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The Starobinsky Model from Superconformal D-Term Inflation

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Abstract

We point out that in the large field regime, the recently proposed superconformal D-term inflation model coincides with the Starobinsky model. In this regime, the inflaton field dominates over the Planck mass in the gravitational kinetic term in the Jordan frame. Slow-roll inflation is realized in the large field regime for sufficiently large gauge couplings. The Starobinsky model generally emerges as an effective description of slow-roll inflation if a Jordan frame exists where, for large inflaton field values, the action is scale invariant and the ratio $\hat{\lambda}$ of the inflaton self-coupling and the nonminimal coupling to gravity is tiny. The interpretation of this effective coupling is different in different models. In superconformal D-term inflation it is determined by the scale of grand unification, $\hat{\lambda} \sim (\Lambda_{\rm GUT}/M_{\rm P})^4$.

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The recently released data from the Planck satellite provide a precise picture of the cosmic microwave background radiation [1]. The observed temperature anisotropies are consistent with a primordial spectrum of density perturbations produced during an inflationary phase [2]. In fact, the data support the simplest version of inflation, the single field slow-roll paradigm [1]. However, several popular inflation models [3] are strongly disfavoured or even ruled out by the data. It is therefore remarkable that the first inflation model, the R^2 model of Starobinsky [4], is fully consistent with the Planck data [1].

Recently, a supergravity model of inflation has been proposed [5], which is based on the superpotential and scalar potential of D-term hybrid inflation [6, 7], and a Kähler potential motivated by the underlying superconformal symmetry of supergravity [8, 9]. For such models, there is a Jordan frame in which the matter part of the Lagrangian takes a particularly simple form, closely resembling global supersymmetry. Depending on gauge and Yukawa couplings, the model allows for small field as well as large field inflation. In this note we point out that in the large field regime the inflaton potential and the spectral indices agree with the predictions of the Starobinsky model.

Superconformal D-term inflation

Let us briefly recall the main ingredients of the model proposed in Ref. [5]. Supersymmetric D-term hybrid inflation models contain two 'waterfall' fields $\phi_\pm$ and and an inflaton field S, with the superpotential

$$W = \lambda S \phi_+ \phi_- \ . \tag{1}$$

In the superconformal version the Kähler potential[1] reads ($z^\alpha = \phi_\pm, S$)

$$\begin{aligned} K(z,\bar{z}) &= 3 \ln \Omega^2(z,\bar{z}) \qquad \text{with} \\ \Omega^{-2} &= 1 - \frac{1}{3}\left(|S|^2 + |\phi_-|^2 + |\phi_+|^2\right) - \frac{\chi}{6}\left(S^2 + \bar{S}^2\right) \ , \end{aligned} \tag{2}$$

where the holomorphic part proportional to χ breaks superconformal symmetry explicitly [8, 9]. In the Einstein frame with metric g, the Lagrangian reads

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - K_{\alpha\bar{\alpha}} g^{\mu\nu} \nabla_\mu z^\alpha \nabla_\nu \bar{z}^{\bar{\alpha}} - V_D - V_F \ ; \tag{3}$$

[1] We use units where $M_{\rm P} = 1/\sqrt{8\pi G} = 1$.

here Q is the matrix of $U(1)$ charges, $\nabla_\mu = \partial_\mu - igA_\mu Q$ is the gauge covariant derivative, $K_{\alpha\bar{\alpha}} = \partial_\alpha \partial_{\bar{\alpha}} K$ and

$$V_D = \frac{g^2}{2}\left(\partial_\alpha K Q z^\alpha + \xi\right)^2 = \frac{g^2}{2}\left(\Omega^2 q(|\phi_+|^2 - |\phi_-|^2) - \xi\right)^2 \tag{4}$$

is the D-term potential for charges 0 and $\pm q$ of the chiral superfields S and $\phi_\pm$, respectively; ξ is the Fayet-Iliopoulos term, and the F-term scalar potential is given by

$$\begin{aligned} V_F &= \Omega^4\left(\delta^{\alpha\bar{\alpha}}\partial_\alpha W \partial_{\bar{\alpha}}\overline{W}\right) \\ &= \Omega^4\lambda^2\left(|S|^2(|\phi_+|^2 + |\phi_-|^2) + |\phi_+\phi_-|^2 - \frac{\chi^2|\phi_+|^2|\phi_-|^2|S|^2}{3 + \frac{1}{2}\chi(S^2 + \bar{S}^2) + \chi^2|S|^2}\right) . \end{aligned} \tag{5}$$

On the inflationary trajectory one has $\phi_\pm = 0$. Hence V_F vanishes identically and V_D provides the vacuum energy $V_0 = g^2\xi^2/2$ which drives inflation.

It is very instructive to also consider the theory in the Jordan frame defined by the metric $g_{J\mu\nu} = \Omega^2 g_{\mu\nu}$. This Weyl transformation yields the Lagrangian [9]

$$\begin{aligned} \frac{1}{\sqrt{-g_J}}\mathcal{L}_J &= \frac{1}{2}\Omega^{-2}R_J - \delta_{\alpha\bar{\alpha}} g_J^{\mu\nu}\nabla_\mu z^\alpha \nabla_\nu \bar{z}^{\bar{\alpha}} - V_J \ , \\ V_J &= \Omega^{-4}\left(V_D + V_F\right) \ . \end{aligned} \tag{6}$$

Contrary to the Einstein frame the kinetic term of the gravitational field is now field dependent whereas the kinetic terms of the scalar fields are canonical. Along the inflationary trajectory one has

$$\Omega_0^{-2} = \Omega^{-2}\big|_{\phi_\pm = 0} = 1 - \frac{1}{3}\left(|S|^2 + \frac{\chi}{2}(S^2 + \bar{S}^2)\right) \ , \quad V_J = \Omega_0^{-4}\ \frac{g^2}{2}\xi^2 \ , \tag{7}$$

i.e., the Fayet-Iliopoulos term is field dependent.

The slope of the inflaton potential is generated by quantum corrections. A straightforward calculation yields for the one-loop potential,

$$\begin{aligned} V_{1l} &= \frac{g^4 q^2 \xi^2}{32\pi^2}\left((x-1)^2\ln(x-1) + (x+1)^2\ln(x+1) - 2x^2\ln x - 1\right) \\ &= \frac{g^4 q^2 \xi^2}{16\pi^2}\left(1 + \ln x + \mathcal{O}\left(\frac{1}{x}\right)\right) \ , \quad x = \frac{\Omega_0^2(S)|S|^2}{\Omega_0^2(S_c)|S_c|^2} \ . \end{aligned} \tag{8}$$

The critical field value S_c, where the mass of the waterfall field ϕ_+ reaches zero, is determined by

$$\Omega_0^2(S_c)|S_c|^2 = \frac{q g^2 \xi}{\lambda^2} \ . \tag{9}$$

The total potential is then given by

$$\begin{aligned} V &= (V_F + V_D + V_{1l})\big|_{\phi_\pm=0} \\ &= \frac{g^2}{2}\xi^2\left(1 + \frac{g^2q^2\xi^2}{8\pi^2}\left(1 + \ln x + \mathcal{O}\left(\frac{1}{x}\right)\right)\right) \ . \end{aligned} \tag{10}$$

Note that on the inflationary trajectory one has $|S| > |S_c|$ and $x > 1$.

Single field slow-roll inflation

Expressing the Lagrangian for the field S in terms of real and imaginary components, $S = (\sigma + i\tau)/\sqrt{2}$,

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}K_{S\bar{S}}(\sigma,\tau)(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\tau\partial^\mu\tau) - V(\sigma,\tau) \ , \tag{11}$$

one obtains the slow-roll equations for the homogeneous fields σ and τ,

$$3K_{S\bar{S}}H\dot{\sigma} = -\frac{dV_{1l}}{d\sigma} \ , \quad 3K_{S\bar{S}}H\dot{\tau} = -\frac{dV_{1l}}{d\tau} \ . \tag{12}$$

One easily verifies that for $\chi < 0$, which we choose w.l.o.g., the trajectory $\sigma \neq 0, \tau = 0$ is an attractor for a sufficiently long phase of inflation before the onset of the final N_* e-folds. Inserting the Kähler metric,

$$K_{S\bar{S}}\big|_{\phi_\pm,\tau=0} = \frac{1}{1 - \frac{1}{6}(1+\chi)\sigma^2}\left(1 + \frac{(1+\chi)^2\sigma^2}{6\left(1 - \frac{1}{6}(1+\chi)\sigma^2\right)}\right) \ , \tag{13}$$

and the one-loop potential (8) into the slow-roll equation (12), one obtains after integrating from σ_* to σ_f,

$$3\ln\left(\frac{1 - \frac{1}{6}(1+\chi)\sigma_*^2}{1 - \frac{1}{6}(1+\chi)\sigma_f^2}\right) - \frac{1}{2}\chi\left(-\sigma_*^2 + \sigma_f^2\right) \simeq -\frac{g^2q^2}{4\pi^2}N_* \ . \tag{14}$$

Here σ_f denotes the value of σ at the end of inflation and σ_* is the value of σ N_* e-folds earlier. Inflation ends when either m_+^2 turns negative,

$$\sigma_f^2 = \sigma_c^2 = \frac{6g^2q\xi}{3\lambda^2 + (1+\chi)g^2q\xi} \ , \tag{15}$$

or when the slow-roll conditions are violated, i.e. $\sigma_f = \sigma_\eta$, for sufficiently large values of λ.

For small couplings, $gq \ll 1$, inflation takes place at small field values. In this case $-(1+\chi)\sigma_*^2/6 < 1$, and Eq. (14) implies

$$\sigma_f^2 < \sigma_*^2 \simeq \frac{g^2q^2}{2\pi^2} N_* < 1 \ . \tag{16}$$

Here we are particularly interested in the large field regime, $-(1+\chi)\sigma_*^2/6 > 1$, which is realized for large couplings gq. As we shall see, for couplings in the perturbative regime, one typically has $-(1+\chi)\sigma_f^2/6 < 1$. From Eq. (14) one then obtains

$$\sigma_f^2 < -\chi\sigma_*^2 \simeq \frac{g^2q^2}{2\pi^2} N_* \left(1 + \mathcal{O}\left(\ln N_*/N_*\right)\right) \ . \tag{17}$$

In order to obtain the spectral index and other observables, we need to evaluate the slow-roll parameters

$$\epsilon = \frac{1}{2}\left(\frac{V'(\hat\sigma)}{V}\right)^2 \ , \qquad \eta = \frac{V''(\hat\sigma)}{V} \ , \qquad \kappa = -\frac{V'V'''(\hat\sigma)}{V^2} \ . \tag{18}$$

Here $\hat\sigma$ is the canonically normalized inflaton field which is determined by (cf. Eq. (11))

$$\frac{d\sigma}{d\hat\sigma} = \frac{1}{\sqrt{K_{S\bar S}}} \ . \tag{19}$$

On the inflationary trajectory the derivatives of the scalar potential with respect to $\hat\sigma$ can be written as $(n = 1, 2, ...)$

$$\frac{d^nV}{d^n\hat\sigma^2} = \frac{d}{d\sigma}\left(\frac{dV^{n-1}}{d\hat\sigma}\right)\frac{d\sigma}{d\hat\sigma} \ , \tag{20}$$

from which one obtains the slow-roll parameters

$$\begin{aligned}
\epsilon &\simeq \frac{1}{2}\left(\frac{g^2q^2}{4\pi^2}\right)^2 \frac{1}{\sigma^2}\frac{1}{1+\frac{1}{6}\chi(1+\chi)\sigma^2} \ , \\
\eta &\simeq -\frac{g^2q^2}{4\pi^2}\frac{1}{\sigma^2}\frac{(1-\frac{1}{6}(1+\chi)\sigma^2)(1+\frac{1}{3}\chi(1+\chi)\sigma^2)}{(1+\frac{1}{6}\chi(1+\chi)\sigma^2)^2} \ , \\
\kappa &\simeq -\left(\frac{g^2q^2}{4\pi^2}\right)^2 \frac{2}{\sigma^4}\frac{(1-\frac{1}{6}(1+\chi)\sigma^2)(1+\frac{1}{2}\chi(1+\chi)\sigma^2(1+\frac{2}{9}\chi(1+\chi)\sigma^2(1-\frac{1}{12}(1+\chi)\sigma^2)))}{(1+\frac{1}{6}\chi(1+\chi)\sigma^2)^4} \ .
\end{aligned} \tag{21}$$

In the large field regime, where $\chi(1+\chi)\sigma_*^2/6 > -(1+\chi)\sigma_*^2/6 > 1$, the connection between σ_* and N_* is given by Eq. (17), which implies

$$\begin{aligned}
\epsilon_* &\simeq 3\left(\frac{g^2q^2}{4\pi^2}\right)^2 \frac{1}{\chi^2\sigma_*^4} \simeq \frac{3}{4N_*^2} \ , \\
\eta_* &\simeq \frac{g^2q^2}{2\pi^2}\frac{1}{\chi\sigma_*^2} \simeq -\frac{1}{N_*} \ , \\
\kappa_* &\simeq -4\left(\frac{g^2q^2}{4\pi^2}\right)^2 \frac{1}{\chi^2\sigma_*^4} \simeq -\frac{1}{N_*^2} \ .
\end{aligned} \tag{22}$$

One easily verifies that these relations hold in the parameter range

$$\frac{1}{-\chi} < \frac{gq}{2\sqrt{3}\pi} < 1 \ . \tag{23}$$

Given these expressions, one then obtains for the scalar spectral index, the tensor-to-scalar ratio and the running of the spectral index ($N_* = 55$)

$$\begin{aligned} n_s &\simeq 1 + 2\eta_* - 6\epsilon_* \simeq 1 - \frac{2}{N_*} \\ &\simeq 0.9636 \qquad [0.963 \pm 0.007] \ , \\ r &\simeq 12\epsilon_* \simeq \frac{12}{N_*^2} \\ &\simeq 0.0040 \qquad [< 0.26] \ , \\ dn_s/d\ln k &\simeq -16\epsilon_*\eta_* + 24\epsilon_*^2 + 2\kappa_* \simeq -\frac{2}{N_*^2} \\ &\simeq -0.00066 \qquad [-0.022 \pm 0.010] \ . \end{aligned} \tag{24}$$

To leading order in $1/N_*$ the expressions agree with those of the Starobinsky model. For comparison, the results obtained by the Planck collaboration [1, 10] are given in brackets. The agreement between predictions and observations is remarkable. The amplitude of the scalar contribution to the primordial fluctuations is given by

$$A_s = \frac{1}{12\pi^2}\frac{V^3}{V'^2}\Big|_{\sigma=\sigma^*} \simeq \frac{V_0}{18\pi^2}N_*^2 \ . \tag{25}$$

For $g^2 = 1/2$ the observed amplitude $A_s = (2.18 \pm 0.05) \times 10^{-9}$ [10] fixes the parameter ξ to a value of order the GUT scale[2], $\sqrt{\xi} \simeq 7.7 \times 10^{15}$GeV. Note that the relative theoretical uncertainty of the slow-roll parameters is $\sim \ln N_*/N_* \sim 0.07$.

Discussion

Let us now discuss in more detail the connection between R^2 inflation and superconformal hybrid inflation. The Starobinsky model

$$\frac{1}{\sqrt{-g}}\hat{\mathcal{L}}_R = \frac{1}{2}\left(R + \frac{1}{6M^2}R^2\right) \tag{26}$$

[2]At the end of hybrid inflation, cosmic strings are formed. For these values of g and ξ, and $q = 8$, the string tension is $G\mu \simeq 3.16 \times 10^{-7}$, which is marginally consistent with the recent Planck limit $G\mu < 3.2 \times 10^{-7}$ [10] (see discussion in Ref. [5]).

Figure 1: Comparison of R^2 inflation (dashed line) and superconformal D-term inflation (solid line) for $gq = 4\sqrt{2}$, $\lambda = 1$, $\chi = -10$ and $N_* = 55$. The slow-roll regimes are $[\phi_\epsilon^R, \phi_*^R]$ and $[\phi_\eta, \phi_*]$, respectively.

is conveniently rewritten as scalar-tensor theory [11],

$$\frac{1}{\sqrt{-g}}\mathcal{L}_R \simeq \frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{3}{4}M^2\left(1 - \exp\left(-\sqrt{\frac{2}{3}}\phi\right)\right)^2 \ . \tag{27}$$

For large values of ϕ one has

$$V_R \simeq \frac{3}{4}M^2\left(1 - 2\exp\left(-\sqrt{\frac{2}{3}}\phi\right)\right) \ . \tag{28}$$

In our discussion of D-term inflation we have used the field σ which has a field dependent kinetic term. In the large field regime the connection with the canonically normalized field $\hat{\sigma}$ is given by

$$\frac{d\sigma}{d\hat{\sigma}} = \frac{1}{\sqrt{K_{S\bar{S}}}} \simeq \frac{\sqrt{6}(1 - \frac{\chi}{6}\sigma^2)}{(-\chi\sigma)} \ , \tag{29}$$

from which one obtains after a convenient choice of integration constant, $\hat{\sigma} = \phi + \phi_0$,

$$\sigma^2 = -\frac{6}{\chi}\left(C\exp\left(\sqrt{\frac{2}{3}}\phi\right) - 1\right) \ . \tag{30}$$

Inserting this relation into the hybrid potential (10) one finds for large field values

$$V \simeq V_0\left(1 - 2\exp\left(-\sqrt{\frac{2}{3}}\phi\right)\right) \ , \tag{31}$$

with

$$V_0 = \frac{g^2}{2}\xi^2\left(1+\mathcal{O}(g^2q^2\ln(g^2q\xi\chi))\right) \ , \quad C = \frac{g^4q^2\xi^2}{32\pi^2V_0} \ , \tag{32}$$

which agrees with the potential (28) after matching the constants, $M^2 = 4V_0/3$.

The two potentials are compared in Figure 1. Although they almost coincide at large ϕ corresponding to $N_* = 55$, they are completely different at small ϕ. Hence, also the two slow-roll regimes, $[\phi_\epsilon^R, \phi_*^R]$ and $[\phi_\eta, \phi_*]$, differ significantly. The slow-roll parameters agree up to higher orders in $1/N_*$, as discussed above, which corresponds to $\phi_*^{\rm R} \approx \phi_*$ and $V_R(\phi_*^{\rm R}) \approx V(\phi_*)$.

Recently, the potential of R^2 inflation has also been derived from a supergravity model with no-scale Kähler potential and Wess-Zumino superpotential with specific couplings [12]. There are also supergravity models with nonminimal couplings to gravity, which have the same behaviour as the Starobinsky model at large field values [13]. Another interesting example is Higgs inflation which, in the Einstein frame, yields the scalar potential (28) with $3M^2 = \lambda/\hat{\chi}^2 \equiv \hat{\lambda}$, where $\hat{\chi}$ is the nonminimal coupling of the Higgs field to gravity [14].

Why do all these models have the same asymptotic behaviour at large fields in the Einstein frame? Consider Higgs or R^2 inflation in the Jordan frame. After a field redefinition $\phi \to h(\phi)$ and a Weyl transformation one obtains [14]

$$\frac{1}{\sqrt{-g_J}}\mathcal{L}_J^{\rm higgs} \simeq \frac{1}{2}(1+\hat{\chi}h^2)R_J - \frac{1}{2}g_J^{\mu\nu}\partial_\mu h\partial_\nu h - \frac{\lambda}{4}h^4 \ . \tag{33}$$

Correspondingly, for superconformal D-term inflation one has

$$\frac{1}{\sqrt{-g_J}}\mathcal{L}_J^{\rm sc} = \frac{1}{2}\Omega_0^{-2}R_J - \frac{1}{2}g_J^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \Omega_0^{-4}\ \frac{g^2}{2}\xi^2 \qquad \text{with}$$
$$\Omega_0^{-2} = 1 - \frac{1}{6}(1+\chi)\sigma^2 \ . \tag{34}$$

In the large field regime the two Lagrangians are identical after the identification $h = \sigma$, $\hat{\chi} = -(1+\chi)/6$ and $\lambda = (1+\chi)^2g^2\xi^2/18$,

$$\frac{1}{\sqrt{-g_J}}\mathcal{L}_J^{\rm sc} \simeq -\frac{1}{12}(1+\chi)\sigma^2R_J - \frac{1}{2}g_J^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{72}(1+\chi)^2g^2\xi^2\sigma^4 \ . \tag{35}$$

It is remarkable that in the large field regime in this Jordan frame, the Lagrangian is universal and scale invariant. The ratio of couplings, $\hat{\lambda} = \lambda/\hat{\chi}^2 = 2g^2\xi^2$ is fixed by observation to 2×10^{-11}. For Higgs inflation, i.e. $\lambda = \mathcal{O}(1)$, this fixes the nonminimal

coupling to $\hat{\chi} \simeq 10^5$. In the case of superconformal D-term inflation the dependence on the nonminimal coupling cancels, and for a GUT gauge coupling, i.e. $g^2 = 1/2$, one obtains for ξ the GUT scale, $\sqrt{\xi} = 7.7 \times 10^{15}$GeV.

It is surprizing that in the large field regime superconformal D-term inflation coincides with the Starobinsky model. As we showed, this is a combined effect of quantum corrections and supergravity corrections to scalar masses, which are determined by the superconformal Kähler potential, and essentially independent of the size of the nonminimal coupling of the inflaton to gravity. It is remarkable that all models showing the asymptotic behaviour of the Starobinsky model are scale invariant at large field values in this Jordan frame, which appears to be the essence of R^2 inflation.

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