PROCEEDINGS OF SCIENCE

DESY 13-133, DO-TH 13/16, MITP/13-038, SFB/CPP-13-51, LPN 13-053, arXiv:1307.7548 [hep-ph]

Recent Results on the 3-Loop Heavy Flavor Wilson Coefficients in Deep-Inelastic Scattering*

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We report on recent progress in the calculation of the 3-loop massive Wilson coefficients in deep-inelastic scattering at general values of N for neutral and charged current reactions in the asymptotic region $Q^2 \gg m^2$.

XXI International Workshop on Deep-Inelastic Scattering and Related Subjects -DIS2013,
22-26 April 2013
Marseilles,France

*This work has been supported in part by DFG Sonderforschungsbereich Transregio 9, Computergestützte Theoretische Teilchenphysik, by Studienstiftung des Deutschen Volkes, by the Austrian Science Fund (FWF) grants P20347-N18, P22748-N18 and SFB F50 (F5009-N15), by the EU Network LHCPHENOnet PITN-GA-2010-264564, by the Reserach Center 'Elementary Forces and Mathematical Foundations (EMG) of J. Gutenberg University Mainz and DFG, and by FP7 ERC Starting Grant 257638 PAGAP.

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1. Introduction

The precision determinations of $\alpha_s(M_Z^2)$, the mass of the charm quark m_c and the parton distribution functions from the world data on deep-inelastic scattering (DIS) require the heavy flavor corrections to 3-loop order [1]. Here the structure function $F_2(x,Q^2)$ provides the highest precision. As has been shown in [2] at scales $Q^2/m_c^2 \gtrsim 10$ the asymptotic representation of the heavy flavor Wilson coefficients provides a representation on the per cent level. [1] They are given in terms of convolutions of massive operator matrix elements (OMEs) and the massless Wilson coefficients [4]. A series of 3-loop Mellin-moments for $F_2(x,Q^2)$ and transversity and the OMEs describing the transition matrix elements in the variable flavor number scheme (VFNS) [5, 6] have been calculated in 2009 in Refs. [7, 8] projecting the respective tensor quantities onto massive tadpoles which could be computed using `MATAD` [9].

A program to compute the massive 3-loop Wilson coefficients at general values of N and their analytic continuation to $N \in \mathbb{C}$ started thereafter. In the unpolarized case, eight Wilson coefficients/OMEs contribute. All logarithmic contributions [10] are available since they rely on the the 2-loop results [2, 11] up to $O(\alpha_s^2 \varepsilon)$ [12]. Two of the eight Wilson coefficients resp. OMEs, $L_{qg,Q}^{(3)}$ and $L_{qq,Q}^{(3),\rm PS}$, were calculated in [13]. We studied the contributions to specific color factors, such as $O(N_F T_F^2 C_{A,F})$, which are completely known now [13, 14]. Further investigations are devoted to diagrams with two fermion lines with finite equal [15] or unequal mass [16, 17]. Genuine 3-loop topologies of the ladder- and V-graph type have been studied in [18, 19]. These calculations were accompanied by mathematical and computer-algebraic developments. In course of this systematic use is made of higher hypergeometric functions, Mellin-Barnes techniques, and modern summation theory [20]. The latter are encoded in the packages `Sigma, EvaluateMultiSums` and `SumProduction` [21]. Extensions of the harmonic sums [22] and polylogarithms [23] to generalized harmonic sums [24, 25] and the associated iterated integrals, the cyclotomic and generalized cyclotomic sums and integrals [26] were developed. Most recently iterated integrals over root-valued letters were systematized. These functions and their relations were encoded in the package `HarmonicSums`, [25, 27], see also [28]. All these developments were necessary to perform the present calculations. They are, however, of much wider use.

In this note we report on progress being obtained during the last year.

2. 3-Loop OMEs with Two Fermion Lines of Equal Mass

A subset of graphs contributing to the 3-loop massive Wilson coefficients contains two fermion lines with equal mass, characterized by the color factor $T_F^2 C_{F,A}$. These graphs may contain new types of sums, which, to a wider extent also emerge in the V-topologies, see Section 5. These are weighted inverse binomial sums. An example is given by the diagram in Figure 1. The diagram is

[1] The corresponding scales are much higher in case of the structure function $F_L(x,Q^2)$ [2], for which the 3-loop heavy flavor corrections for general values of N have been calculated in [3].

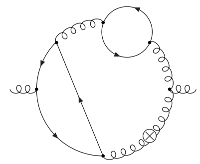

Figure 1: An example for a graph with two massive fermion lines

given by

$$\begin{aligned} I(N) = \frac{1+(-1)^N}{2}\Biggl\{ & \frac{1}{45\varepsilon^2(N+1)} - \frac{1}{\varepsilon}\left[\frac{S_1(N)}{90(N+1)} + \frac{47N^3+20N^2-67N+40}{1800(N-1)N(N+1)^2}\right] \\ & + \frac{105N^3-175N^2+56N+96}{13440(N+1)^2(2N-3)(2N-1)4^N}\binom{2N}{N}\left[\sum_{j=1}^{N}\frac{4^j S_1(j)}{\binom{2j}{j}j^2} - \sum_{j=1}^{N}\frac{4^j}{\binom{2j}{j}j^3} - 7\zeta_3\right] \\ & + \frac{5264N^3-2409N^2-12770N+3528}{100800(N+1)^2(2N-3)(2N-1)}S_1(N) + \frac{S_1^2(N)+S_2(N)+3\zeta_2}{360(N+1)} \\ & + \frac{S_3(N)-S_{2,1}(N)+7\zeta_3}{420(N+1)} + \frac{Q_0(N)}{2268000(N-1)^2N^2(N+1)^3(2N-3)(2N-1)}\Biggr\}. \end{aligned}$$

Here and in the following Q_i denote polynomials in N. The terms $\propto 1/(2N-3), 1/(2N-1)$ deserve special attention. It can be shown that both are removable poles in $I(N)$. It is generally expected that in QCD the rightmost singularity is located at $N = 1$. All basic topologies of this type contributing to the OME $A_{gg}^{(3)}$ have been calculated.

3. 3-Loop OMEs with Two Fermion Lines of Different Mass

From the level of the 3-loop correction onwards, also graphs with two fermion lines of different mass contribute. They require an extension of the renormalization programme of Ref. [7]. It turns out that the equal mass case is better included alongside with the case of two different masses m_c and m_b. The very close values of the charm and bottom quark masses do not allow to treat charm massless at the scale $\mu^2 = m_b^2$ and one has to deal with a two-mass scenario. Yet $\xi = m_c^2/m_b^2 \sim 1/10$ allows an expansion in ξ. For the fixed moments $N = 2,4,6$ the calculation of all OMEs has been performed in [16, 17] after mapping them to tadpoles and using the code `qexp` [29]. First results were derived for general values of N. It is needless to say that also the matching conditions in the variable flavor scheme require these new and no other expressions to stay in accordance with the renormalization group equations inside the correct framework of perturbative QCD. Moreover, the matching scales may vary considerably for different observables [30].

4. Ladder Graphs

First results have been obtained in the calculation of ladder graphs in the massive case, which belong to the genuine 3-loop topologies [19]. Here the class of functions appearing in intermediate and final results extends to generalized harmonic sums, cf. [25]. Let us consider the diagram in Figure 2. The corresponding scalar graph yields

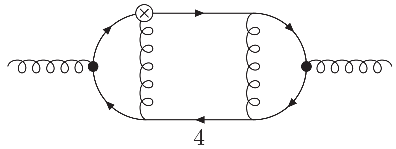

Figure 2: Ladder graph with operator insertion.

$$
\begin{aligned}
\hat{I}_4 \;=\;& \frac{Q_1(N)}{2(1+N)^5(2+N)^5(3+N)^5} + \frac{Q_2(N)}{(1+N)^2(2+N)^2(3+N)^2}\zeta_3 + \frac{(-1)^N\left(65+101N+56N^2+13N^3+N^4\right)}{2(1+N)^2(2+N)^2(3+N)^2}S_{-3} \\
&+\frac{(-24-5N+2N^2)}{12(2+N)^2(3+N)^2}S_1^3 - \frac{1}{2(1+N)(2+N)(3+N)}S_2^2 + \frac{1}{(2+N)(3+N)}S_1^2 S_2 \\
&+\frac{Q_4(N)}{4(1+N)^3(2+N)^2(3+N)^2}S_1^2 - \frac{3}{2}S_5 - \frac{Q_5(N)}{6(1+N)^2(2+N)^2(3+N)^2}S_3 - 2S_{-2,-3} - 2\zeta_3 S_{-2} - S_{-2,1}S_{-2} \\
&+\frac{(-1)^N\left(65+101N+56N^2+13N^3+N^4\right)}{(1+N)^2(2+N)^2(3+N)^2}S_{-2,1} + \frac{(59+42N+6N^2)}{2(1+N)(2+N)(3+N)}S_4 + \frac{(5+N)}{(1+N)(3+N)}\zeta_3 S_1(2) \\
&-\frac{Q_6(N)}{4(1+N)^3(2+N)^2(3+N)^2}S_2 - \zeta_3 S_2 - \frac{3}{2}S_3 S_2 - 2S_{2,1}S_2 + \frac{(99+225N+190N^2+65N^3+7N^4)}{2(1+N)^2(2+N)^2(3+N)}S_{2,1} \\
&+\frac{Q_3(N)}{(1+N)^4(2+N)^4(3+N)^4}S_1 - \frac{(11+5N)}{(1+N)(2+N)(3+N)}\zeta_3 S_1 - \frac{Q_7(N)}{4(1+N)^2(2+N)^2(3+N)^2}S_2 S_1 - S_{2,3} \\
&+\frac{(53+29N)}{2(1+N)(2+N)(3+N)}S_3 S_1 - \frac{3(3+2N)}{(1+N)(2+N)(3+N)}S_1 S_{2,1} + \frac{(-79-40N+N^2)}{2(1+N)(2+N)(3+N)}S_{3,1} - 3S_{4,1} \\
&+S_{-2,1,-2} + \frac{\mathbf{2^{N+1}}\left(-28-25N-4N^2+N^3\right)}{(1+N)^2(2+N)(3+N)^2}S_{1,2}\left(\frac{1}{2},1\right) - \frac{(-7+2N^2)}{(1+N)(2+N)(3+N)}S_{2,1,1} \\
&+5S_{2,2,1} + 6S_{3,1,1} + \frac{\mathbf{2^{N}}\left(-28-25N-4N^2+N^3\right)}{(1+N)^2(2+N)(3+N)^2}S_{1,1,1}\left(\frac{1}{2},1,1\right) \\
&-\frac{(5+N)}{(1+N)(3+N)}S_{1,1,2}\left(2,\frac{1}{2},1\right) - \frac{(5+N)}{2(1+N)(3+N)}S_{1,1,1,1}\left(2,\frac{1}{2},1,1\right)
\end{aligned}
$$

It can be calculated with an extension of the method of hyperlogaritms [31] to the case of massive graphs with operator insertion [19] and is of weight $\mathsf{w} = 5$. One notices the emergence of terms growing individually like $\propto 2^N$, which would potentially imply an instability at large N. However, the asymptotic expansion of the function $\hat{I}_4(N)$ shows that the corresponding terms cancel. In case of this and more involved topologies both in the sum-representation and likewise also in that by iterated integrals the individual entities of the representation, despite spanning the algebraic basis, partly act together forming the physical structures. Individually they may not reflect the properties of the complete diagram.

5. Massive Benz and V-Topologies

The method of hyperlogarithms is also suited to compute non-divergent diagrams of other massive topologies such as Benz-diagrams and the V-topology. This has been done in [18]. The diagram

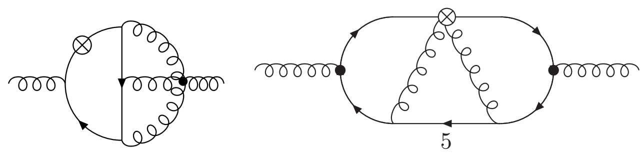

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Figure 3: An example of a diagram with Benz subtopology and a diagram of the V-topology

shown in Figure 3 (left) results in

$$
\begin{aligned}
I(N) = & \frac{1}{(N+1)(N+2)}\Biggl\{\frac{2\left(1-13(-1)^N+(-1)^N 2^{3+N}+N-7(-1)^N N+3(-1)^N 2^{1+N} N\right)}{(1+N)(2+N)}\zeta_3 \\
& +\frac{1}{(2+N)}S_3+\frac{(-1)^N}{2(2+N)}S_1^3-\frac{(-1)^N(3+2N)}{2(1+N)^2(2+N)}S_2+\frac{5(-1)^N}{2}S_2^2 \\
& +\frac{(-1)^N(3+2N)}{2(1+N)^2(2+N)}S_1^2-\frac{(-1)^N}{2}S_2S_1^2+\frac{3(-1)^N(4+3N)}{(1+N)(2+N)}S_3+3(-1)^N S_4+\frac{2}{(2+N)}S_{-2,1} \\
& +2(-1)^N\zeta_3 S_1(2)+\frac{2(-1)^N(3+N)}{(1+N)(2+N)}S_{2,1}-12(-1)^N S_1\zeta_3 \\
& +\frac{(-1)^N(5+7N)}{2(1+N)(2+N)}S_1S_2+3(-1)^N S_1S_3+4(-1)^N S_{2,1}S_1-4(-1)^N S_{3,1} \\
& -\frac{4\left((-1)^N 2^{2+N}-3(-2)^N N+3(-1)^N 2^{1+N} N\right)}{(1+N)(2+N)}S_{1,2}\left(\frac{1}{2},1\right)-5(-1)^N S_{2,1,1} \\
& +\frac{2\left(-(-1)^N 2^{2+N}-13(-2)^N N+5(-1)^N 2^{1+N} N\right)}{(1+N)(2+N)}S_{1,1,1}\left(\frac{1}{2},1,1\right) \\
& -2(-1)^N S_{1,1,2}\left(2,\frac{1}{2},1\right)-(-1)^N S_{1,1,1,1}\left(2,\frac{1}{2},1,1\right)\Biggr\}.
\end{aligned}
$$

Also in this case the asymptotic expansion is regular. The corresponding representation in x-space leads to generalized harmonic polylogarithms. In the case of the massive V-topology, cf. Figure 3 (right), further extensions arise. Here finite nested binomial and inverse binomial sums weighted with generalized harmonic sums contribute. In x-space root-valued letters contribute to the alphabet, extending those of the harmonic polylogarithms by 30 letters in the case of the given graph. An example of a contributing sum is

$$
\begin{aligned}
\sum_{i=1}^{N}\frac{1}{(i+1)\binom{2i}{i}}\sum_{j=1}^{i}\binom{2j}{j}\frac{1}{j}S_2(j) = & \int_0^1 dx\frac{x^N-1}{x-1}\Biggl[\frac{x}{2}\left(H^*_{\mathsf{w}_8,\mathsf{w}_8,1,0}(x)-\zeta_2 H^*_{\mathsf{w}_8,\mathsf{w}_8}(x)\right) \\
& -\frac{x}{\sqrt{x-1/4}}\left(H^*_{\mathsf{w}_8,1,0}(x)-\zeta_2 H^*_{\mathsf{w}_8}(x)\right)\Biggr] \\
& +\frac{2}{3}\zeta_3\int_0^1 dx\frac{\left(\frac{x}{4}\right)^N-1}{x-4}\left[\frac{x}{2}H^*_{\mathsf{w}_3}(x)-\frac{x}{\sqrt{1-x}}\right],
\end{aligned}
$$

with the letters $\mathsf{w}_3, \mathsf{w}_8$ given by

$$\mathsf{w}_3 = \frac{1}{x\sqrt{1-x}}, \qquad \mathsf{w}_8 = \frac{1}{x\sqrt{x-1/4}}.$$

Here the harmonic polylogarithms H^* are defined as iterated integrals w.r.t. the point $x = 1$. In the case of the scalar integral of diagram Figure 3 (right) potential divergencies $\propto 8^N, 4^N$ cancel, while the one $\propto 2^N$ remains. It is expected to cancel for the physical graphs.

6. $\mathbf{O(\alpha_s^2)}$ Charged Current Corrections

Charged current data on heavy flavor production will improve the sea-quark densities. Therefore, here the $O(\alpha_s^2)$ QCD corrections are desirable. In the present analyses [32] the $O(\alpha_s)$ contributions, cf. [33, 34], are used. Since the charged current HERA data are located in the high Q^2 region, the asymptotic form of the $O(\alpha_s^2)$ corrections yields a sufficient representation. It has been studied in Ref. [35] before. Recently these corrections have been derived independently in [36] giving the representations both in Mellin and x-space, extending the former analysis and correcting some errors.

7. Calculation of OMEs containing Benz graphs

Recently we have calculated the massive 3-loop OMEs $A_{qq,Q}^{(3),\rm NS}$ and $A_{qq,Q}^{(3),\rm NS,TR}$ for general values of N and obtained the Wilson coefficient $L_{qq,Q}^{(3),\rm NS}$, cf. [6,8]. The corresponding class of graphs contains also massive Benz diagrams. An extension of the code `Reduze 2` [37, 38] to graphs with local operator insertions allowed to reduce the corresponding integrals to master integrals, which have been calculated using hypergeometric, Mellin-Barnes and advanced summation techniques [20]. In course of this we have also computed the complete 2-loop anomalous dimensions for transversity $\gamma_{\rm NS,TR}^{\pm,(1)}$ [39] and the contributions $\propto T_F$ of the 3-loop anomalous dimensions $\gamma_{\rm NS}^{\pm,(2)}$ and $\gamma_{\rm NS,TR}^{\pm,(2)}$ in an ab initio calculation. In the first case we confirm the results of [40–44] and in the second case our earlier moments [8] and the results in [45, 46]. Details of this calculation are given in [47]. The calculation of further massive OMEs is underway.

8. Conclusions

Recently progress has been made towards the complete calculation of the 3-loop heavy flavor corrections to DIS in the region $Q^2 \gg m^2$, including the matrix elements needed in the variable flavor number scheme at general values of N. The $O(n_f T_F^2 C_{F,A})$ contributions have been completed. The gluonic $O(T_F^2)$ terms are currently calculated, after all principal topologies have been solved. The renormalization in the 2-mass case has been performed and for all OMEs the moments $N = 2, 4, 6$ were calculated. Also the setup for a VFNS in case *both* charm and bottom become massless, has been derived. No hierarchy exists for these terms individually. This scheme is different from the former single mass VFNS. Diagrams of ladder-, V- and Benz-topologies containing no singularities in ε can be systematically calculated. Here new functions occur, including a larger number of root-letters in iterated integrals. All logarithmic contributions to the asymptotic heavy flavor

Wilson coefficients have been determined [10]. After the two Wilson coefficients $L_{qq,Q}^{(3),\rm ps}$ and $L_{qg,Q}^{(3)}$ had been computed in [13] we have calculated $L_{qq,Q}^{(3),\rm NS}$ and $A_{qq,Q}^{(3),\rm NS,TR}$ as well as the associated 2- and 3-loop anomalous dimensions. The calculation of further Wilson coefficients is underway.

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