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Axions as Hot and Cold Dark Matter

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Abstract

The presence of a hot dark matter component has been hinted at 3σ by a combination of the results from different cosmological observations. We examine a possibility that pseudo Nambu-Goldstone bosons account for both hot and cold dark matter components. We show that the QCD axions can do the job for the axion decay constant $f_a \lesssim \mathcal{O}(10^{10})$ GeV, if they are produced by the saxion decay and the domain wall annihilation. We also investigate the cases of thermal QCD axions, pseudo Nambu-Goldstone bosons coupled to the standard model sector through the Higgs portal, and axions produced by modulus decay.

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I. INTRODUCTION

The standard lambda cold dark matter (LCDM) model of cosmology provides an excellent fit to various cosmological observations, and there is no doubt that the current Universe is dominated by dark energy and dark matter, while ordinary matter is only a minor component. Yet this apparent success does not preclude the existence of an extra component in the dark sector.

Recently it has become clear that there is a tension among different cosmological observations, which gives a preference to a hot dark matter (HDM) component [1–4]. According to Ref. [2], a combination of Planck data, WMAP-9 polarization data, measurements of the BAO scale, the HST measurement of the H_0, Planck galaxy cluster counts and galaxy shear data from the CFHTLens survey yields

$$\Delta N_{\rm eff} = 0.61 \pm 0.30, \quad (1)$$

$$m_{\rm HDM}^{\rm eff} = (0.41 \pm 0.13)\,{\rm eV}, \quad (2)$$

at 1σ, where $\Delta N_{\rm eff}$ denotes the additional effective neutrino species and $m_{\rm HDM}^{\rm eff}$ denotes the effective HDM mass (see Eqs. (4) and (5) for the definition). The other groups obtained similar results.

Let us focus on the extension of the LCDM cosmology by adding a HDM component, although the above results do not exclude the existence of massless dark radiation.[1] The important difference of HDM from massless dark radiation is that it has a small but non-zero mass[2], which calls for some explanation. The light mass could be the result of an underlying symmetry such as shift symmetry, gauge symmetry, or chiral symmetry [6]. In the case of shift symmetry, the corresponding Nambu-Goldstone (NG) boson is expected to have a small but non-zero mass as it is widely believed that there is no exact global

[1] If there are both massless dark radiation and HDM, there will be three coincidences of the abundances of $\rho_{\rm baryon} \sim \rho_{\rm CDM}$, $\rho_{\rm photon} \sim \rho_{\rm dark\,radiation}$, and $\rho_{\rm neutrino} \sim \rho_{\rm HDM}$. The solution may be the dark parallel world with particle contents and interactions that are quite similar, if not identical, to the standard model [5].

[2] There are numerous works on dark radiation. See e.g. Refs. [6–8] for thermal production and Refs. [9] for non-thermal production of dark radiation.

symmetry [10]. The effect of mass is twofold. First, the pseudo Nambu-Goldstone (pNG) bosons behave like HDM whose effect on the cosmological observables cannot be mimicked by massless dark radiation [11]. Secondly, the mass enables the pNG bosons to oscillate around the potential minimum, and the coherent oscillations will contribute to CDM if they are stable in a cosmological time scale. Then there is an interesting possibility that the pNG bosons explain both HDM and CDM, thereby providing a unified picture of the two dark components.

One of the well-studied pNG bosons is the QCD axion, which arises in association with the spontaneous breakdown of the Peccei-Quinn (PQ) symmetry, and its mass is assumed to come predominantly from the QCD anomaly [12]. Not only does the axion provide the most elegant solution to the strong CP problem, but it also contributes to dark matter. See Refs. [13] for the review.

The purpose of this paper is to investigate a possibility that pNG bosons, especially the QCD axions, account for both HDM and CDM, the former of which is preferred by the recent observations. We will also discuss whether a pNG boson coupled to the standard model (SM) through the Higgs portal as well as axions in string theory can similarly do the job.

II. HOT DARK MATTER

Here let us summarize the properties of hot dark matter suggested by the observations [1–4]. The HDM component is relativistic and contributes to the total radiation energy density $\rho_{\rm rad}$ after the electron-positron annihilation and (much) before the photon decoupling. It is customary to express $\rho_{\rm rad}$ in terms of the photon energy density ρ_γ and the effective neutrino species $N_{\rm eff}$ as

$$\rho_{\rm rad} \;=\; \left(1 + N_{\rm eff}\frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4\right)\rho_\gamma, \qquad (3)$$

where T_γ and $T_\nu(= (4/11)^{\frac{1}{3}}T_\gamma)$ are the temperature of photons and neutrinos, respectively. While the effective neutrino species $N_{\rm eff}$ is equal to 3.046 in the standard cosmology, it takes a larger value in the presence of extra relativistic degrees of freedom. The additional effective neutrino species $\Delta N_{\rm eff} \equiv N_{\rm eff} - 3.046$ is given by

$$\Delta N_{\rm eff} \;=\; \frac{\rho_{\rm HDM}}{\rho_{\nu 1}}, \tag{4}$$

where $\rho_{\rm HDM}$ is the HDM energy density, and $\rho_{\nu 1} = (7\pi^2/120)\, T_\nu^4$ is the energy density of a single neutrino species (e.g., $\nu_e + \bar{\nu}_e$). Note that $\Delta N_{\rm eff}$ is evaluated when the HDM component is relativistic.

Following Ref. [14], we define the effective HDM mass $m_{\rm HDM}^{\rm eff}$ as

$$\begin{aligned} m_{\rm HDM}^{\rm eff} \;&\equiv\; m_{\rm HDM} \frac{n_{\rm HDM}}{n_\nu} \\ &=\; (94.1\,\Omega_{\rm HDM} h^2)\,{\rm eV}, \end{aligned} \tag{5}$$

where $m_{\rm HDM}$ is the physical HDM mass, $n_\nu = (3\zeta(3)/2\pi^2) T_\nu^3$ is the number density of a single neutrino species, and $\Omega_{\rm HDM}$ is the density parameter for the HDM with h being the present Hubble parameter in the unit of 100 km s^{-1} Mpc^{-1}. The second equation is derived using the density parameter for the ordinary neutrinos, $\Omega_\nu h^2 = (\sum m_\nu)/94.1\,{\rm eV}$.

If the HDM is thermally distributed, $\Delta N_{\rm eff}$ and $m_{\rm HDM}^{\rm eff}$ are given by

$$\Delta N_{\rm eff} \;=\; \frac{4}{7}\, x\, g \left(\frac{T_{\rm HDM}}{T_\nu}\right)^4, \tag{6}$$

$$\begin{aligned} m_{\rm HDM}^{\rm eff} \;&=\; \frac{2}{3}\, y\, g \left(\frac{T_{\rm HDM}}{T_\nu}\right)^3 m_{\rm HDM} \\ &=\; \frac{2y}{3} \left(\frac{7}{4x}\right)^{\frac{3}{4}} g^{\frac{1}{4}} \left(\Delta N_{\rm eff}\right)^{\frac{3}{4}}\, m_{\rm HDM}, \end{aligned} \tag{7}$$

with

$$x \;=\; \begin{cases} 1 & \text{for boson} \\ 7/8 & \text{for fermion} \end{cases}, \tag{8}$$

$$y \;=\; \begin{cases} 1 & \text{for boson} \\ 3/4 & \text{for fermion} \end{cases}, \tag{9}$$

where $T_{\rm HDM}$ is the HDM temperature, g is the internal degrees of freedom: e.g. $g = 1$ for a real scalar and $g = 2$ for a sterile neutrino. The HDM component becomes non-relativistic when $T_{\rm HDM} \sim m_{\rm HDM}$, i.e., $T_\nu \sim m_{\rm HDM}^{\rm eff}/\Delta N_{\rm eff}$.

The effect on cosmological observables is similar when the HDM is non-thermally produced by particle decay [15]. To be concrete, let us suppose that it has a monochromatic spectrum. Then $\Delta N_{\rm eff}$ and $m_{\rm HDM}^{\rm eff}$ are written as

$$\Delta N_{\rm eff} = \frac{E_{\rm HDM}\, n_{\rm HDM}}{\rho_{\nu 1}}, \tag{10}$$

$$m_{\rm HDM}^{\rm eff} = \frac{7\pi^4}{180\zeta(3)} \Delta N_{\rm eff} \left(\frac{T_\nu}{E_{\rm HDM}}\right) m_{\rm HDM}, \tag{11}$$

where $E_{\rm HDM}$ is the energy of the HDM particle. The HDM component becomes non-relativistic when $E_{\rm HDM} \sim m_{\rm HDM}$, i.e., $T_\nu \sim m_{\rm HDM}^{\rm eff}/\Delta N_{\rm eff}$ as in the case of thermal distribution. Note that, as we shall see in the case of axions, $m_{\rm HDM}$ can be significantly different from $m_{\rm HDM}^{\rm eff}$ depending on the production process and the evolution of the Universe. For instance, the effective mass can be of order 0.1 eV even for a much lighter (heavier) physical mass, if the axions are much "colder (hotter)" than the ambient plasma, i.e., $E_{\rm HDM} \ll T_\nu$ ($E_{\rm HDM} \gg T_\nu$) .

Interestingly, a combination of several different observations suggests the existence of the HDM component in the Universe as in (1) and (2). In the next three sections, we consider various scenarios to examine a possibility that pNG bosons account for both HDM and CDM.

III. QCD AXION DARK MATTER

One of the well-studied pNG bosons is the QCD axion. We introduce a PQ scalar ϕ, which develops a non-zero vacuum expectation value (vev), leading to the spontaneous breakdown of the $\mathrm{U}(1)_{\rm PQ}$ symmetry:

$$\phi = \frac{f_a + s}{\sqrt{2}}\, e^{i\theta}, \tag{12}$$

where $f_a \equiv \sqrt{2}\,\langle\phi\rangle$ is the axion decay constant. Throughout this paper the radial component s is called the saxion. The axion appears as a result of the spontaneous $\mathrm{U}(1)_{\rm PQ}$ breaking. Since the kinetic term for ϕ leads to

$$\partial\phi^\dagger\partial\phi = \frac{1}{2}(\partial s)^2 + \frac{f_a^2}{2}(\partial\theta)^2 + f_a s(\partial\theta)^2 + \frac{s^2}{2}(\partial\theta)^2, \tag{13}$$

the canonically normalized axion field is $a \equiv f_a \theta$. The axion is assumed to acquire a mass predominantly from the QCD anomaly:

$$m_a \simeq 6.0\,\mathrm{eV} \left(\frac{f_a/N_{\rm DW}}{10^6\,\mathrm{GeV}} \right)^{-1}, \tag{14}$$

where $N_{\rm DW}$ is the domain wall number. In the following we will set $N_{\rm DW} = 1$ unless otherwise stated.

In the following we consider thermal and non-thermal production of the axion HDM in turn, and then discuss the axion CDM production by the initial misalignment mechanism and the domain wall annihilation.

A. Thermal production of axion HDM

In the early Universe, axions are produced in thermal plasma, and they contribute to HDM. For the decay constant $f_a \lesssim 10^8\,\mathrm{GeV}$, the axions are dominantly produced by the process $\pi + \pi \to \pi + a$, and decouple after the QCD phase transition. The abundance of thermal axions was evaluated in Ref. [16]. Using the results of Ref. [16], one can estimate $\Delta N_{\rm eff}$ and $m_{\rm HDM}^{\rm eff}$ as

f_a [GeV]	$g_*(T_D)$	$\Delta N_{\rm eff}$	$m_{\rm HDM}^{\rm eff}$ [eV]
3×10^6	14.54	0.382	0.99
1×10^7	16.43	0.325	0.26
3×10^7	21.10	0.233	0.068

Here $g_*(T_D)$ denotes the relativistic degrees of freedom at the decoupling temperature T_D, and we have used the expression of $\Delta N_{\rm eff}$ [6],

$$\Delta N_{\rm eff} = \frac{4}{7} \left(\frac{g_{*\nu}}{g_*(T_D)} \right)^{\frac{4}{3}} \leq \frac{4}{7} \simeq 0.57, \tag{15}$$

with $g_{*\nu} = 43/4$. Therefore, the axion decay constant in the range between $f_a = 3 \times 10^6\,\mathrm{GeV}$ and $1 \times 10^7\,\mathrm{GeV}$ seems consistent with the observationally inferred values (1) and (2).

We here note that the decay constant in the above range is in tension with constraints from various astrophysical arguments. One of the most tight constraints comes from the star cooling argument [17, 18]. The limits however rely on the model-dependent axion couplings with photons and electrons, which can be significantly suppressed in a certain set-up [19]. On the other hand, the axion couplings with nucleons are constrained by the energy loss argument of SN1987A, leading to $f_a \gtrsim 4 \times 10^8\,\mathrm{GeV}$ [20, 21]. However the energy loss from the supernova core due to axion emission becomes ineffective for a sufficiently small decay constant, leaving a narrow allowed range at $f_a = \mathcal{O}(10^6)\,\mathrm{GeV}$, called the hadronic axion window. Although the above range of $f_a = 3 \times 10^6 - 1 \times 10^7\,\mathrm{GeV}$ is slightly above the hadronic axion window, it is worthwhile noting that the limits from SN1987A could contain relatively large uncertainties originated from the adopted assumptions and treatment of the nuclear reaction rate and the state of the nuclear matter in the supernovae core.

The cold axions are produced by the initial misalignment mechanism. For f_a in the above range, however, the abundance of axion coherent oscillations is too small to account for the total dark matter abundance. Alternatively, as we shall see later in this section, a right amount of axion CDM can be produced by the domain wall annihilation.

B. Non-thermal production of axion HDM

Here we will show that the axions produced by the saxion decay can contribute to the HDM, and in particular, it can mimic the hot dark matter with $m_{\mathrm{HDM}}^{\mathrm{eff}} \sim \mathcal{O}(0.1)\,\mathrm{eV}$ even for $f_a \gtrsim 4 \times 10^8\,\mathrm{GeV}$ satisfying the limits from SN1987A.

The saxion is produced by coherent oscillations, and its energy density often dominates or comes close to dominating the Universe. For instance, if the saxion is trapped at the origin by thermal effects, it often drives thermal inflation [22–29]. Furthermore, in a supersymmetric theory, the saxion is a flat direction lifted dominantly by the supersymmetry breaking effect, and therefore it is plausible that the saxion starts to oscillate with a large amplitude, contributing to a significant fraction of the energy of the Universe.

The saxion decays into a pair of axions with the rate

$$\Gamma_s = \frac{c}{32\pi}\frac{m_s^3}{f_a^2}, \tag{16}$$

where c depends on the details of the saxion stabilization [30, 31]. Here we take $c = 1$, which is the case for (13) where $\mathrm{U(1)_{PQ}}$ is broken mainly by ϕ, and assume a sudden decay when the Hubble parameter equals to the decay rate, $H = \Gamma_s$. The saxion can decay into gluons (and gluinos) as well as into Higgs bosons in the DFSZ axion model [32, 33], but for the moment we assume that the saxion mainly decays into axions. In the following we consider two cases, in which (*i*) the saxion dominates the Universe before the decay and subsequently entropy production occurs to dilute the axion density to the observationally allowed value; (*ii*) the saxion energy density is subdominant at the decay.

We note that, even if the saxion dominates the Universe, a significant fraction of the saxion coherent oscillations can evaporate into thermal plasma through the dissipation effect, suppressing the abundance of relativistic axions [34].[3] This is an attractive possibility because the saxion can reheat the Universe without late-time entropy production. This scenario can be approximately modelled by our analysis on the case (*ii*).

First let us consider the case (*i*), in which we assume that the Universe was once dominated by the axion radiation and subsequently a late-time entropy occurs to dilute the axion density to the observationally allowed value. The effective neutrino species $\Delta N_{\rm eff}$ receives a contribution from axions according to [26, 35]

$$\Delta N_{\rm eff} = \frac{43}{7}\left(\frac{43/4}{g_{*R}}\right)^{\frac{1}{3}} r, \tag{17}$$

with

$$r \equiv \left(\frac{\rho_a}{\rho_r}\right)_R, \tag{18}$$

where g_{*R} denotes the relativistic degrees of freedom at the entropy production, and r denotes the ratio of the axion energy density ρ_a to the SM radiation energy density ρ_r at

[3] In Ref. [34], the axion contribution to $\Delta N_{\rm eff}$ was evaluated, but the effect of the axion mass was neglected. We point out that such axions naturally explain the HDM suggested by the recent observations.

the entropy production. The subscript R means that the variables are evaluated at the entropy production. Note that $r < 1$ is required for $\Delta N_{\rm eff}$ to be in the allowed range of Eq. (1).

The axion has an energy equal to $m_s/2$ at the production, and it is red-shifted as the Universe expands. What is relevant for the observation is the effective mass $m_a^{\rm (eff)}$ defined by (cf. Eq. (11))

$$m_a^{\rm (eff)} \;=\; \frac{7\pi^4}{180\zeta(3)}\Delta N_{\rm eff} \left.\frac{T_\nu}{E_a}\right|_{\nu\,{\rm dec}} m_a, \tag{19}$$

where T_ν is the neutrino temperature, E_a the axion energy, and the subscript $\nu\,$dec means that the variables are evaluated at the decoupling of neutrinos. We can evaluate $m_a^{\rm (eff)}$ as follows,

$$\begin{aligned} \left.\frac{T_\nu}{E_a}\right|_{\nu\,{\rm dec}} &= \left.\frac{s^{\frac{1}{3}}}{E_a}\right|_R \left.\frac{T_\nu}{s^{\frac{1}{3}}}\right|_{\nu\,{\rm dec}} \\ &= \left.\frac{T_R}{E_a}\right|_R \times \left(\frac{g_{*R}}{g_{*\nu}}\right)^{\frac{1}{3}}, \end{aligned} \tag{20}$$

where s is the entropy density, $g_{*\nu} = 43/4$ is the relativistic degrees of freedom at the neutrino decoupling, and T_R is the temperature of the SM plasma at the entropy production. Here $T_R/E_a|_R$ is given by

$$\left.\frac{T_R}{E_a}\right|_R \;=\; \left(\frac{\pi^2 g_{*R}}{30}\right)^{-\frac{1}{4}} \frac{2}{m_s}\frac{(3\Gamma_s^2 M_p^2)^{\frac{1}{4}}}{r^{\frac{1}{4}}}. \tag{21}$$

Substituting this result into (19) leads to

$$m_a^{\rm (eff)} \;\simeq\; 0.6\,{\rm eV}\left(\frac{\Delta N_{\rm eff}}{0.6}\right)^{\frac{3}{4}}\left(\frac{m_s}{10^4\,{\rm GeV}}\right)^{\frac{1}{2}}\left(\frac{f_a}{10^9\,{\rm GeV}}\right)^{-2}, \tag{22}$$

where we have used (14).

In the case (ii), the effective neutrino species $\Delta N_{\rm eff}$ is similarly given by

$$\Delta N_{\rm eff} \;=\; \frac{43}{7}\left(\frac{43/4}{g_{*d}}\right)^{\frac{1}{3}}\tilde{r}, \tag{23}$$

with

$$\tilde{r} \;=\; \left.\frac{\rho_s}{\rho_r}\right|_{\rm decay}, \tag{24}$$

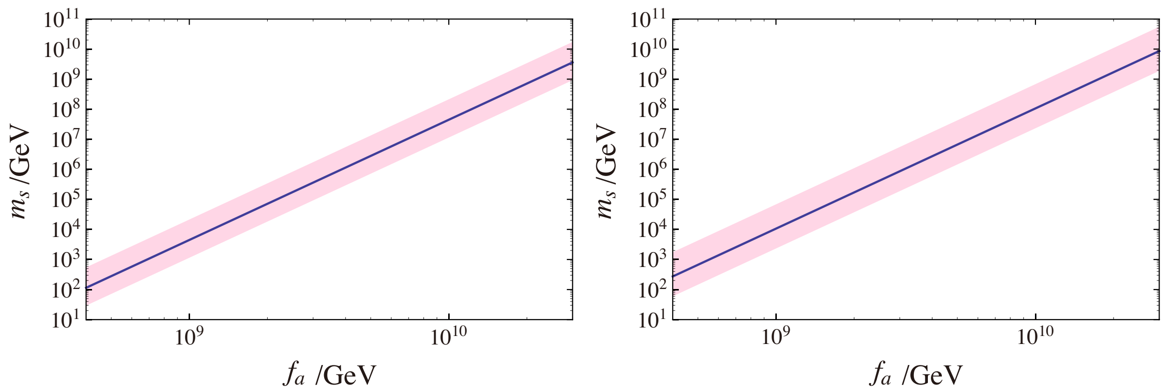

FIG. 1: The HDM abundance and mass $(\Delta N_{\rm eff}, m_a^{\rm eff})$ fall in the 1σ allowed values (1) and (2) in the shaded region on the plane of the axion decay constant f_a and the saxion mass m_s for the case (i) (left) and case (ii) (right). The line inside the shaded region corresponds to the center values of Eq. (1) and (2).

where $\tilde{r}$ denotes the ratio of the saxion energy density to the radiation energy density evaluated at the saxion decay. The effective mass is

$$m_a^{(\rm eff)} \simeq 0.4\,{\rm eV} \left(\frac{g_{*d}}{106.75}\right)^{\frac{1}{12}} \left(\frac{\Delta N_{\rm eff}}{0.6}\right) \left(\frac{m_s}{10^4\,{\rm GeV}}\right)^{\frac{1}{2}} \left(\frac{f_a}{10^9\,{\rm GeV}}\right)^{-2}, \tag{25}$$

where g_{*d} counts the relativistic degrees of freedom in the thermal plasma at the saxion decay, and we have approximated $1+\tilde{r} \simeq 1$ in the above expression. In the numerical estimate we do not use this approximation, but the results are practically the same.

In Fig. 1, we show the 1σ allowed region for $\Delta N_{\rm eff}$ and $m_a^{\rm eff}$ in the (f_a, m_s) plane. One can see that, in both cases (i) and (ii), the axion produced by the saxion decay can account for the HDM for $m_s \approx 10^3 - 10^5\,{\rm GeV}\,(f_a/10^9\,{\rm GeV})^4$ as long as $\Delta N_{\rm eff} \sim \mathcal{O}(0.1)$. Note that the axion decay constant f_a is bounded above, $f_a \lesssim 3 \times 10^{10}\,{\rm GeV}$, for the perturbative stabilization of the PQ scalar.

When the saxion starts to oscillate from the origin after being trapped by thermal effects, the saxon coherent oscillations partially evaporate to form thermal plasma through the dissipation processes, suppressing the axion abundance [34]. Specifically, $\Delta N_{\rm eff} = \mathcal{O}(0.1)$ is realized for the saxion mass ranging from $10^3\,{\rm GeV}$ to $10^4\,{\rm GeV}$ at $f_a = 10^9\,{\rm GeV}$ in a certain set-up. Combined with the above analysis, therefore, we conclude that the axions produced from the saxion decay naturally behave as HDM, and such axion HDM will be a natural outcome of the saxion trapped at the origin.

C. Axion CDM

The axion coherent oscillations are produced by the initial misalignment mechanism, and they contribute to the CDM. Suppose that the PQ symmetry is broken during and after inflation. Then the abundance of axion CDM is approximately given by [36]

$$\Omega_a h^2 \simeq 0.195\,\theta_*^2 F(\theta_*) \left(\frac{f_a/N_{\rm DW}}{10^{12}\,{\rm GeV}}\right)^{1.184}, \tag{26}$$

where $\theta_* \equiv a_*/f_a$ is the initial misalignment angle, and $F(\theta_*)$ represents the anharmonic effect [37],

$$F(\theta_*) = \left[\ln\left(\frac{e}{1-\frac{\theta_*^2}{\pi^2}}\right)\right]^{1.184}, \tag{27}$$

where we have changed the exponent from the original one so as to be consistent with the axion abundance (26). For $\theta_* = \mathcal{O}(1)$, the total CDM abundance can be explained by the axion coherent oscillations with $f_a \approx 10^{11-12}$ GeV. Actually, however, the axion can account for the total CDM abundance even for $f_a \lesssim \mathcal{O}(10^{10})$ GeV, if it initially sits near the hilltop of the potential, thanks to the anharmonic effect. For instance, one needs to fine-tune the initial position near the hilltop at $1\,(0.01)\%$ level for $f_a \simeq 3\times10^{10}\,(10^{10})$ GeV [38].[4]

If the PQ symmetry is restored and becomes spontaneously broken after inflation, topological defects such as cosmic strings and domain walls are produced. Most importantly, the axions are radiated by those topological defects. It depends on the evolution of the string-wall network how many axions are produced. If $N_{\rm DW}$ is equal to unity, strings and domain walls disappear soon after the QCD phase transition due to the tension of the domain walls. The axions radiated by the string-wall network can account

[4] Note that the isocurvature density perturbations are enhanced toward the hilltop initial condition, thereby tightening constraints on the inflation scale [38].

for the total CDM abundance for $f_a \approx (2.0 - 3.8) \times 10^{10}\,\mathrm{GeV}$ [39]. On the other hand, if $N_{\rm DW} \geq 2$, domain walls are stable, leading to the cosmological domain wall problem. To avoid the cosmological catastrophe, one needs to add small PQ symmetry breaking effect, which lifts the degeneracy among different CP conserving vacua. As a result, the domain walls annihilate when the pressure due to the bias becomes comparable to the wall tension [40, 41]. According to Ref. [42], such long-lived domain walls lead to the axion overproduction for $f_a \gtrsim 4 \times 10^8\,\mathrm{GeV}$, unless the CP phase of the PQ symmetry breaking term is finely tuned at more than 1% level. Interestingly, however, the right amount of axions can be produced in the hadronic axion window without fine-tuning of the CP phase of the PQ symmetry breaking term.

In the case where the axion HDM is thermally produced, the required f_a is of order $10^{6-7}\,\mathrm{GeV}$, for which the abundance of the axion coherent oscillations is too small to account for the total DM. As we have seen above, the right amount of axion CDM can be produced by domain wall annihilation without fine-tuning of the CP phase of the PQ symmetry breaking operator. On the other hand, in the case where the axion HDM is non-thermally produced by the saxion decay, the decay constant should be in the range of $f_a = 4 \times 10^8\,\mathrm{GeV} - 3 \times 10^{10}\,\mathrm{GeV}$. For $f_a = \mathcal{O}(10^{10})\,\mathrm{GeV}$, the right amount of axion CDM can be produced by the misalignment mechanism with a hilltop initial condition or by axion radiation from string-wall networks with $N_{\rm DW} = 1$. For a lower f_a, one needs to rely on the domain wall annihilation, which however requires a fine-tuning of the CP phase of the PQ-symmetry breaking at about 1% level. Note that the axion isocurvature perturbation is enhanced at small scales if the axions are produced by domain wall annihilation (see footnote 6).

IV. NAMBU-GOLDSTONE BOSONS THROUGH THE HIGGS PORTAL

We consider a Higgs portal to the global U(1) sector through the interaction,

$$\lambda |\phi|^2 |H|^2, \tag{28}$$

for $\phi = (F+s)e^{ia/F}/\sqrt{2}$ with $F \equiv \sqrt{2}\langle|\phi|\rangle$. Here H is the SM Higgs doublet developing $\langle|H^0|\rangle = v/\sqrt{2}$, and ϕ is the scalar field which breaks spontaneously the U(1) symmetry. Then the radial scalar s and the NG boson a have

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mu v s h + \frac{1}{2}\mu' s^2 h + \frac{1}{2} m_s^2 s^2 + \frac{s}{\Lambda}(\partial a)^2 + \cdots \,, \tag{29}$$

where h is the Higgs boson with mass $m_h \simeq 125$ GeV, and the ellipsis denotes the kinetic terms for s and a, and also the interactions of s and other hidden sector particles if exist. The SM and U(1) sectors are connected via the μ and μ' terms:

$$\mu = \lambda F, \quad \mu' = \lambda v, \tag{30}$$

while the model-dependent parameter Λ is generally of order F. Since the radial scalar couples to $(\partial a)^2$ and mixes with the Higgs boson h, integrating it out gives rise to the effective interaction

$$\frac{\mu}{\Lambda}\frac{m_\psi}{m_h^2 m_s^2}(\partial a)^2 \bar{\psi}\psi, \tag{31}$$

through which the NG bosons can be thermalized with ordinary particles. Here ψ is the SM fermion with mass m_ψ. The contribution of NG bosons to $\Delta N_{\rm eff}$ is not much smaller than 4/7 if they decouple after the QCD phase transition [6, 7]. For this to be the case, the radial scalar should be much lighter than the Higgs boson so that the above interaction is strong enough for Λ around F. The NG bosons remain in thermal equilibrium until the era of muon annihilation if the portal takes place with

$$\frac{\mu}{\Lambda} \approx 10^{-3}\left(\frac{m_s}{200\,\rm MeV}\right)^2, \tag{32}$$

for m_s around or above the muon mass. One should note that the mixing between s and h is suppressed when

$$\mu \ll m_h^2/v \sim 10^2\,\rm GeV, \tag{33}$$

implying that Λ should be lower than about 10^5 GeV. In addition, μ/Λ is constrained to be smaller than 10^{-2} from the requirement that the branching fraction of Higgs decay into NG bosons be smaller than about 0.2 [43]. Combined with the condition (32), this requires the radial scalar to be lighter than about 1 GeV.

On the other hand, if the global U(1) symmetry is only an approximate one, the NG boson acquires a non-zero mass. Such pNG boson may be able to account for both HDM and CDM. We pursue this possibility in the rest of this section.

Suppose that the global U(1) symmetry is explicitly broken to the Z_n subgroup by the following interaction;

$$\Delta\mathcal{L} = \frac{\phi^n}{nM^{n-4}} + \text{h.c.}, \tag{34}$$

with an integer $n \geq 5$, where M is a cut-off scale. Assuming that the above interaction does not change the potential minimum for s, the potential of a reads

$$V = \frac{m_a^2 F^2}{n^2}\left(1 - \cos\left(\frac{na}{F}\right)\right), \tag{35}$$

with the pNG boson mass given by

$$m_a^2 = \frac{n}{2^{n/2-1}}\frac{F^{n-2}}{M^{n-4}}. \tag{36}$$

For instance, the mass is about 1 eV for the case with $F = 50$ TeV, $n = 6$ and $M = M_p$:

$$m_a \simeq 1\,\text{eV}\left(\frac{F}{50\,\text{TeV}}\right)^2\left(\frac{M_p}{M}\right), \tag{37}$$

where $M_p \simeq 2.4 \times 10^{18}\,\text{GeV}$ is the reduced Planck mass.

The pNG bosons are thermalized through the Higgs portal if the radial component s is relatively light. Specifically, one can obtain $\Delta N_{\rm eff} = \mathcal{O}(0.1)$ for $m_s \approx 100$ MeV and $F \lesssim 10^5$ GeV, while satisfying the limit coming from the invisible Higgs decay [7]. The effective HDM mass is calculated as

$$m_a^{\rm eff} \simeq 0.69\left(\frac{\Delta N_{\rm eff}}{0.6}\right)^{\frac{3}{4}} m_a, \tag{38}$$

by using the relation Eq. (7).

One important phenomenon associated with the spontaneous break down of such discrete symmetry is the domain wall formation. We consider the production of pNG bosons from the domain walls in the rest of this section, because a coherent production of the pNG bosons cannot generate the right amount of CDM for the decay constant F that leads to thermalization of pNG bosons through the Higgs portal.

The tension of the domain wall σ is given by

$$\sigma = \frac{8 m_a F^2}{n^2}. \tag{39}$$

According to the numerical simulation [42, 44], the domain-wall network exhibits a scaling behavior. Assuming the radiation dominated Universe, the scaling regime implies

$$\rho_{\rm dw} = 2\mathcal{A}\,\sigma H, \tag{40}$$

where $H = 1/2t$ is the Hubble parameter, and $\mathcal{A} \simeq 2.6$ was obtained in the numerical simulation [42].

The domain walls should disappear before they start to dominate the Universe, as the Universe would be significantly anisotropic. The domination takes place when

$$H_{\rm dom} = \frac{2\mathcal{A}\sigma}{3M_p^2}. \tag{41}$$

Hence $H_{\rm decay} \gg H_{\rm dom}$ must be satisfied, where $H_{\rm decay}$ is the Hubble parameter when the domain walls annihilate. In order to make the domain walls annihilate, we need to introduce a bias that lifts the degeneracy among the n vacua. It is customary to parameterize the bias parameter as

$$\begin{aligned} \delta V &= -\sqrt{2}\xi F^3 \phi e^{i\delta} + \text{h.c.}, \\ &= -2\xi F^4 \cos(\theta - \delta), \end{aligned} \tag{42}$$

where ξ is a dimensionless parameter. The typical difference of the energy density between the adjacent vacua is roughly estimated to be

$$\epsilon \sim \frac{8\xi F^4}{n} \tag{43}$$

or less. Naively, the domain walls start to disappear when the pressure due to the bias ϵ becomes comparable to the energy of the walls. This happens when $\epsilon \sim \rho \simeq 2\mathcal{A}\sigma H$, i.e.,

$$H_{\rm decay} = \frac{1}{2\beta\mathcal{A}} \frac{n\xi F^2}{m_a}, \tag{44}$$

where we have inserted a numerical coefficient β to represent the uncertainty of such naive analytic estimate. According to the numerical simulation [45], it is given by $\beta\mathcal{A} \sim 18$.[5]

There is another important parameter to evaluate the pNG boson abundance. That is the average momentum of the pNG bosons produced by the domain wall annihilation. It was shown that thus produced pNG bosons are marginally relativistic, and the ratio of the averaged momentum to the mass, ϵ_a, is given by

$$\epsilon_a \sim 1.2 - 1.5. \tag{45}$$

Thus, the produced pNG bosons will soon become non-relativistic due to the cosmic expansion. The precise value of ϵ_a is not important, but we will set it to be 1.5 in the following discussion.

The domain walls should annihilate much before the matter-radiation equality, as the dark matter isocurvature perturbations get enhanced at small scales as $\propto k^{\frac{3}{2}}$.[6] In order to be consistent with the primordial density perturbations inferred from various observations [47–49], we require $H_{\rm decay} \gtrsim \mathcal{O}(10^{-22})$ eV, which corresponds to the decay temperature $T_d \gtrsim$ keV. The axion abundance is therefore given by

$$\frac{\rho_a}{s} = \frac{1}{\sqrt{1+\epsilon_a^2}} \frac{2\mathcal{A}\sigma H_{\rm decay}}{\frac{2\pi^2 g_{*s}}{45} T_d^3}, \tag{46}$$

or equivalently,

$$\Omega_a h^2 \simeq 0.1 \left(\frac{6}{n}\right)^2 \left(\frac{m_a}{1\,{\rm eV}}\right) \left(\frac{F}{2\times 10^3\,{\rm TeV}}\right)^2 \left(\frac{T_d}{1\,{\rm keV}}\right)^{-1}. \tag{47}$$

Thus, the decay constant is required to be larger than $\mathcal{O}(10^3)$ TeV for the pNG bosons produced by the domain wall annihilation to comprise the total dark matter. This is slightly too large for the pNG bosons to be thermalized through the Higgs portal at temperature after the QCD phase transition.

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[5] Note that this estimate based on the numerical simulation may contain a relatively large systematic uncertainty, because it relies on extrapolating the results by many orders of magnitude.

[6] This may lead to the formation of ultra-compact mini-halos. If a small fraction of dark matter consists of thermally-produced weakly-interacting massive particles, they may annihilate inside the mini-halos, producing an observable amount of gamma-rays [46].

The tension for obtaining both HDM and CDM can be understood as follows. In order to keep the pNG bosons in thermal equilibrium after the QCD phase transition, its interactions should be strong enough, placing an upper bound on F. On the other hand, one needs a larger value of F to produce the right amount of CDM by domain walls (cf. Eq. (47)).

The crucial assumption in the above argument is that the domain wall network follows the scaling law. We may parameterize the deviation from the scaling law as

$$\rho_{\rm dw} \approx \sigma H_{\rm form} \left(\frac{H}{H_{\rm form}} \right)^p . \tag{48}$$

The scaling regime is recovered for $p = 1$, and the so called frustrated domain wall network correspond to $p = 1/2$ [50]. In the extreme case of the frustrated domain walls, the domain wall abundance can be enhanced by a factor of $T_{\rm form}/T_{\rm decay} \sim 100{\rm MeV}/1{\rm keV} \sim 10^5$. Then we can explain the DM abundance even for $F \sim \mathcal{O}(10)$ TeV, with which the thermalized pNG bosons decouple after the QCD phase transition, leading to $\Delta N_{\rm eff} = \mathcal{O}(0.1)$, for a sufficiently light m_s. Thus, deviation from the scaling law is required for the pNG bosons to account for both HDM and CDM.

Alternatively, if we extend the set-up by introducing additional interactions of ϕ, we may be able to evade this conclusion.[7] For instance, the ϕ may be thermalized while it is trapped at the origin by its additional interactions. Then, after the phase transition, a half of the thermalized ϕ particles will be transformed to the pNG bosons. If this phase transition occurs after the QCD phase transition, $\Delta N_{\rm eff} = \mathcal{O}(0.1)$ will be realized. We may also introduce multiple scalar fields by extending the global U(1) symmetry to a larger group, which relaxes the upper bound on F. Also, if the mass of ϕ is time-dependent, it may affect the evolution of the domain-wall network, alleviating the aforementioned tension.

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[7] One may also consider the Higgs portal implemented by another scalar field, for instance, by a real scalar φ. The effective action for φ at scales below F is read off from (29) by taking the replacement $s \to \varphi$. For $F \gtrsim 10^3$ TeV, one can then obtain $\Delta N_{\rm eff}$ within the range of (1) by taking Λ smaller than F and an appropriate value of μ satisfying the condition (32).

If the pNG bosons are produced non-thermally by the decay of s, we may be able to relax the tension. In particular, the effect of thermal evaporation may also help. We leave the detailed analysis in this case for future work.

V. AXIONS FROM MODULUS DECAY

Moduli fields are ubiquitous in the supergravity/string theory, and they must be successfully stabilized in order to get a sensible low-energy theory. Many of them can be stabilized by the flux compactification [51, 52] or by the KKLT mechanism [53]. In this case the moduli fields have approximately supersymmetric spectrum, and in particular, there are no light axions. However, some of them may be stabilized by supersymmetry breaking effects in such a way that their axionic fields remain light due to the shift symmetry. The corresponding moduli fields tend to be lighter than those stabilized in a supersymmetric fashion, and their masses are comparable to or lighter than the gravitino mass. Such light non-supersymmetric moduli fields tend to dominate the Universe and so play an important cosmological role. Indeed, it was recently pointed out in Ref. [62] that axions are often overproduced by the decay of non-supersymmetric moduli, contributing to $\Delta N_{\rm eff}$. (See also Refs. [63, 64] in the context of LARGE volume scenario.) Here we consider a case in which the produced axions have a small mass and behave as HDM.

Let us suppose that the modulus field ϕ dominates the Universe and decays into axions as well as the standard model particles. The contribution of axions to $\Delta N_{\rm eff}$ is given by [26, 35]

$$\Delta N_{\rm eff} \;=\; \frac{43}{7}\left(\frac{g_{*\nu}}{g_*(T_d)}\right)^{\frac{1}{3}}\frac{B_a}{1-B_a}, \tag{49}$$

where B_a denotes the branching fraction into axions. The 1σ allowed range of $\Delta N_{\rm eff}$ given by Eq. (1) is realized with $B_a = 0.09 \pm 0.04\,(0.18 \pm 0.06)$ for $g_* = 10.75\,(106.75)$. Here T_d is the decay temperature of the moduli defined by

$$T_d \;=\; (1-B_a)^{\frac{1}{4}}\left(\frac{\pi^2 g_*(T_d)}{90}\right)^{-\frac{1}{4}}\sqrt{\Gamma_\phi M_p}\,, \tag{50}$$

with Γ_ϕ being the total decay rate of the modulus ϕ. Let us parameterize the total decay rate by

$$\Gamma_\phi \;=\; \frac{\beta}{4\pi}\frac{m_\phi^3}{M_p^2}, \tag{51}$$

where β is a numerical coefficient of order unity. In order not to spoil the success of big bang nucleosynthesis, the modulus mass should be heavier than about 100 TeV.

The effective axion HDM mass is given by

$$m_a^{(\rm eff)} = \frac{7\pi^4}{180\zeta(3)}\Delta N_{\rm eff}\left.\frac{T_\nu}{E_a}\right|_{\phi{\rm dec}}\left(\frac{g_*(T_d)}{g_{*\nu}}\right)^{\frac{1}{3}} m_a \tag{52}$$

$$\simeq 0.2\,{\rm eV}\,\sqrt{\beta}\left(\frac{g_*(T_d)}{10.75}\right)^{\frac{1}{12}}\left(\frac{\Delta N_{\rm eff}}{0.6}\right)\left(\frac{m_\phi}{100\,{\rm TeV}}\right)^{\frac{1}{2}}\left(\frac{m_a}{1\,{\rm MeV}}\right), \tag{53}$$

where we have approximated $1 - B_a \simeq 1$ for simplicity. Note that the axion mass should be of order MeV for the modulus mass $m_\phi \sim 100\,$TeV. If the modulus field decays into the SM gauge sector, the axion HDM with such a mass can also decay into photons, which is close to the upper limits set by the observed γ-ray flux [65]. If the same axion constitutes CDM, it would contribute to too much diffuse γ-ray. If the modulus mass is heavier than $10^7\,$GeV, one can avoid the observational bound as the axion mass becomes lighter for fixed $m_a^{(\rm eff)}$. On the other hand, if the modulus decays into the Higgs sector through an interaction like $(\phi + \phi^\dagger)H_u H_d$ in the Kähler potential [63, 64], the axion can be stable in a cosmological time scale, and there is no such constraint even for $m_\phi \sim 100\,$TeV.

The axion CDM can be produced by coherent oscillations. The axion CDM abundance is given by

$$\Omega_a h^2 \;\simeq\; 0.3\left(\frac{T_d}{10\,{\rm MeV}}\right)\left(\frac{a_*}{10^{-3}M_p}\right)^2, \tag{54}$$

where a_* is the initial oscillation amplitude. The right amount of CDM can be therefore produced by coherent oscillations if the initial position is sufficiently close to the potential minimum.

In contrast to the case of the QCD axion, the physical mass of the stringy axion should be much heavier than the effective HDM mass, which may enable the axion to decay into photons. While one can avoid the observational limits on the axion decay, it is interesting that γ-ray or X-ray can be a probe of such axion HDM/CDM.

VI. CONCLUSIONS

We have examined a possibility that the pNG bosons, especially the QCD axions, account for both HDM and CDM in the Universe, the former of which has been suggested by the recent observations (cf. Eqs. (1) and (2)). We divide the production process of the axion HDM into thermal and non-thermal ones. In the thermal case, the QCD axion can explain HDM for the decay constant $f_a \approx 3 \times 10^6 - 10^7$ GeV, which however is in tension with the SN1987A limit even for the hadronic axion models. On the other hand, the axion HDM can be naturally produced by the saxion decay. This is possible for the saxion mass ranging from $\mathcal{O}(10^3)$ GeV to $\mathcal{O}(10^{10})$ GeV and $f_a \lesssim 3 \times 10^{10}\,$GeV.

Note that the non-thermally produced axions need to be "colder" than the ambient plasma, in order to explain the hierarchy between the effective HDM mass of $\mathcal{O}(0.1)$ eV and the physical axion mass $m_a = 0.006\,\text{eV}(f_a/10^9\,\text{GeV})^{-1}$ (cf. Eq. (11)). We have discussed two cases in which such axions are produced. In the case (i), there is a late-time entropy production which dilutes the axions produced by the saxion decay, assuming that the saxion dominates the Universe and decays dominantly into a pair of axions. In the case (ii), the saxion decays into a pair of axions when it is subdominant. Our analysis can be also applied to the case where the saxion coherent oscillations partially evaporates into plasma after being trapped at the origin by the thermal effects [6]. We have pointed out that the axion HDM can be a natural outcome of the saxion trapped at the origin. The axion CDM can be produced by either the initial misalignment mechanism or domain wall annihilation.

We have also discussed the pNG bosons coupled through the Higgs portal. While the domain walls associated with the spontaneous U(1) breaking can be the source of the pNG CDM, the required decay constant $F \gtrsim 10^3$ TeV is too large to keep pNG bosons in thermal equilibrium after the QCD phase transition. Therefore it is difficult to explain both HDM and CDM simultaneously with the pNG boson coupled to the SM through the Higgs portal, and some extension of the set-up or deviation from the scaling law of the domain-wall network is required.

Finally we have studied the axions produced by modulus decay, which is considered to take place generically [62]. Such axions can behave like HDM for the axion mass of $\mathcal{O}(1)$ MeV for the modulus mass $m_\phi \sim 100$ TeV. In contrast to the case of the QCD axion, the produced axions are more energetic than the ambient plasma. The right abundance of axion CDM can be generated by the coherent oscillations. Interestingly, the X-ray or gamma-ray can be a probe of such axion dark matter as well as the coupling of the corresponding modulus to the SM sector.

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