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From form factors to generalized parton distributions

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I present an extraction of generalized parton distributions from selected data on the electromagnetic nucleon form factors. The extracted distributions can in particular be used to quantify the contribution to the proton spin from the total angular momentum carried by valence quarks, as well as their transverse spatial distribution inside the proton.

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1. Introduction

Generalized parton distributions (GPDs) contain unique information about the structure of the proton. Most prominently, they quantify the transverse spatial distributions of partons in correlation with their longitudinal momentum [1, 2]. Specific GPDs provide access to the angular momentum carried by partons, as becomes manifest from Ji's sum rule [3] and from spin-orbit correlations such as the change of the spatial parton distribution induced by transverse proton polarization [1].

Sum rules connect the GPDs for unpolarized quarks with the electromagnetic nucleon form factors according to

$$F_1^q(t) = \int_0^1 dx\, H_v^q(x,t)\,, \qquad\qquad F_2^q(t) = \int_0^1 dx\, E_v^q(x,t)\,, \tag{1.1}$$

where we introduced the valence (quark minus antiquark) combinations

$$H_v^q(x,t) = H^q(x,0,t) + H^q(-x,0,t)\,, \qquad E_v^q(x,t) = E^q(x,0,t) + E^q(-x,0,t) \tag{1.2}$$

of the conventional GPDs $H^q(x,\xi,t)$ and $E^q(x,\xi,t)$ defined in [3]. The Dirac and Pauli form factors of the proton are given by charge weighted sums $F_i^p(t) = \sum_q e_q F_i^q(t)$ with $i = 1,2$, and corresponding expressions for the neutron form factors are obtained using isospin symmetry. A Fourier transform to transverse position space turns H_v^q into the probability density for finding a quark with flavor q, longitudinal momentum fraction x and transverse position $\mathbf{b}$ in a proton, minus the corresponding probability density for antiquarks.

This contribution presents the work in ref. [4], where GPDs are determined from nucleon form factor data via the sum rules (1.1). In several ways, such a study is complementary to GPD extractions from hard exclusive processes like deeply virtual Compton scattering or meson production [5]. (i) Form factor data extend to large values of $-t$, much beyond what can ever be reached in hard exclusive processes. (ii) The integrals in (1.1) are sensitive to a wide range of x, from about 10^{-3} to 0.6 and higher in our analysis. (iii) The sum rules (1.1) can be written in terms of GPDs at zero skewness ξ, which after Fourier transform admit a probability interpretation and obey much simpler positivity bounds than their counterparts with $\xi \neq 0$. (iv) On the downside, it is impossible to uniquely reconstruct the x and t dependence of GPDs from (1.1), which leaves us with an irreducible model dependence due to the functional forms assumed for $H_v^q(x,t)$ and $E_v^q(x,t)$.

2. Form factor data

For our analysis we have made a selection from the available data on electromagnetic form factors. The magnetic proton from factor $G_M^p(t)$ offers the largest reach in t, up to $-t \simeq 31$ GeV^2. With experimental uncertainties of only a few % at small to intermediate t, quantitative control of two-photon exchange in the elastic ep cross section is essential for a reliable determination of this quantity. We use the data extracted in the global analysis of [6] and checked that it is compatible with the results of other recent work.

The scaled ratio $R^p = \mu_p G_E^p / G_M^p$ of electric and magnetic form factors can be extracted from elastic ep scattering data where either the polarization of the recoil proton is measured or the proton target is polarized. As is evident from figure 1, recoil polarization data from JLab Hall A published

Figure 1: Left: Selection of recent data on the scaled from factor ratio $R^p = \mu_p G_E^p / G_M^p$. Right: Data on G_M^n, divided by the conventional dipole parameterization. Open symbols denote points that are omitted in our analysis (but clearly dominated a fit by Kelly in 2004, shown as a dashed line). References for the data sets are given in [4].

in 2010 and 2011 are clearly inconsistent with older data from the same hall and with measurements on polarized proton targets. We omit the polarized target data from our fit, while emphasizing that a clarification of the experimental situation is urgent.

Let us now turn to the neutron form factor data. For G_M^n there is a number of data points from the 1990s that are inconsistent with more recent determinations (see figure 1) and have been excluded from our fits. For the electric neutron form factor G_E^n or the ratio $R^n = \mu_n G_E^n / G_M^n$ there is an overall consistent data set, although only up to $-t \simeq 3.4\,\mathrm{GeV}^2$. In our fits we also include the very precise determination of the squared charge radius r_{nE}^2 from neutron scattering on shell electrons in nuclear targets [7].

As a benchmark for our fit using GPDs, we have performed a simple global fit to the form factor data we have selected. We use the functional form

$$F_i^q(t) = F_i^q(0)\,(1 - a_{iq}\,t/p_{iq})^{-p_{iq}}\,(1 - b_{iq}\,t/q_{iq})^{-q_{iq}} \tag{2.1}$$

with $q = u, d$ and $i = 1, 2$, fixing 3 of the 16 parameters in this ansatz and fitting the remaining 13. We obtain a global $\chi^2 = 122.3$ for 178 data points and a uniformly good description of the data for the proton and neutron form factors.

Furthermore, we have interpolated the data on the Sachs form factors and their ratios to a common set of t values, where we can then compute mean values and errors for the form factors $F_i^q(t)$ and for any combination of them (see figure 2 below). Limited by the data on R^n and G_E^n, this set covers a range up to $-t \simeq 3.4\,\mathrm{GeV}^2$.

The electromagnetic form factors receive contributions also from strange quarks, i.e. from F_1^s and F_2^s. We estimate these in a model that is consistent with determinations from lattice QCD and from parity violation in polarized elastic ep scattering (with data up to $-t \simeq 0.6\,\mathrm{GeV}^2$). We find that the estimated size of $F_{1,2}^s(t)$ is comparable to the current *uncertainties* on $F_{1,2}^{u,d}(t)$, so that our poor knowledge of the strangeness sector does not yet have an impact on our determination of the GPDs for u and d quarks.

3. Fit of GPDs

To extract valence quark GPDs from the form factor data we use an ansatz developed in our earlier work [8] (are related analysis can be found in [9]). We take an exponential form

$$H_v^q(x,t) = q_v(x)\exp\bigl[t\,f_q(x)\bigr]\,, \qquad E_v^q(x,t) = e_v^q(x)\exp\bigl[t\,g_q(x)\bigr]\,, \tag{3.1}$$

for the t dependence, with x dependent slopes

$$\begin{aligned} f_q(x) &= \alpha'_q\,(1-x)^3\log(1/x) + B_q\,(1-x)^3 + A_q\,x(1-x)^2\,,\\ g_q(x) &= \alpha'_q\,(1-x)^3\log(1/x) + D_q\,(1-x)^3 + C_q\,x(1-x)^2\,. \end{aligned} \tag{3.2}$$

A Fourier transform turns H_v^q into a Gaussian in transverse space, where the average squared impact parameter $\langle \mathbf{b}^2\rangle_x^q = 4 f_q(x)$ vanishes like $(1-x)^2$ for $x\to 1$ with our ansatz. This ensures that the distance $\mathbf{b}/(1-x)$ between the struck quark and the spectators remains on average finite in this limit, which is plausible for a system subject to confinement [1]. The $t=0$ limit of H_v^q is given by the usual valence quark densities. Although these are relatively well known, there are important differences between different PDF sets at small and at large x. We take the ABM 11 NLO densities [10] at scale $\mu = 2\,\mathrm{GeV}$ as our default and repeat our analysis with a number of other recent parameterizations (CT 10, GJR 08, HERAPDF 1.5, MSTW 2008, NNPDF 2.2) as a cross check. We have verified that the CJ 12 parameterization, which was presented at this conference [11] and differs from other sets in the limiting behavior of d_v at $x\to 1$, gives results within the range spanned by the other parameterizations in our fits. The $t=0$ limit of E_v^q is unknown, and we make the ansatz

$$e_v^q(x) \propto x^{-\alpha_q}(1-x)^{\beta_q}\bigl(1+\gamma_q\sqrt{x}\bigr)\,, \tag{3.3}$$

where the normalization is determined by the anomalous magnetic moments of the proton and neutron. The Fourier transform of E_v^q describes the shift in the transverse spatial distribution of valence quarks that is induced by transverse proton polarization [1]. As a consequence, the size of E_v^q relative to H_v^q is limited by a positivity bound [12]. Imposing this bound in our fit significantly constrains the allowed parameter space, in particular by limiting the large-x power β_q in e_v^q from below. Our best fit results are obtained by taking both β_u and β_d as small as possible, and we typically find $\beta_u \sim 4.5$ to 5 and $\beta_d \sim 5$ to 6 depending on the PDF set used for H_v^q. The precise data on r_{nE}^2 and R^n favor a small flavor dependence with $\alpha'_u > \alpha'_d$ for the shrinkage parameter in H_v^q and E_v^q. Our fitted values of α'_d and α'_u range from 0.68 to $1.0\,\mathrm{GeV}^{-2}$, which is in line with expectations from Regge phenomenology, where the small-x behavior of H_v^q and E_v^q is determined by the ρ and ω trajectories.

We obtain a good overall fit of all form factor data, with a global $\chi^2 = 221.2$ for 178 data points if we use the ABM 11 PDFs. There are small discrepancies between fit and data for G_M^p and R^p below $-t = 1\,\mathrm{GeV}^2$, but only at the level of a few %. In figure 2 we show the ratios F_1^d/F_1^u and F_2^d/F_2^u, both normalized to 1 at $t=0$. As discussed in [8], our ansatz (3.1) and (3.2) reflects the Feynman mechanism by connecting the large-t behavior of F_1^q with the large-x behavior of q_v. The stronger decrease of F_1^d compared with F_1^u seen in the data is thus naturally explained by the faster decrease of d_v compared with u_v. Correspondingly, our fit predicts a faster decrease of $-F_2^d$ compared with F_2^u for $\sqrt{-t}$ above 1.5 GeV. It will be interesting to see whether this is realized in Nature, once there are precise data for G_E^n at higher $-t$.

Figure 2: Normalized ratios of the d and u quark contributions to the Dirac and Pauli form factors of the proton. The band represents our default GPD fit and the dashed line the fit using (2.1). The data points have been obtained by interpolation of data on Sachs form factors and their ratios to a set of common t values.

4. Selected fit results

Having extracted the GPDs H_v^q and E_v^q, we can compute a variety of quantities of interest for the structure of the proton. The left panel in figure 3 shows the average of the distance between the struck parton and the spectators, defined by $d_q(x) = \big[\langle \mathbf{b}^2 \rangle_x^q\big]^{1/2}/(1-x)$, and the right panel shows the average shift of this distance induced by transverse proton polarization, $s_q(x) = \langle \mathbf{b}^y \rangle_x^q/(1-x)$. A clear difference between valence d and u quarks is seen for both quantities.

Figure 3: Results of our default GPD fit for the transverse distribution of valence quarks in the proton. Left: average distance between the struck quark and the spectators. Right: average shift of this distance in the y direction when the proton is polarized in the positive x direction.

We can also evaluate the contribution of the total angular momentum carried by quarks minus the corresponding contribution from antiquarks. According to Ji's sum rule, this is given by

$$J_v^q = \frac{1}{2}\int_0^1 dx\, x\big[q_v(x) + e_v^q(x)\big]\,. \tag{4.1}$$

From our GPD fit we obtain

$$J_v^u = 0.230^{+0.009}_{-0.024}\,, \quad J_v^d = -0.004^{+0.015}_{-0.016}\,, \quad J_v^{u+d} = 0.226^{+0.004}_{-0.026}\,, \quad J_v^{u-d} = 0.233^{+0.020}_{-0.038} \tag{4.2}$$

at the scale $\mu = 2\,\text{GeV}$, where the errors include the parametric uncertainties of our default fit with the ABM 11 PDFs, as well as the variation of results if we employ different PDFs or change other details of the fit.[1] These results are consistent with our previous work [8, 13], while using a much larger data set and giving a more comprehensive estimate of the uncertainty. They also agree with a recent determination [14] based on a phenomenological extraction of the Sivers distributions and a model for chromodynamic lensing, which gives $J_v^u = 0.214^{+0.009}_{-0.013}$ and $J_v^d = -0.029^{+0.021}_{-0.008}$ if we add in quadrature the errors quoted in [14]. We find it encouraging that two model-dependent extractions using entirely different methods obtain consistent results. Subtracting from (4.2) the the helicity carried by valence quarks as obtained from the DSSV set of polarized parton densities [15], we obtain orbital angular momentum contributions

$$L_v^u = -0.141^{+0.025}_{-0.033}\,, \quad L_v^d = 0.114^{+0.034}_{-0.035}\,, \quad L_v^{u+d} = -0.027^{+0.029}_{-0.039}\,, \quad L_v^{u-d} = -0.255^{+0.051}_{-0.061} \tag{4.3}$$

to the proton spin at $\mu = 2\,\text{GeV}$, where we have added in quadrature the errors in (4.2) and those of the DSSV parameterization. In agreement with other analyses, we find a very small net orbital angular momentum, which results from a cancellation between significant individual contributions from u and d quarks. We emphasize that the results (4.2) and (4.3) are for the *difference* between quarks and antiquarks. Using electromagnetic form factors, we cannot access the contribution from antiquarks by itself. Evaluating the usual momentum sum rule for current PDF sets, we find that at $\mu = 2\,\text{GeV}$ the longitudinal momentum carried by sea quarks in the proton is of similar size as the contribution from valence d quarks. If this provides any guidance, then the contribution from sea quarks to Ji's sum rule may not be negligible.

5. Conclusions

The electromagnetic nucleon form factors are among the best known quantities describing proton structure. With the experimental precision that has been reached, several issues require further efforts before a consistent and reliable picture can be established. There are clear inconsistencies between results for the ratio G_E^p/G_M^p, whose resolution appears urgent to us. The quantitative control of two-photon exchange in the extraction of G_M^p, the measurement of G_E^n or G_E^n/G_M^n at higher $-t$, as well as a quantitative determination of the strangeness form factors, remain tasks for the future.

Using our selection of form factor data, we have determined a set of interpolated form factor values up to $-t \simeq 3.4\,\text{GeV}^2$, from which one can compute various form factor combinations of interest. We find that the power-law ansatz (2.1) gives an excellent and economical fit of all data. A good description of the data can be achieved with a fit of u and d valence quark GPDs, in which the positivity conditions relating E and H play an essential role. We have used the resulting GPDs to quantify the transverse distribution of valence quarks in the proton and to evaluate the total angular momentum carried by valence quarks, along with several other applications presented in [4].

[1] In [4] the upper error on J_v^d is erroneously quoted to be $+0.010$ instead of $+0.015$.

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