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Simplified Models for New Physics in Vector Boson Scattering – Input for Snowmass 2013

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ABSTRACT

In this contribution to the Snowmass process 2013 (which is a preliminary version of [1]) we give a brief review of how new physics could enter in the electroweak (EW) sector of the Standard Model (SM). This new physics, if it is directly accessible at low energies, can be parameterized by explicit resonances having certain quantum numbers. The extreme case is the decoupling limit where those resonances are very heavy and leave only traces in the form of deviations in the SM couplings. Translations are given into higher-dimensional operators leading to such deviations. As long as such resonances are introduced without a UV-complete theory behind it, these models suffer from unitarity violation of perturbative scattering amplitudes. We show explicitly how theoretically sane descriptions could be achieved by using a unitarization prescription that allows a correct description of such a resonance without specifying a UV-complete model.

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1 Introduction: extended electroweak symmetry breaking

After the first run of LHC, a light Higgs particle compatible with the predictions from the SM as well as with electroweak precision tests (EWPT) with a mass of $m_H = 125$ GeV has been found. At the moment, all production cross sections and decay rates are compatible with the SM. This, however, is not so astonishing as it is rather difficult to cook up a BSM model that is compatible with EWPT and has deviations in the Higgs properties larger than the experimental uncertainties of the 2011/12 data. So, we got a glimpse on the electroweak symmetry breaking mechanism, but not yet the full answer. We have to check whether longitudinal electroweak vector boson scattering, i.e. scattering of Higgs field components really behaves in the way as expected from the SM. Furthermore we have to look to possible additional states that couple to the electroweak system of the W, Z and the 125 GeV state. Almost all BSM models predict modifications of the EW sector as part of the explanation of the hierarchy problem, namely the stability of a fundamental scalar state under radiative corrections. Examples are extra-dimensional models that comprise Kaluza-Klein recurrences of the of the EW gauge bosons and possibly also the Higgs, SUSY models as more generally two- or multi-doublet Higgs models, Little Higgs models, Twin Higgs models, model of complete or partial compositeness, Technicolor- or Topcolor-like models etc.

Usually, a fine-tuning measure is used for the definition of the little hierarchy problem: if the parameters of a model have to be tuned to a higher precision in order to achieve the correct Higgs mass parameter, the higher the fine tuning is. Though this is not a physical argument per se, it might give a guideline how contrived a model is. Most models seem the most natural for extensions of the electroweak sector "just around the corner", i.e. very close to the EW scale itself (as a prime example, this has been analyzed for Little Higgs models quite recently [2]).

Any model that is believed to solve the hierarchy problem is endowed with some sort of sector of new physics that couples to the EW bosons. The goal of this contribution to the Snowmass white paper is to define a Simplified Model that is able to describe the essence of this new physics sector in an approach as model-independent as possible. We refrain from discussing fermionic resonances here as those would contribute only at the 1-loop order to vector boson scattering, and concentrate on new bosonic resonances. To do so, one needs to supplement the Lagrangian of the EW SM (accounting for the discovery of the 125 GeV state as the SM Higgs boson but maybe allowing its couplings to deviate within the limits of the EWPT from their SM values). As the main signatures to study the EW sector of the SM are diboson, triboson and generically multi-boson production as well as vector-boson scattering (VBS), and here particularly scattering of longitudinal gauge bosons, it is convenient to use an operator basis containing explicitly the longitudinal degrees of freedom (DOFs) of the EW gauge bosons. This effective Lagrangian is basically identical to the chiral EW Lagrangian [3], except that we linearize the Lagrangian by adding the Higgs particle, and all higher-dimensional operators stem from BSM contributions. So we implement $SU(2)_L \times U(1)_Y$ gauge invariance, where the building blocks are the SM fermions, ψ, the EW (transversal) gauge boson fields W^a_μ $(a = 1, 2, 3)$ and B_μ as well as the longitudinal DOFs, $\Sigma = \exp\left[\frac{-i}{v} w^a \tau^a\right]$. Our first goal is to write down the minimal EW Lagrangian, and add then deviations from that Lagrangian in the form of higher-dimensional operators allowed by gauge symmetry as well as CP. Later, we show how precisely such couplings can arise when heavy BSM resonances are in the game. The minimal Lagrangian including gauge interactions is then:

$$\begin{aligned}\mathcal{L}_{\rm min} = &\sum_\psi \overline{\psi}(i\not{D})\psi - \sum_\psi \overline{\psi}_L \Sigma M \psi_R - \frac{1}{2g^2}\,\mathrm{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2g'^2}\,\mathrm{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] \\ &+ (D_\mu\phi)^\dagger D^\mu\phi + V(\phi) + \frac{g_h v}{2}\,\mathrm{tr}\left[\mathbf{V}^\mu\mathbf{V}_\mu\right] h - \sum_\psi \overline{\psi}_L M \psi_R \phi + \frac{v^2}{4}\,\mathrm{tr}\left[(\mathbf{D}_\mu\Sigma)(\mathbf{D}^\mu\Sigma)\right] \end{aligned} \tag{1}$$

Here, bold-faced quantities are always in the vector representation of $SU(2)_L$, D is the corresponding gauge-covariant derivative. $\phi = \frac{1}{\sqrt{2}}(0, v+h)^T$ is the field of the Higgs particle, and $D_\mu\phi = (\partial_\mu + \mathbf{V}_\mu)\phi$ is the gauge-covariant derivative of the Higgs field. $\mathbf{V}$ is the field of longitudinal vectors, $\mathbf{V} = \Sigma(\mathbf{D}\Sigma)^\dagger$ that will be used shortly to write down operators giving rise to modified couplings. $V(\phi)$ contains the trilinear and quadrilinear Higgs self-couplings as well as the Higgs mass term. In order to write down operators projecting out the neutral component, one uses $\mathbf{T} = \Sigma\tau^3\Sigma^\dagger$.

There are two extrem limits, one is the unitary gauge where one chooses a gauge to rotate the Goldstone fields away: $\mathbf{w} \equiv 0$, i.e., $\Sigma \equiv 1$. Here, $\mathbf{V} \longrightarrow -\frac{\mathrm{i}g}{2}\left[\sqrt{2}(W^+\tau^+ + W^-\tau^-) + \frac{1}{c_{\rm w}} Z\tau^3\right]$ and $\mathbf{T} \longrightarrow \tau^3$. On the other hand, there is the gaugeless limit, removing the transversal DOFs by $g, g' \to 0$. This limit makes calculations for scattering processes of longitudinal gauge bosons particularly simple, and was the choice within approaches for Higgsless models or models with strongly interacting Ws and/or (very) heavy Higgs bosons. Here, one has $\mathbf{V} \longrightarrow \frac{\mathrm{i}}{v}\left\{\sqrt{2}\partial w^+\tau^+ + \sqrt{2}\partial w^-\tau^- + \partial z\tau^3\right\} + O(v^{-2})$ and $\mathbf{T} \longrightarrow \tau^3 + 2\sqrt{2}\frac{\mathrm{i}}{v}\left(w^+\tau^+ - w^-\tau^-\right) + O(v^{-2})$.

This minimal (SM) Lagranigan can now be supplemented by additional operators,

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm min} + \beta_1\mathcal{L}'_0 + \sum_i \alpha_i\mathcal{L}_i + \frac{1}{\Lambda}\sum_i \alpha_i^{(5)}\mathcal{L}^{(5)} + \frac{1}{\Lambda^2}\sum_i \alpha_i^{(6)}\mathcal{L}^{(6)} + \ldots \tag{2}$$

where Λ is (up to $\mathcal{O}(1)$ constants) the scale where BSM physics potentially enters.

$$\mathcal{L}'_0 = \frac{v^2}{4}\,\mathrm{tr}\left[\mathbf{T}\mathbf{V}_\mu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right] \tag{3}$$

$$\mathcal{L}_1 = \mathrm{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] \qquad \mathcal{L}_6 = \mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}_\nu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\nu\right] \tag{4}$$

$$\mathcal{L}_2 = \mathrm{i}\,\mathrm{tr}\left[\mathbf{B}_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]\right] \qquad \mathcal{L}_7 = \mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}^\mu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}_\nu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\nu\right] \tag{5}$$

$$\mathcal{L}_3 = \mathrm{i}\,\mathrm{tr}\left[\mathbf{W}_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]\right] \qquad \mathcal{L}_8 = \frac{1}{4}\,\mathrm{tr}\left[\mathbf{T}\mathbf{W}_{\mu\nu}\right]\mathrm{tr}\left[\mathbf{T}\mathbf{W}^{\mu\nu}\right] \tag{6}$$

$$\mathcal{L}_4 = \mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}_\nu\right]\mathrm{tr}\left[\mathbf{V}^\mu\mathbf{V}^\nu\right] \qquad \mathcal{L}_9 = \frac{\mathrm{i}}{2}\,\mathrm{tr}\left[\mathbf{T}\mathbf{W}_{\mu\nu}\right]\mathrm{tr}\left[\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]\right] \tag{7}$$

$$\mathcal{L}_5 = \mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}^\mu\right]\mathrm{tr}\left[\mathbf{V}_\nu\mathbf{V}^\nu\right] \qquad \mathcal{L}_{10} = \frac{1}{2}\left(\mathrm{tr}\left[\mathbf{T}\mathbf{V}_\mu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]\right)^2 \tag{8}$$

For more technical details about this formalism interpreted in that context of simplified models for extended EW symmetry breaking, cf. [4,1]. Indirect information on new physics is encoded in the operator coefficients β_1, α_i.

From EWPT (SLC/LEP/Tevatron measurements), one knows that $\alpha_i \ll 1$, while on the other hand from naive dimensional analysis one would assume $\alpha_i \sim 1/16\pi^2 \approx 0.006$ as they have to renormalize divergencies in an effective field theoretic simplified model of a UV-complete BSM model. Using such a bottom-up approach, it is notoriously difficult, as the usual setup as a ratio of the EW and the BSM scale $\alpha_i = v^2/\Lambda^2$ is only valid upto unknown operator normalization coefficients (that are in general coupling constants of the UV-complete model), furthermore the power counting can be highly nontrivial, producing unexpected scaling behavior of operators.

One way to deal with this in a model-independent is to consider resonances that couple to EWSB sector, which we will do in the next section. For completeness, we repeat the formulae for triple and quartic gauge couplings, and how they depend on the SM parameters as well as on the operator coefficients of the effective Lagrangian above:

$$\begin{aligned}\mathcal{L}_{TGC} = \mathrm{i}e &\left[g_1^\gamma A_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^\gamma W_\mu^- W_\nu^+ A^{\mu\nu} + \frac{\lambda^\gamma}{M_W^2} W_\mu^{-\nu} W_{\nu\rho}^+ A^{\rho\mu}\right] \\ &+ \mathrm{i}e\frac{c_\mathrm{w}}{s_\mathrm{w}}\left[g_1^Z Z_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^Z W_\mu^- W_\nu^+ Z^{\mu\nu} + \frac{\lambda^Z}{M_W^2} W_\mu^{-\nu} W_{\nu\rho}^+ Z^{\rho\mu}\right]\end{aligned} \tag{9}$$

$$\begin{aligned}\mathcal{L}_{QGC} = e^2 &\left[g_1^{\gamma\gamma} A^\mu A^\nu W_\mu^- W_\nu^+ - g_2^{\gamma\gamma} A^\mu A_\mu W^{-\nu} W_\nu^+\right] \\ &+ e^2 \frac{c_\mathrm{w}}{s_\mathrm{w}}\left[g_1^{\gamma Z} A^\mu Z^\nu \left(W_\mu^- W_\nu^+ + W_\mu^+ W_\nu^-\right) - 2 g_2^{\gamma Z} A^\mu Z_\mu W^{-\nu} W_\nu^+\right] \\ &+ e^2 \frac{c_\mathrm{w}^2}{s_\mathrm{w}^2}\left[g_1^{ZZ} Z^\mu Z^\nu W_\mu^- W_\nu^+ - g_2^{ZZ} Z^\mu Z_\mu W^{-\nu} W_\nu^+\right] \\ &+ \frac{e^2}{2 s_\mathrm{w}^2}\left[g_1^{WW} W^{-\mu} W^{+\nu} W_\mu^- W_\nu^+ - g_2^{WW}\left(W^{-\mu} W_\mu^+\right)^2\right] + \frac{e^2}{4 s_\mathrm{w}^2 c_\mathrm{w}^4} h^{ZZ} (Z^\mu Z_\mu)^2\end{aligned} \tag{10}$$

In these equations, the SM values are $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$, and $g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$. The quantity $\delta_Z = \frac{\beta_1 + g'^2\alpha_1}{c_\mathrm{w}^2 - s_\mathrm{w}^2}$ takes into account the definition of the EW scheme as well as the oblique corrections through the ρ parameter. In the presence of the operators Eq. 3, one gets the following shifts:

$$\Delta g_1^\gamma = 0 \qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8) \tag{11}$$

$$\Delta g_1^Z = \delta_Z + \tfrac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8) \tag{12}$$

as well as

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \tfrac{g^2}{c_\mathrm{w}^4}(\alpha_5 + \alpha_7) \tag{13}$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \tfrac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta g_1^{WW} = 2c_\mathrm{w}^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) + g^2\alpha_4 \tag{14}$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \tfrac{g^2}{c_\mathrm{w}^4}(\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2c_\mathrm{w}^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2\left(\alpha_4 + 2\alpha_5\right) \tag{15}$$

$$h^{ZZ} = g^2\left[\alpha_4 + \alpha_5 + 2\left(\alpha_6 + \alpha_7 + \alpha_{10}\right)\right] \tag{16}$$

2 Electroweak Resonances and Translation into Anomalous Couplings

In the previous section, it has been discussed how higher-dimensional operators do lead to deviations of the triple and quartic gauge couplings from their SM values. Here, we want to make the connection how BSM models in their incarnation as EW resonances coupling to the SM EW gauge boson sector (particularly the longitudinal DOFs) generate such anomalous couplings. To be as general as possible, we include weakly interacting cases (e.g. Little Higgs models) where the new resonances are narrow (proper particles), as well as strongly interacting cases (e.g. compositeness or Technicolor models) where the new resonances are rather wide and could even approach the case of a continuum (e.g. unparticles or conformal sectors). As we know from EWPT, $\beta_1 \ll 1$, so the $SU(2)_c$ custodial symmetry of weak isospin (that in the SM is only broken by hypercharge $g' \neq 0$ and the fermion masses) is valid to a very good approximation. From the spin and isospin quantum numbers, only the resonances in the following table can couple to system of two EW vector bosons,

	$J = 0$	$J = 1$	$J = 2$
$I = 0$	σ^0 ("Higgs")	$[\omega^0]$ (γ'/Z')	f^0 (KK graviton)
$I = 1$	$[\pi^\pm, \pi^0]$ (2HDM)	$\rho^\pm, \rho^0$ (W'/Z')	$[a^\pm, a^0]$
$I = 2$	$\phi^{\pm\pm}, \phi^\pm, \phi^0$ (Higgs triplet)	—	$t^{\pm\pm}, t^\pm, t^0$

So only the scalars, vector or tensors can couple, and only the weak isospins $I = 0, 1, 2$ are allowed. The table shows prime examples for the corresponding combinations where a specific choice for the hypercharge has been made. The entries in brackets are combinations that are only possible with $SU(2)_c$-violating couplings, and are not further discussed here. The scalar isoscalar has the same quantum numbers as the SM Higgs boson. The scalar isovector appears in Technicolor models, while the isotensor can be found in the Littlest Higgs model, e.g. Vector resonances appear in extra-dimensional models, Technicolor, Little Higgs models and many more. Tensor resonances without EW quantum numbers can be thought of as a recurrence of the graviton, while the isovector and -tensor are quite exotic and appear only e.g. in extended compositeness models.

As a next step, we relate these resonances from our simplified models to anomalous couplings. Consider any kind of heavy resonance with generic Lagrangian $\mathcal{L}_\Phi = z\Big[\Phi\,(M_\Phi^2 + DD)\,\Phi + 2\Phi J\Big]$. Here, z is a (wavefunction re)normalization constant of the Lagrangian, and D is the gauge-covariant derivative. J is the EW current to which that particular resonance couples. Integrating out the resonance leeds to $\mathcal{L}_\Phi^{\text{eff}} = -\frac{z}{M^2}JJ + \frac{z}{M^4}J(DD)J + \mathcal{O}(M^{-6})$. We now specialize to a scalar isoscalar resonance σ, whose Lagrangian is given by $\mathcal{L}_\sigma = -\frac{1}{2}\Big[\sigma(M_\sigma^2 + \partial^2)\sigma - g_\sigma v\,\mathrm{tr}\,[\mathbf{V}_\mu\mathbf{V}^\mu] - h_\sigma\,\mathrm{tr}\,[\mathbf{T}\mathbf{V}_\mu]\,\mathrm{tr}\,[\mathbf{T}\mathbf{V}^\mu]\Big]$. Integrating out the scalar, leads to the effective Lagrangian

$$\mathcal{L}_\sigma^{\text{eff}} = \frac{v^2}{8M_\sigma^2}\Big[g_\sigma\,\mathrm{tr}\,[\mathbf{V}_\mu\mathbf{V}^\mu] + h_\sigma\,\mathrm{tr}\,[\mathbf{T}\mathbf{V}_\mu]\,\mathrm{tr}\,[\mathbf{T}\mathbf{V}^\mu]\Big]^2$$

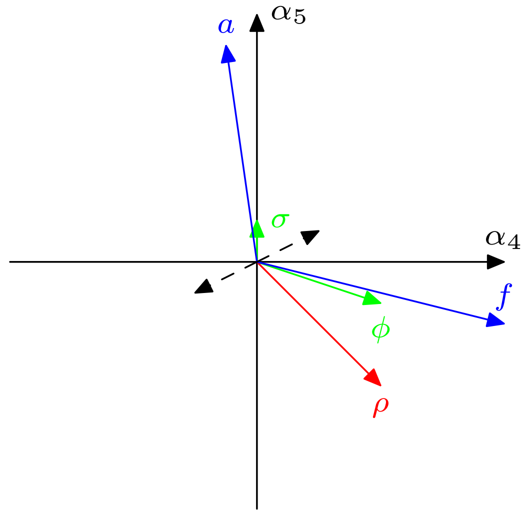

Figure 1: *Shifts in the (α_4, α_5) plane through heavy resonances in different spin-isospin channels. σ and ϕ are scalar resonances with $I = 0, 2$, respectively, ρ is a vector isovector, while f and t are tensor resonances of $I = 0, 2$, respectively. The dashed line shows the corrections to the α parameters from higher orders in the SM perturbative series.*

From this one can read off that integrating out a scalar isoscalar generates the following anomalous quartic couplings

$$\alpha_5 = g_\sigma^2 \left(\frac{v^2}{8M_\sigma^2}\right) \qquad \alpha_7 = 2g_\sigma h_\sigma \left(\frac{v^2}{8M_\sigma^2}\right) \qquad \alpha_{10} = 2h_\sigma^2 \left(\frac{v^2}{8M_\sigma^2}\right) \tag{17}$$

One sees immediately, that a heavy SM Higgs would have fit into that scheme, using the special couplings $g_\sigma = 1$ and $h_\sigma = 0$.

When one tries to turn constraints on anomalous couplings into direct constraints on new physics, one faces the problem that there are too many free parameters to overconstrain the system. There is however one limiting case where one can do that which has been applied in the context of studies of the possible search power of a 1 TeV ILC on anomalous quartic couplings and their interpretation in terms of resonances [5]: In the limit of a very broad resonance (that couples rather strongly to the EW sector), the resonance is close to a broad continuum: $\Gamma \sim M \gg \Gamma(\mathrm{non} - WW, ZZ) \sim 0$. Also, in that case the decays of such a particular resonance into non-W/Zs can be ignored. From the functional relation between the resonance width, its couplings and its mass (again in the case of a scalar isoscalar)

$$\Gamma_\sigma = \frac{g_\sigma^2 + \frac{1}{2}(g_\sigma^2 + 2h_\sigma^2)^2}{16\pi} \left(\frac{M_\sigma^3}{v^2}\right) + \Gamma(\mathrm{non} - WW, ZZ) \tag{18}$$

one can then translate bounds for anomalous couplings directly into those of the effective Lagrangian:

$$\alpha_5 \leq \frac{4\pi}{3} \left(\frac{v^4}{M_\sigma^4}\right) \approx \frac{0.015}{(M_\sigma \text{ in TeV})^4} \quad \Rightarrow \quad 16\pi^2 \alpha_5 \leq \frac{2.42}{(M_\sigma \text{ in TeV})^4} \tag{19}$$

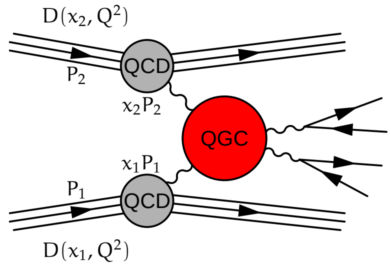

Figure 2: *Signature of vector boson scattering at the LHC as a means to measure quartic gauge couplings.*

Note that because of the different dependence of scalar and tensor widths compared to vector widths, the limits behave differently depending on the spin of the resonance:

$$\boxed{\begin{array}{rll} \text{Scalar:} & \Gamma \sim g^2 M^3,\ \alpha \sim g^2/M^2 & \Rightarrow \quad \alpha_{\max} \sim 1/M^4 \\ \text{Vector:} & \Gamma \sim g^2 M,\ \alpha \sim g^2/M^2 & \Rightarrow \quad \alpha_{\max} \sim 1/M^2 \\ \text{Tensor:} & \Gamma \sim g^2 M^3,\ \alpha \sim g^2/M^2 & \Rightarrow \alpha_{\max} \sim 1/M^4 \end{array}}$$

The following table

Resonance	σ	ϕ	ρ	f	t
$\Gamma[g^2M^2/(64\pi v^2)]$	6	1	$\frac{4}{3}(\frac{v^2}{M^2})$	$\frac{1}{5}$	$\frac{1}{30}$
$\Delta\alpha_4[(16\pi\Gamma/M)(v^4/M^4)]$	0	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{2}$	$-\frac{5}{8}$
$\Delta\alpha_5[(16\pi\Gamma/M)(v^4/M^4)]$	$\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{3}{4}$	$-\frac{5}{8}$	$\frac{35}{8}$

shows the width of the five different possible non-$SU(2)_c$ violating resonances with their widths into longitudinal EW gauge bosons, as well as their contributions to the anomalous quartic couplings parameters α_4 and α_5. Fig. 1 shows how the different resonances would show up in the (α_4, α_5) plane. From this a discrimination of different resonances even slighly below direct production of those resonances would be possible. σ and ϕ are scalar resonances of isospin $I = 0, 2$, respectively, ρ is a vector isovector, while the tensor resonances f and t also have isospins $I = 0, 2$, respectively. Those are just the cases one can write down without violation of the custodial symmetries. The dashed lines in Fig. 1 show shifts due to higher-order corrections from SM longitudinal gauge bosons. Note that the treatment of tensor resonances is notoriously complicated, as their couplings to longitudinal and transversal currents have to be constructed in different ways, as will be shown in [1].

3 Vector Boson Scattering at LHC and Unitarity

In this section, we want to discuss the signatures for vector boson scattering (VBS) at the LHC as well as issues of perturbative tree-level unitarity for our simplified models. At a hadron collider like the LHC, the typical signature for VBS for measuring (anomalous) quartic gauge couplings at the LHC is shown in Fig. 2. When one takes all leptons (including τs) as final-state particles, the cross section at the LHC for $pp \to jj(ZZ/WW) \to jj\ell^-\ell^+\nu_\ell\bar{\nu}_\ell$ is roughly $\sigma \approx 40\,\text{fb}$. The most severe background comes from top pair production, $t\bar{t} \to WbWb$, being three orders of magnitude larger, $\sigma_{tt} \approx 52\,\text{pb}$. Also, single top where one of the jets has been misreconstructed contributes with a cross section of $\sigma_t \approx 4.8\,\text{pb}$ to the background, while the QCD background – though still sizeable – is not that bad: $\sigma_{QCD} \approx 0.21\,\text{pb}$. To separate VBS from the background, a tag of two identified and separated leptons and two jets each is applied, where a large rapidity distance of the two jets is demanded in order to take into account collinear radiation, $|\Delta\eta_{jj}| > 4.4$, and explicit vetos against b jets. The leptons should be in a fiducial volume in the central part of the detector, $\eta_{tag}^{min} < \eta_\ell < \eta_{tag}^{max}$, there is a lower cut on the dijet invariant mass, e.g. $M_{jj} > 1000\,\text{GeV}$, lower cuts on the jet energy (e.g. $E_j > 600, 400\,\text{GeV}$) as well as lower cuts on the two jet $p_{T,j}^1 > 60, 24\,\text{GeV}$ (all values are just rough estimates). Particularly the large dijet invariant mass is a powerful means to discriminate against top contamination. At the moment, it is still unclear, whether vetoing against hadronic activity in the central part of the detector is feasible or not. In general, like those mentioned help to improve the signal-over-background ration by roughly one order of magnitude.

Now, we discuss the issues of perturbative unitarity within one example of our simplified models, a model that contains a scalar resonance explicitly, but also anomalous quartic gauge couplings explicitly. This sounds contrived at first, but such $\mathcal{O}_{4,5}$ can easily arise through other resonances with different spin or different recurrences of scalar resonances, e.g. in extra-dimensional models. Furthermore, they can be generated through higher-order corrections, which in strongly-interacting models can be sizeable. Clearly, in nature, unitarity will never be violated, it is just the truncation of a perturbative series in a simplified model/an effective field theory that leads to a possible violation of lowest-order perturbative unitarity. A UV completion of such a model has to restore unitarity again (possibly through higher orders). For the discussion of the issues of unitarity and an algorithm to a prescription that does not violate unitarity, we review the issue of perturbative unitarity in Goldstone boson scattering, taking the lowest-order EW Lagrangian including the Higgs and anomalous couplings:

$$\mathcal{L} = -\frac{v^2}{4}\,\mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}^\mu\right] + \frac{g_h v}{2}\,\mathrm{tr}\left[\mathbf{V}^\mu\mathbf{V}_\mu\right] h + \alpha_4\,\mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}_\nu\right]\mathrm{tr}\left[\mathbf{V}^\mu\mathbf{V}^\nu\right] + \alpha_5\left(\mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}^\mu\right]\right)^2 \ . \tag{20}$$

Using the standard Mandelstam variables, $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$, this leads to the following amplitudes for the scattering of longitudinal EW vector bosons:

$$\mathcal{A}(s,t,u) =: \qquad \mathcal{A}(w^+w^- \to zz) = \frac{s}{v^2} - \frac{g_h^2}{v^2}\frac{s^2}{s-M_H^2} + 8\alpha_5\frac{s^2}{v^4} + 4\alpha_4\frac{t^2+u^2}{v^4} \tag{21a}$$

$$\mathcal{A}(w^+z \to w^+z) = \frac{t}{v^2} - \frac{g_h^2}{v^2}\frac{t^2}{t-M_H^2} + 8\alpha_5\frac{t^2}{v^4} + 4\alpha_4\frac{s^2+u^2}{v^4} \tag{21b}$$

$$\mathcal{A}(w^+w^- \to w^+w^-) = -\frac{u}{v^2} - \sum_{x=s,t}\frac{g_h^2}{v^2}\frac{x^2}{x-M_H^2} + (4\alpha_4+2\alpha_5)\frac{s^2+t^2}{v^4} + 8\alpha_5\frac{u^2}{v^4} \tag{21c}$$

$$\mathcal{A}(w^+w^+ \to w^+w^+) = -\frac{s}{v^2} - \sum_{x=t,u}\frac{g_h^2}{v^2}\frac{x^2}{x-M_H^2} + 8\alpha_4\frac{s^2}{v^4} + 4\left(\alpha_4+2\alpha_5\right)\frac{t^2+u^2}{v^4} \tag{21d}$$

$$\mathcal{A}(zz \to zz) = -\sum_{x=s,t,u}\frac{g_h^2}{v^2}\frac{x^2}{x-M_H^2} + 8\left(\alpha_4+\alpha_5\right)\frac{s^2+t^2+u^2}{v^4} \tag{21e}$$

The first term in these equations is the so-called low-energy theorem (LET) that constitutes the scattering of longitudinal gauge bosons through themselves, the second term comes from exchange of the Higgs particle (whose coupling in the SM would be $g_h = 1$), while the other terms originate from the higher-dimensional operators.

In order to derive the unitarity limits, one has to decompose this into the corresponding isospin eigenamplitudes according to the following Clebsch-Gordan decomposition:

$$\mathcal{A}(I=0) = 3\mathcal{A}(s,t,u) + \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t) \tag{22}$$

$$\mathcal{A}(I=1) = \mathcal{A}(t,s,u) - \mathcal{A}(u,s,t) \tag{23}$$

$$\mathcal{A}(I=2) = \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t) \tag{24}$$

Following the discussion in [6], the total cross section $\sigma = |\mathcal{A}|^2/(64\pi^2 s)$ due to the unitarity of the S matrix has to obey the optical theorem, $\sigma_{\text{tot}} = \text{Im}\left[\mathcal{A}_{ii}(t=0)\right]/s$, where the Mandelstam variable is $t = -s(1-\cos\theta)/2$. In order to check the scattering wave unitarity, one has to decompose the quantum-mechanical amplitude into partial wave amplitudes, $\mathcal{A}(s,t,u) = 32\pi\sum_\ell(2\ell+1)\mathcal{A}_\ell(s)P_\ell(\cos\theta)$. Assuming only elastic scattering processes and demanding the equality between total cross section and the imaginary part of the forward scattering amplitude for the partial wave results in the condition: $\left|\mathcal{A}(s) - \frac{i}{2}\right| = \frac{1}{2}$. This means that the elastic scattering amplitude has to lie on the Argand circle (Fig. 3, left).

From the Goldstone scattering amplitudes Eq. 21, project out the isospin eigenamplitudes [6], which leads (only for the LET-part) to the three following spin-isopin eigenamplitudes:

$$\mathcal{A}_{I=0} = \frac{s}{16\pi v^2} \qquad \mathcal{A}_{I=1} = \frac{s}{96\pi v^2} \qquad \mathcal{A}_{I=2} = -\frac{s}{32\pi v^2}\ . \tag{25}$$

The condition $|\mathcal{A}_{IJ}| \lesssim \frac{1}{2}$ leads to the famous unitarity bounds, $E \sim \sqrt{\{8,48,16\}\,\pi}v = \{1.2, 3.5, 1.7\}$ TeV, respectively. The Higgs exchange (the second term in Eq. 21) ameliorates

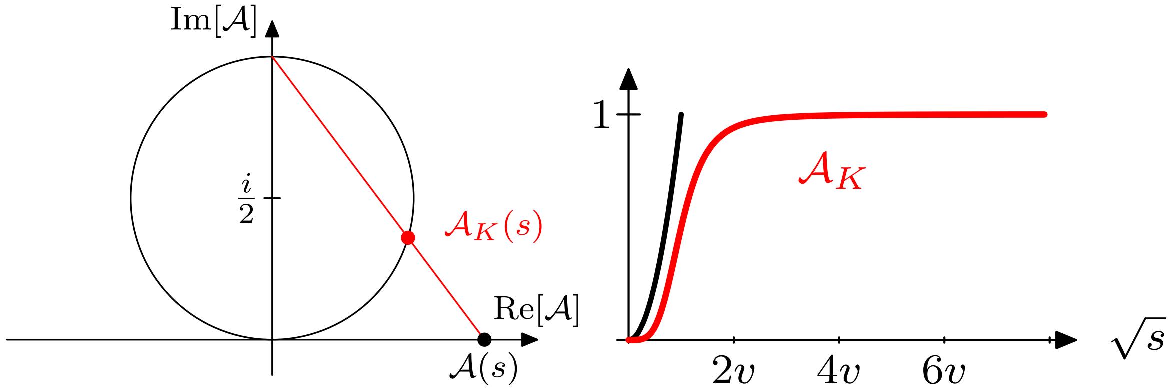

Figure 3: *Left: Argand circle for elastic scattering amplitudes according to the optical theorem. Stereographic K-matrix projection back to the circle for amplitudes violating perturbative unitarity. Right: Saturation of an amplitude rising quadratically with energy due to K-matrix unitarization, reaching a constant value.*

the quadratic rise in energy to $\mathcal{A}(s,t,u) = -(M_H^2/v^2) \times s/(s - M_H^2)$, which leads to the (tree-level) unitarity bound for a heavy SM Higgs boson of $M_H \lesssim \sqrt{8\pi}v \sim 1.2\,\text{TeV}$.

In the SM with a 125 GeV Higgs boson no problem with perturbative unitarity should arise, and no deviations should be visible in VBS from their SM predictions. However, even slight deviations in the Higgs couplings, $g_h = 1$, would lead to uncanceled unitarity violation. Also simplified models to test the spin of the 125 GeV Higgs against are difficult to define in a sane way as one has to take the scalar Higgs out in introduce a tensor resonance in order to exclude spin 2 from data. In all such cases, simplified models could arise that suffer from the issue of perturbative unitarity violations. Particularly, all simplified models endowed with resonances motivated by nearly all BSM models mentioned above do so. In order to get a theoretical description (e.g. for a Monte Carlo simulation), a prescription that leads to a simplified model covering the gross features of such BSM models, but on the other hand yielding amplitudes consistent with unitarity constraints would be highly welcome. Such an algorithm was proposed in [4] and is further refined in [1].

One straightforward prescription is the so-called K-matrix unitarization that has been used for the descrition of pion scattering processes. It consists of using a stereographic projection of an amplitude exceeding the unitarity constraint on the real axis back to the Argand circle (cf. left hand side of Fig. 3):

$$\mathcal{A}_K(s) = \frac{\mathcal{A}(s)}{1 - i\mathcal{A}(s)} = \mathcal{A}(s)\frac{1 + i\mathcal{A}(s)}{1 + \mathcal{A}(s)^2} \quad . \tag{26}$$

Physically, this would correspond to unitarization by an infinitely heavy and infinitely wide resonance (for more technical details, references, and also relations to other unitarization prescriptions cf. [4,1]). This prescription leads ameliorates a e.g. quadratic (or quartic) rise of an amplitude to a constant just saturating the unitarity bound at the very point where unitarity violation would start to set in (right hand side of Fig. 3).

We now show how this unitarization prescription works in the case of a scalar isosinglet resonance. Here, the Lagrangian including the kinetic term and the coupling to the current of the longitudinal EW gauge bosons is given by

$$\mathcal{L}_\sigma = -\tfrac{1}{2}\sigma\left(M_\sigma^2 + \partial^2\right)\sigma + \tfrac{g_\sigma v}{2}\sigma\,\mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}^\mu\right] \quad , \tag{27}$$

which leads to the following Feynman rules: $\sigma w^+ w^-$: $-\frac{2ig_\sigma}{v}(k_+ \cdot k_-)$, σzz : $-\frac{2ig_\sigma}{v}(k_1 \cdot k_2)$. Note that this is in complete analogy to the case of the SM Higgs which has the same quantum numbers, except that now the mass and the coupling are completely arbitrary. Hence, the amplitude for the s-channel exchange is $\mathcal{A}^\sigma(s,t,u) = (g_\sigma^2/v^2) \times s^2/(s - M^2)$. This leads then to the isospin eigenamplitudes that contain explicit resonance poles, and spin-isospin eigenamplitudes that are no longer polynomial in the Mandelstam variables. For e.g.:

$$\mathcal{A}_{00}^\sigma(s) = \; 3\tfrac{g_\sigma^2}{v^2}\tfrac{s^2}{s-M^2} + 2\tfrac{g_\sigma^2}{v^2}\mathcal{S}_0(s) \qquad\qquad \mathcal{S}_0(s) = M^2 - \tfrac{s}{2} + \frac{M^4}{s}\log\frac{s}{s+M^2} \tag{28}$$

For the K-matrix unitarization, the s-channel resonance pole must be treated separately in special way. We define the complete spin-isospin eigenamplitude as the SM amplitude (including the Higgs boson exchange), $A_{IJ}^{(0)}(s)$, then a BSM contribution (due to anomalous couplings or finite pieces of resonance exchange, or due to deviations of the Higgs amplitude from its SM value), $F_{IJ}(s)$, and the explicit resonance pole, $G_{IJ}(s)$:

$$A_{IJ}(s) = A_{IJ}^{(0)}(s) + F_{IJ}(s) + \frac{G_{IJ}(s)}{s - M^2} \quad . \tag{29}$$

The coefficient function $G_{IJ}(s) \propto s$ for vector resonances, and $\propto s^2$ for scalar and tensor resonances, respectively. Applying the K-matrix projection leads to a correction of the amplitude, which can be redefined as an additive correction to the original amplitude:

$$\hat{A}_{IJ}(s) = \frac{A_{IJ}(s)}{1 - \frac{i}{32\pi}A_{IJ}(s)} = A_{IJ}^{(0)}(s) + 32\pi i\Delta A_{IJ}(s), \tag{30}$$

with

$$\Delta A_{IJ}(s) = 32\pi i\left(1 + \tfrac{i}{32\pi}A_{IJ}^{(0)}(s) + \frac{s - M^2}{\frac{i}{32\pi}G_{IJ}(s) - (s-M^2)\left[1 - \frac{i}{32\pi}(A_{IJ}^{(0)}(s) + F_{IJ}(s))\right]}\right) \tag{31}$$

In order to implement this into a Monte Carlo event generator, one has to explicitly take care that the unitarization prescription by means of the K-matrix projection only happens in s-channel like configurations. Hence, such an algorithm breaks crossing symmetry. The formalism described here in form of the simplified models discussed above has been implemented and validated in the event generator WHIZARD [7,8]. Its setup of the color-flow formalism [9] as well as the connection to the parton shower [10] are both compatible with the formalism of the K-matrix prescription. For more technical details (also a validation using the FeynRules interface [11]) cf. [1].

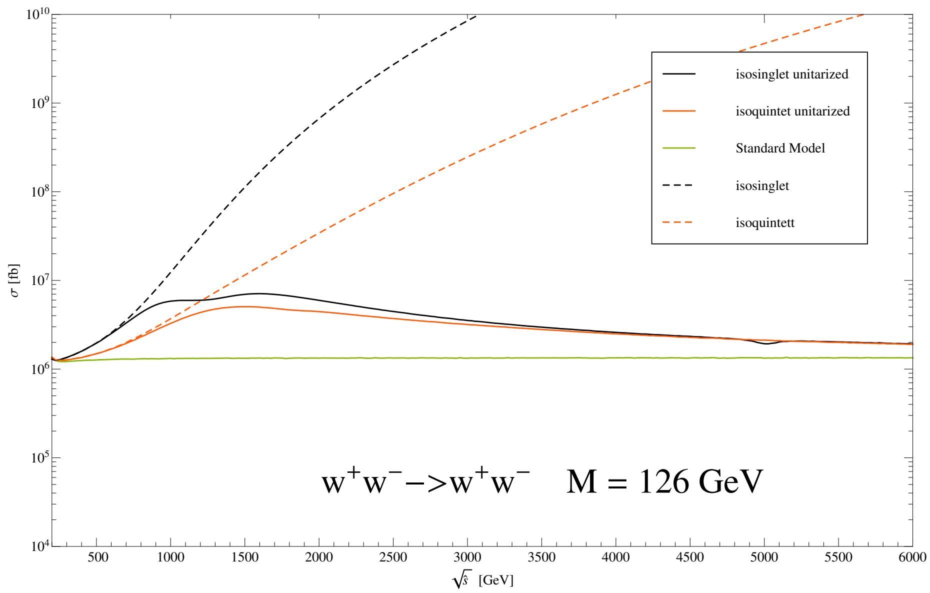

Figure 4: *Scalar resonances the scattering of longitudinal W bosons, $w^+w^- \to w^+w^-$. The green line is the SM cross section which is constant in energy. The dashed black and red line show the cross sections for the scalar isosinglet σ and the isotensor ϕ, respectively. The unitarization violation is clearly visible. The full black and red lines are the ones with K-matrix unitarization.*

Using that implementation, the sensitivity of a 1 TeV ILC with 1 ab^{-1} has been determined in an extensive study [5]. There, the 1 σ sensitivity on the anomalous couplings α_4 and α_5 turn out to be 0.0088 and 0.0071, respectively. This translate into the following reach limits for pure EW resonances in the setup of a 1 TeV ILC (in TeV):

Spin	$I = 0$	$I = 1$	$I = 2$
0	1.39	1.55	1.95
1	1.74	2.67	–
2	3.00	3.01	5.84

For this 1 TeV ILC study, possible unitarity violation issues have not yet played a role, but both 8 and 14 TeV LHC runs as well as a EW physics at a 3 TeV CLIC both have to take this into account. No concise LHC study has as yet been done for these simplified models, but more results will be given in [1]. Generically, one can deduce that the sensitivity on new resonances rises with the number of intermediate (spin) states, such that tensor resonances have higher reach than vectors or even scalar resonances. A first rough estimate from [4] might serve as a guideline for the expectations from 300 fb^{-1} at 14 TeV LHC, where the expected reach is varying from 0.5 TeV to 2 TeV for scalar up to tensor resonances.

4 Summary and Conclusion

In this contribution to the Snowmass White Paper we were constructing simplified models that allow to describe the physics of a modified electroweak sector compared to the SM. Experimental observables for such scenarios are modifications of diboson production, triboson production, and particularly vector boson scattering. Modifications of the EW sector could be just parameterized by deviations in the triple and quartic gauge couplings. In such a case a convenient operator basis is the one from the EW chiral Lagrangian that has now been enlarged by all operators containing the Higgs state observed from the LHC. However, most models have their most natural regions of parameter space where new resonances show up directly in the upcoming 14 TeV of LHC. In such a case, a description with higher-dimensional operators alone is insufficient and not applicable. The simplified models discussed here contain the SM supplemented by all possible resonances that could couple to the sector of EW gauge bosons according to their spin and isospin quantum numbers. Such simplified models cover cases ranging from Two- or Multi-Higgs doublet models, extended scalar sectors, Technicolor models, models of complete or partial compositeness, Little Higgs models, Twin Higgs models and many more. Cases where there is only a single resonance present could be described along these lines as well as cases where there are more resonances (but maybe only one of them accessible to LHC). The resonances are just parameterized by their mass, possibly their width, as well as their couplings to the electroweak sector. As simplified models are like any effective field theory not UV-complete, perturbative unitarity of tree-level amplitudes in that setup are not guaranteed. To give a prescription that can be used by the experiments in a model-independent setup and does not yield overly optimistic results due to unphysical amplitude contributions within exclusion limits, a unitarization formalism has been introduced that projects back on amplitudes that are genuinely unitary. This is insured by giving additive corrections to the SM vector boson scattering augmented by the BSM resonances. A simple implementation has been performed in the event generator WHIZARD [7]. This proceedings contribution is intended as a first gathering of the findings in [1] where also all the technical details can be found. Special emphasis there is also given to tensor resonances that have not been discussed that extensively in the literature. There, particularly a careful treatment of subleading terms in longitudinal and transversal modes of electroweak gauge bosons is crucial.

For a 1 TeV ILC there was an elaborate joint experimental and theoretical study that determined the ILC search reach for anomalous quartic couplings and its re-interpretation in terms of model-independent resonances. The sensitivity rises with number of intermediate states, from scalars over vectors to tensors. At the LHC sensitivity limited in pure EW sector, the projected reach might lie in the range from $0.6 - 2\,\text{TeV}$ compared to $1 - 6\,\text{TeV}$. More studies are urgently needed to find out whether a high-luminosity phase of the 14 TeV LHC or a higher-energy upgrade are the better options for these kinds of extensions of the EW sector. Also, it is not yet clear whether cut-based approaches or multi-variate analyses give the best sensitivities. More kinematic variables have to be investigated in order to optimize the reach of the LHC even with only 300 fb^{-1} for vector boson scattering.

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