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Strangeness of the nucleon from Lattice Quantum Chromodynamics

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We present a non-perturbative calculation of the strangeness of the nucleon y_N within the framework of lattice QCD. This observable is known to be an important cornerstone to interpret results from direct dark matter detection experiments. We perform a lattice computation for $y_{\rm N}$ with an analysis of systematic effects originating from discretization, finite size, chiral extrapolation and excited state effects leading to a value of $y_N = 0.135(46)$ which turns out to be rather small. As a main result of our work, we will demonstrate that the error for $y_{\rm N}$ is dominated by systematic uncertainties.

INTRODUCTION

The question of the exact composition of the nucleon, i.e. what are the the different quark contents of the proton and neutron, is a long standing problem (see for instance [1]) which can be addressed only through non perturbative methods. In this letter, we will resort to lattice QCD techniques to address the calculation of the nucleon strangeness which is important to know not only for addressing the fundamental question of the nucleon composition, but also, because it plays a most important role in the search for dark matter, as will be discussed in more detail below.

A very useful measure for the strange quark content is the $y_{\rm N}$-parameter,

$$y_{\rm N} \equiv \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}, \tag{1}$$

where u, d and s denote respectively the up, down and strange quark fields. We will refer to $y_{\rm N}$ as the *strangeness of the nucleon* in the following.

As mentioned above, the $y_{\rm N}$ parameter plays an important role in the context of Dark Matter searches. Experiments which aim at a direct detection of dark matter [2–6] are based on measuring the recoil energy of a Nucleon hit by a dark matter candidate. Even if in these processes a dark matter particle would not be detected directly, such experiments allow to provide bounds on the nucleon dark matter cross section which can in turn be translated into constraints on models of New Physics. In many supersymmetric scenarios [7] and in some Kaluza-Klein extensions of the standard model[8, 9] the dark matter nucleon interaction is mediated through a Higgs boson. In such a case the theoretical expression of the spin independent scattering amplitude at zero momentum transfer involves the $y_{\rm N}$-parameter. In fact, even rather small changes of the poorly known value of $y_{\rm N}$ can be responsible of a variation of one order of magnitude of the nucleon dark matter cross section. Having a better determination of the $y_{\rm N}$ parameter would thus provide better estimates on the size of the cross-section or more reliable constraints on dark matter models.

The $y_{\rm N}$-parameter is related to the ratio of the pion-Nucleon ($\sigma_{\pi N}$) and the flavour non-singlet (σ_0) σ-terms, defined as

$$\sigma_{\pi N} \equiv m_l\langle N|\bar{u}u + \bar{d}d|N\rangle, \quad \sigma_s \equiv m_s\langle N|\bar{s}s|N\rangle \tag{2}$$

$$\sigma_0 \equiv m_l\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle \tag{3}$$

where m_l denotes the average up and down quark mass, and m_s the strange quark mass and where we also introduced the strange σ-term σ_s. The σ-terms, $\sigma_{\pi N}$ and σ_0 can be estimated within the framework of chiral effective field theories and using the relation

$$y_{\rm N} = 1 - \frac{\sigma_0}{\sigma_{\pi N}} \tag{4}$$

also estimates of $y_{\rm N}$ can be provided.

To be more specific, the value of $\sigma_{\pi N}$ can be extracted from the pion nucleon cross section data at an unphysical kinematics, known as the Cheng-Dashen point. Values for $\sigma_{\pi N}$ extracted in this way read $\sigma_{\pi N} = 45 \pm 8$ MeV from ref. [10] (GLS) and $\sigma_{\pi N} = 64 \pm 7$ MeV from ref. [11] (GWU). A more recent result has been obtained using baryon covariant chiral perturbation theory in ref. [12] (AMO) which gives $\sigma_{\pi N} = 59 \pm 7$ MeV[12]. The value of σ_0 is generally estimated analyzing the breaking of $SU(3)$ in the spectrum of the octet of baryons. An estimate for this quantity is given e.g. in [13] and reads $\sigma_0 = 36 \pm 7$ MeV.

Using the aforementioned values for $\sigma_{\pi N}$ and σ_0 we obtain the following phenomenological estimates of the y_N parameter :

$$y_N^{\rm GLS} = 0.20(21),\ y_N^{\rm GWU} = 0.44(13),\ y_N^{\rm AMO} = 0.39(14). \tag{5}$$

Note that these values from effective field theory (EFT) and phenomenology, although being affected by substantial errors, are rather large leading to a correspondingly large cross-section for dark matter detection.

In this Letter, we want to go beyond EFT and present a first principle computation of the strangeness of the nucleon using lattice QCD techniques. The difficulty of such a computation has been for a long time that the error for $y_{\rm N}$ has been very large and it was not possible to obtain a precise enough value which can be used for calculating the cross-section reliably. A major reason for the uncertainty in the determination of $y_{\rm N}$ is that it involves dis-connected, singlet contributions.

In ref. [14] we were able to make a significant step forward by using a setup of maximally twisted mass fermions which avoids any mixing in the renormalization of the σ-terms and hence $y_{\rm N}$ does not need to be renormalized. In addition, by employing special noise reduction techniques, amenable for our setup, we could achieve a significant improvement in the signal to noise ratio for $y_{\rm N}$. The shortcoming of our result in ref. [14] has been that, being a feasibility study only, $y_{\rm N}$ was obtained at only one value of the lattice spacing, a single finite volume and only one quark mass.

Here we want to extend the calculation of ref. [14] by using different lattice spacings, finite volumes and quark masses such that we can probe effects of the discretization, the finite volume and non-physical quark masses. In addition, we now have available a high statistics analysis of excited state effects which, as we will see below, are potentially very dangerous for the computations of $y_{\rm N}$. Being able to address the systematic uncertainties appearing in a lattice calculation of $y_{\rm N}$, we believe that our computation can provide a reasonable estimate of the $y_{\rm N}$ parameter based on QCD alone and not resorting to effective field theories.

LATTICE QCD CALCULATION

In our computation of $y_{\rm N}$ we use gluon field configurations generated by the European Twisted Mass Collaboration (ETMC) [15] employing maximally twisted mass fermions. In particular, the setup used here includes a mass-degenerate light up and down quark doublet as well as a strange-charm quark pair, a situation which we refer to as the $N_f = 2+1+1$ setup. In our analysis we have used two values of the lattice spacing, $a = 0.082$ fm and $a = 0.064$ fm, to examine lattice cut-off effects. We have a number of light quark masses leading to pseudoscalar meson masses $m_{\rm PS}$ covering the range from 490 to 220 MeV. This mass range allows us to perform the chiral limit with $m_{\rm PS}$ approaching the physical pion mass m_π. We finally remark that we use a mixed action setup with Osterwalder-Seiler quarks in the heavy quark sector which avoids any mixing due to the iso-spin violation otherwise occurring in the twisted mass sea quark action. This mixed action still enjoys the automatic $O(a)$-improvement of twisted mass fermions.

The basic quantity needed for the evaluation of $y_{\rm N}$ is the ratio of correlation functions

$$R(t,t_s) \equiv \frac{\sum_{\mathbf{x}_{\rm s},\mathbf{x}} \langle \bar{J}(x_{\rm s}) \Big(O_{\rm s}(x) - \langle O_{\rm s}(x)\rangle \Big) J(0)\rangle}{\sum_{\mathbf{x}_{\rm s},\mathbf{x}} \langle \bar{J}(x_{\rm s}) \Big(O_{\rm l} - \langle O_{\rm l}(x)\rangle \Big) J(0)\rangle}, \quad (6)$$

where J is an operator with quantum numbers of the nucleon and $O_{\rm l} = \bar{u}u + \bar{d}d$ and $O_{\rm s} = 2\bar{s}s$. The calculation of the ratio of eq. (6) is particularly challenging because of very noisy contributions originating from dis-connected diagrams. In Eq. (6), $x = (t, \mathbf{x})$ and $x_s = (t_s, \mathbf{x}_{\rm s})$ denote the Euclidean time and space coordinates. We will refer to t and t_s as the source-operator separation and the source-sink separation, respectively. We have shown in [14] that $R(t, t_s)$ does not need to be renormalized since no mixing in the renormalization pattern appears. The ratio $R(t, t_s)$ has the following asymptotic behaviour:

$$R(t,t_s) = y_{\rm N} + \mathcal{O}(e^{-\Delta M t}) + \mathcal{O}(e^{-\Delta M (t_{\rm s}-t)}) \quad (7)$$

where we have denoted with ΔM the mass gap between the ground state and the first excited state of the nucleon. Note that the two additional contributions to $y_{\rm N}$ are non-vanishing as long as t and $t_{\rm s}$ are finite. These two contributions are a systematic effect inherent to any lattice calculation and will be referred to as excited state contamination in the following.

More details on our setup and on the technique to evaluate Eq. (6) can be found in [14] where we discuss in particular the crucial points of our improved variance reduction technique and of the non-perturbative renormalization.

RESULTS AND SYSTEMATIC EFFECTS

Our results for $y_{\rm N}$ are shown in Fig. 1 as a function of $m_{\rm PS}^2$. In the graph, we use different values of lattice spacing a, the physical linear extent of the box L and the source-sink separation $t_{\rm s}$. We perform an extrapolation to the physical pion mass employing a linear fit in $m_{\rm PS}^2$ (solid line) and a quadratic one (dashed line). Note that only the points marked by filled symbols are included in the fits. Open symbols are solely used to demonstrate systematic effects which will be discussed in more detail below. The vertical dotted line in Fig. 1 marks the physical value of the pion mass.

In our Osterwalder-Seiler setup the value of the valence strange quark mass has been tuned in order to match the Kaon mass obtained in the unitary setup. In principle, also other matching conditions could be used leading to different values of the valence strange quark mass. By computing $y_{\rm N}$ for valence strange quark masses varying them by about 40% we could not detect any significant change in $y_{\rm N}$ within our statistical error. Hence, below

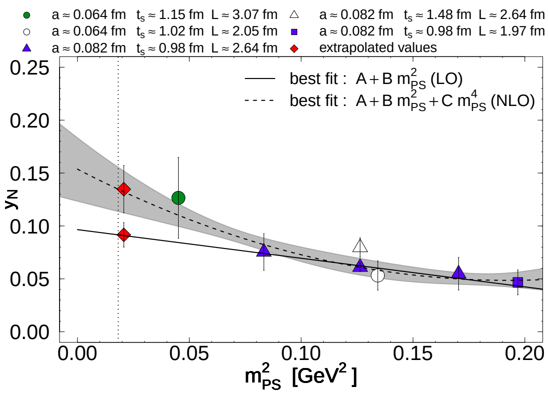

FIG. 1. Our results for y_N as a function of $m_{\rm PS}^2$. The values of the lattice spacing a, the linear extent of the box L and the source-sink separation $t_{\rm s}$ used here are given in the legend. We extrapolate to the physical value of the pion mass (marked by the vertical dotted line) using linear (solid line) and quadratic (dashed line) fits in $m_{\rm PS}^2$. For the quadratic fit, we also show the corresponding error band. Points represented by open symbols are only taken to estimate systematic effects and are not included in our final analysis.

we will not consider the tuning of the valence strange quark mass as a source of systematic errors.

As mentioned above, the excited states contamination needs to be scrutinized carefully in order to obtain reliable results. This is particularly delicate in the case of nucleon matrix elements because the statistical error grows exponentially when t or t_s are increased. Determining the asymptotic regime in t and t_s where the two last terms of Eq. (7) can be safely neglected and a clear plateau behaviour appears is thus often difficult given the typical statistics of lattice calculations for nucleon observables.

In order to study the systematic effects from excited state contamination we computed the ratio $R(t, t_s)$ for $t_s \sim 1.0$ fm (filled triangle in Fig. 1) and $t_s \sim 1.5$ fm (open triangle in Fig. 1) keeping the value of $a = 0.08$ fm and $L = 2.6$ fm fixed. In particular, we find $y_{\rm N} = 0.061(4)$ for $t_s = 1.0$ fm and $y_{\rm N} = 0.081(9)$ for $t_s = 1.5$ fm which indicates a non-negligible excited states contamination of about $\sim 32\%$. We will use the difference between the two values for $y_{\rm N}$ obtained at $t_s = 1.0$ fm and $t_s = 1.5$ fm as an estimate of our systematic error originating from excited states contamination assuming that this systematic effect does not depend strongly on the pseudoscalar meson mass.

We have computed $y_{\rm N}$ also at two different volumes (filled triangles and filled square in Fig. 1). However, we could not detect any significant finite volume effects within the statistical errors such that finite volume effects can be safely neglected as a systematic effect. We also show in Fig. 1 results for $y_{\rm N}$ for two different lattice spacings (filled triangle and empty circle in Fig. 1). The two points are clearly compatible within error bars, indicating that lattice discretization effects are small. Again, we took the difference between the values of these two data points as an estimate of the discretization errors.

In summary our final result reads:

$$y_{\rm N} = 0.135(22)(33)(22)(9) \qquad (8)$$

where the central value is given by the quadratic fit, the first error is statistical, and the last three errors are our estimates of systematic uncertainties, namely the chiral extrapolation, the excited states contamination and the discretization error, respectively. Note that the systematic errors are partly substantially larger than the statistical one and therefore dominate the total error. Adding all errors in quadrature, we find $y_{\rm N} = 0.135(46)$.

DISCUSSION

There are a number of lattice works that concentrate on the independent determination of $\sigma_{\pi N}$ or σ_s (see for instance [16–18] for $N_f = 2+1$ results). Other lattice works for $N_f = 2+1$ provide indirect determinations of y_N [19–22] and our result for y_N is in agreement with these works. We stress, however, that in our work we were able to perform a comprehensive analysis of systematic uncertainties covering lattice spacing and finite volume effects and, in particular, a careful investigation of excited state contamination. Furthermore computing directly the ratio of the matrix element y_N allows us to avoid any assumptions on the domain of validity of EFT relations which is based on $SU(2)$ or $SU(3)$ $HB\chi PT$ expansion and sometimes known only at leading order accuracy.

As in [23], we show in Fig. 2 the $(\sigma_{\pi N}, \sigma_s)$ plane together with vertical colored bands that represent the phenomenological determinations and the corresponding uncertainties of $\sigma_{\pi N}$ mentioned in the introduction. In order to put further constraints on σ_s, we use the following relations,

$$\sigma_s = \frac{1}{2}\frac{m_s}{m_l}\left(\sigma_{\pi N} - \sigma_0\right) = y_{\rm N}\frac{1}{2}\frac{m_s}{m_l}\sigma_{\pi N} \qquad (9)$$

together with the ratio of the quark masses taken from the FLAG group [24]. A first constraint derives from using the phenomenological determination of σ_0 (indicated by σ_0 in Fig. 2). The constraint provided by our work originates the direct computation of $y_{\rm N}$ (gray contour) which includes our estimate of both statistical and systematic errors. As can be seen the result constraints the strange σ-term and suggests an upper bound of ≈ 200 MeV for σ_s.

CONCLUSIONS AND OUTLOOK

In this work, we have performed a direct computation of the strangeness of the nucleon $y_{\rm N}$, including light,

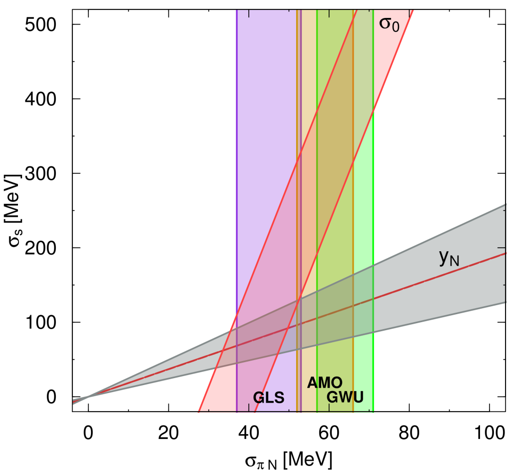

FIG. 2. Constraints on σ_s obtained from our determination of $y_{\rm N}$. The phenomenological determination of $\sigma_{\pi N}$ are represented by colored band as obtained from [10] (GLS), [11] (GWU) and [12] (AMO). We also show the constraint provided by the estimate of σ_0 [13]. As can be seen the value of $y_{\rm N}$ can constrain the value of σ_s to be smaller than about 200 MeV.

strange and charm sea quarks. An emphasis has been laid on the study of systematic effects. Using maximally twisted mass fermions which allow for an efficient noise reduction technique and which avoids mixing under renormalization we have obtained $y_{\rm N} = 0.135(46)$. Our result for $y_{\rm N}$ is compatible with previous determinations [19–22, 25] but includes an analysis of systematic errors originating from discretization, chiral extrapolation and excited states uncertainties. It is worth pointing out that we find a rather low value of the strange $\sigma-$term with corresponding consequences for the nucleon-dark matter cross section.

One important conclusion of our work is that the error we obtain for $y_{\rm N}$ is dominated by systematic uncertainties, in particular the chiral extrapolation and excited state contamination which cannot be neglected. While the error from the chiral extrapolation can be avoided in future calculations which are performed at or very close to the physical value of the pion mass, the excited state contamination cannot be avoided. The error from excited state contamination will hence become the dominating systematic uncertainty which deserves therefore a careful study.

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