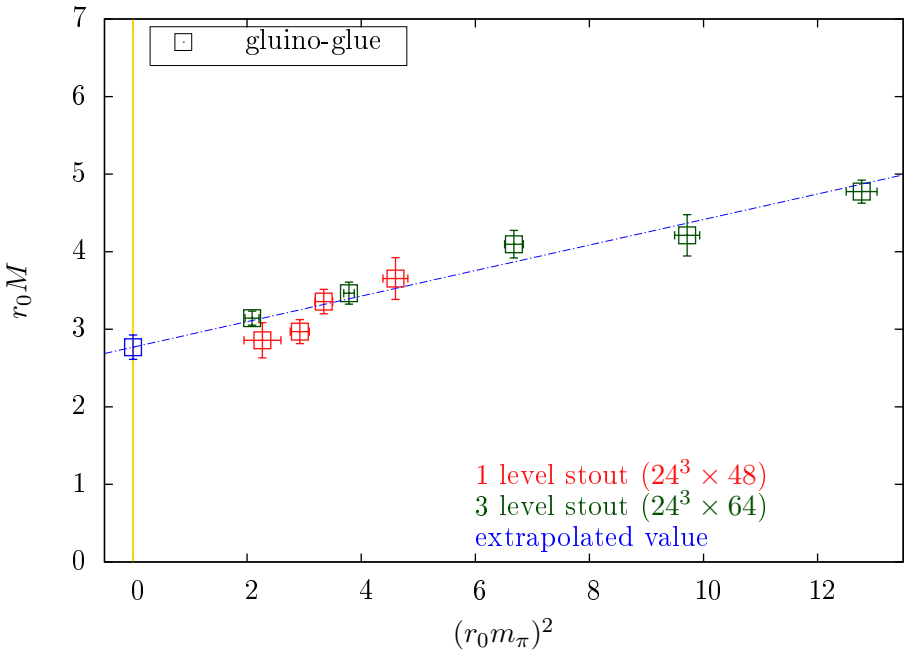

Figure 1: The gluino-glue mass as a function of the squared mass of the adjoint pion in units of the Sommer scale, and the corresponding linear fit.

even-odd preconditioned Hermitian Wilson-Dirac operator. The disconnected contributions are especially significant at smaller adjoint pion masses. The statistical fluctuations in this part are larger than in the connected contribution and lead to a bad signal-to-noise ratio in the correlators. In Fig. 2 the masses of the mesons are displayed together with the extrapolated values at the chiral limit. For comparison, the figures additionally include the linear fit of the gluino-glue mass.

The operator corresponding to the 0^{++} glueball is a combination of gauge links. The correlation functions are as usual afflicted by large statistical fluctuations. We are using variational smearing methods to improve the signal [16]. Smearing the underlying gauge links leads to a basis of different operators with the same quantum numbers. A combination of these operators providing the best overlap with the particle state is estimated numerically. From the correlations of the basic operators the correlation matrix $C(t)$ is obtained. The optimisation corresponds to the solution of the generalised eigenvalue problem

$$C(t)v = \lambda(t, t_0)C(t_0)v \tag{4}$$

for a given fixed t_0, in our case $t_0 = 0$. The resulting eigenvalues $\lambda(t, 0)$ as a function of t are taken as input for a fit procedure analogous to the case of the mesons a–η' and a–f_0. The quality of the signal is improved considerably by the variational method. However, the signal is yet not as good as for the gluino-glue, and the systematic error estimate of the plateau estimation is taken into account. The results for the glueball masses are shown in Fig. 3.

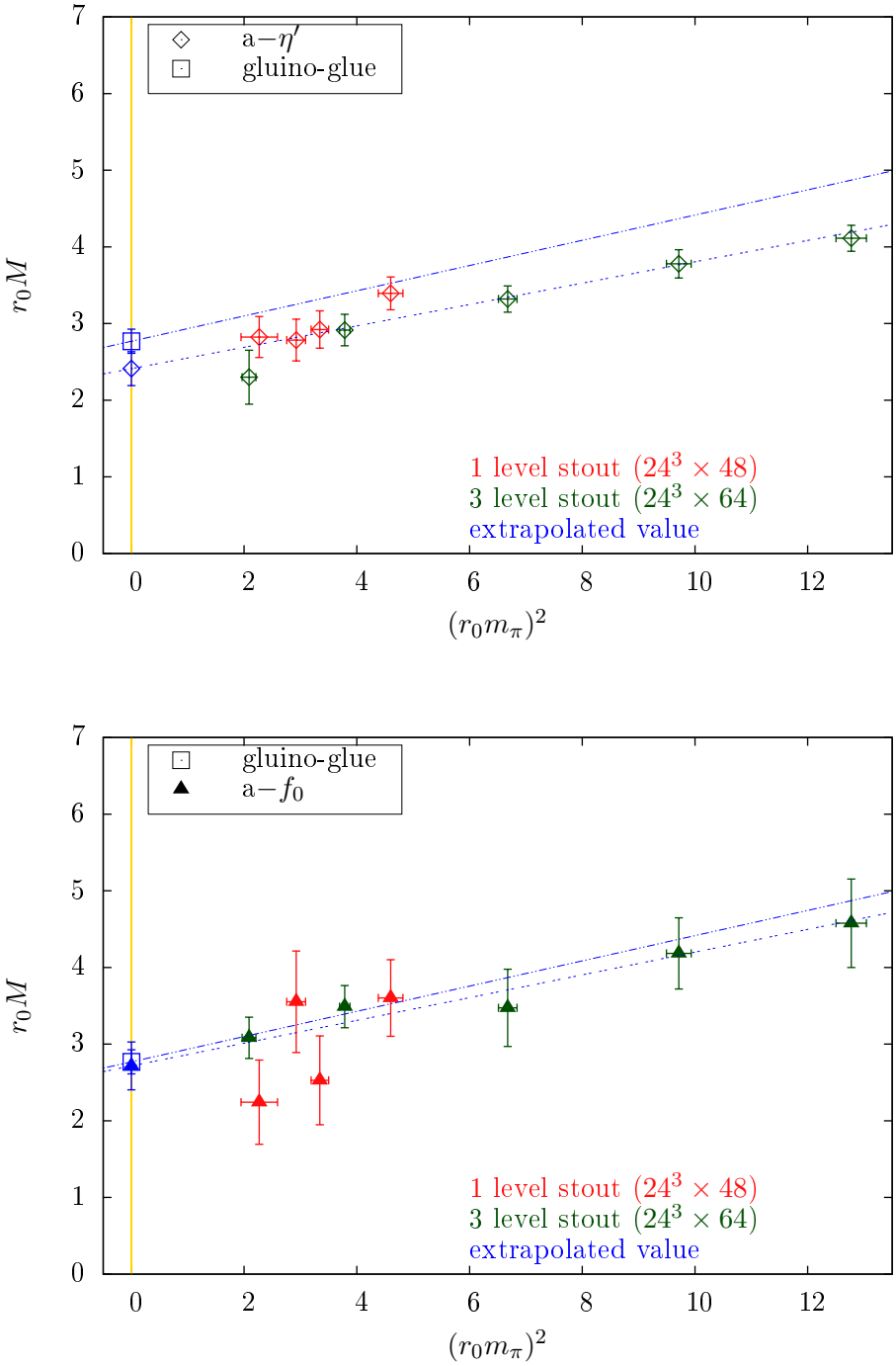

Figure 2: The a–η' mass and the a–f_0 mass as functions of the squared mass of the adjoint pion in units of the Sommer scale, and the corresponding linear fit. Also shown is the fit for the gluino-glue.

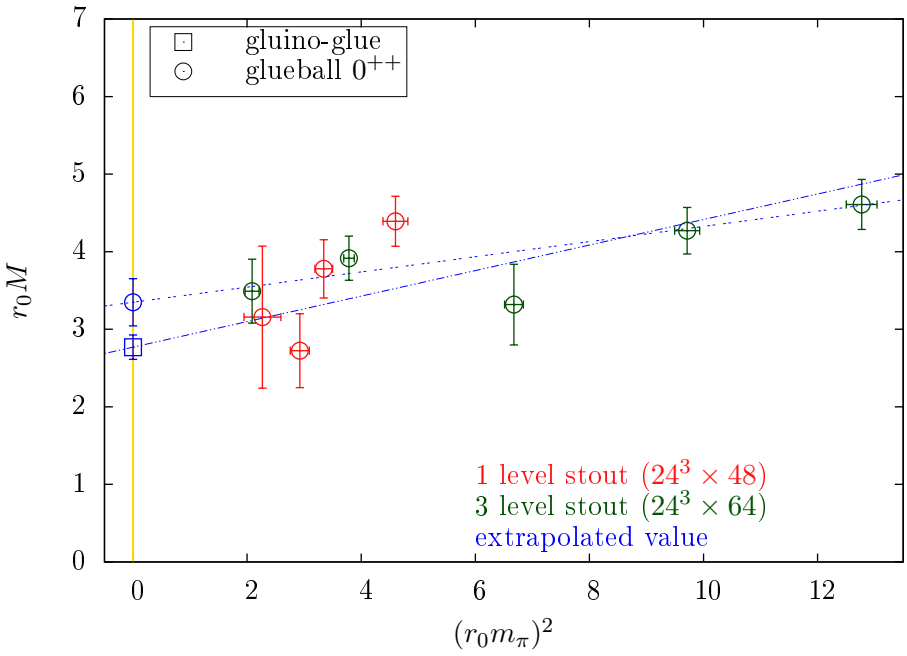

Figure 3: The glueball mass as a function of the squared mass of the adjoint pion in units of the Sommer scale, and the corresponding linear fit. Also shown is the fit for the gluino-glue.

Whereas in previous work a significant gap between the masses of the gluino-glue and its supposed superpartners showed up, the figures presented here display considerably smaller differences. To make the comparison quantitative, we confront the present results with those of our earlier work in [3], done at a larger lattice spacing. The masses are summarised in Table 3. They have been converted into units of MeV using the QCD scale setting $r_0 = 0.5$ fm.

β	a–η'	a–f_0	$\tilde{g}g$	glueball 0^{++}
1.6	670(63)	571(181)	1386(39)	721(165)
1.75	950(87)	1070(123)	1091(62)	1319(120)

Table 3: Comparison of the bound state masses in units of MeV, extrapolated to vanishing gluino mass, at the two values of β. The lattice spacing at $\beta = 1.6$ is between 0.088 and 0.097 fm depending on the level of stout smearing. In the current simulations it is approximately 0.055 fm for $l_s = 1$, and approximately 0.058 fm for $l_s = 3$. All values are obtained using the QCD units by setting $r_0 = 0.5$ fm.

One can observe significant differences between the results at the larger lattice spacing ($\beta = 1.6$) and the current lattice spacing ($\beta = 1.75$). The gluino-glue mass gets smaller and the other masses larger when the lattice spacing is reduced. Taking the statistical and systematic uncertainties into account, the extrapolations towards vanishing pion mass of the

masses of the fermionic and bosonic states in our present calculations are consistent with each other. Note that further away from the chiral limit the difference between fermionic and bosonic masses is expected to grow.

In order to confirm that the remaining differences between the masses are finite lattice spacing effects, the calculations have to be extended to a third, even smaller lattice spacing, and extrapolated to the continuum limit.

4 Conclusions

In this work we present the current results of our simulations of $\mathcal{N} = 1$ supersymmetric Yang-Mills theory on a lattice. Our aim is to obtain a picture of the bound states of this theory in the nonperturbative regime. The results of our preparatory study on finite volume effects [5] are complemented with an analysis of the lattice artifacts. Both, finite volume effects and lattice artifacts increase the mass gap in the spectrum between the bosonic and the fermionic states. Hence the scales need to be chosen carefully for a reliable simulation of the theory.

The parameters of our earlier results [3] have been chosen according to the experience of the QCD simulations. Our new, more detailed study has shown that these settings were on the safe side concerning the finite volume effects. The volume could even be reduced without a considerable systematic error. The supersymmetry breaking due to the discretisation effects or lattice artifacts, on the other hand, has been significant in our earlier results. The difference from the QCD expectations for the best simulation scales is not unexpected. The scale setting is ruled by the Compton wavelength of the particles under consideration. The volume has to be large and the lattice spacing has to be small in comparison to this length scale. In supersymmetric Yang-Mills theory there is no propagating particle corresponding to the lightest particle of QCD, the pion (the corresponding operator has been used only to tune the chiral symmetry restoration). The absence of this light particle induces smaller finite volume effects, but obviously also larger discretisation errors.

With our current parameters at the smaller lattice spacing the mass splitting is of the same order as the statistical and systematic uncertainties. For a quantitative analysis of lattice spacing effects and scaling an extrapolation to the continuum limit is required. In case of the gluino-glue this seems to be possible with the current accuracy of the results. For the other particles a reliable extrapolation requires further improvement of the measurements. The uncertainties of these measurements are in accordance with the expectations from QCD. The corresponding operators in QCD are also known to be hard to measure. In future studies we plan to improve the measurements and test the expected further reduction of the lattice artifacts.

Acknowledgements

This project is supported by the German Science Foundation (DFG) under contract Mu 757/16, and by the John von Neumann Institute for Computing (NIC) with grants of computing time. Further computing time has been provided by the compute cluster PALMA of the University of Münster.

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