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B-physics with $N_f = 2$ Wilson fermions

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We report the final results of the ALPHA collaboration for some B-physics observables: f_B, f_{B_s} and m_b. We employ CLS configurations with 2 flavors of $O(a)$ improved Wilson fermions in the sea and pion masses ranging down to 190 MeV. The b-quark is treated in HQET to order $1/m_b$. The renormalization, the matching and the improvement were performed non-perturbatively, and three lattice spacings reaching $a = 0.048$ fm are used in the continuum extrapolation.

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1. Introduction

Many tests of the Standard Model in the B-physics sector need inputs from the lattice.

First, in experiments such as LHCb and B-factories new physics signals may appear through deviations from the Standard Model predictions of B decays, typically occurring through the weak interaction. The theoretical computations have some hadronic matrix elements as inputs, which due to their intrinsically non-perturbative nature can only be computed from first principles on the lattice.

Second, the b-quark mass (usually in the $\overline{MS}$ scheme) is a necessary input in many perturbative computations of SM processes and in the running of SM parameters. The most precise determination cited by the PDG is obtained by matching the perturbative computation of the moments of the cross section $\sigma(e^+e^- \to b\bar{b})$ with experiment. In comparison, the lattice computation of the RGI b-quark mass M_b by means of step scaling methods, and using the experimental value of the B-meson mass as input, relies on a matching with the 2-loop anomalous dimension and 3-loop β-function in the SF scheme at scales of order ~ 100 GeV. The comparison of these two results obtained with completely different systematics provides an interesting test of the Standard Model.

Third, it is an important test of QCD to reproduce the B-mesons mass spectrum, which has been measured quite precisely. Due to its non-perturbative nature, it can only be computed, in a model independent way, on the lattice.

The aim of this long-standing project is to provide a computation of the above quantities in $N_f = 2$ QCD keeping all systematic uncertainties under control.
In these proceedings I will review the method employed by the ALPHA collaboration to achieve this task and present the results obtained, which I roughly divide into predictions:

$$f_B, \quad f_{B_s}, \quad f_{B_s}/f_B \tag{1.1}$$

and postdictions:

$$m_b^{\overline{MS}}(m_b), \quad m_{B_s} - m_B, \quad m_{B^*} - m_B, \quad m_{B_s^*} - m_{B_s}. \tag{1.2}$$

2. Setup

The results presented in this work are obtained from measurements on the CLS ensembles detailed in Table 1, which have 2 degenerate $O(a)$ improved Wilson quarks in the sea. The improvement and renormalization were performed non-perturbatively [1–3], and the continuum extrapolation is based on three lattice spacings: $0.048, 0.065, 0.075$ fm, where the scale was set using $f_{\rm K}$ [3]. As it can be seen in Table 1, the pion masses to be used in the chiral extrapolation lie in the range $190\,\text{MeV} \lesssim m_\pi \lesssim 450\,\text{MeV}$, while volume effects are expected to be sufficiently suppressed in $m_\pi L$ ($\sim e^{-m_\pi L}$), given that the condition $m_\pi L \geq 4$ is always satisfied.

The finest lattice corresponds to a cutoff $a^{-1} \sim 4$ GeV and therefore cannot be used to simulate the b-quark relativistically. Our approach is to employ HQET, an effective theory of QCD based on an expansion in powers of $1/m_b$. We work at order $1/m_b$ and to this order the HQET Lagrangian is:

$$\begin{aligned}\mathscr{L}_{\rm HQET}(x) &= \mathscr{L}_h^{stat} - \omega_{\rm kin}\mathscr{O}_{kin}(x) - \omega_{\rm spin}\mathscr{O}_{spin}(x) \qquad (2.1)\\ &= \overline{\psi}_{\rm h}(x)\,D_0\,\psi_{\rm h}(x) + m_{\rm bare}\overline{\psi}_{\rm h}(x)\psi_{\rm h}(x) - \omega_{\rm kin}\overline{\psi}_{\rm h}(x)\mathbf{D}^2\psi_{\rm h}(x) - \omega_{\rm spin}\overline{\psi}_{\rm h}(x)\boldsymbol{\sigma}\cdot\mathbf{B}\,\psi_{\rm h}(x)\end{aligned}$$

where $m_{\rm bare}$, $\omega_{\rm kin}$ and $\omega_{\rm spin}$ are parameters to be evaluated through a matching with QCD. In particular $m_{\rm bare}$ is needed to absorb the linear divergence arising from the $\overline{\psi}_{\rm h}(x)D_0\psi_{\rm h}(x)$ term. The appearance of power divergences, which is due to operator mixing under renormalization, is a general feature of HQET. As a consequence, the renormalization has to be carried out non-perturbatively.

Notice that the theory remains renormalizable at every order in the $1/m_b$ expansion. This is so because the exponential of the action is to be expanded in powers of $1/m_b$, so that the operators suppressed by powers of m_b only appear in correlators as insertions:

id	L/a	a [fm]	$m_{\rm PS}$[MeV]	$m_{\rm PS}L$
A4	32	0.0748	380	4.7
A5			330	4.0
B6	48		270	5.2
E5	32	0.0651	440	4.7
F6	48		310	5.0
F7			270	4.3
G8	64		190	4.1
N5	48	0.0480	440	5.2
N6			340	4.0
O7	64		270	4.2

Table 1: CLS ensembles used in this work.

$$\langle \mathscr{O} \rangle = \langle \mathscr{O} \rangle_{stat} + \omega_{\rm kin}\sum_x \langle \mathscr{O}\mathscr{O}_{kin}(x)\rangle_{stat} + \omega_{\rm spin}\sum_x \langle \mathscr{O}\mathscr{O}_{spin}(x)\rangle_{stat} + O(1/m_b^2) \tag{2.2}$$

$$\langle \mathscr{O} \rangle_{stat} = \frac{1}{\mathscr{Z}}\int_{fields} \mathscr{O} \exp\left(-a^4\sum_x [\mathscr{L}_{light} + \mathscr{L}_h^{stat}]\right) . \tag{2.3}$$

The advantage of our approach, as compared to other treatments of the b-quark, such as NRQCD, which are not renormalizable, is that we are able to take the continuum limit, once the power divergencies have been subtracted non-perturbatively. In practice this means that we can compute the discretization error on our final results just relying on our data.

The method of the ALPHA collaboration for the non-perturbative renormalization, matching and improvement of HQET has been described in [4]. Here we just recall the basic ingredients.

The matching with QCD is performed in a small fixed volume ($L_1 \sim 0.4$ fm), such that lattice spacings $a^{-1} \gg m_b$ can be simulated while keeping $z_b = M_b L_1 \gg 1$. In order to determine the observables in eqs. (1.1) and (1.2) without $O(a)$ effects, we need to fix the parameters:

$$\omega(z) = m_{\rm bare}(z),\, Z_{\rm A}^{\rm HQET}(z),\, c_{\rm A}^{(1)}(z),\, \omega_{\rm kin}(z),\, \omega_{\rm spin}(z) \tag{2.4}$$

for a range of z values around z_b, where $Z_A^{\rm HQET}$ and $c_A^{(1)}$ are the HQET parameters of the renormalized time component of the axial current in NLO HQET. We impose the matching conditions

$$\Phi^{\rm HQET}(z,a) = \Phi^{\rm QCD}(z,0) = \lim_{a\to 0}\Phi^{\rm QCD}(z,a)\,, \tag{2.5}$$

for a suitable set of observables Φ_i. Since we match with $O(a)$ improved QCD in the continuum limit, we are able to renormalize and improve HQET in one step. Subsequently the $\omega(z)$ corresponding to the lattice spacings to be used in large-volume simulations can be determined through a step-scaling procedure. To better control discretization effects, this process is repeated for two different static actions, denoted by HYP1 and HYP2 [5].

These parameters are combined with large-volume HQET matrix elements to give physical predictions. One important source of systematic errors is then the contamination from excited states. If E_2 and E_1 are the energies of the first excited state and the ground state respectively, this contamination is suppressed by $e^{-(E_2-E_1)t}$, where t is the sink-source separation. To achieve a better suppression, we solve the Generalized Eigenvalue Problem (GEVP) for a 3×3 correlator matrix, where each entry of the matrix corresponds to a different Gaussian smearing level of the B-meson interpolating field. In this way we achieve a suppression $e^{-(E_4-E_1)t}$ [6]. To ensure that the resulting systematic error is negligible we only take plateau averages where $\sigma_{\rm sys} < \frac{1}{3}\sigma_{\rm stat}$ is satisfied, with $\sigma_{\rm sys}$ estimated from the GEVP results.

The statistical analysis for all observables presented in the following was performed according to the methods described in [7]. This allows us to take all correlations and autocorrelations into account. In particular the errors coming from the fit performed for the continuum and chiral extrapolations are included in our statistical error.

3. Computation of the b-quark mass

At this point the $\omega(z)$ are known for some values of z, namely $z = 11, 13, 15$, but we need them at the physical value for the b-quark mass, z_b, to make physical predictions. To determine z_b we use a physical observable, which we chose to be the B-meson mass, m_B.

We first perform a chiral and continuum extrapolation according to the HMChPT formula at NLO [8][1]:

$$m_B^{\rm sub} \equiv m_B\left(z, m_{\rm PS}, a, {\rm HYPn}\right) + \frac{3\widehat{g}^2}{32\pi}\left(\frac{m_{\rm PS}^3}{f_{\rm PS}^2} - \frac{m_\pi^3}{f_\pi^2}\right) = B(z) + C\left[\tilde{y}_1^{\rm PS} - \tilde{y}_1^{\pi}\right] + D_{\rm HYPn}a^2\,, \qquad (3.1)$$

where $\tilde{y}_1^{\rm PS} \equiv \left(\frac{m_{\rm PS}}{4\pi f_{\rm PS}}\right)^2$, to get the B-meson mass at the physical pion mass $m_{\rm PS} = m_\pi$ for every z. For the $B^*B\pi$ coupling in the chiral limit $\widehat{g}$, we use the value determined in [9], $\widehat{g} = 0.51(2)$. Then we impose:

$$m_B(z, m_\pi, a = 0)|_{z=z_{\rm b}} \equiv m_{\rm B}^{\rm exp} = 5279.5{\rm MeV} \qquad (3.2)$$

to find the physical b-quark mass. This procedure is illustrated in Fig. 1 and we obtain:

$$z_{\rm b} = 13.17(23)(13)_z\,. \qquad (3.3)$$

The second error arises in the procedure of fixing z. It amounts to 1% and, as explained in [4], it has to be propagated into QCD observables only after their extrapolation to the continuum limit.

Using L_1 and the Λ parameter from [3] and the 4-loop perturbative formulas from [10–13], this value can be converted to the b-quark mass in the $\overline{MS}$ scheme. We get:

$$m_{\rm b}^{\overline{MS}}(m_{\rm b}) = 4.23(11)(3)_z\,{\rm GeV}\,. \qquad (3.4)$$

4. Computation of decay constants and B-meson mass splittings

We can now compute the $\omega(z_b)$ by quadratically interpolating the $\omega(z)$. Combining these parameters with the large-volume HQET matrix elements from our ensembles we determine the observables of interest up to $O(1/m_b^2)$ effects. Physical results are obtained after a combined

[1] Here we use the convention $f_\pi \sim 93$ MeV.

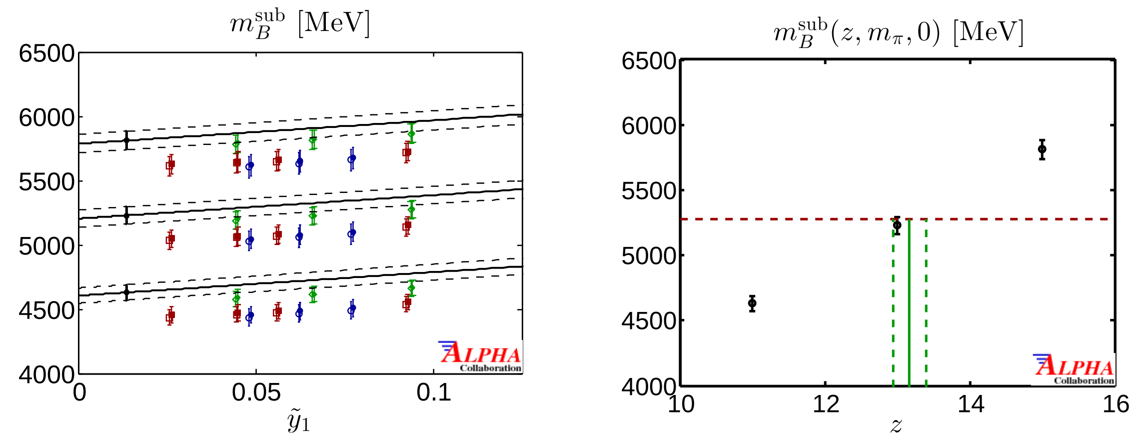

Figure 1: Left: Chiral and continuum extrapolation of $m_B^{\rm sub}$. Right: z_b determination according to eq. (3.2). Green, red, blue points correspond to lattice spacings $a = 0.048$ fm, $a = 0.065$ fm, and $a = 0.078$ fm respectively. Open symbols and dashed lines correspond to the HYP1 static action while filled symbols and continuum lines correspond to HYP2. The band represents the $a = 0$ result.

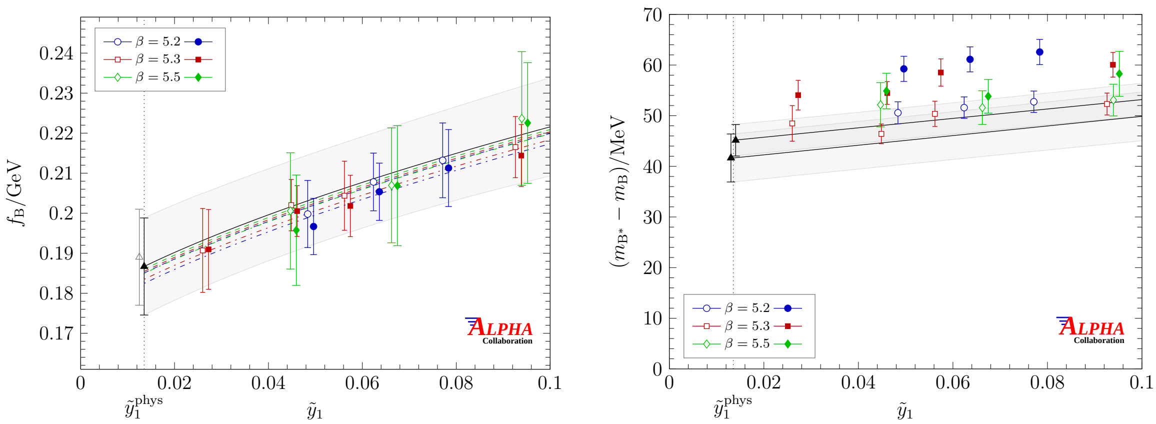

Figure 2: Combined chiral and continuum extrapolation for f_B (left) and $m_{B^*} - m_B$ (right). The same conventions used in Fig. 1 apply. On the left, the extrapolation using HMChPT at NLO (filled triangle) is compared with a linear one (open triangle) to extract the systematic error from truncating HMChPT at NLO. On the right, the extrapolations in a and a^2 are compared to extract the systematic error due to the continuum extrapolation.

chiral and continuum extrapolation. Concerning the chiral extrapolations, if available we take the NLO HMChPT formulas from [8, 14, 15]. Where it is relevant we compare the result from these NLO extrapolations with linear ones in $m_{\rm PS}^2$ and quote the difference as a systematic error from the truncation of the chiral expansion at NLO (subscript *ChPT*).

Since we neglect effects of order $O(a/m_b)$, quantities that are $O(1)$ in HQET are $O(a)$ improved and we expect therefore to reach the continuum limit by extrapolating in a^2. On the other hand, for quantities that are zero in the static limit, such as $m_{B^*} - m_B$, effects of $O(a/m_b)$ might still be relevant. We take this into account by performing one extrapolation linear in a and one in a^2 for these quantities and quote the difference as a systematic error from the continuum limit (subscript a). When no systematic error is quoted, it is because they are found to be negligible compared to

Observable	ALPHA	PDG	Method
m_B[MeV]	input	5279.5	e^+e^- scat.
$m_b^{\overline{MS}}(m_b)$ [GeV]	$4.23(11)(3)_z$	4.18(3)	smeared $\sigma(e^+e^- \to b\bar{b})$+ PT
$m_{B_s} - m_B$ [MeV]	$83.9(6.3)(6.9)_{ChPT}$	87.35(0.23)	pp, $p\overline{p}$ scat.
$m_{B^*} - m_B$ [MeV]	$41.7(4.7)(3.4)_a$	45.3(0.8)	e^+e^- scat.
$m_{B_s^*} - m_{B_s}$ [MeV]	$37.9(3.7)(5.9)_a$	48.7(2.3)	e^+e^- scat.

Table 2: Comparison of ALPHA results for b-quark mass and B-meson mass splittings with Particle Data Group.

Obs.	ALPHA	Lat. Av.	Experiment
f_B [MeV]	$187(12)(2)_{ChPT}$	197(10)	$BR(B \to \tau\nu)_{ALPHA} = 1.065(21) \times 10^{-4}$ $BR(B \to \tau\nu)_{exp} = 1.05(25) \times 10^{-4}$
f_{B_s} [MeV]	224(13)	234(6)	$BR(B_s \to \mu^+\mu^-)_{ALPHA} = 3.15(27) \times 10^{-9}$ $BR(B_s \to \mu^+\mu^-)_{exp} = 2.9(0.7) \times 10^{-9}$
f_{B_s}/f_B	$1.195(61)(20)_{ChPT}$	1.19(05)	

Table 3: Comparison of ALPHA results for decay constants with FLAG (see *http://itpwiki.unibe.ch/flag/index.php*) results and experiment. In the last column we plug our values for f_B and f_{B_s} into the SM predictions for $BR(B \to \tau\nu)$ and $BR(B_s \to \mu^+\mu^-)$ respectively, and compare with experiment. In the first case we need to also input $|V_{ub}|$ from the PDG 12, which is an average of the determination from $BR(B \to \pi l \nu)$ and the one from inclusive decays; in the latter we need to input $|V_{tb}^* V_{ts}|$ from the CKM-fit (see *http://ckmfitter.in2p3.fr/*) , which therefore depends on a large set of observables but mainly the B_s^0 splitting.

present accuracy.

Our results are summarized in Tables 2 and 3 together with a comparison with results from other collaborations and experiment, while the quality of our extrapolations is illustrated in Fig. 2 for some of our observables. More details about our computations, as well as a separation of $1/m_b$ effects, will appear in future publications.

5. Conclusions

As stressed in Table 2 our method, based on NLO lattice HQET for the b-quark, requires just one input from experiment (chosen to be m_B) and allows to make predictions for a wide class of observables. Our computation is the first unquenched one (although with $N_f = 2$) in which renormalization and matching were perfomed entirely in a non-perturbative way. Special attention has been paid to the treatment of correlations and autocorrelations in the statistical analysis, and to the treatment of excited states in the extraction of large-volume matrix elements. The agreement found for the B-meson mass spectrum, albeit with larger errors, gives additional confidence in the correctness of the method and in the values obtained for the decay constants. In the future these methods will be applied to 2+1-flavor simulations.

Concerning the b-quark mass it is quite surprising that our value agrees with the PDG one, considering the different systematics involved ($N_f = 2$ in our case). The error budget analysis shows that nearly 60% of the statistical error comes from the determination of the HQET parameters $\omega(z)$, indicating thus a clear path to further increase the precision of our results.

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