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Heterotic Moduli Stabilization

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Abstract: We perform a systematic analysis of moduli stabilization for weakly coupled heterotic string theory compactified on smooth Calabi-Yau three-folds. We focus on both supersymmetric and supersymmetry breaking vacua of generic (0,2) compactifications obtained by minimising the total (F + D)-term scalar potential. After reviewing how to stabilise all the geometric moduli in a supersymmetric way by including fractional fluxes, non-perturbative and threshold effects, we show that the inclusion of α' corrections leads to new de Sitter or nearly Minkowski vacua which break supersymmetry spontaneously. The minimum lies at moderately large volumes of all the geometric moduli, at perturbative values of the string coupling and at the right phenomenological value of the GUT gauge coupling. However the structure of the heterotic 3-form flux used for complex structure moduli stabilization does not contain enough freedom to tune the superpotential. This results in the generic prediction of high-scale supersymmetry breaking around the GUT scale. We finally provide a dynamical derivation of anisotropic compactifications with stabilized moduli which allow for perturbative gauge coupling unification around 10^{16} GeV.

Contents

1. Introduction

String theory is a candidate for a quantum theory of gravity with full unification of the forces of nature. As such it should be able to describe the patterns of the Standard Models (SMs) of particle physics and cosmology. For this description of 4D physics, string theory needs to compactify its ambient 10D space-time. The multitude of possible compactification choices together with a plethora of massless 4D scalar 'moduli' fields originating from the deformation modes of the extra dimensions, leads to vacuum degeneracy and moduli problems. Recent progress in achieving moduli stabilization points to the possibility of a exponentially large set of cosmologically distinct de Sitter (dS) solutions of string theory with positive but tiny cosmological constant, the 'landscape' (for reviews see [1, 2]).

These results need to be combined with string constructions of viable particle physics. One fruitful region of the string landscape for this purpose is weakly coupled heterotic string theory. Recent works on heterotic compactifications on both smooth Calabi-Yau (CY) manifolds [3] and their singular limits in moduli space, orbifolds [4, 5, 6, 7, 8], provided constructions of 4D low-energy effective field theories matching the minimal supersymmetric version of the SM (MSSM) almost perfectly. However, in contrast to the understanding achieved in type IIB string theory, heterotic CY or orbifold compactifications lack a well controlled description of moduli stabilization, and consequently, of inflationary cosmology as well.

As weakly coupled heterotic string compactification on CYs lack both light D-branes and a part of the 3-form flux available in type IIB, historically moduli stabilization in the heterotic context focused mostly on the moduli dependence of 4D non-perturbative gauge theory contributions to the effective action by gaugino condensation [9, 10, 11]. While this produced models of partial moduli stabilization for the dilaton scalar and some of the CY volume moduli [12, 13, 14, 15], this route generically failed at describing controlled and explicit stabilization of the often $\mathcal{O}(100)$ complex structure moduli of a given CY. Moreover, the resulting vacua tend to yield values for the compactification radius and the string coupling (given by the dilaton) at the boundary of validity for weak coupling and the supergravity approximation. On the other hand the works [16, 17, 18] have included the heterotic H three-form flux to stabilize the complex structure moduli in combination with hidden sector gaugino condensation and worldsheet instanton effects for supersymmetric dilaton and volume stabilization. The absence of Ramond-Ramond three-form flux in the heterotic string generically obliterates the possibility to tune to small values the overall three-form flux contribution to the superpotential, from either cancellations or vicinity to a conifold point.

Since a small flux superpotential is a prerequisite for stabilization at weak string coupling and large volume, one probably needs to use fractional flux from e.g. discrete Wilson lines to achieve $|W_{\rm flux}| = \mathcal{O}(0.1)$. However, this still leaves the resulting vacua close to the boundary of perturbation theory and the supergravity approximation. More importantly, all these vacua are parametrically anti-de Sitter (AdS) solutions, and have difficulty breaking supersymmetry such that it drives 'uplifting' into a near-Minkowski solution.

We organize our analysis along a systematic discussion of quantum corrections from non-perturbative effects, string loops [19, 20, 21], and higher-derivative α'-corrections [22, 23, 24] according to their successive level of suppression by powers of the string coupling and inverse powers of the volume. After introducing the general framework of heterotic CY compactification [25, 26] and the structure of the corrections in Section 2, we start from a discussion of the three-form flux superpotential and its supersymmetric vacua in Section 3. While we verify the result of [27] that a non-trivial three-form flux can stabilize all complex structure moduli supersymmetrically, we find that supersymmetric Minkowski flux vacua with vanishing superpotential are impossible as they force the total three-form flux to vanish entirely[1]. Moreover, we recall the fact that in heterotic compactifications one usually needs $(0,2)$ worldsheet supersymmetry for good MSSM-like 4D physics, which directly produces a non-vanishing contribution to the flux superpotential from the Chern-Simons contributions in the three-form flux. Since the heterotic string lacks Ramond-Ramond fluxes, the freedom to choose the NSNS 3-form flux H is used up mostly to stabilize the complex structure moduli in a controlled vacuum, leaving little freedom the tune the flux superpotential. We also recall that the type IIB avenue towards a small flux superpotential by stabilizing a three-cycle exponentially close to a conifold point in moduli space is unavailable in the heterotic context due to the reality constraint on H. The size of the flux superpotential in $(0,2)$ compactifications thus is controlled by the fractional Chern-Simons contribution to be at least $\mathcal{O}(0.01\ldots0.1)$ in units of M_P. Moreover, in the heterotic string there is an upper bound on the compactification volume driven by perturbativity and gauge coupling unification. Altogether, these constraints generically imply GUT-scale supersymmetry breaking for heterotic CY models.

Next, we include non-perturbative effects into the superpotential in Section 3.2. First, we look at contributions from gaugino condensation in conjunction with threshold corrections to the gauge kinetic function. They give rise to supersymmetric vacua which drive the gauge coupling through strong coupling into negative values in either the visible or hidden sector of heterotic string theory [16]. We then recall [18] how the inclusion of a *single* worldsheet instanton contribution can resolve this difficulty.

At this point it behooves us to clear up the confusion that seems to exist in the literature regarding the inclusion of fluxes in heterotic string theory. The issue goes back to the work of Strominger [28]. By analyzing the classical 10D equations of motion on a space-time of the form $\mathcal{M}\times X$ with a maximally symmetric metric ansatz on the 4D space-time $\mathcal{M}$ with NSNS three-form flux H in X, and demanding $\mathcal{N}=1$ supersymmetry, he showed the following:

1. The 4D space $\mathcal{M}$ is flat.
2. X is a complex manifold with
$$H=\frac{i}{2}(\partial-\bar{\partial})J \tag{1.1}$$
where J is the fundamental (1,1) form on X.
3. The Yang-Mills field strength must satisfy the Hermitian Yang-Mills equations.

[1] This statement is also implicit in [17].

The second item above would in particular imply that X cannot be a Kähler manifold if we turn on a $(2,1)+(1,2)$ H-flux [2]. Furthermore it tells us that the $(3,0)+(0,3)$ flux is zero. In other words, if the manifold is Kähler the entire flux is zero.

One might ask whether these observations have an interpretation in 4D supergravity. As mentioned above, the minimum of the potential (and no runaway dilaton) requires (in the absence of non-perturbative terms) both the $(2,1)$ and the $(3,0)$ flux to be zero and as a consequence (given that the flux is real) the entire flux is zero. This is obviously consistent with Strominger's argument but none of the geometric moduli nor the dilaton are stabilized. To stabilize the dilaton one needs to introduce a non-perturbative term which is then stabilized against the $(3,0)$ part of the H-flux but as argued in [17] 4D supergravity then necessarily generates a non-zero $(2,1)$ flux from the stabilization conditions for the complex structure moduli [3]. What one certainly cannot expect is a simple comparison to the Strominger argument once non-perturbative terms are included, since those arguments are purely classical and it is quite unclear how one could generate a non-perturbative term in the superpotential at the 10D level. One can of course try to force agreement with Strominger's constraint (1.1) by imposing in addition to the minimization condition for the complex structure moduli, the condition $H_{2,1}=0$ as suggested in [18]. However this leads to an over-determined system and a solution can exist only as a result of a lucky accident.

The upshot is that it is not at all clear that the Strominger constraint (1.1) should necessarily be imposed when one is including non-perturbative terms in the 4D superpotential. It would be good to understand the 10D origin of such terms - on the other hand, especially for the gaugino condensate terms which are a consequence of strong coupling IR effects in gauge theories, it is unclear that there exists a 10D argument. In any case the supersymmetry preserving vacua one gets after including both gaugino condensate effects as well as world sheet instanton effects are AdS. This is also in conflict with Strominger's classical result item 1 in the above list.

Emerging from this discussion and bearing its caveats in mind, we next perform a systematic large-volume expansion in the gauge threshold and α'-corrections to the Kähler potential in Section 4. Beyond the threshold correction to the gauge kinetic function [29, 30], the leading terms are $\mathcal{O}(\alpha'^2)$ [24], and $\mathcal{O}(\alpha'^3)$ [22, 23]. We show that their systematic inclusion leads to the generic existence of supersymmetry breaking near-Minkowski vacua. These solutions have the dilaton stabilized at values $\mathrm{Re(S)} \simeq 2$ compatible with gauge coupling unification while allowing us to stabilize the compactification volume $\mathcal{V} \simeq 20$ at the upper limit compatible with string perturbativity. They reside in a well-controlled large-volume expansion of the scalar potential in terms of the threshold correction to the gauge kinetic function and the α'-corrections, which realizes an LVS-type scenario (LARGE Volume Scenario [31, 32]) in the heterotic context.

[2] In fact it cannot even be conformally Kähler.

[3] Furthermore it was argued in [17] that the supergravity equation $W = W_{\mathrm{flux}} + W_{\mathrm{non-pert}}$ is inconsistent with getting the non-perturbative term from some simple 10D argument such as giving an expectation value to a four fermion term as in [16].

In Section 5 we discuss the ensuing pattern of moduli and soft masses from the supersymmetry breaking provided by our LVS-type vacua. Generically, the axionic partner of the 'large' 2-cycle volume modulus turns out to be a potentially viable QCD axion candidate, and we arrive at high-scale supersymmetry breaking with a distinct soft mass pattern with universal scalar masses, A-terms and $\mu/B\mu$-term of $\mathcal{O}(m_{3/2} \sim M_{\rm GUT})$, while gaugino masses appear generically suppressed at the %-level.

Finally, we apply this heterotic LVS scenario to K3- or T^4-fibered CY manifolds in Section 6, and show that this allows for anisotropic CYs where the overall volume $\mathcal{V} \simeq 20$ is controlled by two larger extra dimensions while the remaining four extra dimensions remain smaller. This anisotropic setup is particularly interesting phenomenologically, as it allows one to match the effective string scale to the GUT scale of gauge coupling unification [33, 34], and fits very well with the picture of intermediate 6D orbifold GUTs emerging from heterotic orbifold MSSM constructions [34, 35].

2. Heterotic framework

Let us focus on weakly coupled heterotic string theory compactified on a smooth Calabi-Yau three-fold X. The 4D effective supergravity theory involves several moduli: $h^{1,2}(X)$ complex structure moduli Z_α, $\alpha = 1, ..., h^{1,2}(X)$; the dilaton S and $h^{1,1}$ Kähler moduli T_i, $i = 1, ..., h^{1,1}(X)$ (besides several gauge bundle moduli).

The real part of S is given by the 4D dilaton (see appendix A for the correct normalisation factor):

$$\mathrm{Re}(S) \equiv s = \frac{1}{4\pi}\, e^{-2\phi_4} = \frac{1}{4\pi}\, e^{-2\phi}\, \mathcal{V}\,, \tag{2.1}$$

where ϕ is the 10D dilaton whose vacuum expectation value (VEV) sets the string coupling $e^{\langle\phi\rangle} = g_s$. The imaginary part of S is given by the universal axion a which is the 4D dual of B_2. On the other hand, the real part of the Kähler moduli, $t_i = \mathrm{Re}(T_i)$, measures the volume of internal two-cycles in units of the string length $\ell_s = 2\pi\sqrt{\alpha'}$. The imaginary part of T_i is given by the reduction of B_2 along the basis $(1,1)$-form $\hat{D}_i$ dual to the divisor D_i.

We shall focus on general non-standard embeddings with possible $U(1)$ factors in the visible sector. Hence the gauge bundle in the visible $E_8^{\rm vis}$ takes the form $V_{\rm vis} = U_{\rm vis} \bigoplus_\kappa \mathcal{L}_\kappa$ where $U_{\rm vis}$ is a non-Abelian bundle whereas the $\mathcal{L}_\kappa$ are line bundles. On the other hand the vector bundle in the hidden $E_8^{\rm hid}$ involves just a non-Abelian factor $V_{\rm hid} = U_{\rm hid}$. We shall not allow line bundles in the hidden sector since, just for simplicity, we shall not consider matter fields charged under anomalous $U(1)$s. In fact, if we want to generate a superpotential from gaugino condensation in the hidden sector in order to fix the moduli, all the anomalous $U(1)$s have to reside in the visible sector otherwise, as we shall explain later on, the superpotential would not be gauge invariant.

2.1 Tree-level expressions

The tree-level Kähler potential takes the form:

$$K_{\text{tree}} = -\ln \mathcal{V} - \ln(S+\overline{S}) - \ln\left(\mathrm{i}\int_X \Omega\wedge\overline{\Omega}\right), \tag{2.2}$$

where $\mathcal{V}$ is the Calabi-Yau volume measured in string units, while Ω is the holomorphic $(3,0)$-form of X that depends implicitly on the Z-moduli. The internal volume depends on the T-moduli since it looks like:

$$\mathcal{V} = \frac{1}{6}\,k_{ijk}t_it_jt_k = \frac{1}{48}\,k_{ijk}\left(T_i+\overline{T}_i\right)\left(T_j+\overline{T}_j\right)\left(T_k+\overline{T}_k\right), \tag{2.3}$$

where $k_{ijk} = \int_X \hat{D}_i\wedge\hat{D}_j\wedge\hat{D}_k$ are the triple intersection numbers of X.

The tree-level holomorphic gauge kinetic function for both the visible and hidden sector is given by the dilaton:

$$f_{\text{tree}} = S \qquad \Rightarrow \qquad \mathrm{Re}(f_{\text{tree}}) \equiv g_4^{-2} = s\,. \tag{2.4}$$

The tree-level superpotential is generated by the 3-form flux H_3 and it reads:

$$W_{\text{flux}} = \int_X H_3\wedge\Omega\,, \tag{2.5}$$

with the correct definition of H_3 including α' effects:

$$H_3 = dB_2 - \frac{\alpha'}{4}\left[\mathrm{CS}(A) - \mathrm{CS}(\omega)\right], \tag{2.6}$$

where $\mathrm{CS}(A)$ is the Chern-Simons 3-form for the gauge connection A:

$$\mathrm{CS}(A) = \mathrm{Tr}\left(A\wedge dA + \frac{2}{3}\,A\wedge A\wedge A\right), \tag{2.7}$$

and $\mathrm{CS}(\omega)$ is the gravitational equivalent for the spin connection ω.

The VEV of the tree-level superpotential, W_0, is of crucial importance. Due to the difference with type IIB where one has two 3-form fluxes, H_3 and F_3, which can give rise to cancellations among themselves leading to small values of W_0, in the heterotic case W_0 is generically of order unity. Hence one generically experiences two problems:

1. Contrary to type IIB, the heterotic dilaton is not fixed by the flux superpotential, resulting in a supergravity theory which is not of no-scale type. More precisely, the F-term scalar potential:

$$V_F = e^K\left(K^{I\bar{J}}D_IWD_{\bar{J}}\bar{W} - 3|W|^2\right), \tag{2.8}$$

derived from (2.2) and (2.5) simplifies to:

$$V_F = e^K \left[\sum_Z K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} + \left(K^{S\bar{S}} K_S K_{\bar{S}} + \sum_T K^{i\bar{j}} K_i K_{\bar{j}} - 3 \right) |W|^2 \right]$$
$$= e^K \left(\sum_Z K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} + |W|^2 \right), \qquad (2.9)$$

since $K^{S\bar{S}} K_S K_{\bar{S}} = 1$ and $\sum_T K^{i\bar{j}} K_i K_{\bar{j}} - 3 = 0$. Setting $D_\alpha W = 0 \ \forall \alpha = 1, ..., h^{1,2}(X)$, the scalar potential (2.9) reduces to:

$$V_F = e^K |W_0|^2 = \frac{|W_0|^2}{2s\mathcal{V}}, \qquad (2.10)$$

yielding a run-away for both s and $\mathcal{V}$ if $|W_0| \neq 0$. Given that generically $|W_0| \sim \mathcal{O}(1)$, it is very hard to balance this tree-level run-away against S-dependent non-perturbative effects which are exponentially suppressed in S. One could try to do it by considering small values of $s = g_{4D}^{-2}$ but this would involve a strong coupling limit where control over moduli stabilization is lost. A possible way to lower W_0 was proposed in [16] where the authors derived the topological conditions to have fractional Chern-Simons invariants.

2. If $|W_0| \neq 0$, even if it is fractional, one cannot obtain low-energy supersymmetry. In fact, the gravitino mass is given by $m_{3/2} = e^{K/2}|W_0| M_P$, and so the invariant quantity $e^{K/2}|W_0| = |W_0|/(\sqrt{2s\mathcal{V}})$ has to be of order 10^{-15} to have TeV-scale supersymmetry. As we have seen, the 4D gauge coupling is given by $\alpha_{GUT}^{-1} = g_s^{-2}\mathcal{V}$, and so a huge value of the internal volume would lead to a hyperweak GUT coupling. Notice that a very large value of $\mathcal{V}$ cannot be compensated by a very small value of g_s^{-2} since we do not want to violate string perturbation theory.

Let us briefly mention that in some particular cases one could have an accidental cancellation among the flux quanta which yields a small $|W_0|$ as suggested in [17]. We stress that in the heterotic case, contrary to type IIB, this cancellation is highly non-generic, and so it is not very appealing to rely on it to lower $|W_0|$. Hence it would seem that the most promising way to get low-energy supersymmetry is to consider the case where $|W_0| = 0$ and generate an exponentially small superpotential only at subleading non-perturbative level. This case was considered in [36], where the authors argued that, at tree-level, one can in principle obtain a Minkowski supersymmetric vacuum with all complex structure moduli stabilized and $2(h^{1,1}(X)+1)$ flat directions corresponding to the dilaton S and the Kähler moduli. However, as we shall show in section 3, the conditions $D_{Z^\alpha} W_{\text{flux}} = 0 \ \forall \alpha = 1, ..., h^{1,2}(X)$ and $|W_0| = 0$ imply that no H_3 flux is turned on, resulting in the impossibility to stabilize the Z-moduli. This result already implies that *it is impossible to stabilize the complex structure moduli and the dilaton in two steps* with a Z-moduli stabilization at tree-level and a dilaton stabilization at subleading non-perturbative level. In this case there are two possible way-outs:

1. Focus on the case $D_{Z^\alpha} W = 0 \; \forall \alpha = 1, ..., h^{1,2}(X)$ and $|W_0| \neq 0$ so that H_3 can be non-trivial. In this case one has however a dilaton run-away, implying that no moduli can be fixed at tree-level. One needs therefore to add S-dependent non-perturbative effects which have to be balanced against the tree-level superpotential to lift the run-away. A small $|W_0|$ could be obtained either considering fractional Chern-Simons invariants or advocating accidental cancellations among the flux quanta.

2. Focus on the case with trivial H_3 so that no scalar potential is generated at tree-level. The dilaton and the complex structure moduli could then be fixed at non-perturbative level via a race-track superpotential generating an exponentially small W which could lead to low-energy supersymmetry. However, choosing H_3 trivial is possible only for models with (2,2) worldsheet supersymmetry ($dH_3 = R \wedge R - F \wedge F \neq 0$ for (0,2)-models). For this case there are theorems dictating the absence of purely moduli-dependent worldsheet instanton corrections to the superpotential. Hence, moduli stabilization would have to proceed via a racetrack mechanism involving at least two condensing non-Abelian gauge groups with *all* moduli appearing in the gauge kinetic functions. As this is generically not the case for heterotic compactifications, this avenue will not lead to moduli stabilization except perhaps for a few specific cases.

2.2 Corrections beyond leading order

As explained in the previous section, in smooth heterotic compactifications with trivial B_2 background flux no geometric moduli can be fixed at tree-level. Thus, one has to look at any possible correction beyond the leading order expressions. Before presenting a brief summary of the various effects to be taken into account (perturbative and non-perturbative in both α' and g_s), let us mention two well-known control issues in heterotic constructions:

- *Tension between weak coupling and large volume*: In order to have full control over the effective field theory, one would like to stabilize the moduli in a region of field space where both perturbative and higher derivative corrections are small, i.e. respectively for $g_s \ll 1$ and $\mathcal{V} \gg 1$. However, as we have already pointed out, this can be the case only if the 4D coupling is hyperweak, in contrast with phenomenological observations. In fact, we have:
$$\frac{g_s^2}{\mathcal{V}} = \alpha_{GUT} \simeq \frac{1}{25} \,, \tag{2.11}$$
and so if we require $g_s \lesssim 1$, the Calabi-Yau volume cannot be very large, $\mathcal{V} \lesssim 25$, implying that one can never have a full parametric control over the approximations used to fix the moduli.

- *Tension between GUT scale and large volume*: In heterotic constructions, the unification scale is identified with the Kaluza-Klein scale, $M_{GUT} = M_{KK}$, which cannot be lowered that much below the string scale for $\mathcal{V} \lesssim 25$, resulting in a GUT scale which is generically higher than the value inferred from the 1-loop running of the MSSM gauge couplings.

In more detail, the string scale $M_s \equiv \ell_s^{-1}$ can be expressed in terms of the 4D Planck scale from dimensional reduction as (see appendix A for an explicit derivation):

$$M_s^2 = \frac{M_P^2}{4\pi\alpha_{GUT}^{-1}} \simeq \frac{M_P^2}{100\pi} \simeq \left(1.35 \cdot 10^{17}\,\text{GeV}\right)^2 . \tag{2.12}$$

In the case of an isotropic compactification, the Kaluza-Klein scale takes the form:

$$M_{GUT} = M_{KK} \simeq \frac{M_s}{\mathcal{V}^{1/6}} \gtrsim 8 \cdot 10^{16}\,\text{GeV} \qquad \text{for} \qquad \mathcal{V} \lesssim 25\,, \tag{2.13}$$

which is clearly above the phenomenological value $M_{GUT} \simeq 2.1 \cdot 10^{16}$ GeV. On the other hand, anisotropic compactifications with d large dimensions of size $L = x\ell_s$ with $x \gg 1$ and $(6-d)$ small dimensions of string size $l = \ell_s$, can lower the Kaluza-Klein scale:

$$\text{Vol}(X) = L^d l^{6-d} = x^d \ell_s^6 = \mathcal{V}\,\ell_s^6 \quad \Rightarrow \quad M_{GUT} = M_{KK} \simeq \frac{M_s}{x} \simeq \frac{M_s}{\mathcal{V}^{1/d}}\,. \tag{2.14}$$

For the case $d = 2$, one would get the encouraging result $M_{GUT} = \frac{M_s}{\sqrt{\mathcal{V}}} \gtrsim 2.7 \cdot 10^{16}$ GeV.

2.2.1 Higher derivative effects

Let us start considering higher derivative effects, i.e. perturbative α' corrections to the Kähler potential. In the case of the standard embedding corresponding to (2,2) worldsheet theories, the leading α' correction arises at order $\mathcal{O}(\alpha'^3)\mathcal{R}^4$ [22] and depends on the Calabi-Yau Euler number $\chi(X) = 2\left(h^{1,1} - h^{1,2}\right)$. Its form can be derived by substituting the α' corrected volume $\mathcal{V} \to \mathcal{V} + \xi/2$ into the tree-level expression (2.2) with $\xi = -\zeta(3)\chi(X)/(2(2\pi)^3)$. Given that $\zeta(3) \simeq 1.2$, ξ is of the order $\xi \simeq \left(h^{1,2} - h^{1,1}\right)/200 \simeq \mathcal{O}(1)$ for ordinary Calabi-Yau three-folds with $\left(h^{1,2} - h^{1,1}\right) \simeq \mathcal{O}(100)$. Hence for $\mathcal{V} \simeq \mathcal{O}(20)$, the ratio $\xi/(2\mathcal{V}) \simeq \mathcal{O}(1/40)$ is a small number which justifies the expansion:

$$K \simeq -\ln\mathcal{V} - \frac{\xi}{2\mathcal{V}} \qquad \Rightarrow \qquad K_{\alpha'^3} = -\frac{\xi}{2\mathcal{V}}\,. \tag{2.15}$$

As pointed out in [24] however, this is the leading order higher derivative effect only for the standard embedding since (0,2) worldsheet theories admit α' corrections already at order $\mathcal{O}(\alpha'^2)$ which deform the Kähler form J as:

$$J \to J' = J + \mathcal{O}(\alpha')\,\tilde{h} + \mathcal{O}(\alpha'^2)\,\tilde{h}^{(2)} + \ldots\,, \tag{2.16}$$

where both $\tilde{h}$ and $\tilde{h}^{(2)}$ are moduli-dependent (1,1)-forms which are orthogonal to J, i.e. $\int_X \star J \wedge \tilde{h} = \int_X \star J \wedge \tilde{h}^{(2)} = 0$. Plugging J' into the tree-level expression for K (2.2) and then expanding, one finds that the $\mathcal{O}(\alpha')$ correction vanishes because of the orthogonality between $\tilde{h}$ and J whereas at $\mathcal{O}(\alpha'^2)$ one finds:[4]

$$K_{\alpha'^2} = \frac{1}{2\mathcal{V}} \int_X \star\tilde{h} \wedge \tilde{h} = \frac{||\tilde{h}||^2}{2\mathcal{V}}\,. \tag{2.17}$$

[4]Let us shortly mention here however, that in [24] the correction at $\mathcal{O}(\alpha'^2)$ was guessed from the form of J' at $\mathcal{O}(\alpha')$ and expanding the resulting K up to $\mathcal{O}(\alpha'^2)$. The 10D effective action and the e.o.ms. were

Notice that the correction (2.17) is generically leading with respect to (2.15) since (2.17) should be more correctly rewritten as:

$$K_{\alpha'^2} = \frac{g}{\mathcal{V}^{2/3}} \qquad \text{with} \qquad g \equiv \frac{||\tilde{h}||^2}{2\mathcal{V}^{1/3}} = -\frac{1}{2\mathcal{V}^{1/3}} \int_X J \wedge \tilde{h} \wedge \tilde{h} \geq 0\,, \tag{2.18}$$

where g is a homogeneous function of the Kähler moduli of degree 0 given that J scales as $J \sim \mathcal{V}^{1/3}$ and $\tilde{h}$ does not depend on $\mathcal{V}$. As an illustrative example, let us consider the simplest Swiss-cheese Calabi-Yau X with one large two-cycle t_b and one small blow-up mode t_s so that $J = t_b \hat{D}_b - t_s \hat{D}_s$ and the volume reads:

$$\mathcal{V} = k_b t_b^3 - k_s t_s^3 > 0 \qquad \text{for} \qquad 0 \leq \frac{t_s}{t_b} < \left(\frac{k_b}{k_s}\right)^{1/3} . \tag{2.19}$$

In the limit $k_b t_b^3 \gg k_s t_s^3$, the function g then becomes (considering, without loss of generality, $\tilde{h}$ as moduli-independent):

$$g = c_b + c_s \frac{t_s}{t_b} \geq 0 \qquad \text{with} \quad c_b = -\frac{1}{2\,k_b^{1/3}} \int_X \hat{D}_b \wedge \tilde{h} \wedge \tilde{h} \quad \text{and} \quad c_s = \frac{1}{2\,k_b^{1/3}} \int_X \hat{D}_s \wedge \tilde{h} \wedge \tilde{h}\,. \tag{2.20}$$

The sign of c_b and c_s can be constrained as follows. In the limit $t_s/t_b \to 0$, g reduces to $g = c_b = |c_b| \geq 0$. On the other hand, requiring that g is semi-positive definite for any point in Kähler moduli space one finds:

$$c_s = -\,|c_b| \left(\frac{k_s}{k_b}\right)^{1/3} + |\kappa|\,, \tag{2.21}$$

where $|\kappa|$ is a semi-positive definite quantity.

2.2.2 Loop effects

Let us now focus on g_s perturbative effects which can modify both the Kähler potential and the gauge kinetic function. The exact expression of the string loop corrections to the Kähler potential is not known due to the difficulty in computing string scattering amplitudes on Calabi-Yau backgrounds. However, in the case of type IIB compactifications, these corrections have been argued to be subleading compared to α' effects by considering the results for simple toroidal orientifolds [19] and trying to generalise them to arbitrary Calabi-Yau backgrounds

derived consistently at $\mathcal{O}(\alpha')$. Consequently, only the vanishing correction to K at $\mathcal{O}(\alpha')$ was actually derived self-consistently at this order, as pointed out in [24] in the discussion around equations (4.18) and (4.19) on pp. 18-19 *ibid.* Likewise, one may also wonder whether the fact that the corrected Kähler potential K' can be written in terms of J' as a function of $\int J' \wedge J' \wedge J'$ alone, just the same way as the tree level K in terms of J, may imply that a field redefinition of the Kähler form may actually fully absorb the correction at $\mathcal{O}(\alpha'^2)$. To this end, the observation in [24] that the generically non-vanishing string 1-loop corrections in type IIB appearing at $\mathcal{O}(\alpha'^2)$ are S-dual to the heterotic correction, provides additional evidence for the existence of this term.

[20, 21]. Following [21], we shall try to estimate the behavior of string loop corrections to the scalar potential by demanding that these match the Coleman-Weinberg potential:

$$V_{g_s} \simeq \mathrm{Str} M^2 \Lambda^2 \simeq m_{3/2}^2 M_{KK}^2 \simeq \frac{|W|^2}{2s} \frac{M_P^4}{\mathcal{V}^{2(1+1/d)}} \,, \tag{2.22}$$

where we took the cut-off scale $\Lambda = M_{KK}$ and we considered d arbitrary large dimensions. Notice that these effects are indeed subdominant with respect to the α' ones for large volume since the $\mathcal{O}(\alpha'^2)$ and $\mathcal{O}(\alpha'^3)$ corrections, (2.18) and (2.15), give respectively a contribution to the scalar potential of the order $V_{\alpha'^2} \simeq |W|^2/\mathcal{V}^{5/3}$ and $V_{\alpha'^3} \simeq |W|^2/\mathcal{V}^2$, whereas the g_s potential (2.22) scales as $V_{g_s} \simeq |W|^2/\mathcal{V}^{7/3}$ for the isotropic case with $d = 6$ and $V_{g_s} \simeq |W|^2/\mathcal{V}^3$ for the anisotropic case with $d = 2$. Due to this subdominant behavior of the string loop effects, we shall neglect them in what follows.

String loops correct also the gauge kinetic function (2.4). The 1-loop correction has a different expression for the visible and hidden E_8 sectors:

$$f_{\rm vis} = S + \frac{\beta_i}{2} T_i \,, \qquad f_{\rm hid} = S - \frac{\beta_i}{2} T_i \,, \tag{2.23}$$

where:

$$\beta_i = \frac{1}{4\pi} \int_X (c_2(V_{\rm vis}) - c_2(V_{\rm hid})) \wedge \hat{D}_i \,. \tag{2.24}$$

2.2.3 Non-perturbative effects

The 4D effective action receives also non-perturbative corrections in both α' and g_s. The α' effects are worldsheet instantons wrapping an internal two-cycle T_i. These give a contribution to the superpotential of the form:

$$W_{\rm wi} = \sum_j B_j \, e^{-\, b_{ij} T_i} \,. \tag{2.25}$$

Notice that these contributions arise only for (0,2) worldsheet theories whereas they are absent in the case of the standard embedding. On the other hand, g_s non-perturbative effects include gaugino condensation and NS5 instantons. In the case of gaugino condensation in the hidden sector group, the resulting superpotential looks like:

$$W_{\rm gc} = \sum_j A_j \, e^{-\, a_j \, f_{\rm hid}} = \sum_j A_j \, e^{-\, a_j \left(S - \frac{\beta_i}{2} T_i\right)} \,, \tag{2.26}$$

where in the absence of hidden sector $U(1)$ factors, all the hidden sector gauge groups have the same gauge kinetic function. Finally, NS5 instantons wrapping the whole Calabi-Yau manifold would give a subleading non-perturbative superpotential suppressed by $e^{-\mathcal{V}} \ll 1$, and so we shall neglect them.

2.3 Moduli-dependent Fayet-Iliopoulos terms

As already pointed out, we shall allow line bundles in the visible sector where we turn on a vector bundle of the form $V_{\rm vis} = U_{\rm vis} \bigoplus_\kappa \mathcal{L}_\kappa$. The presence of anomalous $U(1)$ factors induces $U(1)$ charges for the moduli in order to cancel the anomalies and gives rise to moduli-dependent Fayet-Iliopoulos (FI) terms. In particular, the charges of the Kähler moduli and the dilaton under the κ-th anomalous $U(1)$ read:

$$q_{T_i}^{(\kappa)} = 4\, c_1^i(\mathcal{L}_\kappa) \qquad \text{and} \qquad q_s^{(\kappa)} = 2\,\gamma_{(\kappa)} = 2\,\beta_i\, c_1^i(\mathcal{L}_\kappa)\,, \tag{2.27}$$

so that the FI-terms become [30]:

$$\xi_{(\kappa)} = -q_{T_i}^{(\kappa)} \frac{\partial K}{\partial T_i} - q_s^{(\kappa)} \frac{\partial K}{\partial S} = \frac{c_1^i(\mathcal{L}_\kappa)}{\mathcal{V}}\, k_{ijk} t_j t_k + \frac{\gamma_{(\kappa)}}{s}\,. \tag{2.28}$$

Notice that the dilaton-dependent term in the previous expression is a 1-loop correction to the FI-terms which at tree-level depend just on the Kähler moduli. The final D-term potential takes the form:

$$V_D = \sum_\kappa \frac{\xi_{(\kappa)}^2}{\mathrm{Re}\left(f_{(\kappa)}\right)}\,. \tag{2.29}$$

From the expressions (2.27) for the $U(1)$-charges of the moduli, we can now check the $U(1)$-invariance of the non-perturbative superpotentials (2.25) and (2.26). In the absence of charged matter fields, the only way to obtain a gauge invariant worldsheet instanton is to choose the gauge bundle such that all the T_i appearing in $W_{\rm wi}$ are not charged, i.e. $c_1^i(\mathcal{L}_\kappa) = 0$ $\forall\kappa$ and $\forall i$. The superpotential generated by gaugino condensation is instead automatically $U(1)$-invariant by construction since all the anomalous $U(1)$s are in the visible sector whereas gaugino condensation takes place in the hidden sector. Thus, the hidden sector gauge kinetic function is not charged under any anomalous $U(1)$:

$$q_{f_{\rm hid}}^{(\kappa)} = q_s^{(\kappa)} - \frac{\beta_i}{2}\, q_{T_i}^{(\kappa)} = 2\left(\gamma_{(\kappa)} - \beta_i c_1^i(\mathcal{L}_\kappa)\right) = 0\,. \tag{2.30}$$

Before concluding this section, we recall that in supergravity the D-terms are proportional to the F-terms for $W \neq 0$. In fact, the total $U(1)$-charge of the superpotential W is given by $q_W^{(\kappa)} = q_i^{(\kappa)} W_i/W = 0$, and so one can write:

$$\xi_{(\kappa)} = -\,q_i^{(\kappa)} K_i = -\,q_i^{(\kappa)} \frac{D_i W}{W} = -\,q_i^{(\kappa)} \frac{e^{-K/2}}{W}\, K_{i\bar{j}} \bar{F}^{\bar{j}}\,, \tag{2.31}$$

where the F-terms are defined as $F^i = e^{K/2} K^{i\bar{j}} D_{\bar{j}} \bar{W}$. Therefore if all the F-terms are vanishing with $W \neq 0$, the FI-terms are also all automatically zero without giving rise to independent moduli-fixing relations.

3. Supersymmetric vacua

In this section, we shall perform a systematic discussion of heterotic supersymmetric vacua starting from an analysis of the tree-level scalar potential and then including corrections beyond the leading order expressions.

3.1 Tree-level scalar potential

Strominger's analysis of the 10D equations of motion can be recast in terms of an effective potential [37, 38] (see also [39, 40]) all of whose terms except for one are positive definite:

$$V \sim \int_X \frac{1}{4} N \wedge *N + \frac{1}{2}(H + \frac{1}{2} * dJ) \wedge *(H + \frac{1}{2} * dJ) + \dots \tag{3.1}$$

Apart from a linear term which vanishes on the equations of motion, there are other positive definite terms represented by the ellipses, one of which corresponds (at its minimum vanishing value) to the Hermitian Yang-Mills equation. Here J is the complex structure real two-form $J = J_m^n g_{np} dy^m \wedge dy^p$ and N is the Nijenhuis tensor constructed out of it.

The minimum of the first term gives $N = 0$, i.e. X is a complex manifold. Using this in the minimum of the second term gives the condition:

$$H = \frac{i}{2}(\partial - \bar{\partial})J. \tag{3.2}$$

Together with the minimum of the omitted terms in (3.1) we thus have all of Strominger's conditions.

These positive definite terms are suggestive of the no-scale potential and it would be nice to rederive all the Strominger conditions (listed in the introduction) as F-term (or D-term) equations of 4D supergravity. Unfortunately we know of no way of doing so and indeed it is not clear that this can be done in general in supergravity [5]. In [37] and [40] it is suggested that the expression $W = \int_X (H + \frac{i}{2} dJ) \wedge \Omega$ is the appropriate superpotential corresponding to the one in type IIB where one has the RR flux in addition to the H-flux. However for a complex manifold (even if $d\Omega \neq 0$) this term is zero since then dJ is of Hodge type (2,1)+(1,2) and Ω is (3,0). Thus it is unclear that this additional term can play a useful rôle.

On the other hand, the Strominger conditions imply that on a supersymmetric solution to the classical 10D equations of motion, the (3,0)+(0,3) components of the flux should be zero and that in addition if the manifold is Kähler (i.e. $dJ = 0$) then the (2,1)+(1,2) part of H is also zero. Recall that a Calabi-Yau satisfies $dJ = 0$ whereas $dJ \neq 0$ for a non-Kähler manifold. Hence the (1,2)-component of H controls the deformation of the background away from a Calabi-Yau. The moduli space of non-Kähler manifolds is not known exactly, and so one would lose control for $dJ \neq 0$.

In any case the Strominger relations are supersymmetric solutions to the classical equations of motion. So at the classical level at least we should be able to reproduce them in a consistent fashion. In particular this means that the F-term equations of 4D supergravity coming from compactifying the 10D equations on a Calabi-Yau manifold should yield the vanishing of both (3,0) and (2,1) flux and a Minkowski vacuum.

By comparing the dimensional reduction of the 10D coupling of H to the gravitino mass term in the 4D supergravity action Becker et al argued in [41] that even with the Chern-

[5] It has been argued in [39] that the Hermitian Yang-Mills equation can be obtained as a D-term condition.

Simons terms included in H the flux superpotential is given by:

$$W_{\rm flux} = \int_X H \wedge \Omega \equiv {\rm i}\, a(Z)\,. \tag{3.3}$$

Let us now evaluate the complex structure F-terms:

$$D_{Z^\alpha} W_{\rm flux} = \partial_{Z^\alpha} W_{\rm flux} + W_{\rm flux} \partial_{Z^\alpha} K\,. \tag{3.4}$$

Denoting a basis of (2,1)-forms by χ_α and using the fact that (see for example [42]):

$$\partial_{Z^\alpha}\Omega = k_\alpha(Z,\bar Z)\Omega + \chi_\alpha \tag{3.5}$$

$$\Rightarrow \quad \partial_{Z^\alpha} K = -k_\alpha(Z,\bar Z) \quad \text{and} \quad \partial_{Z^\alpha}\partial_{\bar Z^{\bar\beta}} K = \frac{\int_X \chi_\alpha \wedge \bar\chi_{\bar\beta}}{\int_X \Omega \wedge \bar\Omega} \equiv K_{\alpha\bar\beta}\,,$$

we find (normalising $\int_X \Omega \wedge \bar\Omega = -{\rm i}$):

$$D_{Z^\alpha} W_{\rm flux} = \int_X H \wedge \chi_\alpha \equiv {\rm i}\, b_\alpha(Z)\,. \tag{3.6}$$

On the other hand, the dilaton and Kähler moduli F-terms look like:

$$D_S W_{\rm flux} = W_{\rm flux}\partial_S K = -{\rm i}\frac{a}{2s} \qquad \text{and} \qquad D_{T_i} W_{\rm flux} = W_{\rm flux}\partial_{T_i} K = -{\rm i}\frac{a}{4\mathcal{V}}\, k_{ijk} t_j t_k\,. \tag{3.7}$$

Due to the no-scale cancellation, the scalar potential is positive definite and reads:

$$V = e^K \left(\sum_Z K^{\alpha\bar\beta} D_\alpha W D_{\bar\beta}\bar W + |W|^2 \right) = \frac{1}{2s\mathcal{V}} \left(\sum_Z K^{\alpha\bar\beta} b_\alpha \bar b_{\bar\beta} + |a|^2 \right)\,. \tag{3.8}$$

This is a sum of two positive definite terms. Setting the first to zero is equivalent to the condition $D_{Z^\alpha} W = 0$ and means that the (2,1) and (1,2) components of H are zero. This appears to fix the complex structure moduli supersymmetrically since one obtains as many equations, $b_\alpha(Z) = 0$, as the number of unknowns (assuming of course that the $2\,h^{1,2}$ real equations have solutions for some sets of values of the $2\,h^{1,2} + 2$ fluxes). However having fixed the Z-moduli in this way, the resulting potential for the dilaton and the Kähler moduli has a run-away to infinity making the whole system unstable! The only way out is by fine-tuning $W_0 \equiv \langle W_{\rm flux} \rangle = {\rm i}\, a(Z) = 0$ by an appropriate choice of flux quanta. But then the entire flux H is zero and hence not even the complex structure moduli would be determined.[6] In fact what this shows is that at the classical level, setting $dJ = 0$ gives us consistency with Strominger's relations as one should have expected. Therefore no moduli, not even the complex structure ones, can be fixed at tree-level. This implies also that one cannot perform a two-step stabilisation (similarly to type IIB) where the Z-moduli are fixed supersymmetrically at tree-level and the rest of the moduli are lifted by quantum corrections. In particular, this implies that one could not fix the complex structure moduli keeping at the same time the S- and T-moduli as flat directions by tuning $W_0 = 0$.

[6] The corresponding situation in type IIB is very different since there are two types of fluxes and the effective flux G_3 is complex [43].

3.2 Corrections beyond tree-level

Given that no moduli can be fixed at tree-level, let us focus on perturbative and non-perturbative corrections to the scalar potential. We shall proceed in two steps, showing first how to fix the complex structure moduli and the dilaton by the inclusion of an S-dependent gaugino condensate, and then explaining how to stabilise the Kähler moduli by an interplay of world-sheet instantons and threshold corrections to the gauge kinetic function. For the time being, we shall neglect perturbative corrections to the Kähler potential (either α' or g_s) since these generically break supersymmetry, and so we shall include them only in section 4 where we shall study supersymmetry breaking vacua.

3.2.1 Step 1: Z and S stabilization by gaugino condensation

Let us add a single S-dependent gaugino condensate to the superpotential and determine how this term modifies the tree-level picture:

$$W = W_{\rm flux} + W_{\rm gc} = \int_X H \wedge \Omega + A(Z)\, e^{-\lambda S}\,. \tag{3.9}$$

The Kähler-covariant derivatives now become:

$$D_{Z^\alpha} W = \mathrm{i}\, b_\alpha(Z) + e^{-\lambda S}\left[\partial_\alpha A(Z) - k_\alpha(Z,\bar{Z}) A(Z)\right], \tag{3.10}$$

$$D_S W = -\frac{1}{2s}\left[\mathrm{i}\, a(Z) + (2\lambda s + 1) A(Z)\, e^{-\lambda S}\right], \tag{3.11}$$

$$D_{T_i} W = -\frac{\mathrm{i}\, a(Z) + A(Z)\, e^{-\lambda S}}{4\mathcal{V}}\, k_{ijk} t_j t_k\,. \tag{3.12}$$

The potential is again of the no-scale type (i.e. given by the first equality of (3.8)). At the minimum the complex structure moduli will be frozen at the solution to:

$$D_{Z^\alpha} W = 0 \quad \Leftrightarrow \quad \mathrm{i}\, b_\alpha(Z) = e^{-\lambda S}\left[k_\alpha(Z,\bar{Z}) A(Z) - \partial_\alpha A(Z)\right], \tag{3.13}$$

and now the dilaton is not forced anymore to run-away to infinity:

$$D_S W = 0 \quad \Leftrightarrow \quad W_0 \equiv \mathrm{i}\, a(Z) = -(2\lambda s + 1) A(Z)\, e^{-\lambda S}\,. \tag{3.14}$$

The potential for the Kähler moduli is flat, resulting in a Minkowski vacuum with broken supersymmetry since substituting (3.14) into (3.12) one finds:

$$D_{T_i} W = -\left(\frac{2\lambda s}{2\lambda s + 1}\right)\frac{W_0}{4\mathcal{V}}\, k_{ijk} t_j t_k\,. \tag{3.15}$$

The previous expression for $W_0 \neq 0$, finite volume and $t_i > 1\ \forall i$, gives $D_{T_i} W \neq 0$ for a generic point in moduli space.

Let us comment now on the possibility to satisfy (3.14) at the physical point $\langle s \rangle \simeq 2$ that corresponds to $\alpha_{GUT}^{-1} \simeq 25$. Setting $A = 1$ and $\lambda = 8\pi^2/N$ where N is the rank of the $SU(N)$ condensing gauge group, we have (fixing the axion a at $\lambda a = \pi$):

$$W_0 = \left(\frac{16\pi^2 \langle s \rangle}{N} + 1\right) e^{-\frac{8\pi^2}{N}\langle s \rangle}\,. \tag{3.16}$$

As an illustrative example, for $\langle s\rangle \simeq 2$ and $N=5$, the previous expression would give $W_0 \simeq 10^{-12}$, which for $\mathcal{V}\simeq 20$ corresponds to a gravitino mass of the order $m_{3/2} = W_0/(\sqrt{2s\mathcal{V}}) \simeq$ 330 TeV. On the other hand, for $N = 30$ (as in the case of E_8), one would obtain $W_0 \simeq 0.06$ corresponding to a GUT-scale gravitino mass: $m_{3/2} \simeq 10^{16}$ GeV. As we have already pointed out, due to the absence of Ramond-Ramond fluxes, there is in general no freedom to tune the heterotic flux superpotential W_0 to values much smaller than unity. We therefore conclude that *heterotic compactifications generically predict a gravitino mass close to the GUT scale.* A possible way to obtain fractional values of W_0 of the order $0.1 - 0.01$ has been described in [16] where the authors considered a trivial B_2 field and a rigid 3-cycle Σ_3 such that the integral of H over Σ_3 (ignoring the contribution from the spin connection):

$$\int_{\Sigma_3} H \simeq -\int_{\Sigma_3} \mathrm{CS}(A)\,, \tag{3.17}$$

gives rise to a fractional flux [7]. Stabilization of all complex structure moduli would then require scanning the 3-form flux over all cycles to search for VEVs $\langle Z_\alpha\rangle$ such that the overall $(0,3)$-contribution to the superpotential (3.3) is of the order of the fractional CS contribution or smaller [8].

3.2.2 Step 2: T stabilization by worldsheet instantons and threshold effects

The Kähler moduli can develop a potential either by loop corrections to the gauge kinetic function or via worldsheet instantons. Let us start considering the case with just threshold effects.

Threshold effects: The potential generated by gaugino condensation takes the form:

$$W_{\rm gc} = A(Z)\, e^{-\lambda\left(S-\frac{\beta_i}{2}\,T_i\right)}\,, \tag{3.18}$$

lifting the T-moduli and modifying (3.15) into:

$$D_{T_i}W = -\frac{\lambda\, W_0}{2(2\lambda s+1)}\left[\beta_i + \frac{s}{\mathcal{V}}\,k_{ijk}t_jt_k\right] = 0 \quad \Leftrightarrow \quad \beta_i = -\frac{s}{\mathcal{V}}\,k_{ijk}t_jt_k\,. \tag{3.19}$$

[7]Notice that these flux quanta are well-defined quantities even if H is not closed since a rigid homology class admits just one representative.

[8]For the purpose of an explicit demonstration of such vacua one may rely on CYs arising in Greene-Plesser pairs of manifolds related by mirror symmetry [44, 45, 46]. CY mirror pairs arising from the Greene-Plesser construction have their complex structure moduli space partitioned by a typically large discrete symmetry Γ into an invariant subspace and its complement. One can then show that the periods of the invariant subspace depend at higher-order non-trivially on all the Γ-non-invariant complex structure moduli. If the Γ-invariant subspace is of low dimensionality (as is the case e.g. of the CY $\mathbb{CP}^4_{11169}$[18] as discussed in [47, 48]), then turning on the relatively few fluxes on the invariant subspace is enough to stabilize *all* complex structure moduli at an isolated minimum [47, 48]. On such a CY manifold one can therefore stabilize all complex structure moduli by just turning a few fractional CS $(0,3)$-type fluxes on the cycles of the invariant subspace, which can serve to demonstrate the existence of such vacua.

This result, in turn, gives:

$$\mathrm{Re}\left(f_{\mathrm{hid}}^{1-\mathrm{loop}}\right) = -\frac{\beta_i}{2}\,t_i = \frac{s}{2\mathcal{V}}\,k_{ijk}t_it_jt_k = 3\,s = 3\,\mathrm{Re}\left(f_{\mathrm{hid}}^{\mathrm{tree}}\right), \tag{3.20}$$

implying that perturbation theory in the hidden sector is not under control since the one-loop contribution is bigger than the tree-level one. Moreover the gauge kinetic function of the visible sector becomes negative:

$$\mathrm{Re}\,(f_{\mathrm{vis}}) = g_{\mathrm{vis}}^{-2} = s + \frac{\beta_i}{2}\,t_i = -2s < 0\,, \tag{3.21}$$

meaning that the positive tree-level contribution is driven to negative values by threshold effects. Actually, before becoming negative, g_{vis}^{-2} will vanish corresponding to a strong coupling transition whose understanding is not very clear [16]. Notice that we neglected D-terms since, due to the relation (2.31), if present, they would also cause the same problems. Let us see now how these control issues can be properly addressed by including worldsheet instantons.

Threshold effects and worldsheet instantons: The new total non-perturbative superpotential reads:

$$W_{\mathrm{np}} = A(Z)\,e^{-\lambda\left(S-\frac{\beta_i}{2}\,T_i\right)} + B(Z)\,e^{-\mu\,T_*}\,, \tag{3.22}$$

where we included the contribution of a single worldsheet instanton dependent on T_*. In general, one could have more non-perturbative α' contributions, but we shall here show that just one worldsheet instanton is enough to overcome the previous problems. The new Kähler covariant derivatives become:

$$D_{Z^\alpha}W = \mathrm{i}\,b_\alpha(Z) + W_{\mathrm{gc}}\left[\frac{\partial_\alpha A(Z)}{A(Z)} - k_\alpha(Z,\bar{Z})\right] + W_{\mathrm{wi}}\left[\frac{\partial_\alpha B(Z)}{B(Z)} - k_\alpha(Z,\bar{Z})\right], \tag{3.23}$$

$$D_S W = -\frac{1}{2s}\left[W_0 + (2\lambda s+1)W_{\mathrm{gc}} + W_{\mathrm{wi}}\right], \tag{3.24}$$

$$D_{T_p}W = \lambda\frac{\beta_p}{2}\,W_{\mathrm{gc}} - \frac{W_0 + W_{\mathrm{gc}} + W_{\mathrm{wi}}}{4\mathcal{V}}\,k_{pjk}t_jt_k \qquad p \neq *\,, \tag{3.25}$$

$$D_{T_*}W = \lambda\frac{\beta_*}{2}\,W_{\mathrm{gc}} - \mu W_{\mathrm{wi}} - \frac{W_0 + W_{\mathrm{gc}} + W_{\mathrm{wi}}}{4\mathcal{V}}\,k_{*jk}t_jt_k\,. \tag{3.26}$$

The solutions describing supersymmetric vacua with vanishing F-terms are:

$$\mathrm{i}\,b_\alpha(Z) = W_{\mathrm{gc}}\left[k_\alpha(Z,\bar{Z}) - \frac{\partial_\alpha A(Z)}{A(Z)}\right] + W_{\mathrm{wi}}\left[k_\alpha(Z,\bar{Z}) - \frac{\partial_\alpha B(Z)}{B(Z)}\right], \tag{3.27}$$

$$W_0 = -(2\lambda s+1)W_{\mathrm{gc}} - W_{\mathrm{wi}}\,, \tag{3.28}$$

$$\beta_p = -\frac{s}{\mathcal{V}}\,k_{pjk}t_jt_k \qquad p \neq *\,, \tag{3.29}$$

$$\beta_* = -\frac{s}{\mathcal{V}}\,k_{*jk}t_jt_k + 2R\,, \quad R \equiv \frac{\mu W_{\mathrm{wi}}}{\lambda W_{\mathrm{gc}}}\,. \tag{3.30}$$

It is important to note that the total superpotential $W = W_0 + W_{gc} + W_{wi} \neq 0$. Indeed if this were zero the dilaton would not be stabilized (see second equation above). This of course means that the supersymmetric vacua are AdS in contrast to Strominger's classical analysis.

The hidden and visible sector gauge kinetic functions now improve their behavior since they look like:

$$\mathrm{Re}\left(f_{\mathrm{hid}}^{1-\mathrm{loop}}\right) = -\frac{\beta_i}{2}\, t_i = 3\, s - R\, t_* = 3\, \mathrm{Re}\left(f_{\mathrm{hid}}^{\mathrm{tree}}\right) - R\, t_*\,, \tag{3.31}$$

and:

$$\mathrm{Re}\left(f_{\mathrm{vis}}\right) = -2s + R\, t_*\,. \tag{3.32}$$

Thus there is a regime where the hidden sector is weakly coupled and (the real part of) the gauge kinetic function of the visible sector (as well as that of the hidden sector) stays positive for:

$$2\, s \ll R\, t_* \ll 4\, s\,, \tag{3.33}$$

which points towards values $R\, t_* \simeq 3\, s$. In fact, in this regime, not only $\mathrm{Re}\left(f_{\mathrm{vis}}\right) > 0$ and $\mathrm{Re}\left(f_{\mathrm{hid}}\right) > 0$, but also:

$$\left|\frac{\mathrm{Re}\left(f_{\mathrm{hid}}^{1-\mathrm{loop}}\right)}{\mathrm{Re}\left(f_{\mathrm{hid}}^{\mathrm{tree}}\right)}\right| = \left|\frac{\mathrm{Re}\left(f_{\mathrm{vis}}^{1-\mathrm{loop}}\right)}{\mathrm{Re}\left(f_{\mathrm{vis}}^{\mathrm{tree}}\right)}\right| = \left|3 - \frac{R\, t_*}{s}\right| \ll 1\,. \tag{3.34}$$

3.2.3 Tuning the Calabi-Yau condition

As pointed out in [17], in the absence of worldsheet instantons and for $\partial_\alpha A(Z) = 0$, $D_{Z^\alpha} W = 0$ reduces to:

$$\mathrm{i}\, b_\alpha = W_{\mathrm{gc}}\, k_\alpha(Z, \bar{Z}) \neq 0\,. \tag{3.35}$$

This induces a (2,1)-component of H that would, according to Strominger's classical analysis, break the Calabi-Yau condition since $\partial J \propto H^{2,1}$. However from (3.27), one may speculate that the Calabi-Yau condition can be preserved by envisaging a situation where one tunes the flux quanta such that $b_\alpha = 0\ \forall \alpha = 1, ..., h^{1,2}(X)$ corresponding to $H^{2,1} = 0$. The complex structure moduli would then be fixed by:

$$D_{Z^\alpha} W = 0 \quad \Leftrightarrow \quad \frac{W_{\mathrm{wi}}}{W_{\mathrm{gc}}} = -\frac{1 - \frac{\partial_\alpha A(Z)}{A(Z) k_\alpha(Z,\bar{Z})}}{1 - \frac{\partial_\alpha B(Z)}{B(Z) k_\alpha(Z,\bar{Z})}}\,. \tag{3.36}$$

However now we have $4\, h^{1,2}$ real equations determining $2\, h^{1,2}$ real complex structure moduli. Obviously the system has no solution unless we scan over the integer fluxes. However there are only $2\, h^{1,2}+2$ integer fluxes. Thus we have only the freedom to scan over two integers while all $2\, h^{1,2}$ real complex structure moduli as well as all but two of the integers (i.e. $2\, h^{1,2}$ of them) must emerge as solutions to these non-linear equations. It is highly improbable that there would even be one solution of these equations.

Thus we do not think that it is possible to have $b_\alpha = 0$ in the presence of these non perturbative terms. On the other hand, as pointed out in the introduction, this condition emerges only on demanding a supersymmetric solution to the classical 10D equations. The 4D analysis cannot be expected to satisfy these classical conditions once non-perturbative effects are included. As we see from (3.27) and (3.28) the flux terms are related to the non-perturbative terms - in the absence of non-perturbative terms the latter are zero in agreement with Strominger. Indeed as pointed out in the introduction, the Strominger conditions also imply that the classical supersymmetric vacuum is Minkowski. On the other hand, the solutions that we have found are AdS vacua. Clearly there is no sense in imposing these classical 10D conditions on our quantum corrected 4D analysis.

3.3 Flux vacua counting

Let us clarify here a crucial difference which complex structure moduli stabilization with 3-form flux shows between type IIB and heterotic string theory. The F-term conditions (3.4) comprise $2\,h^{1,2}$ conditions for $2\,h^{1,2}$ real variables. Non-trivial H_3-flux supplies us exactly $2\,h^{1,2}$ independent flux quanta (up to the two related to the overall scaling of $\Omega(Z)$) generically supplying the non-linear system of $h^{1,2}$ complex F-term conditions for the $2\,h^{1,2}$ complex structure moduli. However, the existence of a finite number of isolated solutions for such non-linear systems with as many equations as variables (rendering the system 'well behaved') is not guaranteed. One expects therefore that much of available freedom of choice among the $2\,h^{1,2}$ H_3-fluxes is used up to find a relatively small number of isolated solutions for the complex structure moduli where all of them sit safely in the regime of large complex structure. Generically, this precludes the possibility of using the H_3-flux discretuum for tuning a very small VEV of $W_{\rm flux}$.

Note that this is different in the type IIB context. There, the availability of RR 3-form flux F_3 supplies an *additional* set of $2h^{2,1}$ fluxes for an overall discretuum made up from $4\,h^{1,2}$ fluxes. We have therefore an additional set of $2\,h^{1,2}$ discrete parameters available for tuning $W_{\rm flux}$ while keeping a given well-behaved complex structure moduli vacuum. Consequently, after having used $2\,h^{1,2}$ flux parameters to construct a viable complex structure vacuum, we can use the additional $2\,h^{1,2}$ flux quanta to construct a 'discrete $2\,h^{1,2}$-parameter family' of complex structure vacua, which allows for exponential tuning of $W_{\rm flux}$.

We recall also that we cannot use stabilization into the vicinity to a conifold point in complex structure moduli space for generating an exponentially small contribution to $W_{\rm flux}$. This has its root in the reality of H_3-flux in the heterotic context which prevents the existence of solutions with exponentially small complex structure moduli VEVs [18].

Finally we note that in the heterotic case the unavailability of any additional freedom in the choice of fluxes after fixing the complex structure moduli, means that we have to depend on the far more restricted choices that are available in the solution space of the complex structure moduli (which are now determined as functions of the $2\,h^{1,2}+2$ fluxes without the additional freedom that is present in the type IIB case). As mentioned before, one needs to scan over the H flux integers in order to find $2\,h^{1,2}$ acceptable (i.e. in the geometric regime) real solutions to the $2\,h^{1,2}$ non-linear equations $D_\alpha W=0$. The size of the solution set that we get is likely to be much smaller than the size of the original set of flux integers. Thus even if we had started with let us say $h^{1,2}=O(100)$ and let each flux scan over 1 to 10, the number of acceptable fluxes are likely to be far smaller than what is required to tune the cosmological constant. It should also be emphasized here that the only source of tuning that is available after all the low energy contributions to the vacuum energy are included, has to come from these fluxes.

4. Supersymmetry breaking vacua

The strategy is to perform moduli stabilization in two steps as follows:

- Step 1: Fix at leading order some of the moduli supersymmetrically (all the complex structure moduli, the dilaton and some Kähler moduli) at a high scale.
- Step 2: Stabilize the remaining light moduli at a lower scale breaking supersymmetry mainly along the Kähler directions by the inclusion of α' corrections to the Kähler potential in a way similar to type IIB.

In subsection 4.1 we shall consider the contributions to the scalar potential generated by fluxes, non-perturbative effects and threshold corrections showing that there exist no supersymmetry breaking minimum which lies in the regime of validity of the effective field theory. However, in subsection 4.2 we shall describe how this situation improves by the inclusion of α' corrections to the Kähler potential which yield trustworthy Minkowski vacua (see subsection 4.3) where supersymmetry is spontaneously broken by the F-terms of the Kähler moduli [9]. Finally in subsection 4.4 we shall explain what is the rôle played by D-terms in our stabilization procedure.

4.1 Fluxes, non-perturbative effects and threshold corrections

In this section we shall derive the general expression for the scalar potential including fluxes, non-perturbative effects (both gaugino condensation and world-sheet instantons) and threshold corrections for a Calabi-Yau three-fold whose volume is given by:

$$\mathcal{V} = k_b t_b^3 - k_s t_s^3 \,. \tag{4.1}$$

The superpotential and the Kähler potential look like (neglecting a possible Z-dependence of A and B and setting for simplicity $\beta_s = 0$):

$$\begin{aligned} W &= W_{\rm flux}(Z) + A\, e^{-\lambda\left(S-\frac{\beta_b}{2}\,T_b\right)} + B\, e^{-\mu\, T_s}\,, & (4.2)\\ K &= -\ln\mathcal{V} - \ln\left(S+\bar{S}\right) + K_{\rm cs}(Z,\bar{Z})\,. & (4.3)\end{aligned}$$

Performing the following field redefinition:

$$\Phi \equiv S - \frac{\beta_b}{2}\,T_b\,, \tag{4.4}$$

W and K take the form:

$$\begin{aligned} W &= W_{\rm flux}(Z) + A\, e^{-\lambda\,\Phi} + B\, e^{-\mu\, T_s}\,, & (4.5)\\ K &= -\ln\mathcal{V} - \ln\left[\Phi+\bar{\Phi}+\frac{\beta_b}{2}\left(T_b+\bar{T}_b\right)\right] + K_{\rm cs}(Z,\bar{Z})\,. & (4.6)\end{aligned}$$

[9]See [49] for another attempt to fix the heterotic moduli via the inclusion of α' effects.

4.1.1 Derivation of the F-term potential

The F-term scalar potential turns out to be:

$$
\begin{aligned}
V = e^K \Bigg[& \sum_Z K^{\alpha\bar\beta} D_\alpha W D_{\bar\beta} \bar W + K^{\Phi\bar\Phi} D_\Phi W D_{\bar\Phi} \bar W \\
& + \left(K^{\Phi\bar T_b} K_{\bar T_b} + K^{\Phi\bar T_s} K_{\bar T_s} \right) \left(\bar W D_\Phi W + W D_{\bar\Phi} \bar W \right) \\
& + K^{\Phi\bar T_s} \partial_{\bar T_s} \bar W D_\Phi W + K^{T_s\bar\Phi} \partial_{T_s} W D_{\bar\Phi} \bar W \\
& + |W|^2 \left(\sum_T K^{i\bar j} K_i K_{\bar j} - 3 \right) \\
& + \left(K^{T_s\bar T_b} K_{\bar T_b} + K^{T_s\bar T_s} K_{\bar T_s} \right) \left(\bar W \partial_{T_s} W + W \partial_{\bar T_s} \bar W \right) \\
& + K^{T_s\bar T_s} \partial_{T_s} W \partial_{\bar T_s} \bar W \Bigg] .
\end{aligned}
$$

Let us consider the limit:

$$\left| \mathrm{Re}\left(f_{\mathrm{hid}}^{1-\mathrm{loop}} \right) \right| \ll \mathrm{Re}\left(f_{\mathrm{hid}}^{\mathrm{tree}} \right) \quad \Leftrightarrow \quad \frac{\beta_b}{2}\, t_b \ll s \,, \tag{4.7}$$

which implies (defining $\Phi = \phi + \mathrm{i}\psi$):

$$\epsilon_\phi \equiv \frac{\beta_b\, t_b}{2\phi} = \frac{\beta_b\, t_b}{2(s - \frac{\beta_b}{2}\, t_b)} = - \frac{1}{1 - \frac{2s}{\beta_b\, t_b}} \simeq \frac{\beta_b\, t_b}{2s} \ll 1 \,, \tag{4.8}$$

together with:

$$t_b \sim \mathcal{O}(10) > t_s \sim \mathcal{O}(1) \qquad \Rightarrow \qquad \epsilon_s \equiv \frac{k_s t_s^3}{k_b t_b^3} \ll 1 \,. \tag{4.9}$$

We can then expand the relevant terms as:

$$K^{\Phi\bar\Phi} = 4\phi^2 \left(1 + 2\epsilon_\phi + \frac{4\epsilon_\phi^2}{3} + \frac{\epsilon_s \epsilon_\phi^2}{6} \right), \qquad K^{\Phi\bar T_s} = K^{T_s\bar\Phi} = -2\epsilon_\phi \phi t_s \,, \tag{4.10}$$

$$K^{\Phi\bar T_b} K_{\bar T_b} + K^{\Phi\bar T_s} K_{\bar T_s} = \frac{2\epsilon_\phi \phi}{1+\epsilon_\phi} \left(1 + \frac{4\epsilon_\phi}{3} + \frac{\epsilon_s \epsilon_\phi}{6} \right) . \tag{4.11}$$

The no-scale structure gets broken by loop effects:

$$\sum_T K^{i\bar j} K_i K_{\bar j} - 3 = \frac{2\epsilon_\phi}{(1+\epsilon_\phi)^2} \left(1 + \frac{7\epsilon_\phi}{6} + \frac{\epsilon_\phi \epsilon_s}{12} \right) \tag{4.12}$$

Notice that one correctly recovers the no-scale cancellation for $\beta_b = 0 \Leftrightarrow \epsilon_\phi = 0$. Other relevant terms are:

$$K^{T_s\bar T_b} K_{\bar T_b} + K^{T_s\bar T_s} K_{\bar T_s} = -2\, t_s \left(\frac{1 + 3\epsilon_\phi/2}{1+\epsilon_\phi} \right), \qquad K^{T_s\bar T_s} = \frac{2t_s^2}{3\epsilon_s} \left(1 + 2\epsilon_s \right) . \tag{4.13}$$

We shall look for minima in the region $\mathcal{V} \sim W_{\rm flux}\, e^{\mu\, t_s}$ implying that $W_{\rm wi} \sim \epsilon_s\, W_{\rm flux} \ll W_{\rm flux} \sim W_{\rm gc}$. The relevant derivatives scale as:

$$\partial_{Z^\alpha} W \sim W_{\rm flux}\,, \qquad \partial_\Phi W \sim W_{\rm gc} \sim W_{\rm flux}\,, \qquad \partial_{T_s} W \sim W_{\rm wi} \sim \epsilon_s\, W_{\rm flux}\,. \tag{4.14}$$

Therefore the F-term scalar potential can be expanded in the small parameters ϵ_ϕ and ϵ_s as:

$$V = V_0 + \epsilon V_1 + \epsilon^2 V_2 + \ldots \tag{4.15}$$

where (defining $\hat{W} = W_{\rm flux} + W_{\rm gc}$):

$$V_0 = e^K \left(\sum_Z K^{\alpha\bar\beta} D_\alpha \hat{W} D_{\bar\beta} \bar{\hat{W}} + 4\phi^2 D_\Phi \hat{W} D_{\bar\Phi} \bar{\hat{W}} \right) \sim \mathcal{O}\left(e^K |W_{\rm flux}|^2\right),$$

and:

$$\begin{aligned}\epsilon V_1 \,=\, e^K \Bigg[& \sum_Z K^{\alpha\bar\beta} \left(D_\alpha \hat{W} D_{\bar\beta} \bar{W}_{\rm wi} + D_\alpha W_{\rm wi} D_{\bar\beta} \bar{\hat{W}} \right) + 4\phi^2 \left(D_\Phi \hat{W} D_{\bar\Phi} \bar{W}_{\rm wi} + D_\Phi W_{\rm wi} D_{\bar\Phi} \bar{\hat{W}} \right) \\ & + 8\epsilon_\phi \phi^2 D_\Phi \hat{W} D_{\bar\Phi} \bar{\hat{W}} + 2\epsilon_\phi \phi \left(\bar{\hat{W}} D_\Phi \hat{W} + \hat{W} D_{\bar\Phi} \bar{\hat{W}} \right) \\ & + 2|\hat{W}|^2 \epsilon_\phi - 2\, t_s \left(\bar{\hat{W}} \partial_{T_s} W + \hat{W} \partial_{\bar{T}_s} \bar{W} \right) + \frac{2t_s^2}{3\epsilon_s}\, \partial_{T_s} W \partial_{\bar{T}_s} \bar{W} \Bigg] \sim \mathcal{O}\left(\epsilon\, e^K |W_{\rm flux}|^2\right),\end{aligned}$$

and:

$$\begin{aligned}\epsilon^2 V_2 \,=\, e^K \Bigg[& \sum_Z K^{\alpha\bar\beta} D_\alpha W_{\rm wi} D_{\bar\beta} \bar{W}_{\rm wi} + 4\phi^2 D_\Phi W_{\rm wi} D_{\bar\Phi} \bar{W}_{\rm wi} + 8\epsilon_\phi \phi^2 \left(D_\Phi \hat{W} D_{\bar\Phi} \bar{W}_{\rm wi} + h.c. \right) \\ & + \frac{16}{3}\, \epsilon_\phi^2 \phi^2 D_\Phi \hat{W} D_{\bar\Phi} \bar{\hat{W}} + 2\epsilon_\phi \phi \left(\bar{\hat{W}} D_\Phi W_{\rm wi} + W_{\rm wi} D_{\bar\Phi} \bar{\hat{W}} + h.c. \right) + \frac{2\epsilon_\phi^2 \phi}{3} \left(\bar{\hat{W}} D_\Phi \hat{W} + h.c. \right) \\ & - 2\epsilon_\phi \phi t_s \left(\partial_{\bar{T}_s} \bar{W} D_\Phi \hat{W} + h.c. \right) + 2\epsilon_\phi \left(\hat{W} \bar{W}_{\rm wi} + h.c. \right) - \frac{5\epsilon_\phi^2}{3}\, |\hat{W}|^2 \\ & - 2\, t_s \left(\bar{W}_{\rm wi} \partial_{T_s} W + h.c. \right) - t_s \epsilon_\phi \left(\bar{\hat{W}} \partial_{T_s} W + h.c. \right) + \frac{4t_s^2}{3}\, \partial_{T_s} W \partial_{\bar{T}_s} \bar{W} \Bigg] \sim \mathcal{O}\left(\epsilon^2 e^K |W_{\rm flux}|^2\right).\end{aligned}$$

4.1.2 Moduli stabilization

Step 1: We stabilize the Φ and Z-moduli by imposing $D_{Z^\alpha} \hat{W} = D_\Phi \hat{W} = 0$ thus minimizing the leading order term in the potential. We then substitute this solution in the scalar potential obtaining $V_0 = 0$ whereas the other contributions take the form:

$$\epsilon V_1 \,=\, e^K \left[2|\hat{W}|^2 \epsilon_\phi - 2\, t_s \left(\bar{\hat{W}} \partial_{T_s} W + \hat{W} \partial_{\bar{T}_s} \bar{W} \right) + \frac{2t_s^2}{3\epsilon_s}\, \partial_{T_s} W \partial_{\bar{T}_s} \bar{W} \right],$$

and:

$$\begin{aligned}\epsilon^2 V_2 \,=\, e^K \Bigg[& \sum_Z K^{\alpha\bar\beta} D_\alpha W_{\rm wi} D_{\bar\beta} \bar{W}_{\rm wi} + 4\phi^2 D_\Phi W_{\rm wi} D_{\bar\Phi} \bar{W}_{\rm wi} \\ & + 2\epsilon_\phi \phi \left(\bar{\hat{W}} D_\Phi W_{\rm wi} + h.c. \right) + 2\epsilon_\phi \left(\hat{W} \bar{W}_{\rm wi} + h.c. \right) - \frac{5\epsilon_\phi^2}{3}\, |\hat{W}|^2 \\ & - 2\, t_s \left(\bar{W}_{\rm wi} \partial_{T_s} W + h.c. \right) - t_s \epsilon_\phi \left(\bar{\hat{W}} \partial_{T_s} W + h.c. \right) + \frac{4t_s^2}{3}\, \partial_{T_s} W \partial_{\bar{T}_s} \bar{W} \Bigg].\end{aligned}$$

Step 2: We stabilize the T-moduli at order $\mathcal{O}(\epsilon)$ breaking supersymmetry. Writing $T_s = t_s + \mathrm{i}a_s$, $W_0^{\rm eff} = |W_0^{\rm eff}|e^{\mathrm{i}\theta_W}$ and $B = |B|e^{\mathrm{i}\theta_B}$, and setting $e^{\langle K_{\rm cs}\rangle} = 1$, the explicit form of the scalar potential at $\mathcal{O}(\epsilon)$ is:

$$V = \left[\frac{A_1}{\mathcal{V}^{2/3}} + \frac{|A_2|}{|W_0^{\rm eff}|}\cos(\theta_B - \theta_W - \mu a_s)\frac{t_s\, e^{-\mu t_s}}{\mathcal{V}} + \frac{|A_3|}{|W_0^{\rm eff}|^2}\frac{e^{-2\mu t_s}}{t_s}\right]\frac{|W_0^{\rm eff}|^2}{\langle\phi\rangle}\,, \tag{4.16}$$

with:

$$A_1 \equiv \frac{\beta_b}{2\langle\phi\rangle k_b^{1/3}}, \qquad |A_2| \equiv 2\,\mu|B|, \qquad |A_3| \equiv \frac{\mu^2|B|^2}{3k_s}\,, \tag{4.17}$$

where we have defined $W_0^{\rm eff} \equiv \langle\hat{W}\rangle = \langle W_{\rm flux} + W_{\rm gc}\rangle$. The axion a_s is minimised at $\mu\langle a_s\rangle = \theta_B - \theta_W - \pi$ so that (4.16) reduces to:

$$V = \left[\frac{A_1}{\mathcal{V}^{2/3}} - \frac{|A_2|}{|W_0^{\rm eff}|}\frac{t_s\, e^{-\mu t_s}}{\mathcal{V}} + \frac{|A_3|}{|W_0^{\rm eff}|^2}\frac{e^{-2\mu t_s}}{t_s}\right]\frac{|W_0^{\rm eff}|^2}{\langle\phi\rangle}\,. \tag{4.18}$$

Minimising with respect to t_s one finds:

$$\mathcal{V} = \frac{|A_2||W_0^{\rm eff}|}{|A_3|}\frac{(\mu t_s - 1)}{(2\mu t_s + 1)}\,t_s^2\, e^{\mu t_s} \underset{\mu t_s \gg 1}{\simeq} \frac{|A_2||W_0^{\rm eff}|}{2|A_3|}\,t_s^2\, e^{\mu t_s} = \frac{3k_s t_s^2}{\mu|B|}\,|W_0^{\rm eff}|\, e^{\mu t_s}\,, \tag{4.19}$$

which implies:

$$\mu t_s = \ln\left(\frac{\mathcal{V}}{|\lambda_0|}\right) - 2\ln t_s \underset{t_s\sim\mathcal{O}(1)}{\simeq} \ln\left(\frac{\mathcal{V}}{|\lambda_0|}\right) \equiv x(\mathcal{V}) \qquad \text{with} \qquad |\lambda_0| \equiv \frac{3k_s|W_0^{\rm eff}|}{\mu|B|}\,. \tag{4.20}$$

Notice that we can trust our effective field theory when $t_s \geq 1$, that is when $x(\mathcal{V}) \geq \mu = 2\pi$. Substituting (4.19) and (4.20) in (4.18), we end up with:

$$V = \left[A_1\,\mathcal{V}^{4/3} - |C_0|\,x(\mathcal{V})^3\right]\frac{|W_0^{\rm eff}|^2}{\langle\phi\rangle\,\mathcal{V}^2}\,, \qquad \text{where} \qquad |C_0| \equiv \frac{3k_s}{\mu^3}\,. \tag{4.21}$$

The extrema of V are located at:

$$\frac{\partial V}{\partial\mathcal{V}} = 0 \qquad \Leftrightarrow \qquad A_1\mathcal{V}^{4/3} = 3|C_0|\,x(\mathcal{V})^2\left[x(\mathcal{V}) - \frac{3}{2}\right], \tag{4.22}$$

showing that A_1 has to be positive, i.e. $\beta_b > 0$, if we want to have a minimum at large volume, i.e. $x(\mathcal{V}) \geq 2\pi$. Evaluating the second derivative at these points one finds:

$$\frac{\partial^2 V}{\partial\mathcal{V}^2} > 0 \qquad \Leftrightarrow \qquad 4x^2 - 15x + 9 < 0\,. \tag{4.23}$$

Hence the scalar potential has a minimum only for:

$$\frac{3}{4} < x(\mathcal{V}) < 3\,, \tag{4.24}$$

provided one can find values of λ_0 that satisfy (4.22) for this range of values for $\mathcal{V}$. However these minima are not trustworthy since the blow-up mode t_s is fixed below the string scale as $\langle t_s\rangle \simeq x(\mathcal{V})/(2\pi) < 3/(2\pi)$. As the above derivation assumed a regime $x(\mathcal{V}) > 2\pi$ but leads to a condition $x(\mathcal{V}) < 3$ for a minimum to exist, this demonstrates the absence of a minimum for the volume moduli in the controlled region of the scalar potential. This is consistent with a numerical analysis of the scalar potential (4.18) which shows that in the range $3/4 < x < 3$ the only critical point is a saddle point with one tachyonic direction.

4.2 Inclusion of α' effects

Let us now try to improve this situation taking into account also α' corrections to the Kähler potential described in section 2.2.1. Including both the $\mathcal{O}(\alpha'^2)$ and the $\mathcal{O}(\alpha'^3)$ effects, the Kähler potential for the T-moduli receives the following corrections:

$$K \simeq -\ln\mathcal{V} + \frac{|c_b|}{\mathcal{V}^{2/3}} - \frac{\gamma_s t_s + \xi/2}{\mathcal{V}}\,, \quad \text{with} \quad \gamma_s \equiv |c_b|\, k_s^{1/3} - |\kappa| k_b^{1/3}\,, \tag{4.25}$$

where we have used eq. (2.21) for the expression for c_s. These higher-derivative corrections break the no-scale structure as (neglecting threshold effects):

$$\sum_T K^{i\bar{j}} K_i K_{\bar{j}} - 3 \simeq -\frac{2|c_b|}{\mathcal{V}^{2/3}} + \frac{2\gamma_s t_s + 3\xi}{\mathcal{V}}\,. \tag{4.26}$$

The scalar potential (4.18) gets modified and reads:

$$V = \left[\frac{A_1}{\mathcal{V}^{2/3}} - \frac{|c_b|}{\mathcal{V}^{5/3}} - \frac{|A_2|}{|W_0^{\rm eff}|}\frac{t_s}{\mathcal{V}}\, e^{-\mu t_s} + \frac{|A_3|}{|W_0^{\rm eff}|^2}\frac{e^{-2\mu t_s}}{t_s} + \frac{\gamma_s t_s + 3\xi/2}{\mathcal{V}^2}\right]\frac{|W_0^{\rm eff}|^2}{\langle\phi\rangle}\,. \tag{4.27}$$

Minimising with respect to t_s we find:

$$\begin{aligned}\mathcal{V} &= \left(1 \pm \sqrt{1 + \frac{4|A_3|\gamma_s(2\mu t_s + 1)}{|A_2|^2 t_s^2(\mu t_s - 1)^2}}\right)\frac{|A_2||W_0^{\rm eff}| t_s^2(\mu t_s - 1)}{2|A_3|(2\mu t_s + 1)}\, e^{\mu t_s} \\ &\underset{\mu t_s \gg 1}{\simeq} \frac{|A_2||W_0^{\rm eff}|}{4|A_3|}\left(1 + \sqrt{1 + \frac{c}{t_s^3}}\right) t_s^2\, e^{\mu t_s} \qquad \text{with} \qquad c = \frac{8|A_3|\gamma_s}{|A_2|^2\mu} = \frac{2\gamma_s}{3k_s\mu}\,,\end{aligned} \tag{4.28}$$

where we focused only on the solution which for $\gamma_s = 0$ correctly reduces to (4.19) since the other solution can be shown to give rise to a maximum along the t_s direction. Notice that we did not take an expansion for small c/t_s^3 even if this quantity is suppressed by $\mu t_s \gg 1$ since a large denominator might be compensated by a large value of the unknown coefficient γ_s. Performing the following approximation:

$$\mu t_s = \ln\left(\frac{\mathcal{V}}{|\lambda|}\right) - 2\ln t_s \underset{t_s\sim\mathcal{O}(1)}{\simeq} \ln\left(\frac{\mathcal{V}}{|\lambda|}\right) \equiv x(\mathcal{V})\,, \tag{4.29}$$

with:

$$|\lambda| \equiv \frac{|\lambda_0|}{2}\left(1 + \sqrt{1 + \frac{c}{t_s^3}}\right),$$

and substituting (4.28) in (4.27) we end up with (in the regime $x(\mathcal{V}) \gg 1$):

$$V \simeq \left[A_1\,\mathcal{V}^{4/3} - |c_b|\,\mathcal{V}^{1/3}(1 - \delta\, x) - |C|\, x^3 + \frac{3\xi}{2}\right]\frac{|W_0^{\rm eff}|^2}{\langle\phi\rangle\mathcal{V}^2}\,, \tag{4.30}$$

where we have defined:

$$|C| \equiv \frac{|C_0|}{2}\left(1 + \sqrt{1 + \frac{c\mu^3}{x^3}}\right), \qquad \text{and} \qquad \delta \equiv \frac{\gamma_s}{\mu\,|c_b|\,\mathcal{V}^{1/3}}\,. \tag{4.31}$$

Notice that if we switch off the α' corrections by setting $|c_b| = c = \xi = 0$, the scalar potential (4.30) correctly reduces to (4.21) since $|\lambda| \to |\lambda_0|$ and $|C| \to |C_0|$.

Before trying to minimise the scalar potential, let us show two important facts:

- The quantity $\delta\, x$ is always smaller than unity since from (4.25) one finds that:

$$\gamma_s \leq |c_b| k_s^{1/3} \qquad \Rightarrow \qquad \delta\, x \leq \frac{k_s^{1/3} t_s}{\mathcal{V}^{1/3}} \simeq \epsilon_s^{1/3} \ll 1\,. \tag{4.32}$$

  Therefore the term in (4.30) proportional to $|c_b|$ has always a positive sign.

- If the condition $|c|/t_s^3 \ll 1$ is not satisfied, there is no minimum for realistic values of the underlying parameters. In fact, in this case the term proportional to $|C|$ is always subleading with respect to the term proportional to c_b since for $c \geq 0$:

$$R \equiv \frac{|C|\, x^3}{|c_b|\, \mathcal{V}^{1/3}} \leq \left(1 + \sqrt{1 + \frac{c}{t_s^3}}\right) \frac{t_s^3}{c} \frac{\epsilon_s^{1/3}}{x} \ll \left(1 + \sqrt{1 + \frac{c}{t_s^3}}\right) \frac{t_s^3}{c}\,, \tag{4.33}$$

  which for $c/t_s^3 \sim \mathcal{O}(1)$ reduces to $R \ll \mathcal{O}(1)$, whereas for $c/t_s^3 \gg 1$ reduces to $R \ll \sqrt{t_s^3/c} \ll 1$. On the other hand, for $c < 0$, one has $|c|/t_s^3 \leq 1$ but if $|c|/t_s^3 \sim \mathcal{O}(1)$, the ratio R can be shown to reduce again to $R < \epsilon_s^{1/3}/x \ll 1$. Therefore in this case the leading order scalar potential is given by (neglecting the term proportional to δ):

$$V \simeq \left[A_1\, \mathcal{V}^{4/3} - |c_b|\, \mathcal{V}^{1/3} + \frac{3\xi}{2}\right] \frac{|W_0^{\rm eff}|^2}{\langle\phi\rangle \mathcal{V}^2}\,, \tag{4.34}$$

  with:

$$|c| \gtrsim t_s^3 \qquad \Rightarrow \qquad |c_b| \geq \frac{\gamma_s}{k_s^{1/3}} = \frac{3 k_s^{2/3} \mu}{2}\, c \gtrsim \frac{3 k_s^{2/3} \mu}{2}\, t_s^3\,. \tag{4.35}$$

  However the potential (4.34) has a minimum only if:

$$\xi > \frac{5}{12} |c_b| \mathcal{V}^{1/3} \gtrsim \frac{5}{8} k_s t_s^3 \frac{x}{\epsilon_s^{1/3}} \gg 1\,, \tag{4.36}$$

  which is never the case for ordinary Calabi-Yau three-folds with $\xi \sim \mathcal{O}(1)$. As an illustrative example, for $\mathcal{V} = 20$, $t_s = 1.5$ and $k_s = n/6$ with $n \in \mathbb{N}$, one finds $\xi \gtrsim 11\, n^{2/3} \geq 11$, corresponding to Calabi-Yau manifolds with Euler number negative and very large in absolute value: $|\chi| = 2(h^{1,2} - h^{1,1}) \gtrsim 4548$, while most of the known Calabi-Yau manifolds have $|\chi| \lesssim \mathcal{O}(1000)$.

Hence we have shown that in order to have a trustable minimum we need to be in a region where $|c| \ll t_s^3$. In this case, the scalar potential (4.30) simplifies to:

$$V \simeq \left[A_1\, \mathcal{V}^{4/3} - |c_b|\, \mathcal{V}^{1/3} (1 - \delta\, x) - |C_0|\, x^3 + \frac{3\xi}{2}\right] \frac{|W_0^{\rm eff}|^2}{\langle\phi\rangle \mathcal{V}^2}\,, \tag{4.37}$$

where we have approximated $|C| \simeq |C_0|$. Notice that the sign of the numerical coefficient A_1 is a priori undefined and depends on the sign of the underlying parameter β_b.

The new extrema of V are located at:

$$A_1 \mathcal{V}^{4/3} = 3|C_0|\, x^2 \left(x - \frac{3}{2}\right) + \frac{5|c_b|}{2}\, \mathcal{V}^{1/3} \left(1 - \frac{6\delta\, x}{5} + \frac{3\delta}{5}\right) - \frac{9\xi}{2}\,, \tag{4.38}$$

and the second derivative at these points is positive if:

$$u(x) \equiv 12\xi - 5|c_b|\, \mathcal{V}^{1/3} \left(1 - \frac{8\delta\, x}{5} + 2\delta\right) - 2|C_0| x(4x^2 - 15x + 9) > 0\,. \tag{4.39}$$

Notice that for $|c_b| = \delta = \xi = 0$ (4.38) and (4.39) correctly reduce to (4.22) and (4.23) respectively. However we shall now show that including the α' corrections we can find a vacuum with $x \gg 1$ where we can trust the effective field theory.

The value of the vacuum energy is:

$$\langle V \rangle = \frac{|W_0^{\rm eff}|^2}{2\langle\phi\rangle \mathcal{V}^2}\, v(x)\,, \tag{4.40}$$

where:

$$v(x) \equiv -6\xi + |C_0| x^2\, (4x - 9) + 3|c_b| \mathcal{V}^{1/3} \left(1 - \frac{4\delta\, x}{3} + \delta\right). \tag{4.41}$$

Let us perform the following tuning to get a Minkowski vacuum:

$$v(x) = 0 \qquad \text{if} \qquad 6\xi = 3|c_b|\, \mathcal{V}^{1/3} \left(1 - \frac{4\delta\, x}{3} + \delta\right) + |C_0| x^2\, (4x - 9)\,, \tag{4.42}$$

and substitute it in (4.39) obtaining:

$$u(x) \equiv |c_b| \mathcal{V}^{1/3}\, (1 - 4\delta) + 12|C_0| x \left(x - \frac{3}{2}\right) > 0\,, \tag{4.43}$$

which is automatically satisfied for $\delta \ll 1$ and $x \gg 1$. Substituting (4.42) also in the vanishing of the first derivative (4.38), this simplifies to:

$$4A_1\, \mathcal{V}^{4/3} = |c_b| \mathcal{V}^{1/3}\, (1 - 3\delta) + 9|C_0| x^2\,, \tag{4.44}$$

showing that if we want to have a Minkowski minimum A_1 has to be positive, i.e. $\beta_b > 0$.

4.3 Minkowski solutions

Let us first define our use of the term ‘Minkowski solutions’. Owing to the lack of tuning freedom in the heterotic 3-form flux superpotential, we cannot achieve vacua with exponentially small vacuum energy. For us a ‘Minkowski vacuum’ thus shall label a vacuum with a cosmological constant suppressed by at least a 1-loop factor $1/(8\pi^2) \simeq 0.01$ compared the height of the barrier in the scalar potential which will protect the volume moduli from run-away. This barrier height is set by $\sim m_{3/2}^2 M_P^2 \gtrsim M_{\rm GUT}^2 M_P^2$.

The solutions depend on 7 underlying parameters: $k_s, k_b, \beta_b, |B|, |c_b|, |\kappa|$ and ξ. We do not consider $|W_0^{\rm eff}|$ as a free variable at this stage since we fix its value at $|W_0^{\rm eff}| = 0.06$ by the phenomenological requirement of obtaining the right GUT coupling corresponding to $\langle s\rangle \simeq \langle \phi\rangle \simeq 2$.

Let us now describe a strategy to find the values of these underlying parameters which give Minkowski vacua for desired values of the moduli and within the regime of validity of all our approximations.

1. Choose the desired values for $\mathcal{V}$ and t_s (so fixing the value of $x = 2\pi\, t_s$). Then work out the value of $|B|$ as a function of k_s from (4.20).

2. Choose the desired value of t_b and work out the value of k_b as a function of k_s from (4.1).

3. Determine $|c_b|$ as a function of k_s, ξ and $|\kappa|$ from (4.42).

4. Derive the value of β_b as a function of k_s, ξ and $|\kappa|$ from (4.44).

5. Choose the values of k_s, ξ and $|\kappa|$ so that all our approximations are under control, i.e. ϵ_ϕ defined in (4.8) satisfies $\epsilon_\phi \ll 1$, ϵ_s defined in (4.9) gives $\epsilon_s \ll 1$, δ defined in (4.31) satisfies $\delta \ll 1$ and $\epsilon_{\alpha'} \equiv \xi/(2\mathcal{V}) \ll 1$. These values of k_s, ξ and $|\kappa|$ then give the values of $k_b, |B|, |c_b|$ and β_b knowing that this Minkowski vacuum is fully consistent.

As an illustrative example, following this procedure we found a Minkowski vacuum (see Figure (1)) located at:

$$\langle \phi\rangle \simeq \langle s\rangle = 2\,, \qquad \langle \mathcal{V}\rangle = 20\,, \qquad \langle t_b\rangle \simeq 5\,, \qquad \langle t_s\rangle = 1.5\,, \tag{4.45}$$

for the following choice of the microscopic parameters:

$$\begin{aligned} &k_b = k_s = 1/6\,, \quad \beta_b \simeq 0.035\,, \quad |W_0^{\rm eff}| = 0.06\,, \quad c_b = 0.75\,, \quad c_s = -0.75\,, \\ &B \simeq 3\,, \quad \xi \simeq 1.49\,, \quad \mu = 2\pi \quad \Rightarrow \quad \gamma_s \simeq 0.41\,, \quad \kappa = 0\,. \end{aligned} \tag{4.46}$$

Notice that one can get dS or AdS solutions by varying β_b either above or below its benchmark value. Moreover, our approximations are under control since:

$$\epsilon_\phi \simeq 0.043\,, \qquad \epsilon_s \simeq 0.027\,, \qquad \epsilon_{\alpha'} \simeq 0.037\,, \qquad \delta \simeq 0.032 \quad . \tag{4.47}$$

We stress that at the minimum these four quantities are all of the same size: $\epsilon_\phi \simeq \epsilon_s \simeq \epsilon_{\alpha'} \simeq \delta$. This has to be the case since they weight the relative strengths of loops, non-perturbative and higher derivative effects which all compete to give a minimum.

Moreover, we point out that there seem to be problems with the α' expansion since we managed to obtain a minimum by tuning the underlying parameters in order to have the $\mathcal{O}(\alpha'^2)$ term of the same order of magnitude of the $\mathcal{O}(\alpha'^3)$ term, and so higher order α' corrections might not be negligible.

However this might not be a problem if at least one of the following is valid:

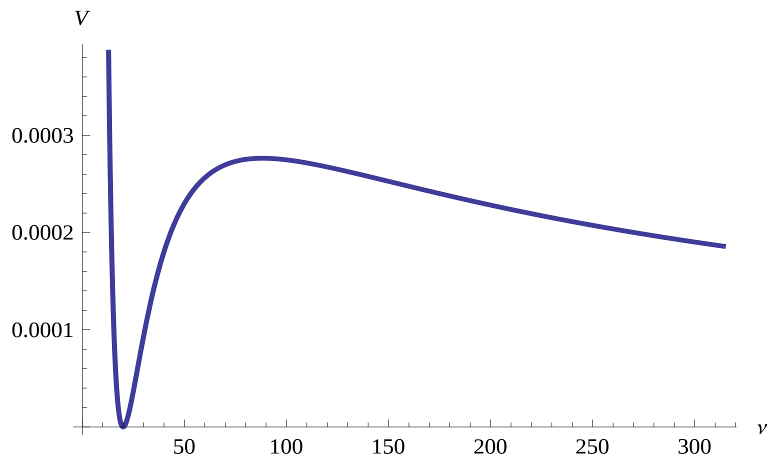

Figure 1: V versus $\mathcal{V}$ assuming the parameters listed in the text which give rise to a near-Minkowski vacuum with $\langle\mathcal{V}\rangle = 20$ and a cosmological constant of small magnitude compared the barrier height set by $m_{3/2}^2 M_P^2 \gtrsim M_{\rm GUT}^2 M_P^2$.

- The $\mathcal{O}(\alpha'^2)$ corrections could be eliminated by a proper redefinition of the moduli.
- The coefficients of higher order α' corrections are not tuned larger than unity, resulting in a α' expansion which is under control. In fact, the α' expansion parameter is of order $q\mathcal{V}^{-1/3}$ with q an unknown coefficient. Thus $\mathcal{O}(\alpha'^4)$ contributions to the scalar potential can be estimated as:

$$\frac{V_{\alpha'^4}}{V_{\alpha'^3}} \simeq \frac{2q}{3\xi\mathcal{V}^{1/3}} \simeq 0.16 \qquad \text{for} \qquad q = 1\,. \tag{4.48}$$

4.4 D-term potential

So far only F-terms have been taken into account. This could be consistent since moduli-dependent D-terms might not be present in the absence of anomalous $U(1)$s, or they might be cancelled by giving suitable VEVs to charged matter fields.

However let us see how D-terms might change the previous picture in the presence of anomalous $U(1)$s but without introducing charged matter fields. Because of the $U(1)$-invariance of the superpotential (4.5), both Φ and T_s have to be neutral. Therefore the only field which can be charged under an anomalous $U(1)$ is T_b with $q_{T_b} = 4\,c_1^b(\mathcal{L}) \neq 0$. From (2.31), this induces an FI-term of the form:

$$\xi = -\,q_{T_b} K_b = q_{T_b}\left(\frac{k_b}{\mathcal{V}}\right)^{1/3}, \tag{4.49}$$

which gives the following D-term potential:

$$V_D = \frac{\xi^2}{\text{Re}(f)} \simeq \frac{p}{\mathcal{V}^{2/3}} \qquad \text{with} \qquad p \equiv \frac{q_{T_b}^2\, k_b^{2/3}}{\phi}\,. \tag{4.50}$$

This term has the same volume scaling as the first term in (4.37) which is the contribution coming from threshold effects. However the ratio between these two terms scales as:

$$\frac{V_{\text{threshold}}}{V_D} = \frac{\epsilon_\phi}{\mathcal{V}^{1/3}} \frac{|W_0^{\text{eff}}|^2}{q_{T_b}^2 \, k_b^{2/3}} \ll 1 \,, \tag{4.51}$$

for $\epsilon_\phi \ll 1$ and $q_{T_b} \sim \mathcal{O}(1)$. As an illustrative example, our explicit parameter choice would give $V_{\text{threshold}}/V_D \simeq 2 \cdot 10^{-4} \, q_{T_b}^{-2}$, showing that V_D is always dominant with respect to the F-term potential (4.37). In this case, V_D would give a run-away for the volume direction and destroy our moduli-stabilisation scenario.

As we have already pointed out, this might not be the case if there are no anomalous $U(1)$s or if the FI-term is cancelled by a matter field VEV. There is however another way-out to this D-term problem which relies on the possibility to fix all the moduli charged under the anomalous $U(1)$s in a completely supersymmetric way, so ensuring the vanishing of the D-term potential. This requires $q_{T_b} = 0$ and the addition of a third Kähler modulus T_c which is charged under the anomalous $U(1)$: $q_{T_c} \neq 0$. Let describe this situation in the next subsection.

4.4.1 D + F-term stabilisation

The Kähler and superpotential now read:

$$W = W_{\text{flux}}(Z) + A \, e^{-\lambda\left(\Phi - \frac{\beta_c}{2} T_c\right)} + B \, e^{-\mu T_s} \,, \tag{4.52}$$

$$K = -\ln \tilde{\mathcal{V}} - \ln \left[\Phi + \bar{\Phi} + \frac{\beta_b}{2} \left(T_b + \bar{T}_b\right)\right] + K_{\text{cs}}(Z) \,, \tag{4.53}$$

with

$$\tilde{\mathcal{V}} = \mathcal{V} - k_c t_c^3 = k_b t_b^3 - k_s t_s^3 - k_c t_c^3 \,. \tag{4.54}$$

Notice that now Φ has to get charged under the anomalous $U(1)$ so that the hidden sector gauge kinetic function $f_{\text{hid}} = \Phi - \frac{\beta_c}{2} T_c$ becomes gauge invariant. In particular we will have $q_\Phi = \frac{\beta_c}{2} q_{T_c}$. From (2.31), the FI-term looks like:

$$\xi = -q_\Phi \frac{D_\Phi W}{W} - q_{T_c} \frac{D_{T_c} W}{W} \,, \tag{4.55}$$

implying that $V_D = 0$ if both Φ and T_c are fixed supersymmetrically. However we have already seen that if all the Kähler moduli are fixed supersymmetrically via threshold effects, then perturbation theory breaks down in the hidden sector and the visible sector gauge kinetic function becomes negative. A way-out proposed in section 3.2.2 was to include worldsheet instantons but, given that we want to break supersymmetry at leading order along T_b and T_s, in order to follow this possibility we should include a fourth modulus with worldsheet instantons. Thus this case does not look very appealing since it requires at least four moduli.

A simplest solution can be found by noticing that the problems with $\text{Re}\left(f_{\text{hid}}^{1-\text{loop}}\right) > \text{Re}\left(f_{\text{hid}}^{\text{tree}}\right)$ and $\text{Re}\left(f_{\text{vis}}\right) < 0$ could be avoided if only some but not all of the Kähler moduli are fixed supersymmetrically by threshold effects. We shall now prove that this is indeed the case if the T-moduli fixed in this way are blow-up modes like t_c. In fact, the solution to $D_{T_c}W = 0$ gives:

$$\beta_c = -\frac{s}{\mathcal{V}}\, k_{cjk} t_j t_k = -\frac{6\, s}{\mathcal{V}}\, k_c t_c^2 \,. \tag{4.56}$$

This result, in turn, gives hidden and visible sector gauge kinetic functions of the form:

$$\frac{\mathrm{Re}\left(f_{\mathrm{hid}}^{1-\mathrm{loop}}\right)}{\mathrm{Re}\left(f_{\mathrm{hid}}^{\mathrm{tree}}\right)} = -\frac{\beta_b t_b}{2s} - \frac{\beta_c t_c}{2s} = -\epsilon_\phi + 3\,\frac{k_c\, t_c^3}{\mathcal{V}} = -\epsilon_\phi + 3\epsilon_s \ll 1\,,$$

and:

$$\mathrm{Re}\left(f_{\mathrm{vis}}\right) = s\left(1 + \frac{\beta_b t_b}{2s} + \frac{\beta_c t_c}{2c}\right) = s\left(1 + \epsilon_\phi - 3\,\frac{k_c t_c^3}{\mathcal{V}}\right) = s\left(1 + \epsilon_\phi - 3\,\epsilon_s\right) \simeq s > 0\,.$$

5. Moduli mass spectrum, supersymmetry breaking and soft terms

Expanding the effective field theory around the vacua found in the previous section, we can derive the moduli mass spectrum which turns out to be (see (4.29) and (4.31) for the definitions of x and δ):

$$\begin{aligned} m_{t_s} &\simeq m_{a_s} \simeq m_{3/2}\, x\,, \\ m_{Z^\alpha} &\simeq m_\Phi \simeq m_{3/2}\,, \\ m_{t_b} &\simeq m_{3/2}\,\delta\,, \\ m_{a_b} &\simeq 0\,. \end{aligned} \tag{5.1}$$

Notice that in the absence of T_b-dependent worldsheet instantons which would give a_b a mass of the order $m_{a_b} \simeq M_P\, e^{-\mu\, t_b} \simeq 10$ TeV for $t_b \simeq 5$, this axion might be a good QCD axion candidate since it could remain a flat direction until standard QCD non-perturbative effects give it a tiny mass.

Moreover, the stabilisation procedure described in the previous sections leads to vacua which break supersymmetry spontaneously mainly along the Kähler moduli directions. In fact, from the general expression of the F-terms and the gravitino mass:

$$F^i = e^{K/2} K^{i\bar{j}} D_{\bar{j}}\bar{W} \qquad \text{and} \qquad m_{3/2} = e^{K/2}|W| \simeq \frac{|W_0^{\mathrm{eff}}|}{\mathcal{V}^{1/2}}\,, \tag{5.2}$$

we find that the Kähler moduli F-terms read:

$$\frac{F^{T_b}}{t_b} = -2\, m_{3/2} \qquad \text{and} \qquad \frac{F^{T_s}}{t_s} \simeq \frac{m_{3/2}}{x}\,. \tag{5.3}$$

On the other hand, the dilaton and the complex structure moduli are fixed supersymmetrically at leading order. However, due to the fact that the prefactor of worldsheet instantons and α' effects are expected to depend on these moduli, they would also break supersymmetry at subleading order developing F-terms whose magnitude can be estimated as (assuming that there are no cancellations from shifts of the minimum due to the subleading corrections):

$$D_{Z^\alpha,\Phi} W \simeq D_{Z^\alpha,\Phi} W_{\rm wi} \simeq \delta\, |W_0^{\rm eff}| \qquad \Rightarrow \qquad F^{Z^\alpha,\Phi} \simeq \delta\, m_{3/2}\,. \tag{5.4}$$

Thus we can see that supersymmetry is mainly broken along the t_b-direction since:

$$\frac{F^{T_b}}{m_{3/2}} \simeq t_b \gg \frac{F^{T_s}}{m_{3/2}} \simeq \frac{t_s}{x} \gg \frac{F^{Z^\alpha,\Phi}}{m_{3/2}} \simeq \delta\,. \tag{5.5}$$

The goldstino is therefore mainly the T_b-modulino which is eaten up by the gravitino in the super-Higgs mechanism.

Soft supersymmetry breaking terms are generated in the visible sector via tree-level gravitational interactions due to moduli mediation. Let us now derive their expression:

- **Gaugino masses**: Their canonically normalised expression is given by:

  $$M_{1/2} = \frac{1}{2{\rm Re}(f_{\rm vis})}\, F^i \partial_i f_{\rm vis} \simeq \frac{F^\Phi}{2\phi} + \delta\, \frac{F^{T_b}}{t_b} \simeq \delta\, m_{3/2}\,, \tag{5.6}$$

  showing that the gaugino masses are suppressed with respect to the gravitino mass by a factor of order $\delta \simeq 0.03$.

- **Scalar masses**: The canonically normalised scalar masses generated by gravity mediation read (assuming that the cosmological constant has been tuned to a value well below the gravitino mass scale):

  $$m^2_{0,\,\alpha} = m^2_{3/2} - F^i \bar{F}^{\bar{j}} \partial_i \partial_{\bar{j}} \ln \tilde{K}_\alpha\,, \tag{5.7}$$

  where $\tilde{K}_\alpha$ is the Kähler metric for matter fields which we assumed to be diagonal. $\tilde{K}_\alpha$ is generically a function of all the moduli but we shall neglect its dependence on the dilaton and the complex structure moduli since they give only a subleading contribution to supersymmetry breaking. Hence we shall consider a Kähler metric for matter fields of the form $\tilde{K}_\alpha \simeq t_s^{-n_s}\, t_b^{-n_b}$, where n_s and n_b are the so-called modular weights. In the type IIB set-up, it is possible to determine the value of n_b by requiring physical Yukawa couplings which do not depend on the large cycle due to the localisation of the visible sector on one of the small cycles [50]. However, in the heterotic framework the situation is different. For instance, in CY compactifications close to the orbifold point the visible sector typically is constructed from split multiplets which partially live in the bulk and partially arise as twisted sector states localized at orbifold fixed points. The value of the modular weights $n_{b,\alpha}$ for the different matter fields is then determined by the requirements of modular invariance. Hence, they cannot be constrained by using an argument similar to the one in [50]. We shall therefore leave it as an undetermined parameter. The scalar masses turn out to be:

  $$m_0^2 = m^2_{3/2} \left(1 - n_b - \frac{n_s}{4x^2}\right)\,, \tag{5.8}$$

showing that for $x \gg 1$, the modular weight n_b has to be $n_b \leq 1$ in order to avoid tachyonic squarks and sleptons. If $n_b = 1$, one has a leading order cancellation in the scalar masses which therefore get generated by the F-terms of the small cycle t_s even if $F^{T_s} \ll F^{T_b}$ (in this case one would need $n_s < 0$). This is indeed the case in type IIB models because of the no-scale structure [51]. Given that the no-scale cancellation holds in the heterotic case as well, we expect a similar cancellation to occur in our case, i.e. $n_b = 1$, with possibly the exception of twisted matter fields at orbifold fixed points , i.e. $n_b < 1$ for twisted states.

- **A-terms**: The canonically normalised A-terms look like:

$$A_{\alpha\beta\gamma} = F^i \left[K_i + \partial_i \ln Y_{\alpha\beta\gamma} - \partial_i \ln \left(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma\right)\right] , \tag{5.9}$$

where $Y_{\alpha\beta\gamma}$ are the canonically unnormalised Yukawa couplings which can in principle depend on all the moduli. Similarly to the Kähler metric for matter fields, we introduce two modular weights, p_b and p_s, and we write the Yukawa couplings as $Y_{\alpha\beta\gamma} \simeq t_b^{-p_b}\, t_s^{-p_s}$. Thus the A-terms take the form:

$$A_{\alpha\beta\gamma} = 3m_{3/2}\left(1 + p_b - n_b + \frac{p_s}{2x} - \frac{n_s}{2x} - \frac{\delta}{3x}\right) . \tag{5.10}$$

In the type IIB case, there is again a leading order cancellation (since $n_b = 1$ and $p_b = 0$ given that the Yukawa couplings do not depend on the Kähler moduli due to the axionic shift-symmetry and the holomorphicity of W) which is again due to the no-scale structure [51]. Similarly to the scalar masses, we expect this leading order cancellation also in the heterotic case for matter fields living in the bulk.

- **μ and Bμ-term**: The μ-term can be generated by a standard Giudice-Masiero term in the Kähler potential $K \supset \tilde{K}(t_s, t_b) H_u H_d$ which gives again $\mu \simeq m_{3/2}$ and $B\mu \simeq m_{3/2}^2$.

Summarizing, we obtained a very specific pattern of soft terms with scalars heavier than the gauginos and universal A-terms and $\mu/B\mu$-term of the order the gravitino mass:

$$m_0 \simeq A_{\alpha\beta\gamma} \simeq \mu \simeq B \simeq m_{3/2} \gg M_{1/2} \simeq \delta\, m_{3/2} . \tag{5.11}$$

We stress again that the soft mass scale is expected to be generically of order $m_{3/2} \sim M_{\rm GUT}$ due the fact that in the heterotic string there is not enough freedom to tune the flux superpotential below values of the order $0.1 - 0.01$.

6. Anisotropic solutions

In this section we shall show how to generalise the previous results to obtain anisotropic compactifications with 2 large and 4 small extra dimensions which allow for a right value of the GUT scale [10]. For this purpose, we shall focus on Calabi-Yau three-folds whose volume looks like [55]:

$$\mathcal{V} = k_b t_b t_f^2 - k_s t_s^3 \,. \tag{6.1}$$

This Calabi-Yau admits a 4D K3 or T^4 divisor of volume t_f^2 fibered over a 2D $\mathbb{P}^1$ base of volume t_b with an additional del Pezzo divisor of size t_s^2. We shall now show how to fix the moduli dynamically in the anisotropic region $t_b \gg t_f \sim t_s$. We shall consider a hidden sector gauge kinetic function of the form:

$$f_{\rm hid} = S - \frac{\beta_b}{2}\,T_b - \frac{\beta_f}{2}\,T_f \equiv \Phi \,, \tag{6.2}$$

with $\beta_s = 0$.

The superpotential looks exactly as the one in (4.5) whereas the Kähler potential reads:

$$K = -\ln \mathcal{V} - \ln\left[\Phi + \bar{\Phi} + \frac{\beta_b}{2}\left(T_b + \bar{T}_b\right) + \frac{\beta_f}{2}\left(T_f + \bar{T}_f\right)\right] + K_{\rm cs}(Z)\,. \tag{6.3}$$

Focusing on the limit where 1-loop effects are suppressed with respect to the tree-level expression of the gauge kinetic function:

$$\epsilon_b \equiv \frac{\beta_b\, t_b}{2\phi} \ll 1 \qquad \text{and} \qquad \epsilon_f \equiv \frac{\beta_f\, t_f}{2\phi} \ll 1\,, \tag{6.4}$$

the dilaton is again fixed at leading order by requiring $D_\Phi W = 0$. On the other hand the Kähler moduli develop a subdominant potential via non-perturbative contributions, α' corrections and threshold effects which break the no-scale structure as:

$$\sum_T K^{i\bar{j}} K_i K_{\bar{j}} - 3 = 2\left(\epsilon_b + \epsilon_f\right) + \mathcal{O}(\epsilon^2)\,. \tag{6.5}$$

The scalar potential has therefore the same expression as (4.16) but with a different expression of the coefficient A_1 which is now moduli-dependent and looks like:

$$A_1(\mathcal{V}, t_f) = \frac{\mathcal{V}^{2/3}}{2\langle\phi\rangle}\left(\frac{\beta_b}{k_b t_f^2} + \frac{\beta_f\, t_f}{\mathcal{V}}\right)\,, \tag{6.6}$$

where we have traded t_b for $\mathcal{V}$. This is the only term which depends on t_f since the rest of the potential depends just on $\mathcal{V}$ and t_s. Hence we can fix t_f just minimising $A_1(\mathcal{V}, t_f)$ obtaining:

$$t_f = \left(\frac{2\beta_b}{k_b \beta_f}\right)^{1/3} \mathcal{V}^{1/3} \qquad \Leftrightarrow \qquad t_f = \frac{2\beta_b}{\beta_f}\, t_b\,. \tag{6.7}$$

Substituting this result in (6.6) we find that A_1 becomes:

$$A_1 = \frac{3\beta_b}{2\langle\phi\rangle k_b^{1/3}}\left(\frac{\beta_f}{2\beta_b}\right)^{2/3}\,, \tag{6.8}$$

[10] For anisotropic solutions in the type IIB case for the same kind of fibred CY manifolds see [52, 53, 54].

which is not moduli-dependent anymore and takes a form very similar to the one in (4.17). We can therefore follow the same stabilisation procedure described in the previous sections but now with the additional relation (6.7) which, allowing the moderate tuning $\beta_f \simeq 20\,\beta_b$, would give an anisotropic solution with $t_b \simeq 10\,t_f$. For example for $\mathcal{V} \simeq 20$ and $k_b = 1/2$, one would obtain $t_b \simeq 16 \gg t_f \simeq 1.6$.

We finally mention that this kind of fibred CY manifolds have been successfully used in type IIB for deriving inflationary models from string theory where the inflaton is the Kähler modulus controlling the volume of the fibre [56]. It would be very interesting to investigate if similar cosmological applications could also be present in the heterotic case.

7. Conclusions

The heterotic string on a Calabi-Yau manifold (or its various limiting cases such as orbifolds and Gepner points), has been studied since the late eighties as a possible UV complete theory of gravity that can realize a unified version of the Standard Model. In the last decade there has been much progress towards the goal of getting a realistic model with the correct spectrum. However the major problem in getting phenomenologically viable solutions for the heterotic string is that the gauge theory, and hence phenomenology, resides in the bulk and getting an acceptable model cannot be decoupled from the problem of moduli stabilization. Unfortunately the stabilization of the various moduli that arise upon compactification, has remained a difficult problem for the reasons that we discussed at various points in this paper.

Let us recapitulate these difficulties and the resolutions that we have proposed.

- The classical 10-dimensional equations of motion imply that there is no supersymmetric solution of the form $\mathcal{M}_4 \times X$ where the first factor is a maximally symmetric space and X is a Calabi-Yau space, if H-flux is turned on. However, without turning on fluxes (at least in the sense discussed in [16]) there is no way to stabilize complex structure moduli and it is impossible to tune the cosmological constant.

In section 3 the 4D counterpart to Strominger's argument given in [16, 17] was reviewed. Essentially the point was that the (no-scale) positive definite potential which is obtained in the absence of non-perturbative terms, would determine the complex structure moduli by the condition that the $(2,1)+(1,2)$ flux vanishes, but the dilaton (and Kähler moduli) direction is runaway unless the entire flux vanishes in agreement with Strominger's analysis. On the other hand one can add non-perturbative (gaugino condensate) terms which stabilize the dilaton (with the Kähler moduli being flat directions). However this generates $(2,1)+(1,2)$ fluxes which violate the 10D Strominger analysis. However the point is that there is no classical 10D analog of the addition of the non-perturbative terms so that actually there is no contradiction here. In fact the low energy theory (below the scale of gaugino condensation) develops a mass gap and has different degrees of freedom as in QCD. One should not expect to see this in the classical 10D theory.

- In the heterotic string there is only one type of flux. So generically integral fluxes are going to give us $\geq O(1)$ flux generated superpotential W_{flux}.

By going to fractional Chern-Simons invariants as in [16] we avoided this problem and stabilized the dilaton at the phenomenologically acceptable value. Next we discussed supersymmetric solutions obtained via gaugino condensate terms (including also threshold corrections) and worldsheet instanton terms. We also included for generality line bundle terms in the gauge bundle. In this case one also has field dependent FI-terms leading to D-terms. Since the total superpotential is non-vanishing at the minimum these do not give extra constraints, and the minimum of the F-term potential gave us stable supersymmetric minima with all geometric moduli and the dilaton stabilized.

Finally we have to admit that at the end of the day there are two unresolved issues which do not have any obvious resolution: tuning the cosmological constant and getting TeV-scale supersymmetry.

In fact, once the complex structure moduli are fixed there is no further freedom to tune the fluxes. Thus it is unclear how a sufficiently fine Bousso-Polchinski type discretuum can be obtained. This is a generic problem in heterotic string theory whether or not it is in weak coupling. In the case that we studied we got $2\,h^{1,2}+2$ flux integers to fix the $2\,h^{1,2}$ (real) complex structure moduli leaving just two free fluxes. This is certainly not enough to get a sufficiently fine discretuum so that one has to scan over the possible values of the stabilized complex structure moduli in order to tune the cosmological constant. Unfortunately there is no guarantee that this will enable us to get a cosmological constant at the observed scale.

Regarding low-energy supersymmetry, the problem is the value of the flux superpotential which cannot be much less than $O(1/10)$, so that the gravitino mass is close to the GUT scale. This gives a supersymmetry breaking phenomenology with soft masses close to the GUT scale though there can be a no scale cancellation for the scalar masses. Thus weak coupling heterotic string theory appears to predict that supersymmetry will neither be observed at the LHC nor in any realistic future machine! Of course this is an issue only if one believes in a supersymmetric solution to the gauge hierarchy problem.

Perhaps the difficulties that we have discussed and the somewhat speculative nature of their resolution, really means that the only way to discuss moduli stabilization in the heterotic string is to work with non-Kähler manifolds (i.e. with $dJ \neq 0$). The problem in this case is that it is not clear how to recast the resulting 4D theory as an $\mathcal{N}=1$ supergravity. The moduli spaces of such $SU(3)$ structure non-Kähler manifolds is not well understood, and there is no clear way of finding the correct superpotential. As we pointed out the interesting guess for this in [37] will reduce to the standard flux superpotential when the manifold is complex (as opposed to just being almost complex). In other words one would then have to replace one of Strominger's classical 10D equations, namely $N=0$, by something else. By contrast our approach has been to remain as close a possible to these equations, except when the effects of non-perturbative terms are to be included in which case we see no reason to impose them.

In this paper, we have organized our analysis along a systematic discussion of quantum corrections from non-perturbative effects, string loops [19, 20, 21], and higher-derivative α'-corrections [22, 23, 24] according to their successive level of suppression by powers of the string coupling and inverse powers of the volume. To this end, we first gave a detailed discussion of the 3-form flux superpotential and it supersymmetric vacua, summarized above. While we verified the result of [27] that non-trivial 3-form flux can stabilize all complex structure moduli supersymmetrically, we found that supersymmetric Minkowski flux vacua with vanishing superpotential are impossible as they force the total 3-form flux to vanish entirely.

Next, we used the structure of the flux-induced moduli potential coming from the above discussion as the starting point for the analysis of the quantum corrections which determine the scalar potential for the dilaton and the volume moduli. Following the above discussion this structure is summarized by:

- the fact that in heterotic compactifications one usually needs $(0, 2)$ worldsheet supersymmetry for good MSSM-like 4D physics, which directly produces a non-vanishing contribution to the flux superpotential from the Chern-Simons contributions in the 3-form flux;
- that since the heterotic string lacks Ramond-Ramond fluxes, the freedom to choose NSNS 3-form flux H_3 is used up mostly to stabilize the complex structure moduli in a controlled vacuum, which leaves little freedom the tune the flux superpotential;
- that the type IIB avenue towards a small flux superpotential by stabilizing a 3-cycle exponentially close to conifold point in moduli space is unavailable due to the reality constraint on H_3 in the heterotic context.

The size of the flux superpotential in $(0, 2)$ compactification thus is controlled by the fractional Chern-Simons contribution to be at least $\mathcal{O}(0.01 - 0.1)$ in units of M_P. Moreover, in the heterotic string there is an upper bound on the compactification volume driven by perturbativity and gauge coupling unification. Altogether, these constraints *generically imply GUT-scale supersymmetry breaking for heterotic orbifold/CY models.*

Next, we have included non-perturbative effects into the superpotential. First, we looked at contributions from gaugino condensation in conjunction with threshold corrections to the gauge kinetic function. They provide for supersymmetric vacua which drive the gauge coupling into negative values in either the visible or hidden sector of heterotic string theory [16]. We then improved this situation by including a *single* worldsheet instanton contribution following [18].

The resulting vacua are AdS. Consequently, we proceeded to perform a systematic large-volume expansion in the loop-corrections to the gauge kinetic function and α'-corrections to the Kähler potential. Their systematic inclusion leads to the generic existence of supersymmetry breaking near-Minkowski vacua, which is one of the main results of the paper. These solutions have the dilaton stabilized at phenomenologically interesting values which fit with perturbative gauge coupling unification in a conventional GUT-picture, albeit with deviations corresponding to the high gaugino mass scale. At the same time they allow us to achieve stabilization at moderately large compactification volume in the context of K3- or T^4-fibered CY manifolds, allowing for anisotropic CYs at the upper limit compatible with string perturbativity. This is beneficial as it allows to match the effective string scale to the GUT scale of gauge coupling unification [33, 34].

We discussed the ensuing pattern of moduli and soft masses from the supersymmetry breaking provided by our LVS-type vacua. Generically, there is a potentially viable QCD axion candidate present in the axionic partner of the 'large' 2-cycle volume modulus. Moreover, we are led to high-scale supersymmetry breaking. The soft mass pattern show certain distinct features such as universal scalar masses, A-terms and $\mu/B\mu$-term of $\mathcal{O}(m_{3/2} \sim M_{\rm GUT})$ while gaugino masses appear generically suppressed at the %-level.

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A. Dimensional reduction of 10D heterotic action

The 10D heterotic supergravity action in string frame for energies below the mass of the first excited string state $M_s = \ell_s^{-1}$ with $\ell_s = 2\pi\sqrt{\alpha'}$ contains bosonic terms of the form:

$$\begin{aligned} S \supset\; & \frac{1}{(2\pi)^7\alpha'^4}\int d^{10}x\sqrt{-G}\,e^{-2\phi}\left(\mathcal{R}-\frac{\alpha'}{4}\mathrm{Tr}F^2\right) \\ =\; & \frac{M_{10}^8}{2}\int d^{10}x\sqrt{-G}\,e^{-2\phi}\mathcal{R}-\frac{1}{2g_{10}^2}\int d^{10}x\sqrt{-G}\,e^{-2\phi}\mathrm{Tr}F^2\,. \end{aligned} \tag{A.1}$$

Comparing the first with the second line in (A.1), we find:

$$M_{10}^8 = \frac{2}{(2\pi)^7\alpha'^4} = 4\pi M_s^8 \qquad \text{and} \qquad g_{10}^2 = \frac{4}{\alpha' M_{10}^8} = 4\pi M_s^{-6}\,. \tag{A.2}$$

Compactifying on a 6D Calabi-Yau three-fold X, the 4D Planck scale M_P turns out to be:

$$M_P^2 = e^{-2\langle\phi\rangle}M_{10}^8\mathrm{Vol}(X) = 4\pi\, g_s^{-2}\mathcal{V}M_s^2\,, \tag{A.3}$$

where we measured the internal volume in units of M_s^{-1} as $\mathrm{Vol}(X) = \mathcal{V}\,\ell_s^6$ and we explicitly included factors of the string coupling $g_s = e^{\langle\phi\rangle}$. On the other hand, the 4D gauge coupling constant becomes:

$$\alpha_{GUT}^{-1} = 4\pi g_4^{-2} = \frac{4\pi \mathrm{Vol}(X)}{g_{10}^2\, e^{2\langle\phi\rangle}} = g_s^{-2}\mathcal{V}\,. \tag{A.4}$$

The tree-level expression of the gauge kinetic function $f = S$ requires $\mathrm{Re}(S) = g_4^{-2}$, implying the following normalisation of the definition of the dilaton field:

$$S = \frac{1}{4\pi}\left(e^{-2\phi}\mathcal{V} + \mathrm{i}\,a\right). \tag{A.5}$$

From (A.4), we immediately realise that there is a tension between large volume and weak coupling for the physical value $\alpha_{GUT}^{-1} \simeq 25$:

$$\mathcal{V} = g_s^2\alpha_{GUT}^{-1} \simeq g_s^2 25 \lesssim 25 \qquad \text{for} \qquad g_s \lesssim 1\,. \tag{A.6}$$

On top of this problem, isotropic compactifications cannot yield the right value of the GUT scale $M_{GUT} \simeq 2.1 \cdot 10^{16}$ GeV which is given by the Kaluza-Klein scale $M_{KK} = M_s/\mathcal{V}^{1/6}$. In fact, combining (A.4) with (A.3), one finds that the string scale is fixed to be very high:

$$M_s^2 = \frac{M_P^2}{4\pi\alpha_{GUT}^{-1}} \simeq \frac{M_P^2}{100\pi} \simeq \left(1.35 \cdot 10^{17}\,\mathrm{GeV}\right)^2\,. \tag{A.7}$$

In turn, for $\mathcal{V} \lesssim 25$, the GUT scale becomes too high: $M_{GUT} = M_{KK} \gtrsim 8 \cdot 10^{16}$ GeV. The situation can be improved by focusing on anisotropic compactifications with d large extra dimensions of size $L = x\ell_s$ with $x \gg 1$ and $(6-d)$ small dimensions of string size $l = \ell_s$. The internal volume then becomes $\mathrm{Vol}(X) = L^d l^{(6-d)} = x^d \ell_s^6 = \mathcal{V}\,\ell_s^6$, implying that the Kaluza-Klein scale now becomes $M_{KK} = M_s/x = M_s/\mathcal{V}^{1/d}$. Clearly, for the case $d = 6$, we recover the isotropic situation. The case with $d = 1$ is not very interesting since Calabi-Yau manifolds do not admit non-trivial Wilson lines to perform the GUT breaking. We shall therefore focus on the case $d = 2$ where we get the promising result:

$$M_{GUT} = M_{KK} = \frac{M_s}{\sqrt{\mathcal{V}}} \gtrsim 2.7 \cdot 10^{16}\,\mathrm{GeV}. \tag{A.8}$$

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