Simplified Models for Vector Boson Scattering at ILC and CLIC

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ABSTRACT

Quasi-elastic scattering of the vector bosons W and Z is a sensitive probe of the details of electroweak symmetry breaking, and a key process at future lepton colliders. We discuss the limitations of a model-independent effective-theory approach and describe the extension to a class of Simplified Models that is applicable to all energies in a quantitative way, and enables realistic Monte-Carlo simulations. The framework has been implemented in the Monte-Carlo event generator WHIZARD.

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1 Introduction

After the discovery of a Higgs-like particle at the LHC, and without clear signals of new physics, the focus of particle physics has moved towards a detailed study of the electroweak symmetry breaking sector [1]. Regarding observable processes at colliders, the most striking effect of the Higgs boson is a delicate cancellation in the scattering of on-shell electroweak vector bosons (VBS). This cancellation would be spoiled by any deviation from the minimal Standard Model (SM). Therefore, the study of high-energy electroweak vector-boson scattering is an important part of the physics program of a future lepton collider [2,3,4,5].

At high energies, the electroweak gauge bosons effectively decompose into four gauge bosons $W_\mu^{\pm,0}, B_\mu^0$ with two transverse polarization components, and three scalar Goldstone bosons $w^{\pm,0}$ which correspond to the longitudinal polarization components of W and Z. Consequently, the high-energy scattering amplitudes of W and Z bosons can be broken down into scattering of Goldstone bosons, gauge bosons, and a mixed mode, respectively.

Gauge-boson interactions respect unitarity. The calculated scattering amplitudes of scalar Goldstone bosons are unitary only if they are part of a gauge multiplet which transforms linearly. Without any Higgs boson, there is no linear realization, and therefore the calculated scattering amplitudes of Goldstone bosons rise with energy, in violation of the unitarity limit [6, 7]. Conversely, the SM Higgs boson implements a linear realization, so it cancels this rise. This is also true for any other Higgs-sector incarnation in a linear representation. Actually, with the measured small Higgs mass of $m_H = 125$ GeV, the asymptotic value of the Goldstone-boson scattering amplitude is rather suppressed, and the quasi-elastic scattering of W and Z bosons in the SM is dominated by the transversal degrees of freedom.

The ILC and CLIC colliders will probe the VBS processes

$$VV \to VV \qquad \text{with } V = W^+, W^-, Z \tag{1}$$

embedded in the class of processes

$$e^-e^+ \to \ell\ell'VV \qquad \text{with } \ell, \ell' = e^\pm, \nu, \tag{2}$$

where the initial vector bosons of process (1) are represented, in (2), by virtual vector bosons radiated from the incoming electron and positron, respectively [8,9,2]. The virtual particles are space-like with a typical off-shellness $p_T(\ell) \sim m_W$. The final-state vector bosons decay into leptons or quarks and are off-shell only by $\Gamma_{W,Z}$.

The ILC/CLIC environment essentially fixes the total energy of the leptonic process. This results in a spectrum of the VV invariant mass m_{VV}, which falls down with increasing value of m_{VV} [10,11,12]. This situation is more favorable than at the LHC, where the analogous processes are further suppressed by the steeply falling parton distribution functions.

The physical question is whether the SM is correct, as an effective theory, up to the energy scale which is accessible in the collider experiment. VBS processes are especially important, because they directly probe the Higgs sector, and because the SM prediction is strongly suppressed. Any small deviation can result in a large effect, if sufficiently high energies can be reached.

2 Effective Theory

Traditionally, deviations from the SM prediction are parameterized by an effective Lagrangian which contains, in addition to the renormalizable (SM) part, a series of higher-dimensional effective operators with prefactors of order $1/\Lambda^{d-4}$. Here, d is the dimension of the operator. The scale Λ is unknown. If the operator is the result of tree-level exchange of a new particle, Λ is of the order of the particle mass m. If it is the result of a radiative correction, Λ is rather of the order $4\pi m$. If it is the result of a strong interaction, e.g., a composite nature of the SM particles, Λ indicates the compositeness scale.

In the effective Lagrangian of gauge bosons, Goldstone bosons, and Higgs, the leading dimension d is six. If we only consider operators which do not affect vector-boson self-energies or trilinear couplings, they start at $d = 8$. Such interactions are generated by tree-level exchange of new particles, if they exist.

We write the SM Lagrangian, excluding fermions, in the form

$$\mathcal{L}_{\min} = -\frac{1}{2}\operatorname{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2}\operatorname{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] \tag{3}$$

$$-\frac{1}{2}\operatorname{tr}\left[(\mathbf{D}_\mu\mathbf{H})^\dagger\mathbf{D}^\mu\mathbf{H}\right] - \frac{\mu^2}{4}\operatorname{tr}\left[\mathbf{H}^\dagger\mathbf{H}\right] + \frac{\lambda}{16}\left(\operatorname{tr}\left[\mathbf{H}^\dagger\mathbf{H}\right]\right)^2, \tag{4}$$

where we use the notation

$$\mathbf{D}_\mu\mathbf{H} = \partial_\mu\mathbf{H} + \mathrm{i}g\mathbf{W}_\mu\mathbf{H} - \mathrm{i}g'\mathbf{H}\mathbf{B}_\mu \tag{5}$$

$$\mathbf{W}_{\mu\nu} = \partial_\mu\mathbf{W}_\nu - \partial_\nu\mathbf{W}_\mu + \mathrm{i}g\left[\mathbf{W}_\mu, \mathbf{W}_\nu\right] \tag{6}$$

$$\mathbf{B}_{\mu\nu} = \partial_\mu\mathbf{B}_\nu - \partial_\nu\mathbf{B}_\mu \tag{7}$$

with the fields

$$\mathbf{W}_\mu = W_\mu^a\frac{\tau^a}{2}, \qquad \mathbf{B}_\mu = B_\mu^a\frac{\tau^3}{2}, \qquad \mathbf{H} = \frac{1}{2}\begin{pmatrix} v + h - \mathrm{i}w^3 & -\mathrm{i}\sqrt{2}w^+ \\ -\mathrm{i}\sqrt{2}w^- & v + h + \mathrm{i}w^3 \end{pmatrix}. \tag{8}$$

The Higgs field, which includes the Goldstone bosons, is represented here as a 2×2 hermitian matrix[1].

The possible anomalous effective interactions which directly influence VBS processes contain higher powers and higher derivatives of these building blocks, for instance the two dimension-eight operators

$$\mathcal{L}_{S,0} = F_{S,0}\operatorname{tr}\left[(\mathbf{D}_\mu\mathbf{H})^\dagger(\mathbf{D}_\nu\mathbf{H})\right] \times \operatorname{tr}\left[(\mathbf{D}^\mu\mathbf{H})^\dagger(\mathbf{D}^\nu\mathbf{H})\right]$$

$$\mathcal{L}_{S,1} = F_{S,1}\operatorname{tr}\left[(\mathbf{D}^\mu\mathbf{H})^\dagger(\mathbf{D}_\mu\mathbf{H})\right] \times \operatorname{tr}\left[(\mathbf{D}^\nu\mathbf{H})^\dagger(\mathbf{D}_\nu\mathbf{H})\right]$$

which modify the pure Goldstone-boson part of VBS.

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[1]The matrix form allows for simply relating the higher-dimensional operators to corresponding operators in the no-Higgs scenario [13,14,15,16]. Alternatively, by breaking down the matrix into columns, we can relate to the operators in Higgs-doublet notation [17,18,19,20]

Augmenting the SM Lagrangian by a complete (and minimal) set of additional operators with unknown coefficients, we can model the behavior of the collider processes in a certain energy range, represented by m_{VV}. Experiment would determine or limit the values of those coefficients [2,21,3,1].

In VBS processes, we are in the special situation that the interesting operators have dimension eight, and the SM contribution is small. This is in contrast to the analogous situation in W pair production [18,22] or VBS without light Higgs [9]. Just slightly above the point where the anomalous effect becomes detectable, the squared amplitude rises steeply ($\sim s^4$) in relation to the SM reference distribution, and crosses the unitarity bound. This severely limits the use of the effective theory.

3   Beyond the Limit

This situation is rather unsatisfactory. In essence, there is no model-independent description of high-energy VBS that is both different from the SM and phenomenologically useful. It would be possible to switch to a top-down description where all observables are calculated within a specific model. Unfortunately, while we know some non-SM models of the Higgs sector such as multi-Higgs doublet models, the really interesting cases of strong interactions or compositeness where large effects are possible, are rather uncertain.

We therefore propose to model the physics of VBS by an improved effective theory, as an attempt to make the effective theory consistent with general principles of quantum theory, and to cover a wide class of possible extensions into the energy range where no definite predictions exist [24,25].

The first step is the unitarization of the low-energy effective theory. Given an arbitrary approximation to the true S matrix, we may resum the scattering in such a way that the result is unitary, without losing information or introducing arbitrary new parameters. A simple application is Dyson resummation, which turns an intermediate on-shell particle into a Breit-Wigner resonance. This recipe, which is known as the K-matrix [23] or inverse-amplitude unitarization procedure, is particularly simple if we are dealing with a $2 \to 2$ scattering process. If we diagonalize the $VV \to VV$ scattering amplitudes in the high-energy limit, VBS satisfies this condition. Given a correctly normalized real-valued diagonal amplitude $\mathcal{A}(s)$, the unitarized amplitude reads

$$\mathcal{A}^K(s) = \mathcal{A}(s)/(1 - \mathrm{i}\mathcal{A}(s)). \tag{9}$$

If, as in VBS with anomalous contributions, $\mathcal{A}(s)$ rises without bound for increasing energy, the asymptotic value is always $\mathcal{A}^K(\infty) = \mathrm{i}$, which is the maximum absolute value consistent with unitarity. The unitarized amplitude asymptotically saturates the unitarity limit. For low energies where $|\mathcal{A}(s)| \ll 1$, it coincides with the effective-theory result. The coefficients of the higher-dimension operators thus describe the *slope* of the approach to saturation.

Of course, the real physics may behave in a completely different way – as long as the saturation limit is not exceeded. In accordance with unitarity, the elastic amplitude which must lie on the Argand circle $|\mathcal{A}(s) - 1/2| = 1/2$, may actually pass the point $\mathcal{A} = \mathrm{i}$. This is a resonance, which can be described by a mass and width parameter.

We therefore include resonances in the description. In the Higgs-Goldstone sector, the interacting particles $h, w^{\pm}, z$ are scalars and have definite quantum numbers regarding weak isospin (custodial symmetry), so the scattering is diagonalized in terms of spin-isospin eigenamplitudes $\mathcal{A}_{IJ}(s)$, where I, J is in the range $0, 1, 2$. Furthermore, with exact isospin, accessible resonances have even values of $I + J$. There are five possibilities: scalar ($I = 0$ or $I = 2$), vector ($I = J = 1$), or tensor ($I = 0$ or $I = 2$). This amounts to five mass and five coupling parameters, which then determine the resonance widths.

This discussion is not just academic – at sub-GeV energies, QCD behaves in exactly this way. There, the ρ vector resonance dominates scattering amplitudes, but this need not be the case for electroweak interactions. In any case, it is reasonable to assume that this approach covers the dominant effects both in weakly interacting scenarios (such as multi-Higgs models, a special case where all resonances are scalar) and in the presence of strong interactions and compositeness.

In summary, we propose to model the physics of vector boson by a Simplified model of Strong interactions and Compositeness, which we will denote as **SSC**. At low energies, it smoothly approaches the SM, and at intermediate energies below the first resonance (if any), it is equivalent to the generic effective theory. The ingredients are:

1. All particles of the SM, including the Higgs boson

2. The leading higher-dimensional operators, possibly restricted to those that affect the process in question, with arbitrary coefficients

3. The full set of resonances accessible in Goldstone- (and Higgs) scattering, parameterized by independent mass and coupling parameters

We emphasize that the resonances, if integrated out, do contribute a shift to the low-energy anomalous couplings. We do not assume that they are the only contribution.

4 Implementation

While the model is formulated in terms of the high-energy degrees of freedom, it is straightforward to extend it to the full range of energies [24]. The SM, effective operators, and resonances, are expressed as a Lagrangian and can thus be evaluated in terms of Feynman rules with off-shell particles. The model is useful as long as the off-shellness is small compared to the relevant energy scale. However, at low energies where this is not the case, the unitarization corrections and the resonance effects disappear. The corrections due to unitarization are not expressible as a Lagrangian, but they behave as well defined, Lorentz-invariant and gauge-covariant form factors, so they can be extrapolated off-shell and included in the calculation in a straightforward way.

The unitarization corrections violate crossing symmetry. Thus, results obtained for $VV \to VV$ processes cannot directly be translated into predictions for the crossed process $V^* \to VVV$, or vice versa, as it would be possible in the pure SM.

We have implemented this framework in the Monte-Carlo event generator WHIZARD [26,27]. In the upcoming version 2.2 of this generator, the relevant physics models are

SM : The pure Standard Model as reference

SM_ac : Standard Model with anomalous interactions, as a consistent effective theory but violating unitarity

SSC : The Simplified Model with anomalous interactions, resonances, and K-matrix unitarization, as described above

For convenience, there is also the alternative model

AltH : The Simplified Model without the light Higgs boson

This model allows for connecting to older VBS studies in the literature, where the focus was on the no-Higgs scenario.

5 Conclusions

The SSC approach enables a reasonable evaluation of the sensitivity of a future lepton collider, regarding the physics of high-energy VBS. It should also be useful for the analysis of real data, and in particular for combining ILC and CLIC with hadron collider results. If non-SM effects are actually found, the model should be extended to describe physics phenomenologically in more detail. For instance, one should allow for weak-isospin violating corrections and independent couplings to transversal vector bosons. The setup for the Higgs sector will be embedded in the context of potential new physics in the fermion and gauge sectors, and it should be connected with specific models of the new interactions at a more fundamental level.

In the opposite, but not unlikely case that no deviations can be detected, the traditional effective-theory approach loses its value for data analysis at the energies that we want to access at the ILC and, in particular, at CLIC. Instead, the SSC framework extends the effective theory and allows us to quantify the precision with which the SM will be verified.

Acknowledgments

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