The value of H_0 in the inhomogeneous Universe

Ido Ben-Dayan[1], Ruth Durrer[2], Giovanni Marozzi[2] and Dominik J. Schwarz[3]
[1] *Deutsches Elektronen-Synchrotron DESY, Theory Group, D-22603 Hamburg, Germany*
[2] *Université de Genève, Département de Physique Théorique and CAP, 24 quai Ernest-Ansermet, CH-1211 Genève 4, Switzerland*
[3] *Fakultät für Physik, Universität Bielefeld, Postfach 100131, 33501 Bielefeld, Germany*
(Dated: February 6, 2014)

Local measurements of the Hubble expansion rate are affected by structures like galaxy clusters or voids. Here we present a first fully relativistic treatment of this effect, studying how clustering modifies the mean distance (modulus)-redshift relation and its dispersion. The best estimates of the local expansion rate stem from supernova observations at small redshifts ($0.01 < z < 0.1$). It is interesting to compare these local measurements with global fits to data from cosmic microwave background anisotropies. In particular, we argue that cosmic variance (i.e. the effects of the local structure) is of the same order of magnitude as the current observational errors and must be taken into account in all future local measurements of the Hubble expansion rate.

PACS numbers: 98.80.-k, 95.36.+x, 98.80.Es

The Hubble constant, H_0, determines the present expansion rate of the Universe. For most cosmological phenomena a precise knowledge of H_0 is of utmost importance. In a perfectly homogeneous and isotropic world H_0 is defined globally. But the Universe contains structures like galaxy clusters and voids. Thus the local expansion rate, measured by means of cepheids and supernovae at small redshifts, does not necessarily agree with the expansion rate of an isotropic and homogeneous model that is used to describe the Universe at the largest scales.

Recent local measurements of the Hubble rate [1, 2] are claimed to be accurate at the few percent level, e.g. [1] finds $H_0 = (73.8 \pm 2.4)$ km s^{-1}Mpc^{-1}. In the near future, observational techniques will improve further, such that the local value of H_0 will be determined at 1% accuracy [3], competitive with the current precision of indirect measurements of the global H_0 via the cosmic microwave backgound anisotropies [4].

The observed distance modulus μ is related to the bolometric flux Φ and the luminosity distance $d_{\rm L}$ by ($\log \equiv \log_{10}$)

$$\mu = -2.5 \log[\Phi/\Phi_{10\ \rm pc}] = 5 \log[d_{\rm L}/(10\ \rm pc)]. \qquad (1)$$

The relation between the intrinsic luminosity, L, the bolometric flux, Φ, and the luminosity distance d_L of a source is $\Phi = L/4\pi d_L^2$. In a flat ΛCDM universe with present matter density parameter Ω_m the luminosity distance as a function of redshift z is then given by

$$d_L(z) = \frac{1+z}{H_0/c} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + 1 - \Omega_m}}. \qquad (2)$$

As long as we consider only small redshifts, $z \le 0.1$, the dependence on cosmology is weak, $d_L(z) \simeq c[z + (1 - 3\Omega_m/4)z^2]/H_0$ and the result varies by about 0.2% when Ω_m varies within the 2σ error bars determined by Planck [4]. However, neglecting the model dependent quadratic term entirely induces an error of nearly 8% for $z \simeq 0.1$.

The observed Universe is inhomogeneous and anisotropic on small scales and the local Hubble rate is actually expected to differ from its global value for two reasons. First, any supernova (SN) sample is finite (sample variance) and, second, we observe only one realization of a random configuration of the local structure (cosmic variance). Thus, even for arbitrarily precise measurements of fluxes and redshifts, the local H_0 will differ from the global H_0. Sample variance is fully taken into account in the literature, but cosmic variance is usually not considered.

In the context of Newtonian cosmology, cosmic variance of the local H_0 has been estimated in [5–7]. First attempts to estimate cosmic variance of the local Hubble rate in a relativistic approach can be found in [8, 9] (see also [10]), they are based on the idea of calculating the ensemble variance of the spatial volume averaged expansion rate. It has been shown that this approach agrees very well with the Newtonian one [8] and it predicts a sampling volume dependent cosmic variance from the sub-per cent to per cent level. However, this approach still ignores that observers probe the past light-cone and not a spatial volume. Also, the measured quantity is not an expansion rate, but the bolometric flux with the redshift.

In this letter, we present the first fully relativistic estimation of the effects of clustering on the local Hubble parameter. In particular, considering only the directly measured quantities, we study the effect of a stochastic background of inhomogeneities on the determination of H_0 performed using local measurements, i.e., taking light propagation effects fully into account. Let us stress that we do not make any special hypothesis about how the fluctuations are distributed around us, unlike [11, 12], where a "Swiss cheese" model and a "Hubble bubble" model are assumed respectively. We simply consider the cosmological standard model with Gaussian fluctuations.

We shall find that the mean value of the Hubble parameter is modified at sub-percent level, while the contribution from clustering to the error budget is larger, typically 2 to 3% and hence as large as the observational errors quoted in the literature [1]. As we shall see, the small modification of the mean can be reduced by a factor of 3 by using the flux, which is less prone to bias, instead of the distance modulus to infer the Hubble parameter. On the other hand, the cosmic variance induced by inhomogeneities on H_0 is independent of the observable used. Finally, we find that even for an infinite number of SNIa within $0.01 < z < 0.1$ with identical redshift distribution compared to a finite sample considered, clustering induces a minimal error of about 2% for a local determination of H_0.

Following [13, 14] we use cosmological perturbation theory up to second order with an almost scale-invariant initial power spectrum to determine the mean perturbation of the bolometric flux (and of the distance modulus) from a standard candle and its variance.

Let us first consider the fluctuation of the mean on a sphere at fixed observed redshift z. We denote the light-cone average [15] over a surface at fixed redshift by $\langle \cdots \rangle$, and a statistical average by $\overline{\cdots}$. Using the results of [16, 17] (see also [18]) the fluctuation of the flux $\Phi \propto d_L^{-2}$, away from its background value in the Friedmann-Lemaître Universe (denoted by $(d_L^{\rm FL})^{-2}$), is given by

$$d_L^{-2} = (d_L^{\rm FL})^{-2} \left[1 + \Phi_1/\Phi_0 + \Phi_2/\Phi_0\right] . \qquad (3)$$

Averaging over the sphere at fixed redshift the first order perturbation does not contribute at first order, while it gives a contribution at second order that has to be added to the second order flux perturbation (see, e.g. [19]). In the end we obtain

$$\overline{\langle d_L^{-2} \rangle}(z) = (d_L^{\rm FL})^{-2} \left[1 + f_\Phi(z)\right] \qquad (4)$$

where

$$f_\Phi(z) \simeq -\left(\frac{1}{\mathcal{H}(z)\Delta\eta}\right)^2 \overline{\langle (\vec{v}_s \cdot \vec{n})^2 \rangle}, \text{ for } z \ll 1 , \qquad (5)$$

$\vec{n}$ denotes the direction to a given SNIa and $\vec{v}_s$ is its peculiar velocity. Due to the nonlinear function connecting the flux and the distance modulus, for the distance modulus one obtains a different second order contribution,

$$\overline{\langle \mu \rangle} - \mu^{\rm FL} = -\frac{2.5}{\ln(10)} \left[f_\Phi - \frac{1}{2} \overline{\langle (\Phi_1/\Phi_0)^2 \rangle} \right] , \qquad (6)$$

with

$$\overline{\langle (\Phi_1/\Phi_0)^2 \rangle} \simeq -4 f_\Phi , \qquad (7)$$

at $z \ll 1$. We use similar notations to [13, 14], η denotes conformal time, $\Delta\eta = \eta_0 - \eta(z)$, and $\mathcal{H}(z)$ is the conformal Hubble parameter at redshift z. The approximate equalities in Eqs. (5) and (7) are valid only for $z \ll 1$, where the first order squared contribution of the peculiar velocity terms dominates over the other second order contributions. We have already removed the observer velocity since observations are usually quoted in the CMB frame, corresponding to $\vec{v}_o = 0$. For $z \sim 0.3$ and larger, additional contributions notably due to lensing become also relevant, see [13, 14].

For measurements of the Hubble parameter, low redshift SNe are used in order to minimize the dependence of the result on cosmological parameters. As a consequence, Eqs. (5) and (7) are good approximations for the aim of this Letter.

In Fourier space we then obtain [14]

$$\overline{\langle (\Phi_1/\Phi_0)^2 \rangle} \simeq 4 \left(\frac{1}{\mathcal{H}(z)\Delta\eta}\right)^2 \frac{\tau^2(z)}{3} \int_{H_0}^{k_{\rm UV}} \frac{dk}{k} k^2 \mathcal{P}_\psi(k), \qquad (8)$$

where

$$\tau(z) = \int_{\eta_{in}}^{\eta_s} d\eta \frac{a(\eta)}{a(\eta_s)} \frac{g(\eta)}{g(\eta_o)} ,$$

$g(\eta)$ is the growth factor and the source and the observer times are indicated with the suffix s and o. Furthermore, $\mathcal{P}_\psi(k) = (k^3/2\pi^2)|\Psi_k(\eta_o)|^2$ is the dimensionless power spectrum of the Bardeen potential $\Psi(\mathbf{x}, \eta)$ at present time. Hereafter we use the cosmological parameters from Planck [4], the linear transfer function given in [20] taking baryons into account, and $k_{\rm UV} = 0.1\, h\, {\rm Mpc}^{-1}$, see [14] for details.

As an illustration, we plot in Fig. 1 the average $\overline{\langle d_L^{-2} \rangle}(z)$ and its variance (as defined in [19]), obtained using Eqs. (4), (5), (7) and (8).

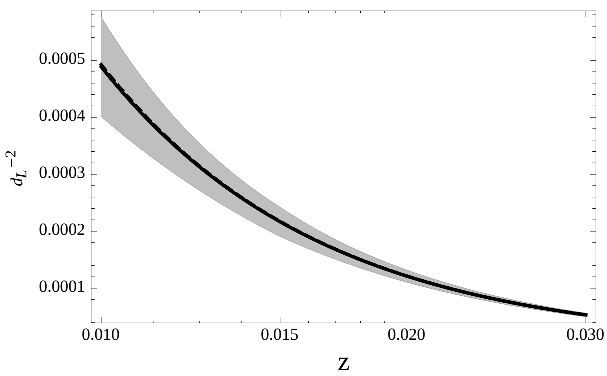

FIG. 1: The average $\overline{\langle d_L^{-2} \rangle}(z)$ of Eq. (5) in units of ${\rm Mpc}^{-2}$ (thick solid curve), its dispersion (shaded region), and the homogeneous value (dashed curve) are computed within a range $z = 0.01$ and $z = 0.03$. We have used a UV cutoff of $k_{\rm UV} = 0.1\, h\, {\rm Mpc}^{-1}$ and Planck parameters [4].

As already stressed in [13, 14], Fig. 1 clearly shows how at low redshift the dispersion of the flux is much more important than the shift of the average.

Comparing Eqs. (4) and (6), and using (7), we see that the flux averaged over a sphere at constant redshift, experiences a different effect than the distance modulus. Of course in practice, observers do not have at their disposal many SNe at the same redshift, so the average over a sphere cannot be performed in a straightforward way.

On the other hand, the induced theoretical dispersion on the bare value of H_0, which is due entirely to squared first order perturbations, is independent of the observable considered to infer H_0. To determine the dispersion of H_0 from a sample of SNe we consider that at small redshift $H_0^2 \simeq c^2z^2/d_L^2$ so that the variance of H_0 from a single SN at redshift $z \ll 1$ can be approximated by [19]

$$(\Delta H_0)^2 = \frac{H_0^2}{4}\overline{\langle(\Phi_1/\Phi_0)^2\rangle}\,. \tag{9}$$

For a sample of N SNe at positions $(z_i, \vec{n}_i)$ we have to take into account the full covariance,

$$\begin{aligned}\left(\frac{\Delta H_0}{H_0}\right)^2 &= \frac{1}{4N^2}\sum_{ij}\overline{\frac{\Phi_1(z_i,\vec{n}_i)}{\Phi_0(z_i)}\frac{\Phi_1(z_j,\vec{n}_j)}{\Phi_0(z_j)}} \\ &= \frac{1}{N^2}\sum_{ij}\frac{V_{ij}}{\mathcal{H}(z_i)\Delta\eta_i\mathcal{H}(z_j)\Delta\eta_j}\end{aligned} \tag{10}$$

with

$$V_{ij} = \tau(z_i)\tau(z_j)\int_{H_0}^{k_{\rm UV}}\frac{dk}{k}k^2\mathcal{P}_\psi(k)I\big(k\Delta\eta_j, k\Delta\eta_i, (\vec{n}_i\cdot\vec{n}_j)\big), \tag{11}$$

and

$$\begin{aligned}I(x,y,\mu) &= \frac{1}{4\pi}\int d\Omega_{\hat{k}}e^{ix(\hat{k}\cdot\vec{n}_i)}e^{-iy(\hat{k}\cdot\vec{n}_j)}(\hat{k}\cdot\vec{n}_j)(\hat{k}\cdot\vec{n}_i) \\ &= \frac{xy(1-\mu^2)}{R^2}j_2(R) + \frac{\mu}{3}\big[j_0(R) - 2j_2(R)\big],\end{aligned} \tag{12}$$

where $\mu = (\vec{n}_i\cdot\vec{n}_j)$ and $R = \sqrt{x^2+y^2-2\mu xy} = kd$. Here d is the comoving distance between the SNe at $(z_i,\vec{n}_i)$ and $(z_j,\vec{n}_j)$, j_ℓ denotes the spherical Bessel function of order ℓ and $\hat{k}$ is the unit vector in direction $\vec{k}$. To arrive at (12) we have used some well known identities like $e^{ix\mu} = \sum(2\ell+1)i^\ell j_\ell(x)P_\ell(\mu)$ and the addition theorem of spherical harmonics. Note that with $I(x,x,1) = 1/3$ and (8), the auto-correlation term reproduces (9).

If the fluxes are perfectly coherent for all SNe so that $\overline{\Phi_1(z_i,\vec{n}_i)\Phi_1(z_j,\vec{n}_j)} = 4\sigma^2\Phi_0(z_j)\Phi_0(z_i)$, for all correlations, we obtain $(\Delta H_0/H_0)^2 = \sigma^2$, while in the incoherent case, $\overline{\Phi_1(z_i,\vec{n}_i)\Phi_1(z_j,\vec{n}_j)} = \delta_{ij}4\sigma^2\Phi_0(z_j)\Phi_0(z_i)$ we obtain $(\Delta H_0/H_0)^2 = \sigma^2/N$. The reality will lie somewhere in-between, wavelength with $kd < 1$ being rather coherent while those with $kd > 1$ are rather incoherent.

In order to estimate the effect of the cosmic (co)variance for a realistic sample of SNe, we consider the following set up. We calculate $\Delta H_0/H_0$ from Eqs.(10) to (12) considering a sample of 155 SNe given by the CfA3 sample together with the OLD sample and considering only the SNe in the redshift range $0.01 \leq z \leq 0.1$, see [21, 22], but ignoring their exact positions on the sky. We then also study the limiting case of infinitely many SNe. The redshift distribution of the sample is shown in Fig 2.

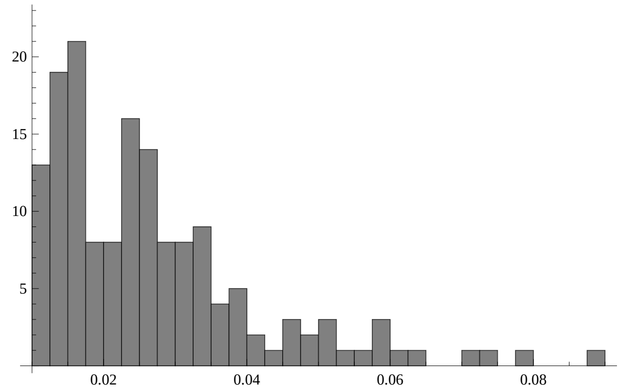

FIG. 2: The redshift distribution of the 155 SNe of the CfA3+OLD sample [21, 22] with redshift within 0.01 and 0.1 considered here.

For the redshift distribution of the 155 SNe of the sample considered here, Eq. (10) yields a dispersion induced by inhomogeneities between 2.2 and 3.3%. From which we infer an error of

$$\Delta H_0 = (1.6 - 2.4)\ \mathrm{km\ s^{-1}Mpc^{-1}}\,, \tag{13}$$

for H_0 as given in [1]. To obtain these values we have considered different angular distributions for the SNe. We have kept μ constant to different values and we have chosen a random distribution of directions over one hemisphere. The different choices give rise to the mentioned error in the range of $(2.2-3.3)\%$. The smallest error corresponds to the case where we take a random distribution of directions over one hemisphere, while the largest one to the case where all the SNe are inside a narrow cone ($\mu \simeq 1$). In the real case one should take the true angular distribution of the sample into consideration, this will be included in a forthcoming publication.

Let us also calculate the effect of inhomogeneities on the measured value of H_0 itself for this sample. In [1] the analysis is performed using the distance modulus and considering the Universe as homogeneous. A partial reconstruction of the peculiar velocity field has been applied in [1], which we ignore for a moment. To determine the effect of inhomogeneities on the final result, we have to consider backreaction from inhomogeneities on the distance modulus, correct for it and evaluate the real local value of H_0. In a homogeneous Universe with Hubble parameter $\hat{H}_0$ we would have

$$\mu(z \ll 1) \simeq 5\log\frac{cz}{\hat{H}_0} + C\,, \tag{14}$$

with C a constant. Considering the real Universe as homogeneous only $\hat{H}_0$ can be measured and not the true underlying background value H_0. Comparing Eqs.(6) and (14) we obtain, to leading order, the following relation between $\hat{H}_0$ and H_0:

$$H_0 \simeq \hat{H}_0\left(1 - \frac{3}{2}f_\Phi\right). \tag{15}$$

We now consider the 155 SNe of the sample here used and generate the mean value of the corrected H_0, starting from a value of $\hat{H}_0$ and for the given redshift distribution. The final result is about 0.3% higher than $\hat{H}_0$. A similar global shift has already been included in the analysis of [1] as a consequence of the partial reconstruction of the peculiar velocity field [23]. Let us underline that the correction to H_0 would be three times smaller if we would consider the backreaction on the *flux* instead of the one on the *distance modulus*. In this case Eq. (15) should be replaced by $H_0 \simeq \hat{H}_0\left(1 - \frac{1}{2}f_\Phi\right)$.

Adding the quoted experimental error of 2.4 km/s/Mpc [1] to the theoretical error (13), we obtain

$$\begin{aligned} H_0 \;&=\; (73.8 \pm \Delta)\ \text{km s}^{-1}\text{Mpc}^{-1}\,, \\ &2.9 \le \Delta \le 3.4 \end{aligned} \tag{16}$$

The tension with the Planck measurement [4] is reduced when taking into account the larger error. In [4] a value $(H_0)_{\rm CMB} = 67.3 \pm 1.2$ km s^{-1}Mpc^{-1} is reported, considering the result (16) this now deviates by about 2.2 to $1.9\,\sigma$ from the local value of H_0, while the difference is 2.7σ when using the error quoted in [1].

Before concluding, we want to determine the ultimate error for an arbitrarily large sample when the SNe are distributed isotropically over directions. In this case we can integrate $I(x, y, \mu)$ over directions. With

$$\frac{1}{2}\int_{-1}^{1} d\mu I(x, y, \mu) = j_1(x)j_1(y)$$

we obtain, for a normalized redshift distribution $s(z)$,

$$\left(\frac{\Delta H_0}{H_0}\right)^2 = \int\frac{dk}{k}k^2\mathcal{P}_\psi(k)\left(\int dz\tau(z)s(z)\frac{j_1(k\Delta\eta(z))}{\mathcal{H}(z)\Delta\eta(z)}\right)^2 \tag{17}$$

with $\int dzs(z) = 1$. Approximating the redshift distribution of our sample using an interpolating function of the histogram in Fig 2, integrating from $z = 0.01$ to 0.1, we obtain a dispersion of about 1.8% which corresponds to an error of

$$\Delta H_0 = 1.3\ \text{km s}^{-1}\text{Mpc}^{-1}\,. \tag{18}$$

This is the minimal dispersion of a SN sample with a redshift space distribution given by the one in Fig 2. It is not much smaller than the value obtained for the real sample. The error from the very nearby SNe with small $\Delta\eta(z)$ is the largest contribution. Therefore, the dispersion can be reduced by considering higher redshift SNe for which, however, the model dependence becomes more relevant. If we consider higher redshifts (close to or larger than 0.3), we also have to take into account the other contributions to the perturbation of the luminosity distance, see [16–18] for the full expression. As it is well known (see, for example, [13, 14]), at redshift $z > 0.3$, the lensing term begins to dominate.

In [24] the peculiar velocity field has been reconstructed using the IRAS PSCz catalog [25]. As already mentioned above, this is subtracted in the analysis of [1]. It is clear that this procedure also modifies the expected mean and its variance in our method, but is beyond the scope of this work. As the (minimal) cosmic variance Eq. (18) receives mainly contributions from scales that are coherent within the considered volume, we expect that it still has to be taken into account, besides reconstructing peculiar velocities.

To conclude, in this Letter we have estimated the impact of stochastic inhomogeneities on the local value of the Hubble parameter and on its error budget when given an initial sample of standard candles. In Eqs. (10) to (12) we have provided a general formula for computing the cosmic variance contribution to ΔH_0 from a sample of SNe with $z \lesssim 0.2$. In particular, we have found that for samples presently under consideration, this error is not negligible but of the same order as the experimental error, i.e. between 2.2 and 3.3%. We have also considered different samples, with smaller and larger numbers of SNe in the range $0.01 < z < 0.1$. The results are similar to the ones presented here. This cosmic variance is a fundamental barrier on the precision of a local measurement of H_0. It has to be added to the observational uncertainties and it reduces the tension with the CMB measurement of H_0 [4].

Let us note that our results for the cosmic variance of H_0 are not affected if we consider a higher UV cutoff $k_{\rm UV}$. On the other hand, the impact of structure on the expected local value of the Hubble parameter depends somewhat on the UV-cutoff.

Finally, even when the number of SNe is arbitrarily large, an irreducible error remains due to cosmic variance of the local Universe. We have estimated this error and found it to be about 1.8% for SNe with redshift $0.01 < z < 0.1$ and a distribution given by the one in Fig.2. This error can only be reduced by considering SNe with higher redshifts, but if too high redshifts are included the result becomes strongly dependent on other cosmological parameters like Ω_m and curvature.

We wish to thank Ulrich Feindt, Benedict Kalus, Marek Kowalski, Martin Kunz, Lucas Macri, Adam Riess, Mickael Rigault, Marco Tucci and Alexander Wiegand for helpful discussion. IB-D is supported by the German Science Foundation (DFG) within the Collaborative Research Center (CRC) 676 Particles, Strings and the Early Universe. RD acknowledges the Swiss National Science Foundation. GM is supported by the Marie Curie IEF, Project NeBRiC - "Non-linear effects and backreaction in classical and quantum cosmology". DJS thanks the Deutsche Forschungsgemeinschaft for support within the grant RTG 1620 "Models of Gravity".

[1] A. G. Riess, L. Macri, S. Casertano, H. Lampeitl, H. C. Ferguson, A. V. Filippenko, S. W. Jha and W. Li *et al.*, Astrophys. J. **730**, 119 (2011) [Erratum-ibid. **732**, 129 (2011)].
[2] W. L. Freedman, B. F. Madore, V. Scowcroft, C. Burns, A. Monson, S. E. Persson, M. Seibert and J. Rigby, Astrophys. J. **758**, 24 (2012).
[3] S. H. Suyu, T. Treu, R. D. Blandford, W. L. Freedman, S. Hilbert, C. Blake, J. Braatz and F. Courbin *et al.*, arXiv:1202.4459 [astro-ph.CO].
[4] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO].
[5] X. -D. Shi and M. S. Turner, Astrophys. J. **493**, 519 (1998).
[6] Y. Wang, D. N. Spergel and E. L. Turner, Astrophys. J. **498**, 1 (1998).
[7] R. Wojtak, A. Knebe, W. A. Watson, I. T. Iliev, S. Hess, D. Rapetti, G. Yepes and S. Gottloeber, arXiv:1312.0276 [astro-ph.CO].
[8] N. Li and D. J. Schwarz, Phys. Rev. D **78**, 083531 (2008).
[9] A. Wiegand and D. J. Schwarz, Astron. Astrophys. **538**, A147 (2012).
[10] C. Clarkson, K. Ananda and J. Larena, Phys. Rev. D **80**, 083525 (2009); O. Umeh, J. Larena and C. Clarkson, JCAP **1103**, 029 (2011).
[11] P. Fleury, Hln. Dupuy and J. -P. Uzan, Phys. Rev. D **87**, 123526 (2013); Phys. Rev. Lett. **111**, 091302 (2013).
[12] V. Marra, L. Amendola, I. Sawicki and W. Valkenburg, Phys. Rev. Lett. **110**, 241305 (2013).
[13] I. Ben-Dayan, M. Gasperini, G. Marozzi, F. Nugier and G. Veneziano, Phys. Rev. Lett. **110**, 021301 (2013).
[14] I. Ben-Dayan, M. Gasperini, G. Marozzi, F. Nugier and G. Veneziano, JCAP **06**, 002 (2013).
[15] M. Gasperini, G. Marozzi, F. Nugier and G. Veneziano, JCAP **07**, 008 (2011).
[16] I. Ben-Dayan, G. Marozzi, F. Nugier and G. Veneziano, JCAP **11**, 045 (2012).
[17] G. Fanizza, M. Gasperini, G. Marozzi and G. Veneziano, JCAP **11**, 019 (2013).
[18] C. Bonvin, R. Durrer and M. A. Gasparini, Phys. Rev. D **73**, 023523 (2006) [Erratum-ibid. D **85**, 029901 (2012)].
[19] I. Ben-Dayan, M. Gasperini, G. Marozzi, F. Nugier and G. Veneziano, JCAP **04**, 036 (2012).
[20] D. J. Eisenstein, W. Hu, Astrophys. J. **496**, 605 (1998).
[21] M. Hicken, P. Challis, S. Jha, R. P. Kirsher, T. Matheson, M. Modjaz, A. Rest and W. M. Wood-Vasey, Astrophys. J. **700**, 331 (2009).
[22] S. Jha, A. G. Riess and R. P. Kirshner, Astrophys. J. **659**, 122 (2007).
[23] A. Riess, private communication.
[24] J. D. Neill, M. J. Hudson and A. J. Conley, Astrophys. J. **661**, L123 (2007).
[25] E. Branchini,*et al.*, Mon. Not. Roy. Astron. Soc. **308**, 1 (1999).